A Nonhomothetic Globally Concave Cost Function with the General Index of Technical Change and Its Application to Panel Data

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Abstract

A globally concave version of the *GO* cost function, a nonlinear nonhomothetic flexible cost function due to Nakamura 1990, is introduced. The cost fuction is applied to a panel data set of firms in the Japanese paper & pulp industry. We use the general index of technical change due to Baltagi and Griffin (1987) instead of the standard quadratic function of time trend. Empirical results indicate that concavity is automatically satisfied whereas homotheticity is decisively rejected.

1 Introduction

Homotheticity of the production function, if it holds, is an extremely useful property that greatly simplifies the analysis of producer behavior. In particular, this property is fundamental to the feasibility of aggregation over producers and to the existence of aggregates over subsets of inputs which are consistent with multi-stage optimization procedures (see Lau (1982) and Blackorby, Primont and Russell (1978), among others).

However useful homotheticity is in simplifying our models, the issue of its consistency with real data is a different one. In fact, recent empirical studies based on micro data by Baltagi and Griffin (1987), Atkinson and Cornwell (1994), and Norsworthy and Jang (1992), among others, report that homotheticity is strongly rejected.

Nakamura (1990) introduced a nonhomothetic flexible cost function, the generalized Ozaki (*GO*) function, and showed an empirical example where the *GO* turned out to

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be superior to the well known translog and generalized Leontief (GL) cost functions. A distinguishing feature of the GO consists in that it includes as a special case the nonlinear factor limitational cost function considered by Komiya (1962) and Ozaki (1969).

Many flexible cost functions including the translog and GL cannot satisfy global concavity without losing flexibility in the price space. This applies to the GO as well, since it is a nonhomothetic extension of the GL. In order to render global concavity to the GO without losing flexibility, we need to replace its price substitution term by that of globally concave flexible function(s). Fortunately, the set of cost functions with these properties is not empty; the generalized McFadden cost function, GM, (Diewert and Wales 1987) is such a function.

We derive a globally concave version of the GO by replacing its price substitution term by that of GM while leaving the nonhomothetic part unchanged. The resulting cost function, the generalized Ozaki-McFadden (GOM for short) cost function, can be globally concave in the price space without losing flexibility, and maintains the original nonlinear nonhomothetic form.

The proposed GOM cost function is empirically illustrated by applying it to a panel data set composed of twenty-six firms of the Japanese paper & pulp industry for the period of 1976-87. In the empirical literature on production functions it is a standard practice to specify technical change as a quadratic function of time trend (see Jorgenson (1986)). However standard this practice is, it reflects our ignorance. We therefore use the alternative way proposed by Baltagi and Griffin (1987) of directly estimating a general index of technical change using time dummies and a panel data set.

2 The Model

2.1 The GOM cost function

Our starting point is the GO cost function introduced by Nakamura (1990):

$$c(p,y) = \left(\sum_{j\neq j} b_{ij}\sqrt{p_i p_j} + \sum_i b_{ii} p_i y^{\beta_i}\right) h(y) \tag{1}$$

where p is a vector of input prices, and y is a scalar output [1]. Except for the nonlinear-nonhomothetic term, this is simply the GL form due to Diewert (1971). Unfortunately, the GL has a disadvantage that it cannot be globally concave unless all inputs are mutually substitutable. The same holds to the GO as well. In order to render global concavity to the GO without losing flexibility we need to replace its GL term by that of globally concave flexible function(s).

[1] y can be an aggregate of multiple outputs. In that case we implicitly assume separability of outputs from inputs

Fortunately, the set of cost functions with these properties is not empty; the generalized McFadden cost function, GM, (Diewert and Wales 1987) is such a function. One disadvantage of GM consists in its asymmetric treatment of inputs; one of the inputs that is used as a normalizer is treated differently from the remaining $n-1$ inputs [2]. The symmetric version of GM is free of this disadvantage, and is given by [3] :

$$c(p,y) = \left(\frac{1}{2}p^T Sp/\theta^T p + \sum_i b_i p_i\right) h(y) \tag{2}$$

where θ is a given $n \times 1$ vector of non-negative constants, not all equal to zero, and $S = [s_{ij}]$ is a symmetric negative semidefinite matrix with

$$\sum_j s_{ij} = 0, \forall i \tag{3}$$

The negativity condition of S can be imposed upon it by representing it as $S = -AA^T$, with A^T being an upper triangular matrix (Diewert and Wales 1987, Theorem 9). This reparameterization does not reduce the number of parameters, and preserves flexibility.

Replacing the GL term of (1) by that of (2) and introducing the general index of technical change A (Baltagi and Griffin 1987) to account for disembodied technical change, we obtain the following generalized Ozaki McFadden, GOM for short, cost function:

$$c = \left[\frac{1}{2}p^T Sp/\theta^T p + \sum_i b_i p_i y^{\beta_i} e^{\gamma_i A(t)}\right] y^{\beta} e^{A(t)} \tag{4}$$

The corresponding demand function for the $i^{th}, i = 1, \cdots, n$ input per unit of output is then given by

$$\begin{aligned} \frac{\partial c}{\partial p_i} &= \frac{x_i}{y} \\ &= \left[S_i p/\theta^T p - \frac{\theta_i}{2}p^T Sp/(\theta^T p)^2 + b_i y^{\beta_i} e^{\gamma_i A(t)}\right] y^{\beta} e^{A(t)} \end{aligned} \tag{5}$$

where S_i refers to the i^{th} row of S. Estimation of (5) would be a simple matter if $A(t)$ were observable. Following Baltagi and Griffin (1987), however, we can estimate (5) utilizing dummy variables and a pooled data set as

$$\frac{x_i}{y} = \left[S_i p/\theta^T p - \frac{\theta_i}{2}p^T Sp/(\theta^T p)^2 + b_i y^{\beta_i} e^{\sum_t \gamma_{it}^* D_t}\right] y^{\beta} e^{\sum_t \gamma_t^* D_t} \tag{6}$$

where D_t is a time specific dummy ($t = 2, \cdots, l$) and D_k is a firm specific dummy ($k = 2, \cdots, m$). We take the initial year as the base year for $A(t)$ and set $A(1) = 0$. (6) is

[2] Nakamura (1995) reports an empirical example where the use of different inputs as the normalizer yields different results for curvature conditions

[3] Still, even this symmetric GM is not almighty; It can be flexible only for the price vector p^* satisfying $Sp^* = 0$. See Diewert and Wales (1987) p.54.

identical to (5) iff

$$\begin{aligned} \gamma_{it}^* &= \gamma_i A(t) && (7) \\ \gamma_t * &= A(t) && (8) \end{aligned}$$

which implies

$$\gamma_{it}^* = \gamma_i \gamma_t^* \tag{9}$$

With (9) imposed, (6) becomes

$$\frac{x_i}{y} = \left[S_i p/\theta^T p - \frac{\theta_i}{2} p^T S p/(\theta^T p)^2 + b_i y^{\beta_i} e^{\gamma_i \sum_t \gamma_t^* D_t} \right] y^\beta e^{\sum_t \gamma_t^* D_t} \tag{10}$$

Estimation of the system of equations (10) with a panel data set will enable us to obtain estimates of $A(t)$.

2.2 Technical Change, Scale, and Substitution Effects

We now turn to economic implications of the *GOM* cost function. Since the use of the general index of technical change is a distinguishing feature of our model, we start from implications of technical change. The dual rate of technical change (the growth rate of adjusted *TFP*) is given by

$$\begin{aligned} \dot{T} &= \frac{\partial \ln c}{\partial A} \frac{dA}{dt} \\ &\approx \left\{ 1 + \frac{\sum_i \gamma_i b_i p_i y^{\beta_i} e^{\gamma_i A}}{\Gamma} \right\} \Delta A && (11) \end{aligned}$$

where Γ refers to the expression inside the square brackets of (4) and Δ to the first order difference operator. (11) shows decomposition of technical change into two components consisting of the term referring to pure technical change (ΔA) and of that referring to the effects of nonneutral technical change and scale augmentation. Note that in contrast to the model that uses time trend as a proxy to the state of technology, $\dot{T}$ in our model can be zero when there is no change in A.

When time trend is used, the unit cost is given by (see Nakamura 1990)

$$c = \left[\frac{1}{2} p^T S p/\theta^T p + \sum_i b_i p_i y^{\beta_i} e^{\gamma_i t} \right] y^\beta e^{\gamma t + 1/2\delta t^2} \tag{12}$$

where γ and δ are unknown parameters and t is the time trend. The rate of technical change then becomes

$$\dot{T} = \left\{ \gamma + \delta t + \frac{\sum_i \gamma_i b_i p_i y^{\beta_i} e^{\gamma_i t}}{\bar{\Gamma}} \right\} \tag{13}$$

where $\bar{\Gamma}$ refers to the expression inside the square brackets of (12). Thus, with time trend the rate of technical change can never be zero unless all the parameters referring to technical change are zero, which corresponds to the case of no technical change at all.

If the technology was subject to constant returns to scale (CRS), the rate of technical change would be identical to the traditional TFP measure given by

$$T\dot{F}P = -\dot{c} + \sum_i w_i \dot{p} \tag{14}$$

where $\dot{}$ refers to the growth rate and w_i to the share of the i^{th} input in the total cost of production with $\sum w_i = 1$. Otherwise, the following relationship between the two measures would hold (Baltagi and Griffin 1988, p.25)

$$T\dot{F}P = -\dot{T} - \epsilon_{cy}\dot{y} \tag{15}$$

where ϵ_{cy} is the elasticity of unit cost with respect to output. The latter is given by

$$\epsilon_{cy} = \beta + \frac{\sum_i \beta_i b_i p_i y^{\beta_i} e^{\gamma_i A}}{\Gamma} \tag{16}$$

Partial derivatives of (11) with respect to input prices gives the bias of technical change

$$\frac{\partial \dot{T}}{\partial \ln p_i} = \frac{\Delta A}{\Gamma} \left[\gamma_i b_i p_i y^{\beta_i} e^{\gamma_i A} - \left(\sum_j \gamma_j b_j p_j y^{\beta_j} e^{\gamma_j A} \right) w_i \right] \tag{17}$$

where w_i is the share of the i^{th} input in the total cost of production. It is easy to see that

$$\sum \frac{\partial \dot{T}}{\partial \ln p_i} = 0 \tag{18}$$

Similarly, the effect of output augmentation on the growth of TFP is given by

$$\frac{\partial \dot{T}}{\partial \ln y} = \frac{\Delta A}{\Gamma} \left[\sum_i \beta_i \gamma_i b_i p_i y^{\beta_i} e^{\gamma_i A(t)} - \left(\sum_j \gamma_j b_j p_j y^{\beta_j} e^{\gamma_j A} \right) \epsilon_{cy} \right]. \tag{19}$$

We next turn to issues of scale effects. Under nonhomotheticity factor combinations change with a change in the size of production. In other words, a given change in the size of production can have different effects on different inputs. This input specific effect of the size of output is given by the partial input elasticity of output:

$$\frac{\partial \ln a_i}{\partial \ln y} = \beta + \frac{\beta_i b_i y^{\beta_i} e^{\gamma_i A}}{\Gamma_i} \tag{20}$$

where Γ_i refers to the the expression inside the square brackets of (5). In the special case of factor limitationality ($S = [0]$), this simplifies to

$$\frac{\partial \ln a_i}{\partial y} = \beta + \beta_i \tag{21}$$

The relationship between the partial input elasticity of output (20) and ϵ_{cy} (16) is given by

$$\epsilon_{cy} = \sum \frac{\partial \ln a_i}{\partial \ln y} w_i \tag{22}$$

ϵ_{cy} is thus a share weighted mean of partial input elasticities of output. On the other hand, the overall scale elasticity, es, is given by

$$es = (1 + \epsilon_{cy})^{-1} \tag{23}$$

Substituting from (22) we obtain the following representation of es :

$$es = \left(1 + \sum \frac{\partial \ln a_i}{\partial \ln y} w_i\right)^{-1} \tag{24}$$

An important implication of this representation will be that the mere presence of non-homotheticity itself does by no mean imply the presence of CRS; share weighted partial elasticities of different signs can cancel each other resulting in es close to unity.

Finally, we turn to issues of substitution among inputs. The Allen Uzawa partial elasticity of substitution between inputs i and j, σ_{ij}, is given by

$$\begin{aligned} \sigma_{ij} &= c\left(\frac{\partial c}{\partial p_i}\frac{\partial c}{\partial p_j}\right)^{-1} \frac{\partial^2 c}{\partial p_i \partial p_j} \\ &= \frac{\Gamma}{\Gamma_i \Gamma_j}\left[S_{ij}(\theta^T p) - \theta_i S_i p - \theta_j S_j p + \frac{\theta_i \theta_j p^T S p}{\theta^T p}\right]\frac{1}{(\theta^T p)^2} \end{aligned}$$

3 Stochastic Specification and Estimation

We apply the GOM cost model to a panel KLM data set on 26 firms in the Japanese paper & pulp industry for the period of 1976 to 87. Data Appendix gives details of the data. The KLM version of (10) is given by

$$\begin{aligned} a_K &= e^{\sum_k a_{Kk} D_k}\left[\frac{1}{\theta^T p}\left(s_{KK}(p_K - p_M) + s_{KL}(p_L - p_M)\right) - \frac{\theta_K}{2} p^T S p/(\theta^T p)^2 \right. \\ &\quad \left. + b_K y^{\beta_K} \exp\left(\gamma_K \sum_t \gamma_t^* D_t\right)\right] y^{\beta} \exp\left(\sum_t \gamma_t^* D_t\right) \end{aligned} \tag{25}$$

$$\begin{aligned} a_L &= e^{\sum_k a_{Kk} D_k}\left[\frac{1}{\theta^T p}\left(s_{KL}(p_K - p_M) + s_{LL}(p_L - p_M)\right) - \frac{\theta_L}{2} p^T S p/(\theta^T p)^2 \right. \\ &\quad \left. + b_L y^{\beta_L} \exp\left(\gamma_L \sum_t \gamma_t^* D_t\right)\right] y^{\beta} \exp\left(\sum_t \gamma_t^* D_t\right) \end{aligned} \tag{26}$$

$$\begin{aligned} a_M &= e^{\sum_k a_{Kk} D_k}\left[\frac{1}{\theta^T p}\left((s_{KK} + s_{KL})(p_M - p_K) + (s_{KL} + s_{LL})(p_M - p_L)\right) \right. \\ &\quad \left. - \frac{\theta_M}{2} p^T S p/(\theta^T p)^2 + b_M y^{\beta_M} \exp\left(\gamma_M \sum_t \gamma_t^* D_t\right)\right] y^{\beta} \exp\left(\sum_t \gamma_t^* D_t\right) \end{aligned} \tag{27}$$

where the quadratic form $p^T Sp$ takes the form

$$p^T Sp = s_{KK}(p_K - p_M)^2 + s_{LL}(p_L - p_M)^2 + 2s_{KL}(p_K - p_M)(p_L - p_M) \tag{28}$$

and θ is set equal to the sample means of a_K, a_L, a_M. Adding the stochastic error term $u_{i,kt}, i = K, L, M$ to the right hand side of (25),(26), and (27), respectively, we obtain the estimating system of equations for the k^{th} firm in year t as follows

$$\begin{aligned} a_{K,kt} &= f_{K,kt} + u_{K,kt} \\ a_{L,kt} &= f_{L,kt} + u_{L,kt} \\ a_{M,kt} &= f_{M,kt} + u_{M,kt} \end{aligned}$$

where $a_{i,kt}$ and $f_{i,kt}$ refer to the left and right hand side of (25)-(27), or after having stacked the three inputs in a 3×1 vector form by

$$a_{kt} = f_{kt} + u_{kt} \tag{29}$$

Let the 3×3 matrix Σ_{kt} be the variance covariance matrix of u_{kt}. We first assume that the errors are homoscedatsic, that is Σ_{kt} is the same for all k and t

$$E\left(u_{kt}u_{kt}^T\right) = \Sigma_{kt} = \Sigma, \forall k, t \tag{30}$$

Note that our dependent variable is the input per unit output and not the input level itself. To the extent that the variance of the latter is proportional to the level of output, our assumption of homoscedasticity will be a reasonable one. We further assume that the errors of different firms are mutually uncorrelated and that there is no serial correlation, that is

$$E\left(u_{kt}u_{lt}^T\right) = 0, \forall k \neq l \tag{31}$$

$$E\left(u_{kt}u_{ks}^T\right) = 0, \forall t \neq s \tag{32}$$

Under these assumptions, we estimate the system of 3 equations (29) for the pooled data of 312 observations by using the iterative SUR [4].

If the estimated matrix S does not satisfy negative semi-definiteness, this condition could be imposed upon the model without losing flexibility by re-parameterizing the model based on the following representation

$$\begin{aligned} S &= -\begin{pmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix} \\ &= -\begin{pmatrix} a_{11}^2 & a_{11}a_{12} & a_{11}a_{13} \\ a_{11}a_{12} & a_{12}^2 + a_{22}^2 & a_{12}a_{13} + a_{22}a_{23} \\ a_{11}a_{13} & a_{12}a_{13} + a_{22}a_{23} & a_{13}^2 + a_{23}^2 + a_{33}^2 \end{pmatrix} \end{aligned} \tag{33}$$

[4]RATS version 4.2 was used.

4 Empirical Results

4.1 Estimation Results and Hypothesis Testing

Table 1 shows the iterative SUR estimates of the parameters. The estimates of S_{ij}, although statistically insignificant, indicate that global concavity is automatically satisfied. In contrast, the estimates of parameters that refer to nonhomothetic scale effects, $\beta_i, i = K, L, M$, and nonbiased technical change, $\gamma_i, i = K, L, M$, are highly significant. The estimates of $\gamma_t, t = 2, \cdots, 12$ take negative values except for γ_5 which refers to 1980. Since γ_1 is normalized to zero at 1977, a negative $\gamma_t, t > 1$, indicates an increase in TFP relative to its level in 1977 [5]. While statistically not significant, a positive estimate of γ_5 appears to indicate a decline in the TFP level itself. This is a feature of the general index of technical change that is not shared by the model based on time trend. The latter, by definition, can not accommodate a sudden decline or increase in TFP (see (13)).

For reference purposes Table 3 shows estimates with the restriction of neutral technical change. The estimate of γ_5 remains negative while increasing its statistical significance. Our finding of the decline of TFP in 1980 is thus robust to the assumption of neutral technical change.

Table 2 shows test results of the significance of parameters based on the Wald principle. While factor limitationality is not rejected, homotheticity, the absence of technical change as well as neutral technical change are all strongly rejected. It appears that nonhomothetic scale effects and biased technical change were major determinants of factor proportions in the Japanese paper & pulp manufacturers.

4.2 Scale Effects

Figure 1 shows the estimated overall scale elasticity es (23) for all the 26 firms for 1977, 1980, 1983 and 1986. Note that in all the subsequent figures the firms are ordered by the size of production with the smallest firm in the far left side and the largest in the far right side for each year. We find:

1. There exist significant economies of scale within the range of output of the 10 smallest firms. Overall, however, the elasticity tends to decrease with the size of production toward unity.

2. For a majority of the case, the estimate is close to unity indicating CRS.

3. The cross sectional pattern remains stable over time.

Recall that nonhomotheticity does not imply economies of scale (see (24) and the discussion below it). Nakamura (1990) reports similar results estimating the GO cost function with a pooled data set.

[5] Here we implicitly assume that the level of output remains constant. See (15)

Figures 2a and 2b show the estimates of partial scale elasticity (20) for each of the three inputs for all the 26 firms for 1977 and 1980, and for 1983 and 1986, respectively. We find:

1. Capital input per unit of output increases with the size of production.

2. Labor input per unit of output decreases with the size of production.

3. Materials input per unit of output is not affected by the size of production.

4. The cross sectional pattern remains stable over time.

Thus, with other things being equal, the capital to labor ratio increases with an increase in output. The finding that the materials input coefficient is homogeneous of degree zero in output will provide a support to the traditional practice in input output analysis.

4.3 Substitution Effects

Figures 3a, 3b, 3c and 3d show estimates of Allen Uzawa elasticity of substitution (25) evaluated at actual prices and output. We find:

1. Capital(K) and labor(L) are complements

2. K-M(materials) and L-M are substitutes.

3. cross sectional pattern is stable over time

We, however, should not place too much significance on these estimates because of the low significance of the underlying S_{ij} parameters. Still, it seems safe to say that the change in capital labor ratio was not due to factor substitution based on changes in relative prices. Together with above findings about nonhomotheticity, we conclude that the size of production but not the level of relative factor prices was the primary determinant of factor ratios.

4.4 Effects of Technical Change

4.4.1 Generalized index model

Figure 4 shows the estimated growth rate of TFP (15) for all the 26 firms for 1977,1980,1983 and 1986. We find

1. TFP growth rate tends to be positively correlated with the size of firms in each cross section. In particular, for all but one of the ten smallest firms the growth rate is negative for each of the four years.

2. The cross sectional pattern remains stable over time.

In our model the size of pure technical change ΔA can change over time but is the same across firms for a given year. Possible factors of the observed difference in TFP growth over cross section include biased technical change (17) and scale augmentation (19).

Figures 5a, 5b and 5c shows the biase of technical change. We find

1. TFP growth is increasing in the price of capital and labor
2. TFP growth is decreasing in the price of materials.
3. The cross sectional pattern remains stable over time

Since an increase in the price of a factor of input decreases the demand for it, the above result indicates that TFP growth is capital and labor saving, and materials using (this terminology is due to Jorgenson and Fraumeni (1980)). Since the price of capital is negatively correlated with the size of production, it follows that the TFP growth rate would be negatively correlated with the size. On the other hand, however, the positive correlation of the price of labor services with the size of production implies a positive correlation of the growth rate with the size. The biase of TFP growth is thus incapable of explaining the positive correlation of TFP growth with the size of production.

Figure 6 shows scale augmentation effects on TFP growth. We find that the scale of production has positive effects on TFP growth, the size of which does not vary over cross section. The positive correlation of TFP growth with the size of production can thus be attributed to the positive scale augmenting effects.

4.4.2 Time trend model

For a comparison, Figure 7 shows our estimate of the growth rate of TFP based on the conventional time trend model. TFP growth rate shows a significant negative correlation with the size of production. The smallest firms have remarkably high growth rates higher than .30, whereas the largest ones have low growth rates below 0.01 or even negative. These results appear strange, and are hard to accept. The estimate of es is .92 and stable both over time and firm, indicating the presence of significant diseconomies of scale. We conclude that the GOM with time trend yielded hardly acceptable results for both TFP growth and scale elasticity, whereas with the general index we obtained acceptable results.

4.4.3 General index and time trend at the aggregate level

In order to obtain an aggregate picture at the industry level we computed an industry aggregate using varying output weights over time (see Figure 8). Figure grptfpind contrasts the TFP growth rate implied by the standard time trend model with that implied by the general technical index model. The two models yield significantly different pictures of TFP growth in several respects. First, the range of fluctuations of the growth rate implied by the trend model is much smaller (±0.01) than that implied by the general technical index model (±0.04). Secondly, the time trend model yields a monotonically increasing growth rates, and cannot track the year to year fluctuations in 1978-81 and in 1983-86.

4.5 Pure Effects of Scale and Technical Change

In order to isolate the effects of scale of production and of technical change from those based on changes in relative prices, we computed theoretical values of the unit cost corresponding to the levels of scale and TFP of 1977, 80, 83 and 86, setting all the factor prices at unity. Figure 9 shows the results. We find:

1. The TFP level declined around 1980.
2. There is a clear sign of economies of scale, the size of which decreases with the size of production, reaching CRS in the rage of output of the largest five firms.

The cross-sectional fluctuation in es in Figure 2 can thus be attributed to the cross-sectional variation in factor prices.

4.6 Translog Cost Function

For a comparison, I also estimated a nonhomothetic translog TL model with nonneutral technical change and five size dummies [6]. The cost function resembles that used by Baltagi and Griffin (1987) [7]

$$\begin{aligned}\ln c &= \alpha + \sum_k \alpha_k D_k + \sum_i \beta_i \ln p_i + \beta_y \ln y + A + 1/2 \sum_{i,j;i\neq j} \beta_{ij} \ln p_i \ln p_j \\ &\quad + 1/2\beta_{yy} y^2 + \sum_i \beta_{yi} \ln p_i \ln y + A \sum_i \beta_{Ai} \ln p_i + \beta_{yA} A \ln y \qquad (34)\end{aligned}$$

Table 5 shows iterative SUR estimates of the parameters obtained by estimating (34) together with the two share equations. Note that conversion of iterations was only possible subject to the restriction $\beta_{yA} = 0$. With $\beta_{K,L}$ both positive, global concavity condition is not satisfied. Furthermore, the local curvature condition was violated at 38 of 312 observation points. Except for this, however, we find that the estimation results are very similar to that based on GOM. In particular, the two models produce the same results in the following points:

1. γ_5 takes a positive value indicating a decline in TFP in 1980

[6] See next subsection for our choice of size dummies.
[7] See Baltagi and Griffin for the estimating system of equations with time dummies.

2. technology is nonhomothetic with the capital labor ratio increasing with the size of production: $\beta_K > 0, \beta_L < 0$

3. TFP growth is increasing in the price of capital (β_{KT} and labor ($\beta_{LT} < 0$), and decreasing in the price of materials.

These findings are thus robust to alternative functional specifications consisting of GOM and TL.

Table 6 shows formal test results of several hypothesis. In sharp contrast to the test results obtained for the GOM in Table 2, the null of factor limitationality is decisively rejected. Still, β_{KL} takes a negative value indicating $K-L$ complementarity. Furthermore, homotheticity is strongly rejected.

4.7 Firm Specific Effects

Up until now, we have not introduced any firm specific effects which are usually the case with studies using panel data. In our data set, the factor that mostly differentiates individual firms is its size either in terms of employees or of production. Allowing for nonhomotheticity, therefore, we have tried to incorporate this most differentiating feature of firms into the model. Still, it is likely that there are elements of firm specific effects which cannot be accounted for by nonhomotheticity alone.

Specific factors can be introduced into the GOM in several different ways, input specific or non-specific, among others. Our first attempt to using firm specific (but input non-specific) dummies ended up with highly significant estimates of firm specific effects and statistically insignificant but numerically huge estimates of structural parameters. A substantial portion of the variation of our sample is explained by firm dummies, leaving no enough variation for estimating nonlinear parameters. It was thus necessary to use a smaller number of specific dummies. Our choice was to use dummies referring to the size of firms in terms of employees. Figure 10 shows that the firms in our sample differ substantially in terms of the employee size, ranging from 200 to more than 6000. We therefore chose to use dummies referring to five ranges of the size of employees: (1) below 500, (2) 500 to 1000, (3) 1000 to 3000, (4) 3000 to 4000, and (5) over 4000.

First, we introduced four size dummies into GOM in a multiplicative and input neutral manner:

$$\begin{aligned} c &= c(p, y, \alpha, A) \\ &= e^{\sum_k \alpha_k D_k} \left[\frac{1}{2} p^T S p / \theta^T p + \sum_i b_i p_i y^{\beta_i} e^{\gamma_i A(t)} \right] y^{\beta} e^{A(t)} \qquad (35) \end{aligned}$$

We treat α_k as fixed effects. This specification of firm specific efficiency corresponds to input technical efficiency, which corresponds to an over-utilization of inputs given output and the input mix (see Atkinson and Cornwell (1994)). The estimating system of demand equations is then given by

$$\frac{x_i}{y} = e^{\sum_k \alpha_k D_k} \left[S_i p/\theta^T p - \frac{\theta_i}{2} p^T Sp/(\theta^T p)^2 + b_i y^{\beta_i} e^{\gamma_i A(t)} \right] y^{\beta} e^{A(t)} \tag{36}$$

Table 7 shows estimates of the parameters by the iterative SUR. Major findings such as rejection of homotheticity and of neutral technical change as well as a positive estimate of γ_5 remain unaltered with the introduction of size specific effects. TFP growth rate, however, becomes extremely volatile with its values mostly two digits numbers and even exceeding 100 percent for some points.

We also tried to incorporate input specific size effects by adding dummies to the diagonal elements:

$$c = \left[\frac{1}{2} p^T Sp/\theta^T p + \sum_i p_i \left(\sum_k \alpha_k D_k + b_i \right) y^{\beta_i} e^{\gamma_i A(t)} \right] y^{\beta} e^{A(t)} \tag{37}$$

Conversion of the iterative SUR was possible only when the restriction of neutral technical change was imposed. Table 8 shows the estimates. While input specific size effects turned out to be mostly significant, major findings remained unchanged: The estimates still indicate nonhomotheticity, with capital-labor ratios increasing with the size of production, and a decline in the TFP level in 1980.

5 Concluding Remarks

In this paper we introduced a globally concave version of the GO cost function by replacing the term referring to price substitution effects by that of the Generalized McFadden cost function. We also replaced the standard specification of technical change based on time trend with the general index of technical change due to Baltagi and Griffin. The resulting GOM cost function was applied to a panel data set of firms in the Japanese paper & pulp industry.

The estimated GOM cost function satisfied global concavity automatically. From the estimates we found, among others, that:

1. homotheticity was decisively rejected with the capital-labor ratio increasing with the size of production

2. there was a decline in the TFP level around 1980

3. technical change was capital and labor saving and materials using.

These results turned out to be robust to the introduction of size specific effects of several different forms and also to the specification by the GOM or translog.

While highly nonlinear and cumbersome to estimate compared with more standard specifications, it seems worth trying the *GOM* as an alternative specification when a panel data set is available. In particular, this will be the case when standard specifications happen to violate curvature conditions: concavity could be restored or imposed by using a *GOM* type specification.

The general index of technical change is also cumbersome to estimate compared with standard specifications based on time trend. Still, it is capable of conveying rich information about the fluctuation of the TFP level, that is entirely missing in the monotone picture the standard time trend model generates. Whenever a panel data set is available, one should use the general index.

The model in this paper is a static one, and assumes instantaneous adjustments for each of the factors of production. This is a testable hypothesis. An important future directions for research will therefore be to allow for possible quasi-fixedness of some factors, and to investigate if our finding of nonhomotheticity is robust to general dynamic specifications [8].

A Data

The set of data used in this study is based on annual financial reports of corporations taken from the NEEDs tape, and was kindly provided to me by Professor Ichiro Tokutsu, Kobe University. In the following, we give a brief summary of how the data set was constructed. Tokutsu (1995) gives a full detail of the data set.

A.1 Price indices of output and materials

In order to obtain real values of output from nominal values in the financial report, we used the price index at the industry level that was obtained from national income accounts supplemented by the wholesale price index published by the Bank of Japan. This implies the absence of any cross-sectional difference in the price of output.

To obtain real values of intermediate inputs, we also used the price indices of materials and energy at the industry level. These indices were obtained as weighted averages of the price indices of output mentioned above and the price indices of imports with weights being given by input coefficients from input-output tables. These price indices at the industry level were further Divisia aggregated to obtain the price index of intermediate inputs, using value shares of materials and energy of individual firms. Accordingly, the aggregate price index of intermediate inputs can be different over firms reflecting the difference in value shares among them.

[8] Nonseparability of multiple outputs from inputs does not produce seemingly nonhomothetic relationships of otherwise homothetic relationships. See Färe and Mitchell (1993)

A.2 Labor input

Labor is measured by the number of employees times hours of work. The wage rate was obtained by dividing the nominal labor compensation by labor input. We implicitly assume homogeneity of labor both over time and over cross section.

A.3 Capital input

The time series of real capital stock in 1980 prices was obtained by applying the perpetual inventory method to the benchmark value of real capital stock for 1965 and a fixed rate of depreciation. The user cost of capital p_K was obtained for each firm from

$$p_K = q(r + \delta) - \dot{q}$$

where q is the price index of investment goods, r is the mean interest rate of firm's loans, δ is the mean of annual rates of depreciation $\delta_t = (K_{t-1} - K_t + I_t)/K_{t-1}$.

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Table 1: Estimates of the Model Parameters:GOM
Generalized Index of Technical Change

parameter	estimates	s.e.	p-values
S_{KL}	-0.329042	0.246088	0.1811
S_{KK}	-0.934786	0.525124	0.0750
S_{LL}	-3.790750	2.145434	0.0772
β_{KK}	0.010905	0.003410	0.0013
β_{LL}	0.607475	0.203224	0.0027
β_{VV}	0.869724	0.059773	0.0000
β	-0.299338	0.057126	0.0000
β_K	0.485321	0.062770	0.0000
β_L	0.146262	0.072947	0.0449
β_V	0.289651	0.056029	0.0000
γ_2	-0.001246	0.038267	0.9740
γ_3	-0.096671	0.104511	0.3549
γ_4	-0.090123	0.098883	0.3620
γ_5	0.078982	0.088135	0.3701
γ_6	-0.014091	0.040492	0.7278
γ_7	-0.042618	0.057277	0.4568
γ_8	-0.086547	0.093660	0.3554
γ_9	-0.066751	0.076717	0.3842
γ_{10}	-0.063102	0.073722	0.3920
γ_{11}	-0.159232	0.158721	0.3157
γ_{12}	-0.144230	0.144931	0.3196
γ_K	-1.428351	0.505291	0.0047
γ_L	-1.694495	0.545106	0.0018
γ_V	-0.410711	0.535608	0.4431

Table 2: Test Results of Hypothesis: GOM

hypothesis	test statistics	degrees of freedom	P value
homotheticity	165.31	3	0.000
factor limitationality	3.35	3	0.339
no technical change	40.86	11	0.000
no biased technical change	39.38	11	0.00

Table 3: Estimates of the Model Parameters:GOM
Generalized Index of Technical Change, Neutral Technical Change

parameter	estimates	s.e.	t-values	p-values
S_{KL}	-0.203712	0.417080	-0.48	0.6252
S_{KK}	-1.776167	1.188284	-1.49	0.1349
S_{LL}	-8.052490	5.373600	-1.49	0.1339
β_{KK}	0.014816	0.004305	3.44	0.0005
β_{LL}	0.668535	0.208311	3.20	0.0013
β_{VV}	0.830427	0.054607	15.20	0.0000
$\beta_{\cdot}$	-0.387214	0.067058	-5.77	0.0000
β_K	0.553497	0.071706	7.71	0.0000
β_L	0.235108	0.082075	2.86	0.0041
β_V	0.380579	0.065898	5.77	0.0000
γ_2	-0.001329	0.016617	-0.08	0.9362
γ_3	-0.040574	0.016970	-2.39	0.0168
γ_4	-0.054122	0.017187	-3.14	0.0016
γ_5	0.020848	0.016455	1.26	0.2051
γ_6	-0.013527	0.016715	-0.80	0.4183
γ_7	-0.026345	0.016841	-1.56	0.1177
γ_8	-0.042686	0.017018	-2.50	0.0121
γ_9	-0.034521	0.016945	-2.03	0.0416
γ_{10}	-0.027561	0.016911	-1.62	0.1031
γ_{11}	-0.063788	0.017276	-3.69	0.0002
γ_{12}	-0.062309	0.017283	-3.60	0.0003

Table 4: Estimates of the Model Parameters:GOM Time Trend

parameter	estimates	s.e.	t-values	p-values
S_{KL}	-0.200292	0.262791	-0.76	0.4459
S_{KK}	-1.419431	1.114408	-1.27	0.2027
S_{LL}	-3.176397	2.553106	-1.24	0.2134
β_{KK}	0.012363	0.003735	3.30	0.0009
β_{LL}	1.186977	0.321000	3.69	0.0002
β_{VV}	0.756557	0.052575	14.38	0.0000
β	-0.374327	0.083838	-4.46	0.0000
γ	0.076378	0.029491	2.58	0.0096
β_K	0.548292	0.087412	6.27	0.0000
β_L	0.186213	0.093525	1.99	0.0464
β_V	0.372168	0.083231	4.47	0.0000
γ_K	-0.063351	0.030896	-2.05	0.0403
γ_L	-0.094643	0.033964	-2.78	0.0053
γ_V	-0.071143	0.027832	-2.55	0.0105
γ_2	-0.001047	0.000712	-1.47	0.1412

Table 5: Estimates of the Model Parameters:TL

parameter	estimates	s.e.	p-values
β_K	-0.387980	0.075186	0.0000
β_L	0.251840	0.030379	0.0000
β_Y	-0.070504	0.100980	0.4850
β_T	1.039216	0.507893	0.0407
β_{KK}	0.005004	0.010172	0.6227
β_{LL}	0.033590	0.013620	0.0136
β_{YY}	-0.000014	0.010010	0.9988
β_{KL}	-0.076445	0.008645	0.0000
β_{KY}	0.023065	0.001892	0.0000
β_{LY}	-0.021558	0.002405	0.0000
β_{KT}	-1.141304	0.387555	0.0032
β_{LT}	-0.624515	0.245615	0.0110
γ_1	-0.180100	0.077440	0.0200
γ_2	0.008180	0.006040	0.1756
γ_3	-0.009551	0.006293	0.1291
γ_4	0.003513	0.005818	0.5460
γ_5	0.023757	0.008010	0.0030
γ_6	-0.007149	0.006290	0.2557
γ_7	-0.008310	0.006372	0.1922
γ_8	-0.019001	0.007798	0.0148
γ_9	-0.010682	0.006623	0.1067
γ_{10}	-0.011933	0.006727	0.0761
γ_{11}	-0.027542	0.009189	0.0027
γ_{12}	-0.020089	0.007898	0.0109
α_2	-0.068697	0.018513	0.0002
α_3	-0.128512	0.032023	0.0000
α_4	-0.206206	0.035803	0.0000
α_5	-0.266917	0.035687	0.0000

violation of con cavity = 38 of 312 observation points

Table 6: Test Results of Hypothesis: TL

hypothesis	test statistics	degrees of freedom	P value
homotheticity	208.84	2	0.0000
factor limitationality	39.83	3	0.0000

Table 7: Estimates of the Model Parameters: GOM
Generalized Index of Technical Change, exponential size dummies (input un-specific)

parameter	estimates	s.e.	p-values
S_{KL}	-0.523471	0.388934	0.1783
S_{KK}	-1.656304	0.849958	0.0513
S_{LL}	-7.002750	3.558484	0.0490
β_{KK}	0.023347	0.007594	0.0021
β_{LL}	1.031104	0.362186	0.0044
β_{VV}	1.912411	0.225072	0.0000
β	-0.336450	0.050625	0.0000
β_K	0.463099	0.057002	0.0000
β_L	0.142581	0.066886	0.0330
β_V	0.264090	0.049441	0.0000
γ_2	-0.000132	0.051202	0.9979
γ_3	-0.112120	0.100096	0.2626
γ_4	-0.079044	0.080310	0.3250
γ_5	0.118728	0.099395	0.2322
γ_6	0.007069	0.050491	0.8886
γ_7	-0.027403	0.055081	0.6188
γ_8	-0.073711	0.074785	0.3243
γ_9	-0.045273	0.061438	0.4611
γ_{10}	-0.043867	0.060929	0.4715
γ_{11}	-0.162113	0.125319	0.1958
γ_{12}	-0.134879	0.108809	0.2151
γ_K	-1.484238	0.441368	0.0007
γ_L	-2.020674	0.598597	0.0007
γ_V	-0.497627	0.336618	0.1393
α_2	-0.056065	0.013683	0.0000
α_3	-0.101926	0.017272	0.0000
α_4	-0.169430	0.022421	0.0000
α_5	-0.233519	0.029020	0.0000

Table 8: Estimates of the Model Parameters: GOM

Generalized Index of Technical Change, additive input specific size dummies

parameter	estimates	s.e.	t-values	p-values
S_{KL}	0.052364	0.116227	0.45	0.6523
S_{KK}	-0.678982	0.629740	-1.07	0.2809
S_{LL}	-1.386640	1.379771	-1.00	0.3149
β_{KK}	0.045571	0.034373	1.32	0.1849
β_{LL}	7.579905	1.524555	4.97	0.0000
β_{VV}	0.479859	0.100169	4.79	0.0000
β	-0.255240	0.093990	-2.71	0.0066
β_K	0.333617	0.117516	2.83	0.0045
β_L	-0.079842	0.104169	-0.76	0.4433
β_V	0.291494	0.089087	3.27	0.0010
γ_2	0.000382	0.014978	0.02	0.9796
γ_3	-0.034081	0.015340	-2.22	0.0263
γ_4	-0.046403	0.015710	-2.95	0.0031
γ_5	0.017143	0.014903	1.15	0.2500
γ_6	-0.016201	0.015200	-1.06	0.2865
γ_7	-0.027287	0.015355	-1.77	0.0755
γ_8	-0.039852	0.015628	-2.54	0.0107
γ_9	-0.030572	0.015547	-1.96	0.0492
γ_{10}	-0.023964	0.015517	-1.54	0.1225
γ_{11}	-0.054471	0.015968	-3.41	0.0006
γ_{12}	-0.052472	0.016084	-3.26	0.0011
α_{K2}	-0.024277	0.007715	-3.14	0.0016
α_{K3}	-0.005820	0.011231	-0.51	0.6042
α_{K4}	-0.008388	0.014174	-0.59	0.5540
α_{K5}	-0.052230	0.016986	-3.07	0.0021
α_{L2}	-0.011121	0.008330	-1.33	0.1818
α_{L3}	-0.052015	0.008538	-6.09	0.0000
α_{L4}	-0.108310	0.010242	-10.57	0.0000
α_{L5}	-0.165668	0.014762	-11.22	0.0000
α_{V2}	0.002628	0.014995	0.17	0.8608
α_{V3}	0.049876	0.021458	2.32	0.0201
α_{V4}	0.070058	0.026911	2.60	0.0092
α_{V5}	0.081929	0.032709	2.50	0.0122

Figure 1

Overall Scale Elasticity by Firm Over Time

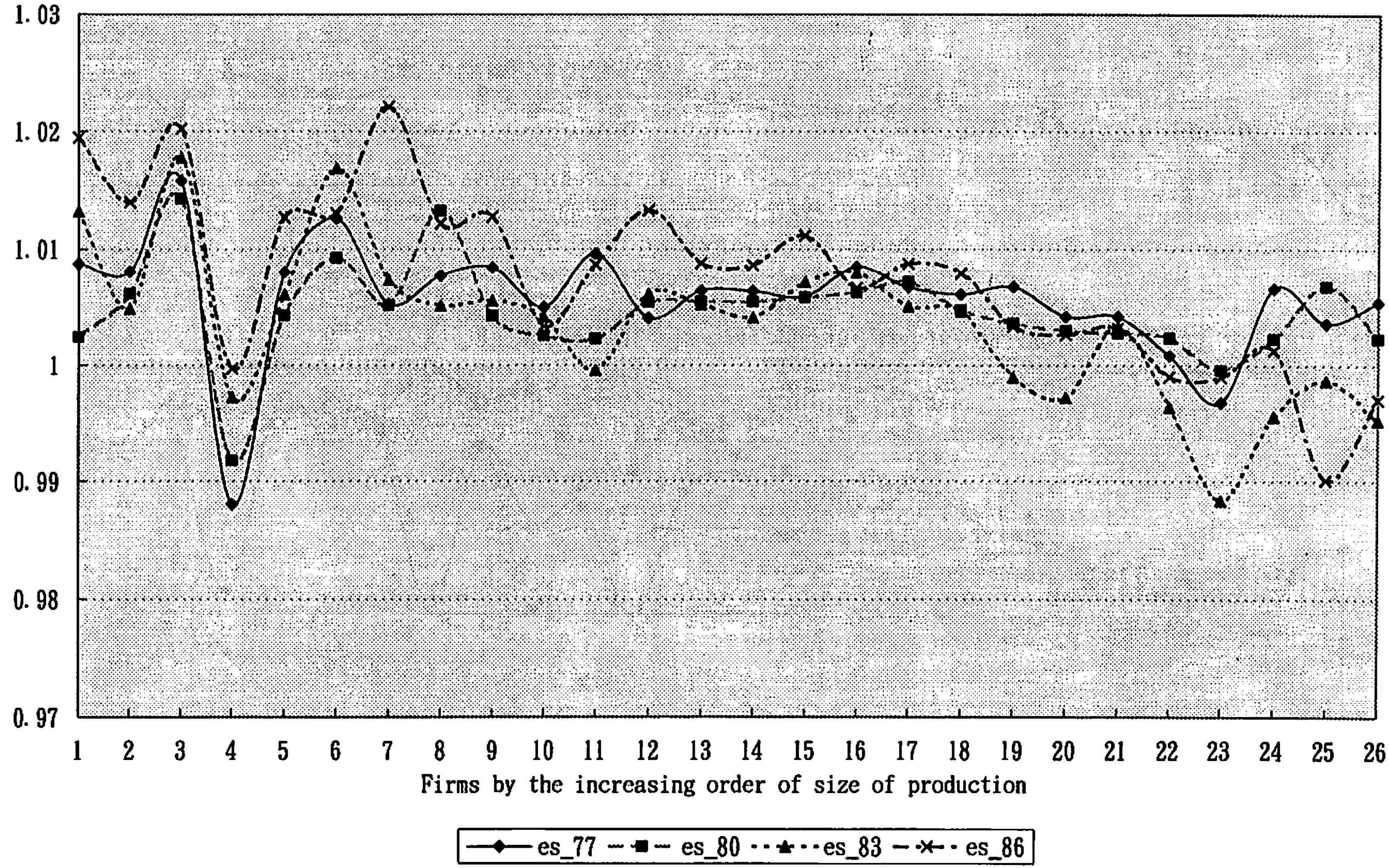


Figure 2a

Partial Scale Elasticity

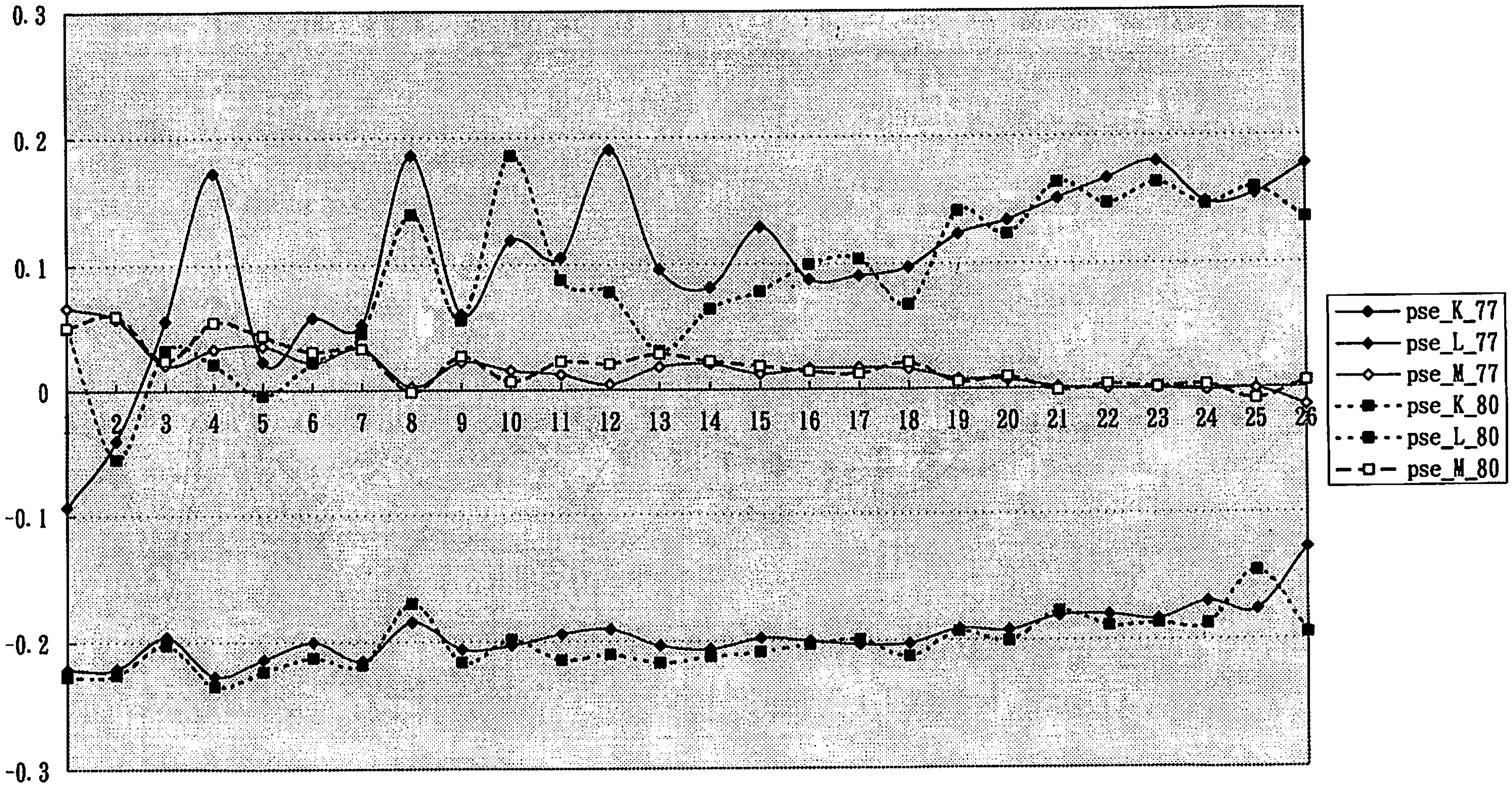


Figure 2b

Partial Scale Elasticity by Firm Over Time

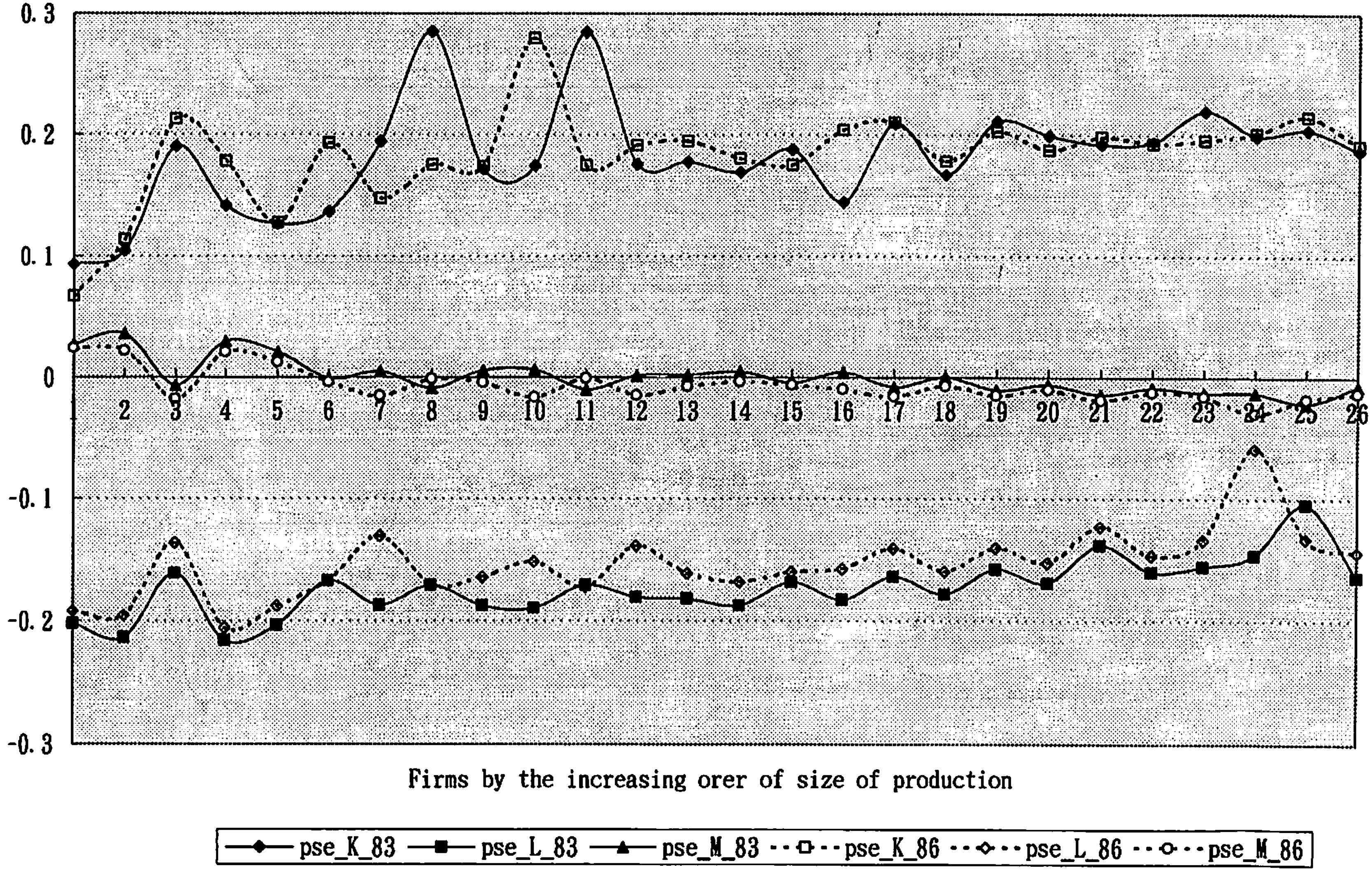


Figure 3a

Allen Uzawa Elasticity of Substitution-77

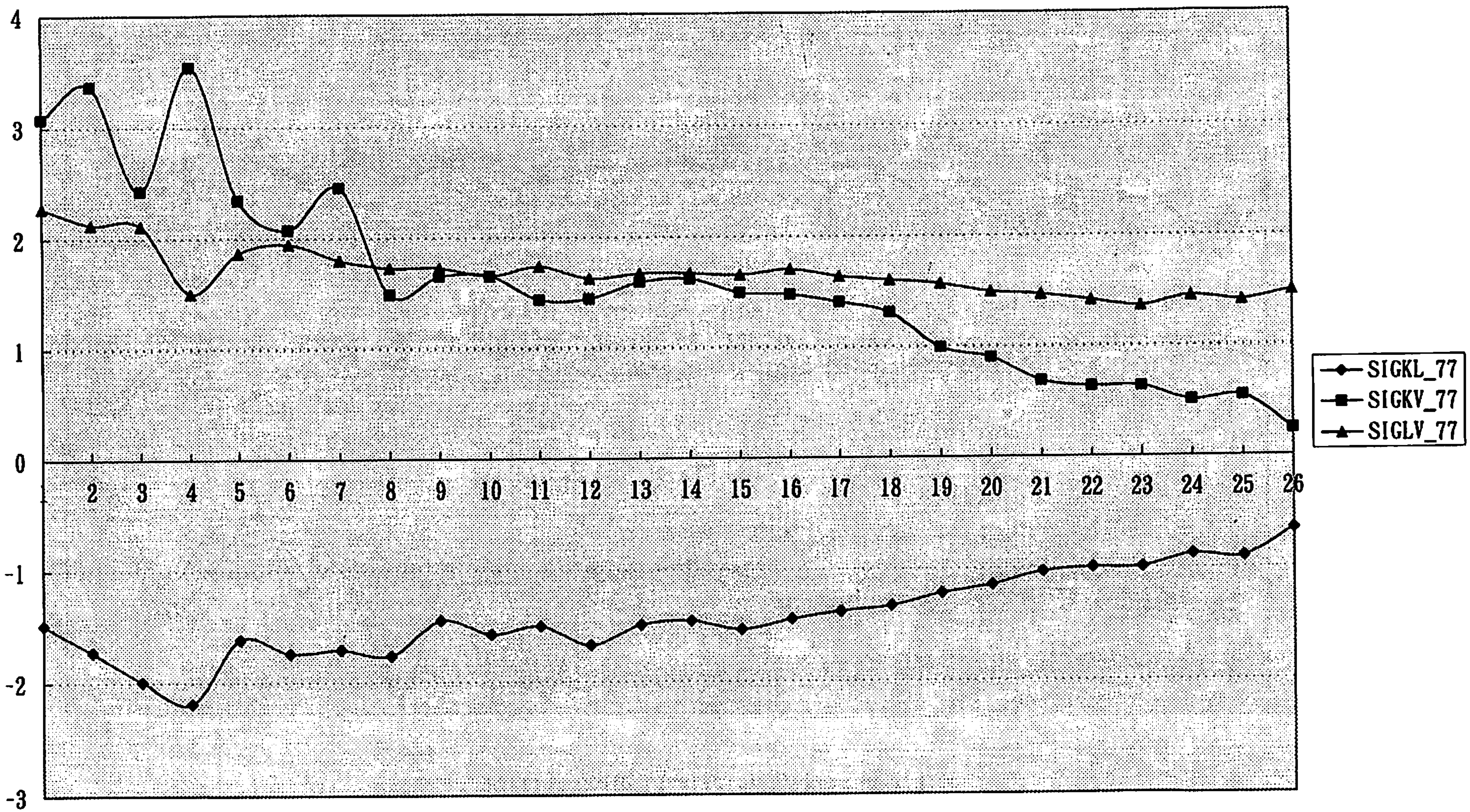
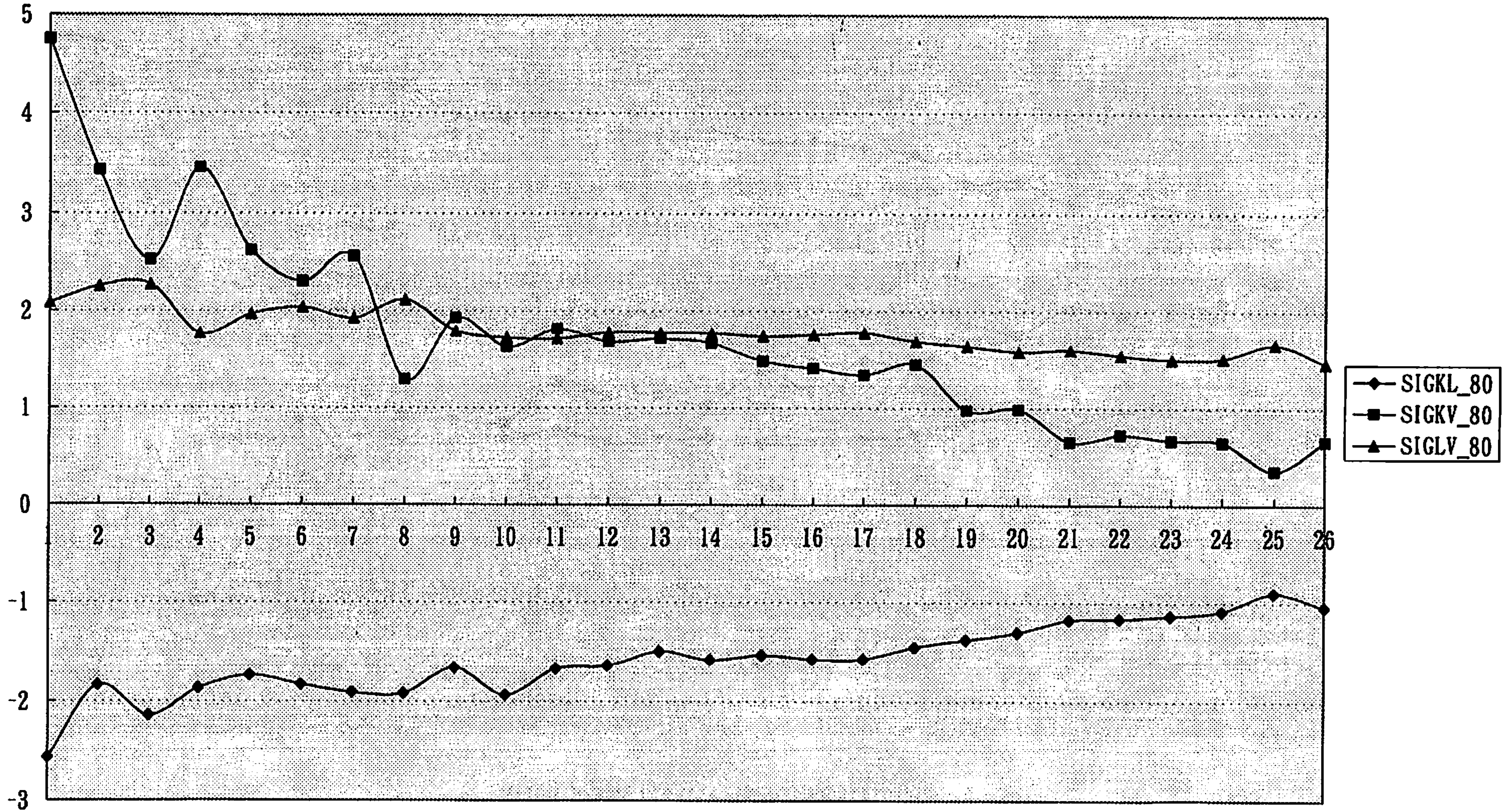


Figure 3b

Allen Uzawa Elasticity of Substitution-80



Fifure 3c

Allen Uzawa Elasticity of Substitution-83

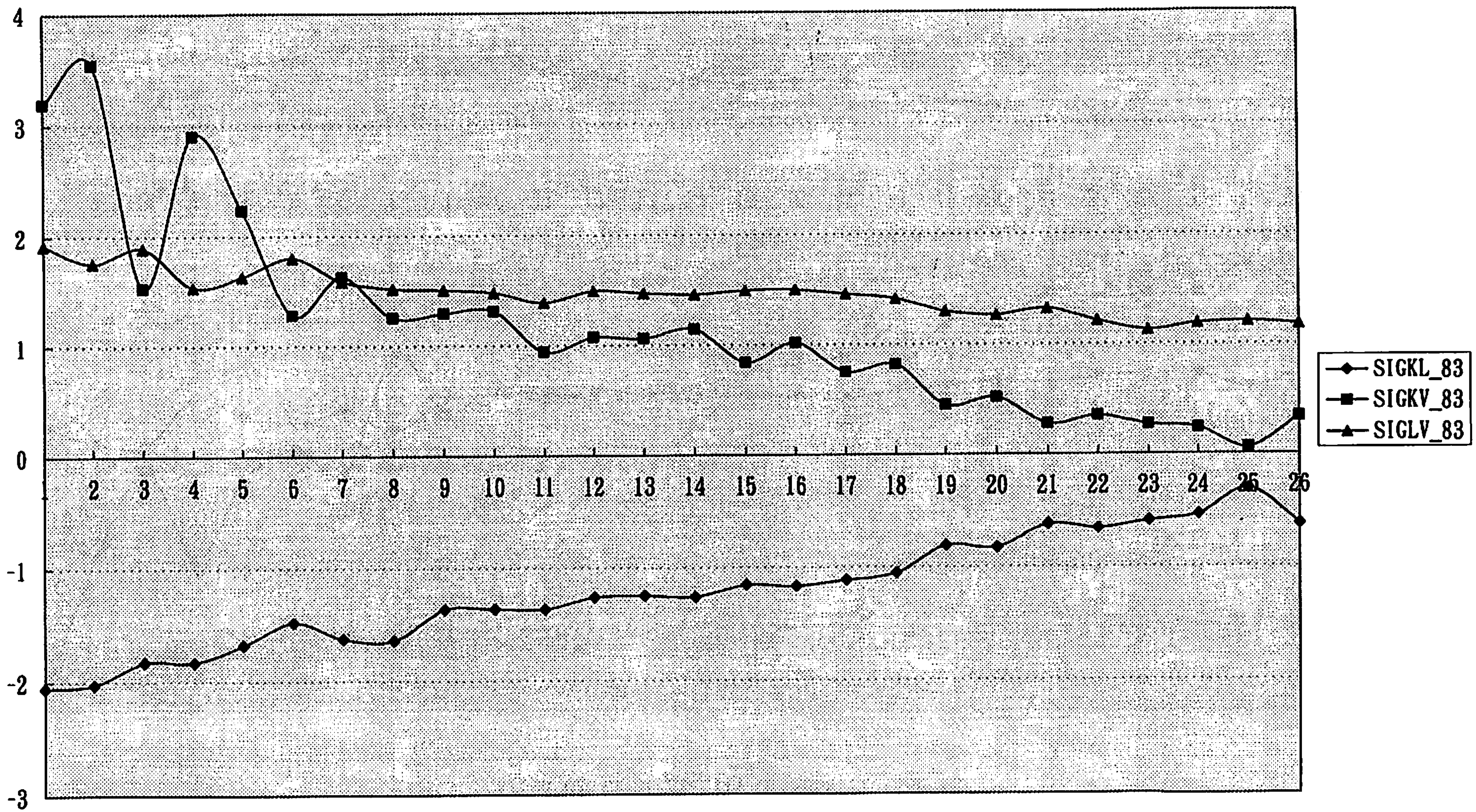


Figure 3d

Allen Uzawa Elasticity of Substitution-86

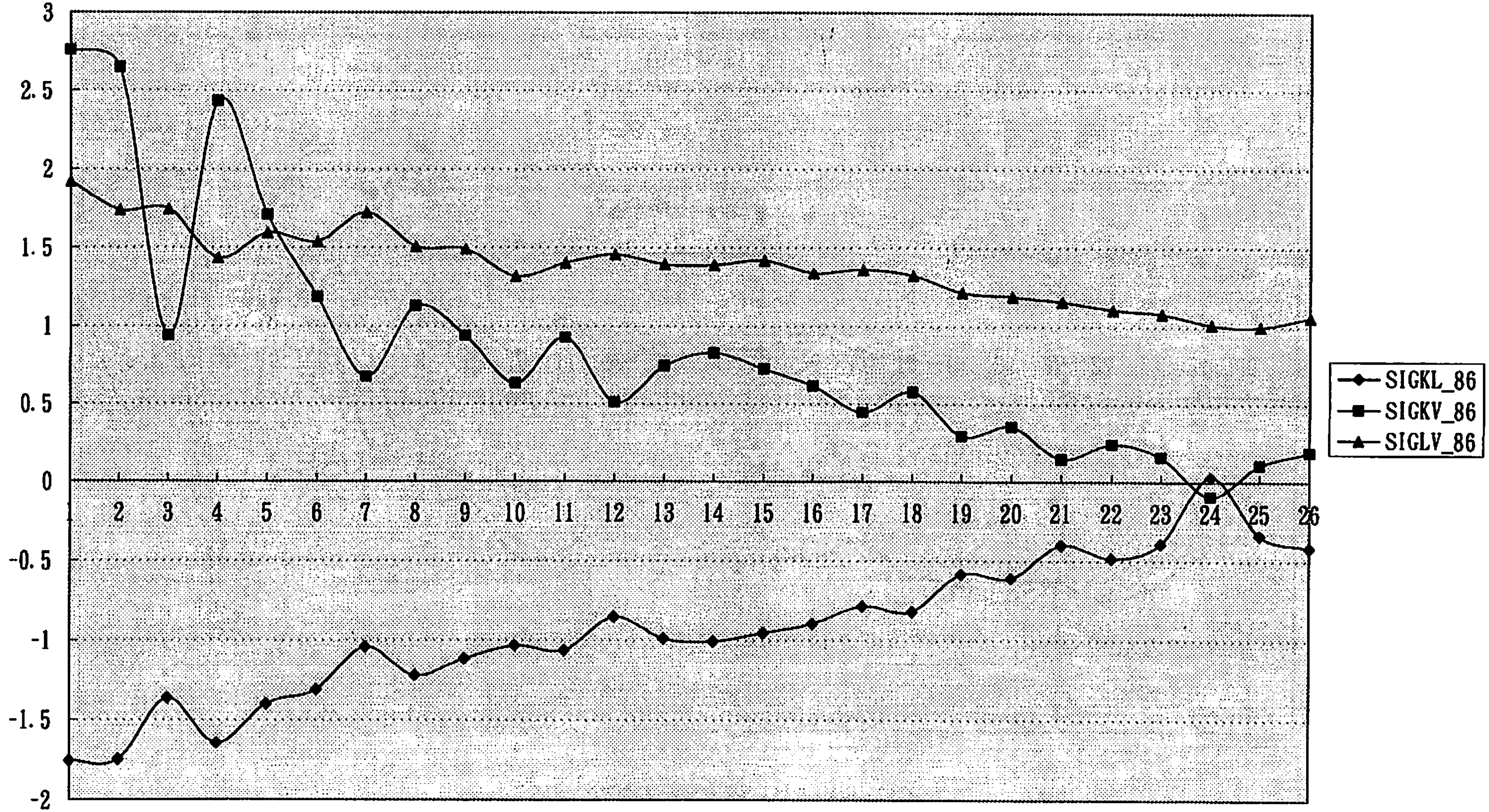


Figure 4

TFP growth rate by firm over time

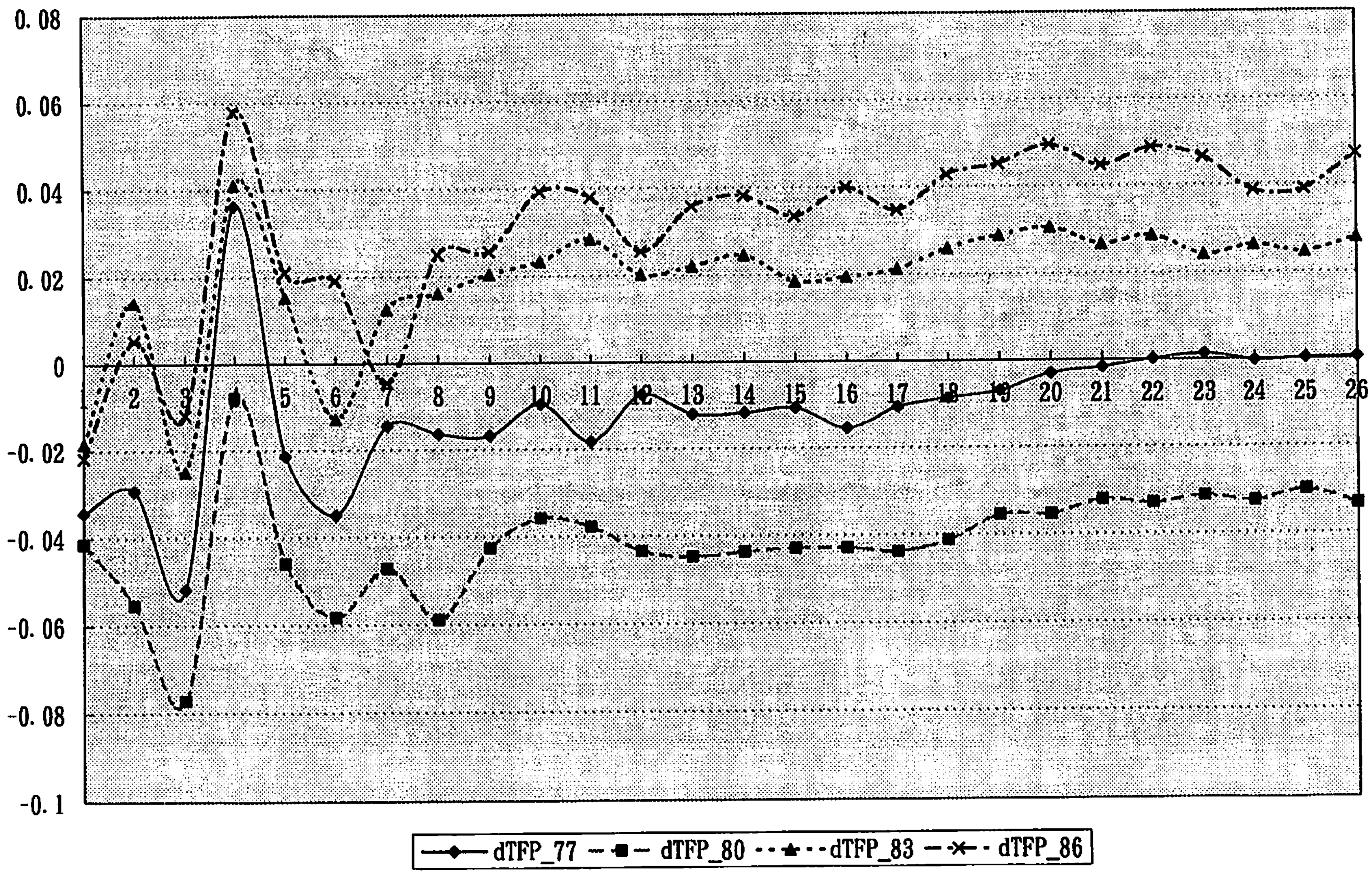


Figure 5a

Biase of TFP growth

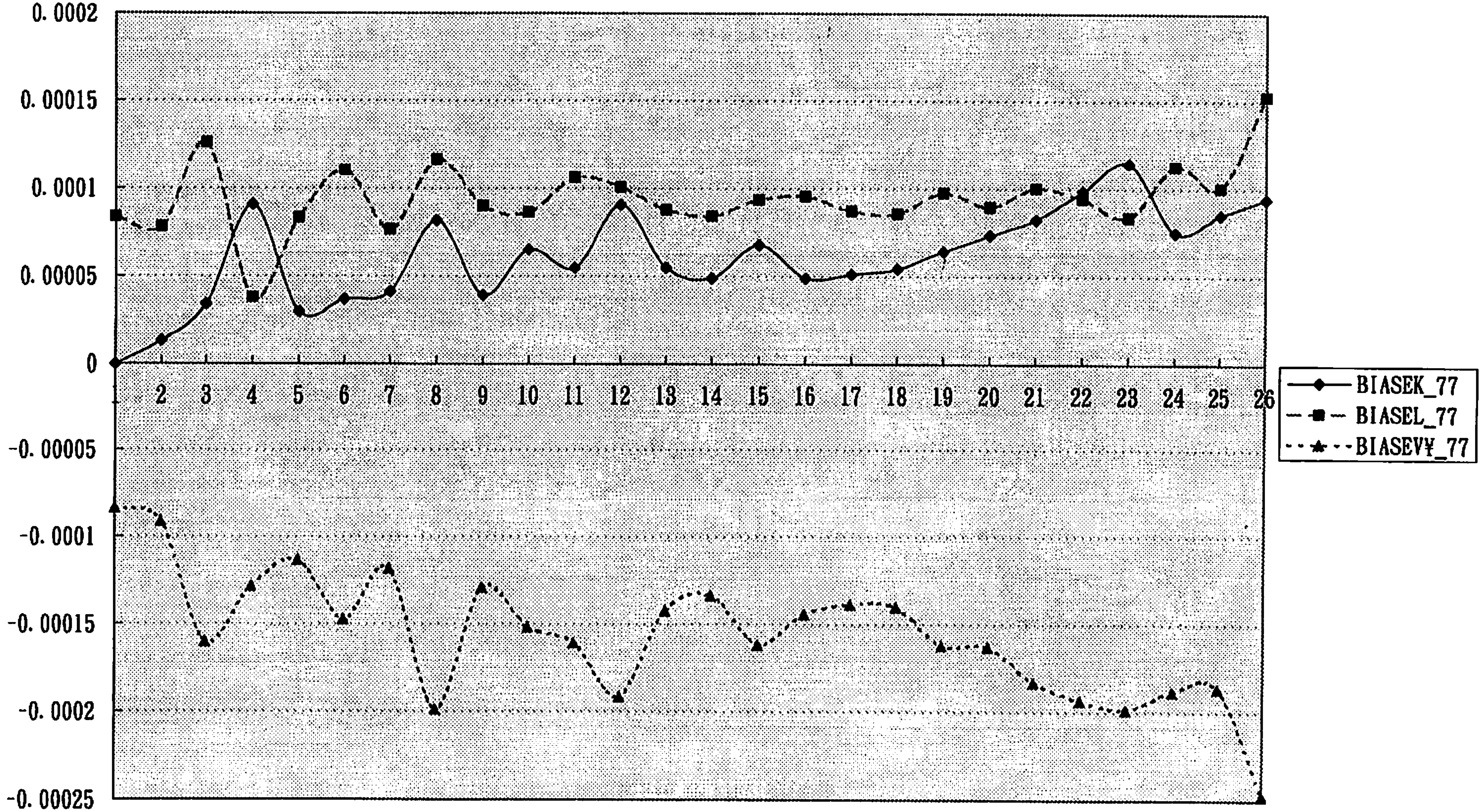


Figure 5b

Biase of TFP growth

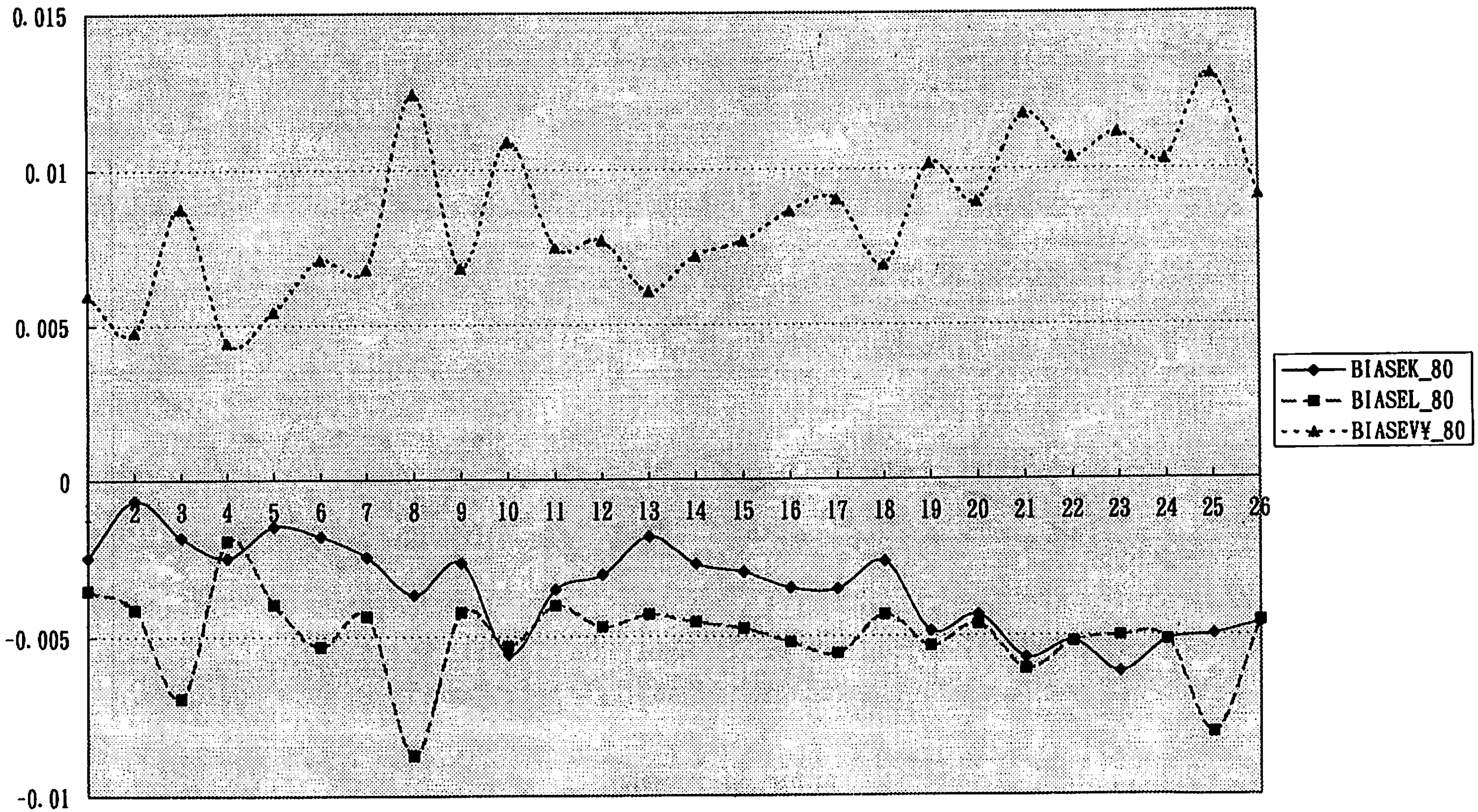
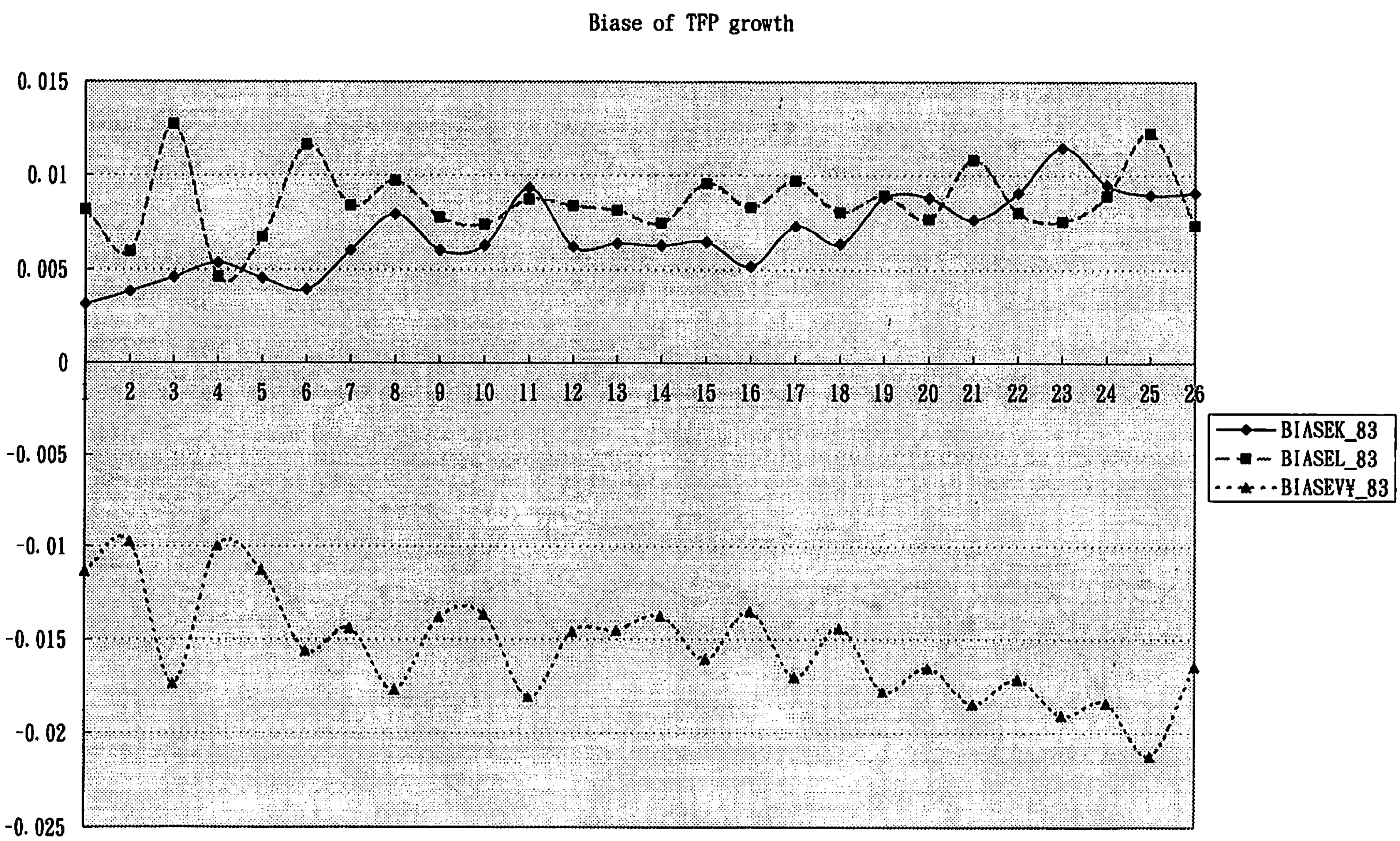


Figure 5c


Biase of TFP growth
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-0.01
-0.015
-0.02
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Figure 5d

Biase of TFP growth

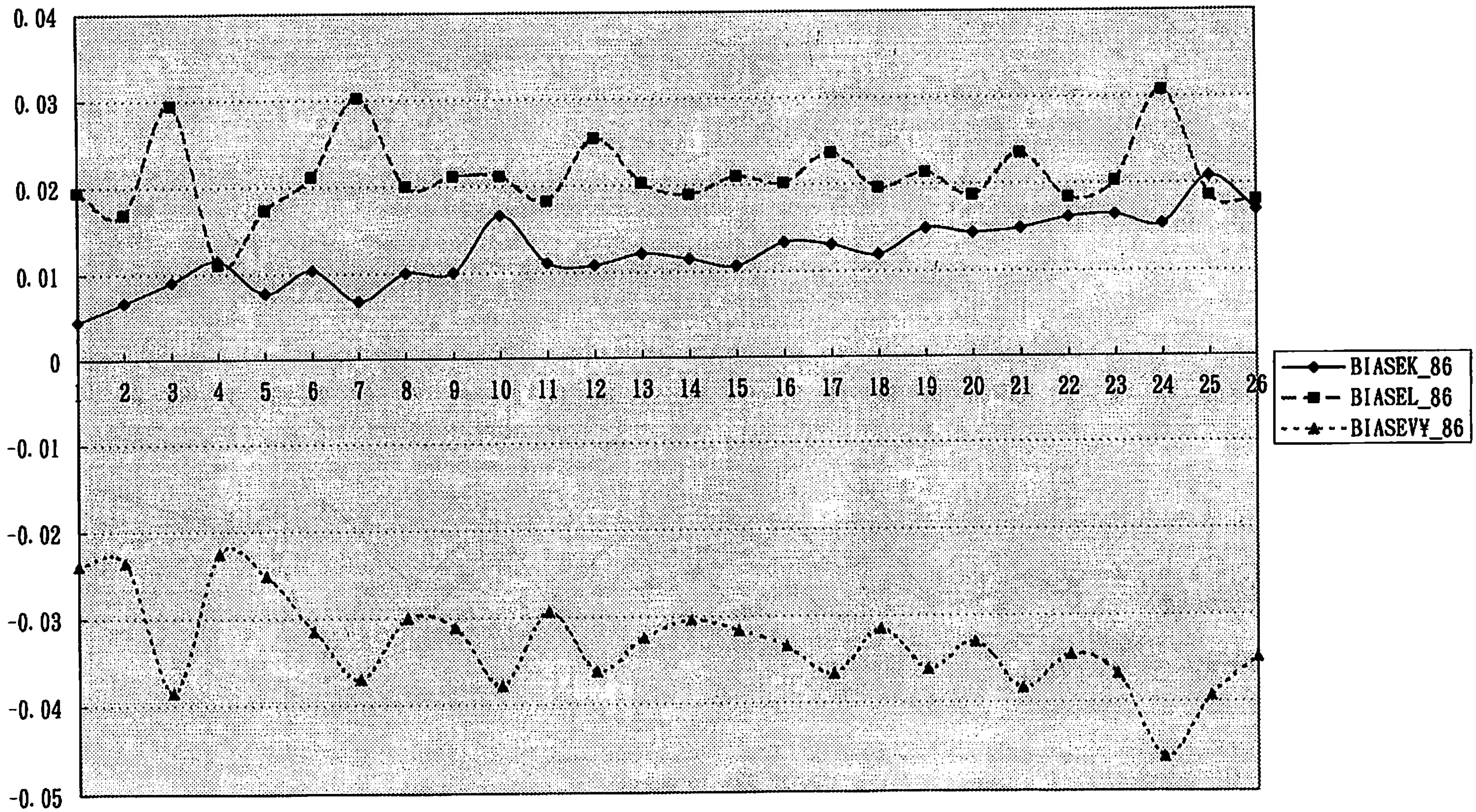

Figure 6

Effects of output on TFP growth

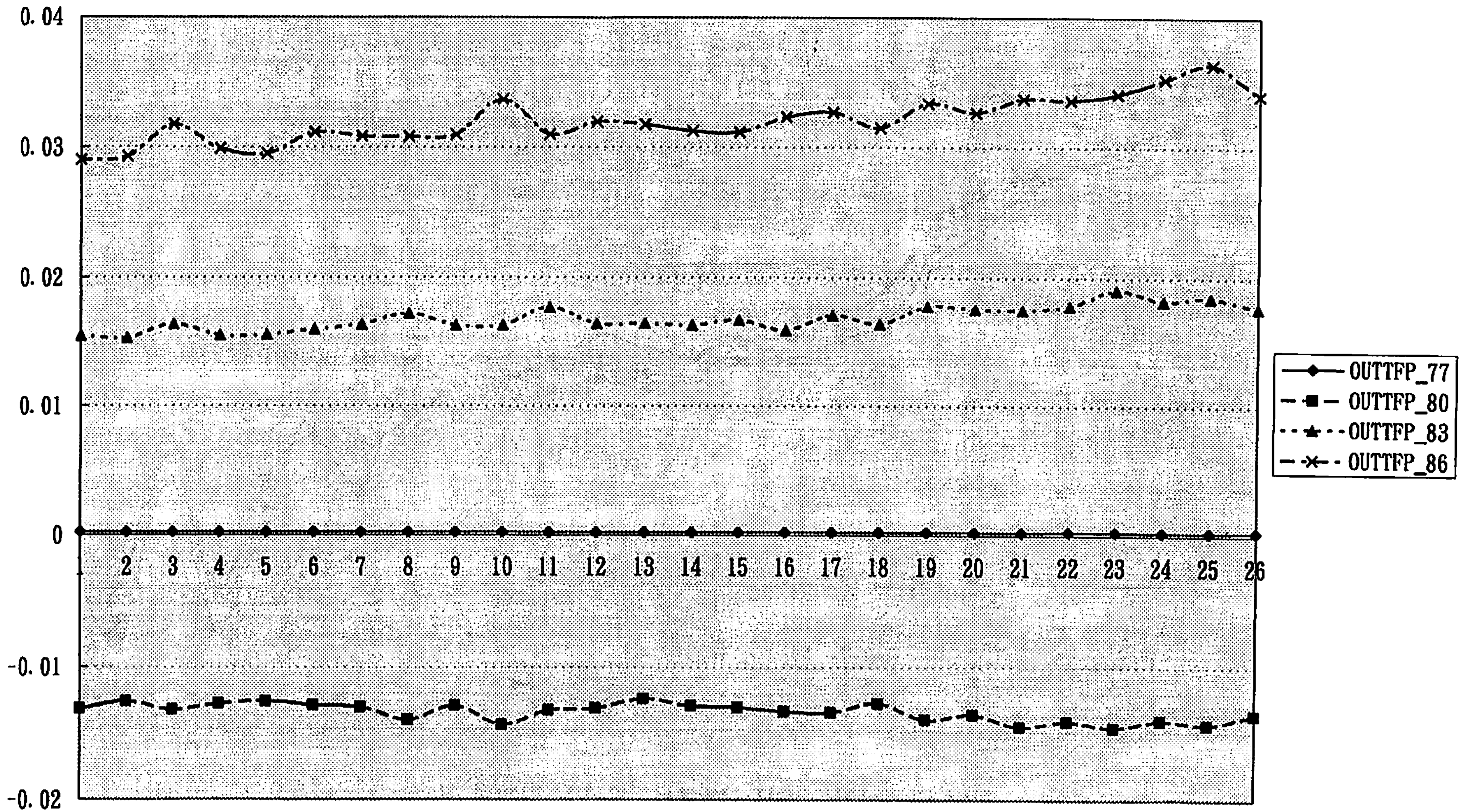

Figure 7

TFP growth rate by firm over time: time trend model

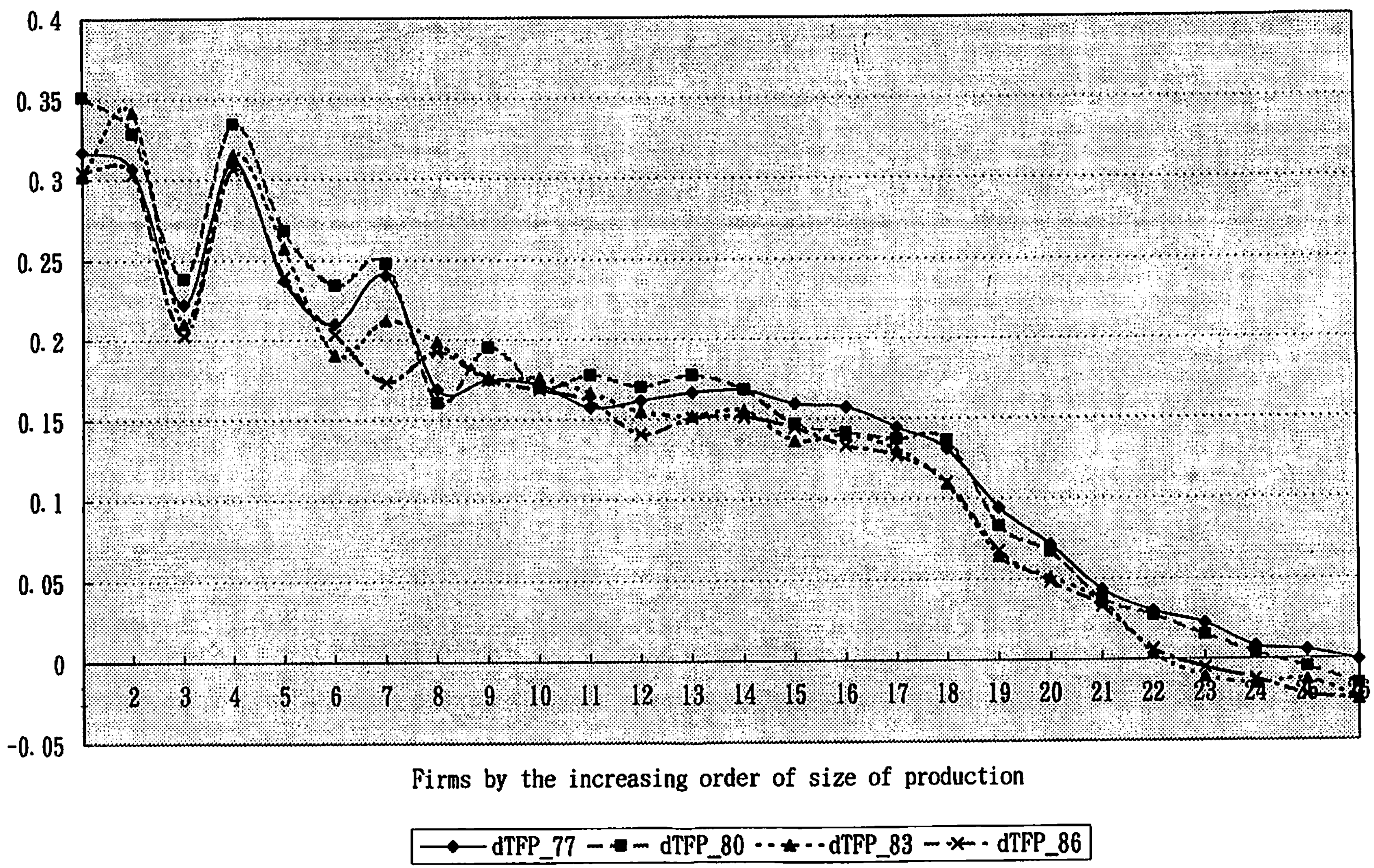

Figure 8

TFP growth rate at aggregate level; generalized index and time trend

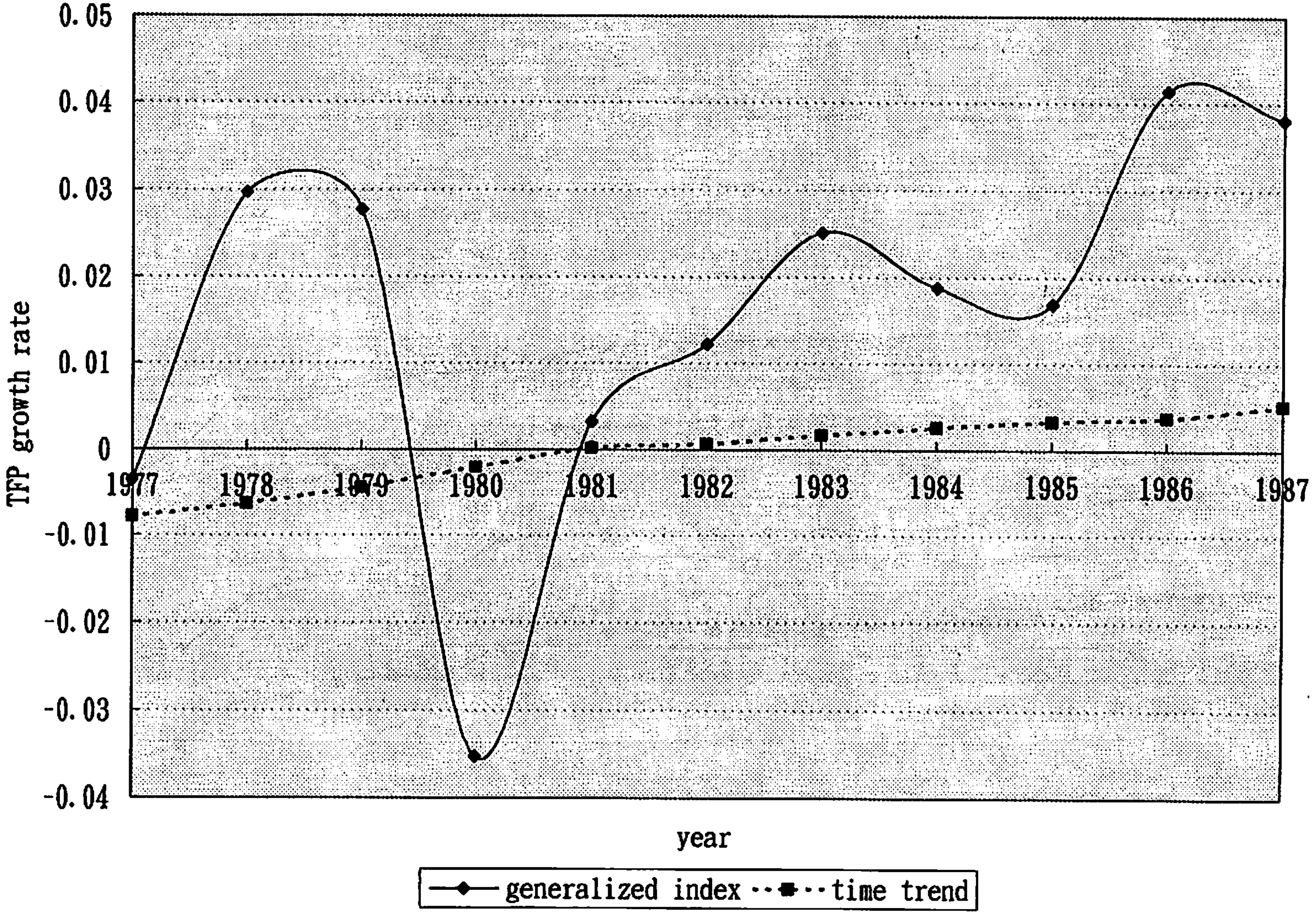

Figure 9

Unit Cost and SIze of Produciton: evaluated at prices equal unity

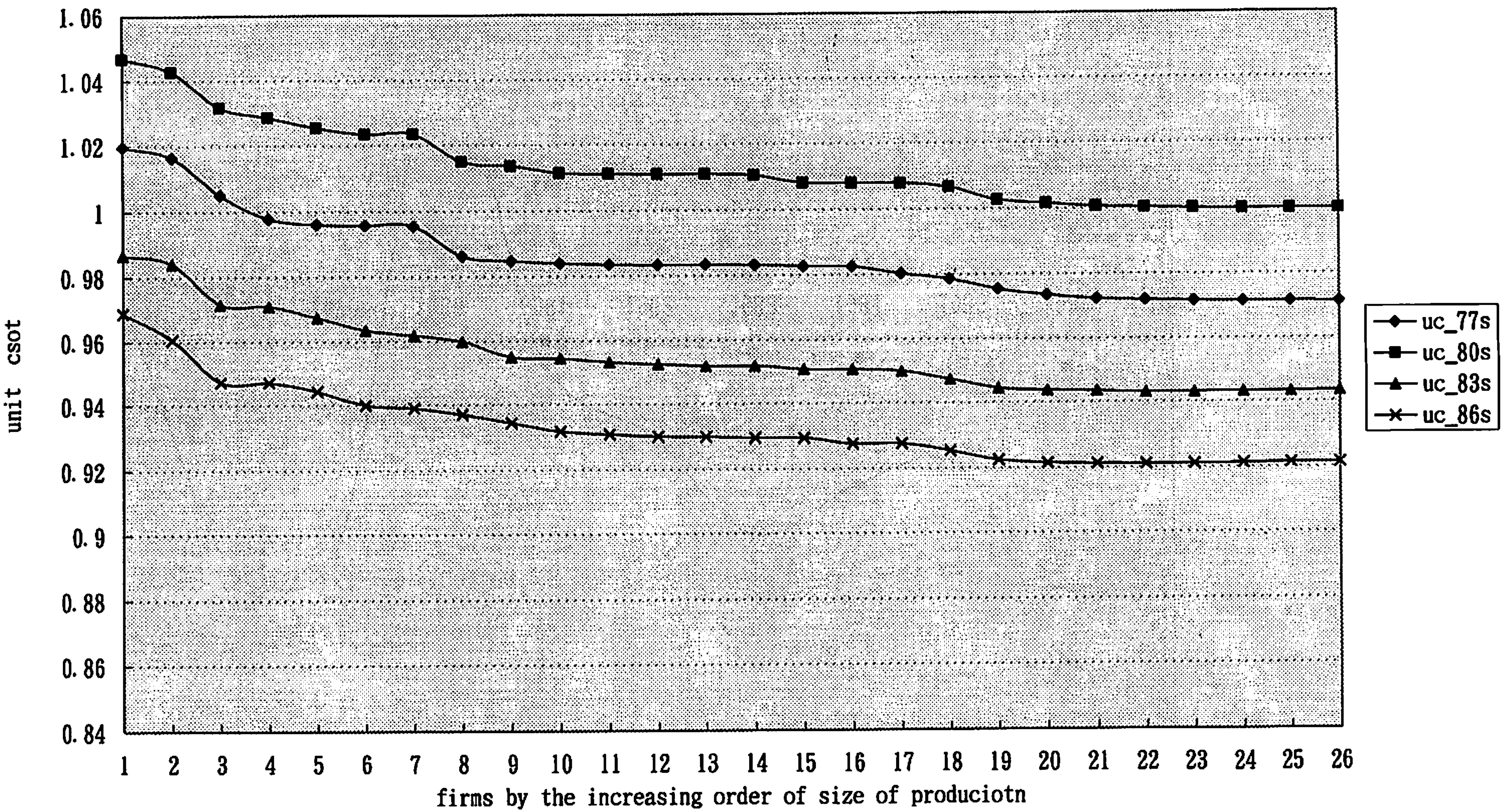

Figure 10

Size of employment by firm: Japanese paper & pulp industry