Bayesian Tests of Serial Correlation in Regression Analysis

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ABSTRACT

The Bayesian tests of serial correlation in a linear regression model are shown and their power is compared with that of the Durbin-Watson test through Monte Carlo simulations. The experiments show that the Bayes tests are superior to the Durbin-Watson test in many cases and that the former can compete with the latter even when the latter is uniformly most powerful. Moreover, the Bayes tests retain their power when the power of the Durbin-Watson test is poor.

1. Introduction

Detection of serial correlation among error terms has been one of the main concerns in econometrics. This is because autocorrelation has adverse effects on the standard least-squares method in regression analysis. For example, the true size of the conventional t tests for regression coefficients tends to be higher than the nominal one when the explanatory variables are positively autocorrelated (Johnston (1984), pp. 310-313). It is therefore relevant to detect serial correlation before conducting regression analysis.

Many test statistics have been proposed and compared in this regard (See King's (1987) survey). A result from such studies is that no test can be uniformly most powerful. However, the Durbin-Watson (DW) ratio, among others, is easy to compute and uniformly most powerful under certain conditions (Sawa (1970) pp. 129-130). Therefore, the test is frequently used in econometric research.

However, few work has been done on the Bayesian tests of serial correlation. A reason for this is that Bayesian statisticians have focused on estimation, not testing, in correlated-error models. For instance, Zellner and Tiao (1969) deal with the posterior probability density function (pdf) of the autocorrelation parameter to calculate the posterior pdf of regression parameters. Griffiths and Dao (1980) use the posterior probability of the null hypothesis of uncorrelated errors and the alternative hypothesis of autocorrelated errors as weights for the Bayesian pre-test estimator. On the other hand, O'Brien (1970) and Tsurumi and Kan (1991) indicate a possibility of using the posterior pdf as a test for dependent errors, and yet they do not compare the Bayesian procedures with the DW test.

The aim of this paper is to compare the power of the Bayesian tests with that of the DW ratio through Monte Carlo simulations. In the next section, we outline the Bayesian tests of autocorrelation in a linear regression model. In Section 3, the design of our experiments is described. Section 4 evaluates the results of the simulations. Some concluding remarks are made in the final section.

2. Bayesian Tests of Serial Correlation

2.1 Model

Consider a linear regression model with the first order autoregressive (AR(1)) errors,

$$y = X\beta + u, \tag{1}$$

where y is an (nx1) observation vector, X is an (nxk) matrix of non-stochastic variables, β is a (kx1) coefficient vector, and u is an (nx1) error vector, each elements of which follows an AR(1) process,

$$u_t = \rho u_{t-1} + v_t, \quad v_t \sim \mathrm{NID}(0, \sigma^2), \quad |\rho| < 1. \tag{2}$$

Using Jeffreys' non-informative prior pdf,

$$p(\beta, \sigma, \rho) \propto (1-\rho)^{-1/2}, \tag{3}$$

and integrating out β and σ, we obtain

$$f(\rho|y) \propto [(y^* - X^* b^*)'(y^* - X^* b^*)]^{-(n-k)/2} |X^{*\prime}X^*|^{-1/2}, \tag{4}$$

where

$$y^* = Py, \quad X^* = PX, \quad \text{and} \quad b^* = (X^{*\prime}X^*)^{-1}X^{*\prime}y^* \tag{5}$$

with

$$P = \begin{bmatrix} c & 0 & 0 & \cdots & 0 \\ -\rho & 1 & 0 & & 0 \\ 0 & -\rho & 1 & & \\ & \cdots & \cdots & & \\ 0 & \cdots & & -\rho & 1 \end{bmatrix} \quad \text{and} \quad c = (1-\rho^2)^{1/2} \tag{6}$$

(Judge, *et al.* (1984), pp. 291-293). The posterior pdf of ρ given by (4) plays a key role in testing serial correlation in a Bayesian approach. For one thing, it yields the posterior odds to test autocorrelation. For another, it provides the highest probability density (HPD) region to detect serial dependence in disturbances. The details of these formulae are shown in the rest of this section.

2.2 Hypothesis Testing

We denote the null and the alternative as follows.

$$H_0: \quad \rho = 0 \tag{7}$$

$$H_+: \quad \rho > 0 \tag{8}$$

$$H_-: \quad \rho < 0 \tag{9}$$

$$H_{+-}: \quad \rho \neq 0 \tag{10}$$

Since negative correlation is unlikely in economic data, H- will not appear in the remainder of this paper.

From a Bayesian viewpoint, the test for correlated disturbances can be constructed by the posterior odds or the posterior pdf.

2.1.1 Posterior Odds

First, the posterior odds ($P[H_0|y]/P[H_x|y]$, $x= +,\pm$) can be regarded as a test statistic. The higher the value, the more plausible the null hypothesis is. The ratio is given by the following equation.

$$K_{0x} = \underbrace{\left(\frac{\pi_0}{\pi_x}\right)}_{\text{(prior odds)}}\underbrace{\left(\frac{[(y-Xb)'(y-Xb)]^{-(n-k)/2}|X'X|^{-1/2}}{c_x \int_{(H_x)} f(\rho\,|y)\,d\rho}\right)}_{\text{(Bayes factor)}}, \tag{11}$$

where π_x is the prior probability of H_x,

$$b=(X'X)^{-1}X'y, \text{ (OLS estimator)} \tag{12}$$

$$c_x = \pi \text{ (for } x = +\text{)}, \ \pi/2 \text{ (for } x = +-\text{), and} \tag{13}$$

$$[H_x] = (0, 1) \text{ (for } x = +\text{)}, \ (-1, 1) \text{ (for } x = +-\text{)} \tag{14}$$

(Griffiths and Dao (1980), pp. 390-392). If we assume that type I and type II errors cause an equal loss, the reference value $K_{0x}=1$ ($x=+,\pm$) produces a minimum expected loss (Suzuki (1978), pp. 149-152). It follows that when the posterior odds is less than unity, the null is rejected. In addition, according to Jeffreys (1961, p. 246), it is acceptable to assume the prior odds to be unity since we suppose a non-informative situation about autocorrelation. Consequently, the null is rejected when the Bayes factor is less than one.

Before explaining the second test, we point out the connection between the posterior odds and the DW ratio. Consider the point alternative hypothesis,

$$H_r: \quad \rho=\rho_r. \tag{15}$$

After some algebra, we have the posterior odds of H_0 against H_r,

$$K_{0r} = \left(\frac{\pi_0}{\pi_r}\right)(1-\rho_r^2)^{-1/2} \times \frac{[(y-Xb)'(y-Xb)]^{-(n-k)/2}|X'X|^{-1/2}}{[(y_r^*-X_r^*b_r^*)'(y_r^*-X_r^*b_r^*)]^{-(n-k)/2}|X_r^{*\prime}X_r^*|^{-1/2}}, \tag{16}$$

where y_r^*, X_r^*, and b_r^* are obtained by (5) and (6) with $\rho=\rho_r$. The Bayes factor in equation (16) coincides with the necessary condition for the locally most powerful invariant test of H_0 against H_+. Moreover, this factor leads to the DW test (Durbin and Watson (1971), p. 10, and Kariya (1979), pp. 132-136). This can is shown as below. First, we note

$$(y_r^*-X_r^*b_r^*)'(y_r^*-X_r^*b_r^*)=(y-Xb_r^*)'P_r'P_r(y-Xb_r^*) \tag{17}$$

where P_r is given by (6) substituted ρ_r with ρ. Next, $P_r'P_r$ can be rewritten as

$$P_r'P_r = (1-\rho_r)^2 I + \rho_r A + \rho_r(1-\rho_r)L, \qquad (18)$$

where A is a matrix such that $z'Az = \Sigma(z_t - z_{t-1})^2$ and L is a matrix that has one in its top-left and bottom-right, and zero elsewhere. Following Durbin and Watson (1971, p. 9) and Kariya (1979, p. 129), we ignore the last term on the right-hand side of (18). Also, we assume that ρ_r is in the neighborhood of $\rho=0$. According to Durbin and Watson (1971, pp. 10-11), this means that b_r^* is close to b. Thus, we obtain

$$(y_r^* - X_r^* b_r^*)'(y_r^* - X_r^* b_r^*)/(y-Xb)'(y-Xb)$$
$$\fallingdotseq (1-\rho_r)^2 + \rho_r d, \qquad (19)$$

where d is the DW ratio (Durbin and Watson (1971), p. 10). Substituting (19) into (16) yields

$$K_{0r} = \left(\frac{\pi_0}{\pi_r}\right)\frac{|X_r^{*\prime}X_r^*|^{1/2}}{(1-\rho_r^2)^{1/2}|X'X|^{1/2}}[(1-\rho_r)^2 + \rho_r d]^{(n-k)/2} \qquad (20)$$

or

$$d = \frac{1}{\rho_r}\{[(\frac{K_{0r}}{L_{0r}})^2 \frac{(1-\rho_r^2)|X'X|}{|X_r^{*\prime}X_r^*|}]^{1/(n-k)} - (1-\rho_r)^2\}, \qquad (21)$$

where $L_{0r} = \pi_0/\pi_r$, the prior odds of H_0 against H_r.

In consequence, from a Bayesian viewpoint, the DW test differs from the posterior odds in two ways.

First, a statistic for the point null against the point alternative is applied to the composite alternative. While (11) contains the posterior pdf of ρ within (-1,1), (20) and (21) use the posterior density at $\rho=\rho_r$. As a result, the DW test does not cover the whole information about ρ obtained from data.

Second, even when the point alternative (15) is in concern, Non-Bayesians examine the sample distribution of (21) under the null hypothesis whereas Bayesians evaluate the posterior odds (20). For this reason, even though they are related by (20) or (21), the Durbin-Watson test and the posterior odds might lead to the opposite conclusions.

2.1.2 Posterior Distribution

The second Bayesian test of AR(1) errors in a regression model is based on the posterior pdf. In the case of the size-α one-sided test against positive correlation, the null hypothesis is rejected when the upper ($1-\alpha$) quatile of the posterior pdf given by (4) does not contain $\rho=0$. Similarly, in the case of the α-size two-sided test, the null is rejected when the ($1-\alpha$) HPD region does not include the origin.

The above-mentioned Bayesian tests of serial correlation do not depend on the repeated sampling principle. However, to make a comparison with the DW test possible, we undertake a Monte Carlo study in the next section.

3. Monte Carlo Study

3.1 Aim

The aim of this section is to compare the power of the Bayesian tests with that of the DW test via Monte Carlo analysis. The DW test can serve as a bench mark because its power is scrutinized by many authors, such as Kuriyama (1972a) and Kuriyama (1972b). Therefore, it is of interest to examine power of the Bayes tests, given the performance of the DW test.

3.2 Design

3.2.1 Method of Comparison

We compare the power of the Bayes tests with that of the DW test by completing a table whose format is shown in Appendix. $N_{\cdot\cdot}$ is the number of iteration. $N_{\cdot 0}$ and $N_{\cdot x}$ ($N_{0\cdot}$ and $N_{x\cdot}$), respectively, show the number of cases where the Bayesian (the DW) test accepts H_0 and H_x ($x=+,\pm$). N_{ij} shows the number of cases in which the DW test accepts H_i while the Bayesian test accepts H_j ($i,j=0,x$). The diagonal elements (N_{00} and N_{xx}) figure the number of cases where the two tests accept the same hypothesis whereas the off-diagonal elements (N_{0x} and N_{x0}) account for cases where they accept the opposite ones.

3.2.2 Parameter Values

In model (1) with the error structure (2), we set $\sigma^2=1.0$ and $\beta=0$ in all trials. However, this particular setup of parameter values is irrelevant to the power functions of the Bayesian tests and the DW test (See Breush (1980)). The autocorrelation parameter ρ is set at 0.0, 0.2, 0.4, 0.8 and 0.98 since positive correlation is likely to occur in economic analysis.

3.2.3 The Independent Variables

The sample size (n=21) and the number of independent variables (k=4, including the constant term) are common in all experiments.

The selected cases of independent variables are:

1) X_{dL} (The DW ratio achieves its lower bound, and its power is maximum against H_+.);

2) X_{dU} (The DW ratio attains its upper bound.);

3) X_{LE} (The power of the DW test is minimum against H_+.); and

4) The independent variables of the consumption function in Klein Model I (the current corporate profit, its lagged value, and the sum of personal wage payments).

The first three cases are artificial but substantial because the DW test functions as a bench mark in our Monte Carlo study. The variables concerned are given as follows.

$$X_{dL}: \quad X=[1 \;\; a_n \;\; a_{n-1} \;\; \cdots \;\; a_{n-k+2}], \tag{23}$$

$$X_{dU}: \quad X=[1 \;\; a_2 \;\; a_3 \;\; \cdots \;\; a_k], \text{ and} \tag{24}$$

$$X_{LE}: \quad X=[1 \;\; a_2+a_n \;\; a_3+a_{n-1} \;\; \cdots \;\; a_k+a_{n-k+2}], \tag{25}$$

where

$$a_j{}'=[\cos\{(j-1)\pi/2n\} \;\; \cos\{3(j-1)\pi/2n\} \;\; \cdots \;\; \cos\{(2n-1)(j-1)\pi/2n\}], \; j=1,2,\cdots,n. \tag{26}$$

As to effects of these variables on the DW test, see Kuriyama (1972b, p. 26).

The last case, on the other hand, is a sample from empirical studies. This may be helpful to examine the Bayesian tests in a practical situation.

3.2.3 Critical Values

We try to hold the significance level 5.0% in our simulation study. However, since Bayesian inference does not rely on the sampling theory, we cannot control the size (the probability that a test rejects the null when it is true) of the Bayes tests *a priori*. Nevertheless, we aim at the size 0.05 throughout our experiments. The critical values for the DW test, the posterior odds test, and the posterior pdf test are as follows.

We use 5.0% and 2.5% critical values of the DW ratio for a one-sided and a two-sided test, respectively. These figures are obtained by means of the $(a+bd_U)$ approximation proposed in Durbin and Watson (1971). Note that a two-sided test with the 2.5% critical value does not always convey the size 0.05.

Both the prior odds and the reference value for the Bayesian tests based on the posterior odds are set at unity for the reason mentioned in the previous section. Notice that this may have the size of the Bayes tests depart from the target level (5.0%). However, we have no choice but finding the size of the Bayesian tests through Monte Carlo simulations because the sampling theory does not explain the critical values for the Bayesian tests.

In dealing with a one-sided test based on the posterior pdf, we reject the null if $P[\rho>0|y]>0.95$. In conducting a two-sided test, H_0 is rejected when the 0.95 HPD region does not contain $\rho=0$. However, the size of these tests may differ from 5.0%, and it should be investigated by our Monte Carlo experiments.

3.2.4 Computation

The program is coded with Quick BASIC Ver.4.5. The DE formula (Mori (1987), pp. 168-186) is employed to carry out numerical integration. The normal random variables are stocked in a file and picked up at random from there to generate disturbances. The number of iteration is a hundred in all experiments.

4. Results and Discussion

Tables 1 to 4 report the results for X_{dL}, X_{dU}, X_{LE}, and the consumption function in Klein Model I, respectively. For the sake of space, only the cases of uncorrelated errors (ρ =0.0), moderately correlated errors (ρ =0.4) and highly correlated errors (ρ =0.8) are tabulated. The rest of the outcomes is available on request.

The main results are summarized as follows.

4.1 Size

First, we examine the size of the Bayes tests. On the whole, they reject the null at most 22 times out of 100 trials. In particular, the size of those based on the posterior pdf is close to 5.0% (the minimum is 2 and the maximum 12). Moreover, the size of the Bayesian two-sided tests are more stable, on the one hand, and closer to 5.0% on the average, on the other hand, than the counterparts of the DW test. These observations are remarkable since the Bayesian tests have no foundations on the sampling theory.

It is worth noticing that when the size of the Bayesian tests is much larger than that of the DW test (namely, in Tables 2 and 3), the independent variables contain trended elements and hence reduce the power of the DW test against positive correlation. From a Bayesian point of view, it seems reasonable to be sceptical in judging $\rho=0$ when data does not contain enough information. In this regard, the larger actual size in Tables 2 and 3 at $\rho=0$ could be thought of as a safegurd against positive autocorrelation.

We may also point out that the DW test can be misleading when used as a two-sided test. It is a common practice to reject the null hypothesis when the DW ratio d falls in $\{d|d<d_{a/2}$ or $d>4-d_{a/2}\}$, where $d_{a/2}$ is the lower $\alpha/2$ quantile of the DW ratio under the null. We suppose that this procedure should provide a size-α two-sided test. However, the size of the test may differ from the expected one, depending on regressors. For instance, the size is lower than its nominal value in part (A) of Table 1-2 (or 1-4), and larger in part (A) of Table 2-2 (or 2-4). The reason for this is obvious. Since the DW ratio achieves its lower bound in Table 1, $P\{d|d<d_{a/2}\}=\alpha/2$ but $P\{d|d>4-d_{a/2}\}<\alpha/2$, and the actual size is lower than the nominal one. Likewise, it is readily shown that the same test strategy causes a larger actual size in Table 2. While these two are trivial examples, a heuristic case is illustrated in Table 4, where the size of a two-sided test based on the DW ratio is larger than the expected one. A larger actual size of the two-sided DW test might be common in economic studies since, according to Kuriyama (1972b. p. 26), X_{dU} defined as (24) stands for trended variables observed as typical components in economic data.

4.2 Power

In most comparative tables, the Bayesian tests outperform the DW test. This is shown by the off-diagonal elements in Tables 1 to 4 when error terms are serially correlated. In short, N_{0x} is larger than N_{x0} in 30 out of 32 cases for ρ =0.4, 0.8 in Tables 1 to 4. In addition, in the two exceptions (parts (B) and (C) in Table 1-3), the difference N_{x0}-N_{0x} is at most 1 and hence negligible. Furthermore, Table 1-3 deals with a condition which is the most advantageous to the DW test. As far as the independent variables examined here are concerned, we may conclude that the Bayes tests are superior to the DW test in terms of power.

One of the most outstanding features from the comparison is that the Bayes tests can retain its power when the DW is the least powerful (Table 3). When disturbances are strongly correlated (ρ =0.8), the Bayesian tests reject the null hypothesis more than 80% on the basis of the posterior odds and more than 58% on the basis of the posterior pdf while the power of the DW test is less than 25%. When errors are moderately correlated (ρ =0.4), the Bayesian procedures reach the correct conclusion in 25 to 45 times out of 100 trials whereas the DW ratio detects autocorrelation at most 20%. The difference N_{0x}-N_{x0} is the largest when we refer to the one-sided test based on the posterior odds (part (B) in Table 3-1). However, we should not exaggerate this strength of the posterior odds Bayes test because of its larger size than that of the DW ratio (part (A) in Table 3-1).

To summarize, the Bayesian tests are more powerful than the DW test in the situations examined here. When the DW test is powerful, the Bayes tests can compete with it. When the power of the DW test is poor, the Bayes tests maintain their power. In addition, the Bayes tests may perform better than the DW test when we deal with economic data. In consequence, we may use the Bayesian tests in place of the DW test if we emphasize the risk of type II error.

5. Conclusion

We found that Bayesian inference can be powerful in detecting serial correlation in a linear regression model. Our Monte Carlo experiments proved that the Bayesian tests of autocorrelation can compete with the DW test when the latter achieves its maximum power, and that the Bayesian tests remain powerful when the power of the DW test is poor. Although we cannot control their size, the Bayesian tests may compensate the power of the DW test.

Last, we describe some points which need further research. First, we employed Jeffreys' prior pdf given by (3) throughout this paper, but did not try other types, such as a flat prior. Next, we should control the size of the Bayesian tests to make a comparison meaningful. Finally, our simulation experience was simply preliminary. First of all, the number of repetition was too small as a reliable Monte Carlo study. Second, the normal random variables were generated in a rather primitive way, and it should be improved.

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Appendix

The Foramt of a Comparative Table

DW	Bayes H_0	Bayes H_x	Total
H_0	N_{00}	N_{0x}	$N_{0\cdot}$
H_x	N_{x0}	N_{xx}	$N_{x\cdot}$
Total	$N_{\cdot 0}$	$N_{\cdot x}$	$N_{\cdot\cdot}$

Notaion:

H_0: The null ($\rho = 0$);
H_x: The alternative ($\rho > 0$ when x=+; $\rho \neq 0$ when x=$\pm$);
N_{ij}: The number of cases in which the Durbin-Watson test accepts H_i while the Bayes test accepts H_j ($i,j=0,x$);
$N_{i\cdot}$: The number of cases in which the Durbin-Watson test accepts H_i ($i=0,x$);
$N_{\cdot j}$: The number of cases in which the Bayes test accepts H_j ($j=0,x$); and
$N_{\cdot\cdot}$: The number of iteration.

Table 1: X_{dL} (The power of the DW test is maximum (n=21, k=4).)

Table 1-1: One-Sided Test Based on Posterior Odds

(A) ρ =0.0

	Bayes		
DW	H_0	H_+	Total
H_0	95	3	98
H_+	0	2	2
Total	95	5	100

(B) ρ =0.4

	Bayes		
DW	H_0	H_+	Total
H_0	45	14	59
H_+	0	41	41
Total	45	55	100

(C) ρ =0.8

	Bayes		
DW	H_0	H_+	Total
H_0	6	3	9
H_+	0	91	91
Total	6	94	100

Table 1-2: Two-Sided Test Based on Posterior Odds

(A) ρ =0.0

	Bayes		
DW	H_0	H_{+-}	Total
H_0	89	10	99
H_{+-}	0	1	1
Total	89	11	100

(B) ρ =0.4

	Bayes		
DW	H_0	H_{+-}	Total
H_0	58	16	74
H_{+-}	0	26	26
Total	58	42	100

(C) ρ =0.8

	Bayes		
DW	H_0	H_{+-}	Total
H_0	10	2	12
H_{+-}	0	88	88
Total	10	90	100

Table 1-3: One-Sided Test Based on Posterior Distribution

(A) ρ =0.0

	Bayes		
DW	H_0	H_+	Total
H_0	98	0	98
H_+	0	2	2
Total	98	2	100

(B) ρ =0.4

	Bayes		
DW	H_0	H_+	Total
H_0	59	0	59
H_+	1	40	41
Total	60	40	100

(C) ρ =0.8

	Bayes		
DW	H_0	H_+	Total
H_0	8	1	9
H_+	1	90	91
Total	9	91	100

Table 1-4: Two-Sided Test Based on Posterior Distribution

(A) ρ =0.0

	Bayes		
DW	H_0	H_{+-}	Total
H_0	90	9	99
H_{+-}	0	1	1
Total	90	10	100

(B) ρ =0.4

	Bayes		
DW	H_0	H_{+-}	Total
H_0	62	12	74
H_{+-}	0	26	26
Total	62	38	100

(C) ρ =0.8

	Bayes		
DW	H_0	H_{+-}	Total
H_0	10	2	12
H_{+-}	1	87	88
Total	11	89	100

Table 2: X_{du} (The DW ratio achieves its upper bound (n=21, k=4).)

Table 2-1: One-Sided Test Based on Posterior Odds

(A) ρ =0.0

DW	Bayes H_0	Bayes H_+	Total
H_0	78	16	94
H_+	0	6	6
Total	78	22	100

(B) ρ =0.4

DW	Bayes H_0	Bayes H_+	Total
H_0	31	35	66
H_+	0	34	34
Total	31	69	100

(C) ρ =0.8

DW	Bayes H_0	Bayes H_+	Total
H_0	20	23	43
H_+	0	57	57
Total	20	80	100

Table 2-2: Two-Sided Test Based on Posterior Odds

(A) ρ =0.0

DW	Bayes H_0	Bayes H_{+-}	Total
H_0	48	7	55
H_{+-}	37	8	45
Total	85	15	100

(B) ρ =0.4

DW	Bayes H_0	Bayes H_{+-}	Total
H_0	40	27	67
H_{+-}	8	25	33
Total	48	52	100

(C) ρ =0.8

DW	Bayes H_0	Bayes H_{+-}	Total
H_0	27	23	50
H_{+-}	2	48	50
Total	29	71	100

Table 2-3: One-Sided Test Based on Posterior Distribution

(A) ρ =0.0

DW	Bayes H_0	Bayes H_+	Total
H_0	90	4	94
H_+	0	6	6
Total	90	10	100

(B) ρ =0.4

DW	Bayes H_0	Bayes H_+	Total
H_0	49	17	66
H_+	0	34	34
Total	49	51	100

(C) ρ =0.8

DW	Bayes H_0	Bayes H_+	Total
H_0	30	13	43
H_+	0	57	57
Total	30	70	100

Table 2-4: Two-Sided Test Based on Posterior Distribution

(A) ρ =0.0

DW	Bayes H_0	Bayes H_{+-}	Total
H_0	48	7	55
H_{+-}	40	5	45
Total	88	12	100

(B) ρ =0.4

DW	Bayes H_0	Bayes H_{+-}	Total
H_0	45	22	67
H_{+-}	8	25	33
Total	53	47	100

(C) ρ =0.8

DW	Bayes H_0	Bayes H_{+-}	Total
H_0	30	20	50
H_{+-}	2	48	50
Total	32	68	100

Table 3: X_{LE} (The power of the DW test is minimum (n=21, k=4).)

Table 3-1: One-Sided Test Based on Posterior Odds

(A) ρ =0.0

	Bayes		
DW	H_0	H_+	Total
H_0	82	13	95
H_+	0	5	5
Total	82	18	100

(B) ρ =0.4

	Bayes		
DW	H_0	H_+	Total
H_0	55	27	82
H_+	0	18	18
Total	55	45	100

(C) ρ =0.8

	Bayes		
DW	H_0	H_+	Total
H_0	11	64	75
H_+	0	25	25
Total	11	89	100

Table 3-2: Two-Sided Test Based on Posterior Odds

(A) ρ =0.0

	Bayes		
DW	H_0	H_{+-}	Total
H_0	83	10	93
H_{+-}	2	5	7
Total	85	15	100

(B) ρ =0.4

	Bayes		
DW	H_0	H_{+-}	Total
H_0	69	16	85
H_{+-}	2	13	15
Total	71	29	100

(C) ρ =0.8

	Bayes		
DW	H_0	H_{+-}	Total
H_0	16	68	84
H_{+-}	1	15	16
Total	17	83	100

Table 3-3: One-Sided Test Based on Posterior Distribution

(A) ρ =0.0

	Bayes		
DW	H_0	H_+	Total
H_0	92	3	95
H_+	1	4	5
Total	93	7	100

(B) ρ =0.4

	Bayes		
DW	H_0	H_+	Total
H_0	73	9	82
H_+	1	17	18
Total	74	26	100

(C) ρ =0.8

	Bayes		
DW	H_0	H_+	Total
H_0	41	34	75
H_+	1	24	25
Total	42	58	100

Table 3-4: Two-Sided Test Based on Posterior Distribution

(A) ρ =0.0

	Bayes		
DW	H_0	H_{+-}	Total
H_0	88	5	93
H_{+-}	4	3	7
Total	92	8	100

(B) ρ =0.4

	Bayes		
DW	H_0	H_{+-}	Total
H_0	74	11	85
H_{+-}	1	14	15
Total	75	25	100

(C) ρ =0.8

	Bayes		
DW	H_0	H_{+-}	Total
H_0	25	59	84
H_{+-}	1	15	16
Total	26	74	100

Table 4: Consumption Function in Klein Model 1 (n=21, k=4).

Table 4-1: One-Sided Test Based on Posterior Odds

(A) ρ =0.0

	Bayes		
DW	H_0	H_+	Total
H_0	90	8	98
H_+	0	2	2
Total	90	10	100

(B) ρ =0.4

	Bayes		
DW	H_0	H_+	Total
H_0	38	31	69
H_+	0	31	31
Total	38	62	100

(C) ρ =0.8

	Bayes		
DW	H_0	H_+	Total
H_0	12	11	23
H_+	0	77	77
Total	12	88	100

Table 4-2: Two-Sided Test Based on Posterior Odds

(A) ρ =0.0

	Bayes		
DW	H_0	H_{+-}	Total
H_0	74	1	75
H_{+-}	22	3	25
Total	96	4	100

(B) ρ =0.4

	Bayes		
DW	H_0	H_{+-}	Total
H_0	60	15	75
H_{+-}	1	24	25
Total	61	39	100

(C) ρ =0.8

	Bayes		
DW	H_0	H_{+-}	Total
H_0	19	13	32
H_{+-}	1	67	68
Total	20	80	100

Table 4-3: One-Sided Test Based on Posterior Distribution

(A) ρ =0.0

	Bayes		
DW	H_0	H_+	Total
H_0	96	2	98
H_+	0	2	2
Total	96	4	100

(B) ρ =0.4

	Bayes		
DW	H_0	H_+	Total
H_0	55	14	69
H_+	0	31	31
Total	55	45	100

(C) ρ =0.8

	Bayes		
DW	H_0	H_+	Total
H_0	21	2	23
H_+	0	77	77
Total	21	79	100

Table 4-4: Two-Sided Test Based on Posterior Distribution

(A) ρ =0.0

	Bayes		
DW	H_0	H_{+-}	Total
H_0	74	1	75
H_{+-}	22	3	25
Total	96	4	100

(B) ρ =0.4

	Bayes		
DW	H_0	H_{+-}	Total
H_0	59	16	75
H_{+-}	1	24	25
Total	60	40	100

(C) ρ =0.8

	Bayes		
DW	H_0	H_{+-}	Total
H_0	21	11	32
H_{+-}	1	67	68
Total	22	78	100

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