A Gradual Switching Regression Model with Gradual Switching Autocorrelation among Errors

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ABSTRACT

A gradual switching regression model with gradual switching autocorrelation among errors is discussed. The maximum likelihood inference of this model is proposed and applied to a consumption function in Japan as well as money demand functions in the U.S., Canada, and Japan. The author concludes that the likelihood ratio test for change points with a chi-square distribution is unreliable and that the exact test procedure should be developed.

1. Introduction

Switching regressions have been frequently used to investigate parameter shifts in linear models. In particular, it is believed that a gradual switching regression model with autocorrelated errors is pertinent to detect an economic structural change for several reasons. For example, absorption of new technology in industries tends to be gradual. In addition, many economic models appear with serial correlation among disturbances. In consequence, a number of methods in this field have been developed in econometric research. Nonetheless, very few work can be found dealing with gradual changes in the autocorrelation coefficient of error terms.

Many econometricians consider a changing regression model with correlated errors, assuming the autoregressive parameter of residuals to remain constant during estimation. For instance, Salazar, et al. (1981) and Ohtani (1982) examine abrupt switching regressions while Ohtani and Katayama (1986) as well as Tsurumi (1976) discuss gradual shifts in regression coefficients. Similarly, from a Bayesian point of view, Salazar (1982) compares abrupt and gradual changing models on the same assumption of errors. Furthermore, Tsurumi and Kan (1991) study a gradual switching regression with correlated and heteroskedastic disturbances. However, all of them suppose the stability of autocorrelation among residual terms.

In contrast, Ilmakunnas and Tsurumi (1984, 1985) as well as Cadsby and Stengos (1986) test a two-phase regression model where autocorreltated errors obey two regimes, presuming the simultaneous chnage in regression coefficients and autocorrelation. In fact, they detect a change in correlation among error terms. As a result, Cadsby and Stengos (1986) conclude that "There is rarely a justification for assuming ρ [the autocorrelation parameter] to be stable a priori (p. 32)." However, not only Ilmakunnas and Tsurumi (1984, 1985) but Cadsby and Stengos (1986) as well suppose that regression coefficients and the autoregressive parameter abruptly switch at the same join point.

Nevertheless, Inaba (1990) asserts that changes in stochastic properties of error terms may occur not only gradually but also differently from those in regression coefficients. He observes that the variance of error terms may first increase gradually prior to shifts in regression coefficients and then revert to the previous level. In general, he implies that changes in statistical characteristics of disturbances may be gradual and that they begin at a different time from parameter shifts in regression equations. Consequently, in a switching model with autocorrelated errors, it is relevant to assume that the autocorrelation parameter of disturbances can gradually switch in a different time pattern from regression coefficients.

However, very little research has been done on this possibility. Although Salazar (1982) explores a gradual switching autoregressive time series model, his idea has not been applied to gradual shifts in autocorrelation of errors in switching regressions.

The aim of this study is to examine a gradual switching regression model with gradual switching autocorrelation among errors. The next section outlines the model. Section 3 describes the maximum likelihood inference on the model. In Section 4, the model is applied to economic data. Some concluding remarks are made in the final section.

2. Model

Consider a regression model

$$y_t = \mathbf{x}'_t(\boldsymbol{\beta} + \lambda_t \boldsymbol{\delta}) + u_t, \qquad t=1,2,\cdots,T, \tag{1}$$

where y_t is the scalar dependent variable; $\mathbf{x}_t$ is a k-dimensional vector of the independent variables; $\boldsymbol{\beta}$ and $\boldsymbol{\delta}$ are k-dimensional coefficient vectors; λ_t is a scalar transition path for regression coefficients defined afterwards; and u_t follows

$$u_t = (\rho + \mu_t \omega) u_{t-1} + v_t, \qquad v_t \sim NID(0, \sigma^2), \tag{2}$$

where μ_t is a scalar transition path for the autocorrelation parameter specified later and

$$-\infty < \rho < \infty, \quad -\infty < \omega < \infty. \tag{3}$$

Both y_0 and $\mathbf{x}_0$ are assumed to be given. Condition (3) means that model (1) covers an explosive case. Although σ^2 is constant in each observation, model (1) contains heteroscedastic errors (See appendix).

Following Ohtani and Katayama (1986), we assume a linear transition path for both regression coefficients and the autocorrelation parameter, i.e.:

$$\lambda_t = \begin{cases} 0 & for \ t=1,2,\cdots,r_1, \\ \dfrac{t-r_1}{r_2-r_1} & for \ t=r_1+1,\cdots,r_2, \\ 1 & for \ t=r_2+1,\cdots,T, \end{cases} \quad k \le r_1 < r_2 \le T-k, \tag{4}$$

where $\underline{r}_1$ and $\underline{r}_2$ are the unknown start and end points of gradual switches in regression coefficients; and

$$\mu_t = \begin{cases} 0 & for \ t=1,2,\cdots,s_1, \\ \dfrac{t-s_1}{s_2-s_1} & for \ t=s_1+1,\cdots,s_2, \\ 1 & for \ t=s_2+1,\cdots,T, \end{cases} \quad 2 \le s_1 < s_2 \le T-2 \tag{5}$$

where $\underline{s}_1$ and $\underline{s}_2$ denote the unknown start and end points of gradual shifts in the autocorrelation parameter. We follow Worsley (1983) in defining the lower and upper limits of these change points. According to Ohtani and Katayama (1986), a linear transition path, such as (4) and (5), enables a researcher to clarify both the beginning and the end of gradual changes in parameters. Moreover, Tsurumi (1980) points out that the choice of a transition formula is irrelevant to inference when the sample size is large. Therefore, we use this specification throughout this study.

3. Statistical Inference

3.1 Estimation

Let γ_t denote

$$\gamma_t = \rho + \mu_t \omega. \tag{6}$$

Subtracting the (t-1)-th observation multiplied by γ_t from each side of equation (1), we obtain

$$\boldsymbol{y}^* = \boldsymbol{X}^*\boldsymbol{\theta} + \boldsymbol{v}, \tag{7}$$

where

$$\begin{aligned}
\boldsymbol{y}^* &= [y_t - \gamma_t y_{t-1}],\\
\boldsymbol{X}^* &= [\boldsymbol{x}_t' - \gamma_t \boldsymbol{x}_{t-1}' \quad \lambda_t \boldsymbol{x}_t - \gamma_t \lambda_{t-1} \boldsymbol{x}_{t-1}'],\\
\boldsymbol{\theta} &= \begin{pmatrix} \boldsymbol{\beta} \\ \boldsymbol{\delta} \end{pmatrix}.
\end{aligned} \tag{8}$$

Equation (7) yields the likelihood function conditioned on $\underline{y}_0$ and $\underline{x}_0$

$$\begin{aligned}
&L(r_1, r_2, s_1, s_2, \boldsymbol{\theta}, \rho, \omega, \sigma^2)\\
&= \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^T \exp\left[-\frac{1}{2\sigma^2}(\boldsymbol{y}^* - \boldsymbol{X}^*\boldsymbol{\theta})'(\boldsymbol{y}^* - \boldsymbol{X}^*\boldsymbol{\theta})\right].
\end{aligned} \tag{9}$$

Note that both $\underline{y}^*$ and $\underline{X}^*$ are functions of $\underline{r}_1$, $\underline{r}_2$, $\underline{s}_1$, $\underline{s}_2$, ρ and ω. Therefore, the log likelihood function is

$$\begin{aligned}
&l(r_1, r_2, s_1, s_2, \boldsymbol{\theta}, \rho, \omega, \sigma^2)\\
&= const. - \frac{T}{2}\log\sigma^2 - \frac{1}{2\sigma^2}(\boldsymbol{y}^* - \boldsymbol{X}^*\boldsymbol{\theta})'(\boldsymbol{y}^* - \boldsymbol{X}^*\boldsymbol{\theta}).
\end{aligned} \tag{10}$$

The maximum likelihood estimators (MLE's) for θ and σ^2 conditional on $\underline{r}_1$, $\underline{r}_2$, $\underline{s}_1$, $\underline{s}_2$, ρ and ω in equation (10) are expressed, respectively, as:

$$\begin{aligned} \hat{\theta} &= (\boldsymbol{X}^{*\prime}\boldsymbol{X}^{*})^{-1}\boldsymbol{X}^{*\prime}\boldsymbol{y}^{*}, \\ \hat{\sigma}^2 &= \frac{1}{T}(\boldsymbol{y}^{*} - \boldsymbol{X}^{*}\hat{\theta})'(\boldsymbol{y}^{*} - \boldsymbol{X}^{*}\hat{\theta}). \end{aligned} \tag{11}$$

Note that $\underline{X}^{*\prime}\underline{X}^{*}$ is non-invertible at $\rho=1$ and $\omega=0$ when model (1) has the constant term. In this case, the first difference should be applied to the data matrix with the constant term omitted.

The MLE's (11) provide the concentrated log likelihood function

$$l_{\max}(r_1,\ r_2,\ s_1,\ s_2,\ \rho,\ \omega) = const. - \frac{T}{2}\log\hat{\sigma}^2. \tag{12}$$

Non-linear equation (12) in the parameters ρ and ω requires numerical optimization to obtain the MLE's.

Moreover, this equation contains discrete elements, $\underline{r}_1$, $\underline{r}_2$, $\underline{s}_1$ and $\underline{s}_2$. However, according to Quandt (1958), these integers could be regarded as approximately continuous if a large number of observations are available. As a result, the MLE's can be described as a set of continuous and discrete parameters which maximizes the value of equation (12).

3.2 Test

One might test a hypothesis in model (1) based on the likelihood ratio (LR). For instance, to test a hypothesis of no change in the error structure (i.e., $\omega=0$), the following critical region could be used

$$l_{\max}(\tilde{r}_1,\ \tilde{r}_2,\ \tilde{s}_1,\ \tilde{s}_2,\ \tilde{\beta},\ \tilde{\omega}) - l_{\max}(\hat{r}_1,\ \hat{r}_2,\ \hat{\beta}\,|\,\omega=0) > \chi_3^2(\alpha)/2, \quad (13)$$

where the parenthesized parameters with tildes of the first term on the left hand side represent the MLE's of the full model whereas those with hats of the second term denote the MLE's of the restricted models, and $\chi_3^2(\alpha)$ denotes the upper α percent critical value of the chi-square distribution with degrees of freedom 3.

However, one should not emphasize a result of LR tests in two-phase regressions. Not only Quandt (1960) but also Beckman and Cook (1979) report that the LR test with a chi-square distribution conveys the inappropriate size of tests in detecting a join point of regression coefficients. Even though Chu and White (1991) shows the limiting distribution of the LR test statistic for a change point in some simple cases, it is not applicable to such tests as (13). Therefore, the testing procedure given by (13) is unreliable in terms of size.

4. Some Applications

In this section, we apply model (1) to (a) a consumption function in Japan and (b) money demand functions in the U.S., Canada and Japan. To obtain the MLE's, we use the Hooke and Jeeves method (Iwata and Kuroda (1968)) with respect to both continuous and discrete parameters. This procedure requires several initial values to examine the reliability on results.

4.1 Consumption Function in Japan

The consumption function in Japan used here is written as

$$\left(\frac{C}{PC}\right)_t = (\beta_1+\lambda_t\delta_1)+(\beta_2+\lambda_t\delta_2)\left(\frac{YD}{PC}\right)_t+(\beta_3+\lambda_t\delta_3)\left(\frac{W_{-1}}{PC}\right)_t+u_t, \qquad (14)$$

where $\underline{C}$ is the household consumption; $\underline{PC}$ is the deflator of $\underline{C}$; $\underline{YD}$ is the household disposable income; and $\underline{W}_{-1}$ is the household net financial asset at the beginning of the current period. The error terms have the same structure as model (1). The data used in this study are annual data from 1967 to 1990. The results are summarized in Table 1.

From the upper half of Table 1, we see that (i) regression coefficients changed quickly between 1977 and 1979: the constant term decreased to 0, the marginal propensity to consume (MPC) almost tripled and the marginal effect of financial asset reduced to almost 0; and that (ii) the autocorrelation parameter diminished abruptly within 1981.

The lower half of Table 1 exhibits the result of the conditional model on ω=0. The start and the end points of gradual shifts in regression coefficients in this model are the same as those in the full model. The autoregressive parameter in the constraned model is within the range calculated by the complete model. The estimates of regression coefficients are slightly different. The test statistic given by formula (13) is 2.14 and insignificant at 5 percent level: The autocorrelation of errors has not switched during estimation. Still, the sample size $\underline{T}$=23 requires caution in interpreting this conclusion.

A three-fold change in MPC combined with an annihilated effect of assets within a few years seems strange to economists.

One possibility is that the consumption function in this nation is still shifting to a new stage. That is, a sudden and remarkable change in regression parameters could result from ongoing parallel shifts in the consumption function. Japan's consumption expenditure stimulated by soaring land prices in the late 1980's might justify this explanation, but further research is necessary regarding this point.

4.2 Money Demand Functions in the U.S., Canada, and Japan

The money demand function used in this section is

$$\begin{aligned}\ln\left(\frac{M_{1t}}{P_t}\right)=(\beta_1+\lambda_t\delta_1)+(\beta_2+\lambda_t\delta_2)\ln\left(\frac{GNP_t}{P_t}\right)+(\beta_3+\lambda_t\delta_3)\ln(RMS_t)\\+(\beta_4+\lambda_t\delta_4)\ln\left(\frac{M_{1\,t-1}}{P_{t-1}}\right)+u_t,\end{aligned} \tag{15}$$

where $\underline{M_1}$ is currency and demand deposit; $\underline{GNP}$ is gross national product; $\underline{P}$ is GNP deflator; and $\underline{RMS}$ is short term market rate of interest. This model is employed by Tsurumi and Kan (1991) to estimate money demand functions in the U.S., Canada, and Japan.

Tables 2a, 2b and 2c present the estimated results for the three nations. The upper and lower halves of each table show the results from the full and the constrained (i.e., ω=0) models, respectively. The source of data, the definition of $\underline{RMS}$, and the estimation period are given in the caption of each table.

The results for the U.S. show that in the unrestricted model, regression coefficients changed abruptly from the first quarter of 1980 to the next quarter while the autoregression in errors increased gradually from -0.19 to 0.42 between the fourth quarter of 1969 and the first quarter of 1986. This leads to gradual changes in the variance of errors around a shift in economic structures. From the lower half of Table 2a, we see that in the restricted model, the start and end points of shifts in regression coefficients coincide with those in the former model while the correlation parameter 0.19 is positioned within the ranges between -0.19 and 0.42. Due to the difference in autocorrelation, the estimates of regression coefficients are different, but the deviation do not seem to be significant. The test statistic given by formula (13) is 3.00 and smaller than the critical value at 5 percent. However, the final conclusion should be postponed because of possible invalidity of test size.

The results for Canada present that shifts in regression parameters began at the second quarter of 1970 and finished at the first quarter of 1980, followed by abrupt changes in autoregression among errors from the third quarter of 1983 to the adjacent period. In contrast to the results for the U.S., the change points of regression coefficients in the restricted and unrestricted model are not the same and the hypothesis of no change in the error structure is rejected at 5 percent (The test statistic is 9.03). Still, it seems unpersuasive that a sudden change in the correlation among errors occurred much later than gradual shifts in regression parameters. This may indicate misspesification of model (15). However, it is not our goal to obtain an acceptable money demand function in this nation, thus no further details will be discussed here.

The outcomes for Japan represent that regression coefficients changed gradually during the 21 years from the fourth quarter of 1968 to the fourth quarter of 1989 whereas the autocorrelation among errors shifted step-by-step from 0.47 to

0.04 between the third quarter of 1961 and the fourth quarter of 1973. The lower part of Table 2c exhibits that the shift points in regression coefficients of the limited model are the same as those of the full model and that errors are positively correlated. The test statistic proposed by (13) is 1.75 and insignificant at 5 percent. However, the same caution as in the case of the U.S. should be referred to in assessing this result.

To summarize the results in this subsection, the shifts points of regression coefficients are relatively stable irrespective of the error structure. However, estimated regression parameters are different, particularly for the lagged dependent variable. This may be due to correlation between errors and the lagged dependent variable, although the maximum likelihood estimators are still consistent under this circumstance.

5. Conclusion

In this paper, we proposed the maximum likelihood procedure for a gradual switching regression model with gradual switching autocorrelation among errors, and applied it to a consumption function in Japan and money demand equations in the U.S., Canada, and Japan. It was observed that correlation among disturbances may gradually change at the different timing from that of parameter shifts in regression lines. Therefore, it was concluded that researchers should assume this possibility when they employ a gradual switching regression with autoregressive errors.

However, in this study, the significance level of tests containing discrete parameters were not examined at all. Further research is required concerning this point. One solution is a Bayesian procedure which yields posterior distributions for continuous parameters and posterior mass functions for discrete elements. The strengths of Bayesian methods may be remarkable in a gradual switching regression with gradual switching correlation among disturbances.

APPENDIX

In this appendix, we derive the variance of error terms defined by (2) and (5). With the notation (6), the disturbance terms can be rewritten as

$$\begin{aligned} u_t &= v_t + \gamma_t u_{t-1} \\ &= v_t + \gamma_t v_{t-1} + \gamma_t \gamma_{t-1} u_{t-2} \\ &\cdots \\ &= v_t + \sum_{i=1}^{t-1} \left\{ \left(\prod_{j=1}^{i} \gamma_{t-i+j} \right) v_{t-i} \right\} + \left(\prod_{j=1}^{t} \gamma_j \right) u_0 . \end{aligned} \tag{A.1}$$

Therefore, we have

$$\mathrm{E}(u_t) = \begin{cases} 0 & (u_0 \ \mathit{random}), \\ \left(\prod_{j=1}^{t} \gamma_j \right) u_0 & (u_0 \ \mathit{given}), \end{cases} \tag{A.2}$$

and

$$\mathrm{V}(u_t) = \begin{cases} \sigma^2 \left\{ 1 + \sum_{i=1}^{t-1} \left(\prod_{j=1}^{i} \gamma_{t-i+j}^2 \right) \right\} + \left(\prod_{j=1}^{t} \gamma_j^2 \right) \sigma_0^2 & (u_0 \ \mathit{random}), \\ \sigma^2 \left\{ 1 + \sum_{i=1}^{t-1} \left(\prod_{j=1}^{i} \gamma_{t-i+j}^2 \right) \right\} & (u_0 \ \mathit{given}). \end{cases} \tag{A.3}$$

From (6), we have

$$
\prod_{j=1}^{i}\gamma^{2}_{t-i+j}
= \begin{cases}
\rho^{2i} & 2 \le t \le s_1, \\
\left.\begin{array}{ll}
\displaystyle\prod_{j=t-i+1}^{t}\left(\rho+\frac{j-s_1}{s_2-s_1}\omega\right)^{2} & t-i > s_1, \\
\displaystyle\rho^{2(i-t+s_1)}\prod_{j=s_1+1}^{t}\left(\rho+\frac{j-s_1}{s_2-s_1}\omega\right)^{2} & t-i \le s_1,
\end{array}\right\} & s_1+1 \le t \le s_2, \\
\left.\begin{array}{ll}
(\rho+\omega)^{2i} & t-i > s_2, \\
\displaystyle\left\{\prod_{j=i-t+s_2}^{s_2}\left(\rho+\frac{j-s_1}{s_2-s_1}\omega\right)^{2}\right\}(\rho+\omega)^{2(t-s_2)} & s_1+1 \le t-i \le s_2, \\
\displaystyle\rho^{2(i-t+s_1+1)}\left\{\prod_{j=s_1+1}^{s_2}\left(\rho+\frac{j-s_1}{s_2-s_1}\omega\right)^{2}\right\}(\rho+\omega)^{2(t-s_2)} & t-i \le s_1,
\end{array}\right\} & s_2+1 \le t.
\end{cases}
\tag{A.4}
$$

Thus, the stochastic process given by (2) and (5) yields heteroscedasticity.

However, as is readily shown, the variance for $\underline{t} \le \underline{s}_1$ and $\underline{t} \ge \underline{s}_2$ is almost stable if the absolute values of ρ and $\rho+\omega$ are smaller than unity. For example, given $\underline{T}=30$, $\underline{s}_1=10$, $\underline{s}_2=20$, $\rho=0.5$ and $\omega=0.3$ as well as non-random $\underline{u}_0$, the variance of error terms is depicted in Figure A.1.

In conclusion, the error process given by (2) and (5) describes heteroscedasticity during a gradual switch of regimes of disturbances. Therefore, the formulation reflects Inaba's (1990) assertion that the variance of errors may gradually shift along with economic structural change.

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Table 1 Consumption Function in Japan

	r_1	r_2	s_1	s_2	ρ	ω	β_1	δ_1	β_2	δ_2	β_3	δ_3
Full	11	13	14	15	0.775	-0.864	43.509	-58.537	0.315	0.581	0.338	-0.325
	$\bar{\sigma}^2 = 0.897$		(Standard Deviation)				9.862	15.651	0.074	0.105	0.046	0.048
Null	11	13			0.559		30.079	-55.336	0.397	0.554	0.315	-0.310
	$\bar{\sigma}^2 = 1.081$		(Standard Deviation)				5.100	14.895	0.057	0.102	0.049	0.052

(1) The model used is given by (14).

(2) Full means the unrestricted model.

(3) Null means the model conditioned on $\omega = 0$.

(4) The standard deviations are conditionded on the change points and the autoregressive parameter(s).

(5) Annual data from 1967 to 1990.

(6) Data source:

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p. 20(1976), p. 24(1977), p. 26(1978), p. 30(1979), p. 30(1980),
p. 32(1981), p. 32(1982), p. 32(1983), p. 32(1984), p. 220(1985),
p. 220(1986), p. 220(1987), p. 220(1988), p. 222(1989).

Table 2a Money Demand Function in the U.S.

	r_1	r_2	s_1	s_2	ρ	ω	β_1	δ_1	β_2	δ_2	β_3	δ_3	β_4	δ_4
Full	75	76	33	89	-0.190	0.614	0.437	0.277	0.085	0.055	-0.002	-0.060	0.845	-0.081
	$\bar{\sigma}^2 = 0.000107$			(Standard Deviation)			0.173	0.415	0.025	0.081	0.005	0.012	0.049	0.079
Null	75	76			0.186		0.744	-0.003	0.125	-0.008	-0.005	-0.057	0.760	0.029
	$\bar{\sigma}^2 = 0.000113$			(Standard Deviation)			0.232	0.384	0.033	0.070	0.006	0.012	0.065	0.083

(1) The model used is given by (15).

(2) Full means the unrestricted model.

(3) Null means the model conditioned on $\omega = 0$.

(4) The standard deviations are conditionded on the change points and the autoregressive parameter(s).

(5) Quarterly data from 1961.3 to 1990.4.

(6) RMS is the rate of certificates deposit.

(7) Data source:
Department of Economics and Statistics, OECD,
Main Economic Indicators, 1960-1990, 1991.
(Transrated into Japanese by Yoshitomi, M. (ed.), Tokyo: Hara-Shobo, 1992, pp. 48, 85, 88.)

Table 2b Money Demand Function in Canada

	r_1	r_2	s_1	s_2	ρ	ω	β_1	δ_1	β_2	δ_2	β_3	δ_3	β_4	δ_4
Full	41	81	94	95	-0.182	0.894	1.708	2.081	0.132	0.067	-0.022	-0.045	0.667	-0.267
	$\bar{\sigma}^2 = 0.000275$				(Standard Deviation)		0.434	0.428	0.036	0.055	0.009	0.016	0.066	0.100
Null	41	76			-0.087		1.133	1.047	0.152	-0.029	-0.026	-0.019	0.702	-0.056
	$\bar{\sigma}^2 = 0.000319$				(Standard Deviation)		0.403	0.407	0.042	0.055	0.011	0.015	0.071	0.097

(1) The model used is given by (15).

(2) Full means the unrestricted model.

(3) Null means the model conditioned on $\omega = 0$.

(4) The standard deviations are conditionded on the change points and the autoregressive parameter(s).

(5) Quarterly data from 1960.2 to 1990.4.

(6) RMS is the call rate.

(7) Data source: Department of Economics and Statistics, OECD,
Main Economic Indicators, 1960-1990, 1991.
(Transrated into Japanese by Yoshitomi, M. (ed.), Tokyo: Hara-Shobo, 1992, pp. 2, 34, 37.)

Table 2c Money Demand Function in Japan

	r_1	r_2	s_1	s_2	ρ	ω	β_1	δ_1	β_2	δ_2	β_3	δ_3	β_4	δ_4
Full	34	117	6	54	0.473	-0.506	0.692	5.352	-0.002	-0.158	0.003	-0.037	0.935	-0.282
	$\hat{\sigma}^2 = 0.000262$				(Standard Deviation)		0.213	1.015	0.051	0.146	0.011	0.020	0.052	0.114
Null	34	117			0.142		0.659	4.864	-0.030	-0.024	0.004	-0.043	0.970	-0.388
	$\hat{\sigma}^2 = 0.000270$				(Standard Deviation)		0.230	0.882	0.051	0.129	0.012	0.022	0.045	0.119

(1) The model used is given by (15).

(2) Full means the unrestricted model.

(3) Null means the model conditioned on $\omega = 0$.

(4) The standard deviations are conditionded on the change points and the autoregressive parameter(s).

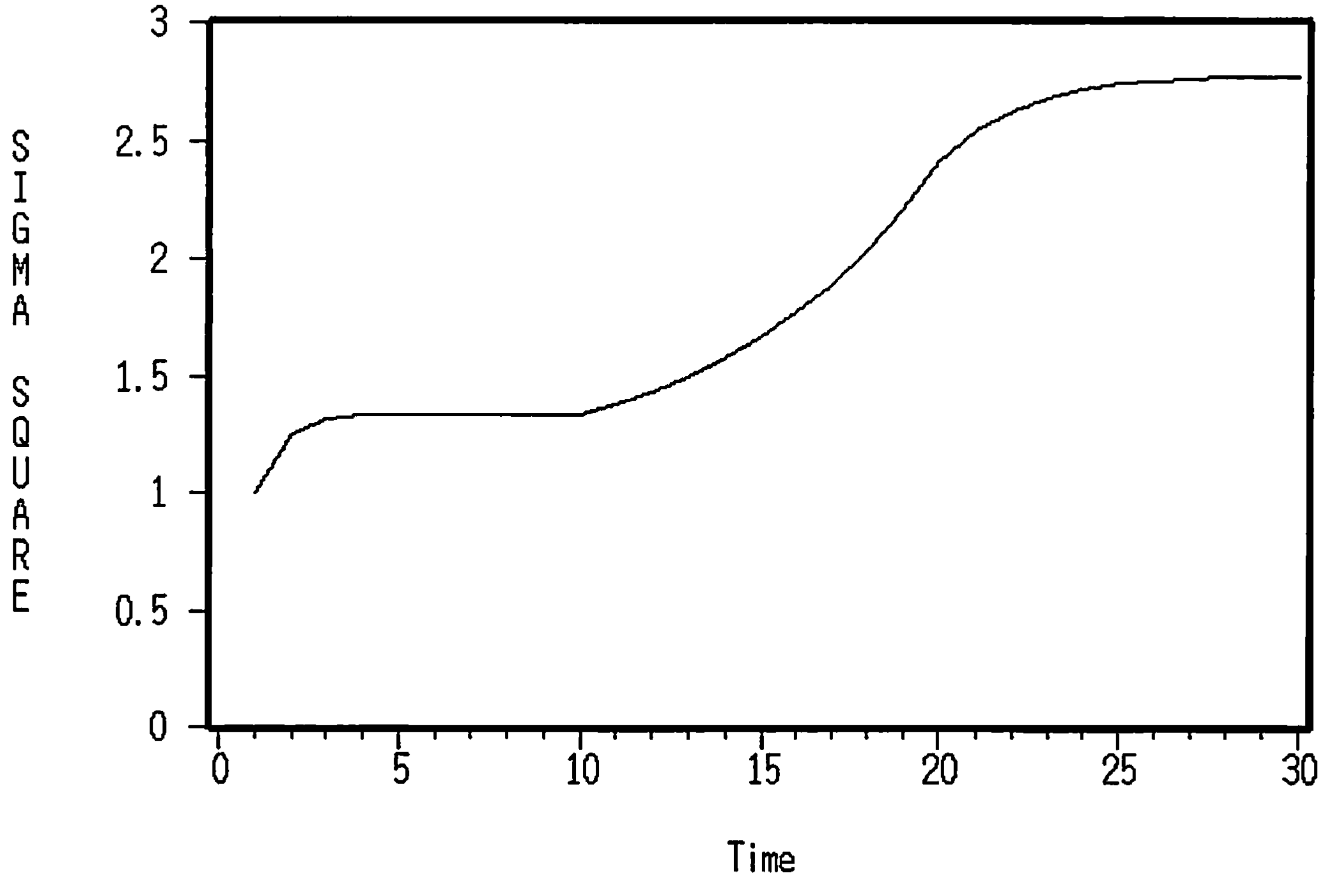
(5) Quarterly data from 1960.2 to 1990.4.

(6) RMS is the call rate.

(7) Data source: Department of Economics and Statistics, OECD,
Main Economic Indicators, 1960-1990, 1991.
(Transrated into Japanese by Yoshitomi, M. (ed.), Tokyo: Hara-Shobo, 1992, pp. 98, 135, 138.)

Figure A.1. The Variance of Errors

T=30, s1=10, s2=20, rho=0.5, omega=0.3



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