Evolution of Attitudes towards Risk*

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Abstract

This paper models the evolution of attitudes towards risk in a finite capacity setting. It will be shown that non-expected utility maximizing behavior will evolve if risk is correlated, while expected utility maximizing behavior will evolve if risk is idiosyncratic. In the case of correlated risk, the fraction of the population that is simultaneously affected by a shock determines the dominant attitude. The dominant attitude converges to the expected utility maximizing behavior as the fraction approaches to zero. These results provide a new evolutionary foundation with mixed strategy. Further, the specific model we use suggests that costly thinking behavior will emerge even if its cost is higher than its benefits from the viewpoint of individuals. The result also offers a possible explanation to the seemingly extraordinarily high degree of risk aversion reported in the empirical literature.

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1 Introduction

Expected utility theory is regarded as one of the central pillars of economic theory. However, severe criticism has been voiced against it by Allais[1] and others. Many researchers have reported the results of their attempts to test its validity both in laboratories and in the field. The experimental evidences has been mixed, with some authors expressing serious doubt about the validity of the theory[1]. Those who believe that the theory had been falsified by the experiments looked for alternatives and many new theories were proposed. Some of them gained popularity, but none of them succeeded in outperforming expected utility theory convincingly in the laboratories. In fact, it is repeatedly reported that expected utility theory fits better, at least in some contexts, than any of the newly proposed theories[2]. On the other hand, the field data, which it is more difficult to falsify the theory, reports 'anomalies.' For example, we need to accept an extraordinarily high degree of risk aversion to reconcile the data on portfolio selection with expected utility theory[3]. In this paper, we do not try to judge which is the right theory nor to add another new theory to the already long list of 'theories' on the decision under risk. Rather, we ask why expected utility theory is so capricious. Why does the expected utility theory serve as a good approximation to people's behavior in some contexts and not in others?

Robson[6] investigated the problem using a model motivated by biology. He concluded that an expected utility maximizer will be dominated by a particular type of non-expected utility maximizer when risk is correlated across the population, while the expected utility maximizer becomes dominant when risk is independent across all individuals. However, the model assumed that the size of the populations of all types diverges to infinity to exploit some of the nice results of branching process. This paper will investigate the same problem as Robson but with a finite capacity model, which is more realistic both in the biological and socio-economic settings. It will be shown that the type which dominates the population in the Robson's model will evolve in our finite population setting if risk is correlated and *simultaneously* affects the *whole* population. The size of the correlated risk is crucial to the result, and other attitudes may evolve if only a certain fraction of the population is

[1] See Mashina[5].

[2] See Camerer[3].

[3] Rust[9] contains a survey on the issue and offers an explanation of the so-called risk premium paradox.

affected by a correlated risk.

The following section explains how we build a relation between expected utility theory and biological fitness. Section 3 summarizes the Robson's model and the derivation of his main results. In section 4, a discrete finite population model which captures truly idiosyncratic risk is introduced and its long-run implications are examined. In section 5, a continuous population model with a capacity limit which deals with correlated risk is introduced and its long-run behavior is analyzed. Section 6 discusses some of the implications of the results obtained in the previous sections. In particular, we compare two attitudes towards risk: following rules of thumb (or biological instinct) and conducting costly thinking before taking an action. Section 7 concludes the paper.

2 Biological Interpretations of Expected Utility Theory

Utility as Biological Fitness. Recent developments in evolutionary theory allow us to reinterpret some economic concepts which were originally derived using an axiomatic approach. For example, the Nash bargaining solution is given an evolutionary foundation by Young[10]. Expected Utility theory, which has a firm axiomatic foundation, deserves the same treatment. Why do people behave *as though* they have a utility function and maximize its expected value? In particular, why would a von Neumann-Morgenstern utility function, in particular, evolve? We follow Becker and others in considering the biological basis of preference represented by a von Neuman Morgenstern utility function. The following view expressed by Robson[7] is persuasive, "it is hard to see why preferences would be fixed at all if biological evolution did not fix them." We adopt this hypothesis and assume that the utility level of each individual can be directly related to his biological fitness.

There are at least two ways to put utility functions on a biological basis. One approach is to define utility function as a mapping from consumption space to the expected number of offspring. Another approach is to interpret the function as a mapping from consumption space to the expected survival rate. Throughout this paper, we assume that each type of individual is characterized by the choice they make over actions vis-à-vis risky situations. An action corresponds to a lottery over consumption goods. We will identify the dominating behavior which evolves in the long-run by tracing the evolution of the population mix of types.

The Expected Number of Offspring Interpretation. Robson[6] relates the maximization of offspring to the maximization of expected utility by the following argument. Suppose that each individual has to choose a lottery over commodity bundles. Denote the consumption space of commodities by R^n_+ and a bundle by $x \in R^n_+$ (hence we assume that there are n kinds of commodity). Bundle x^i_k is obtained in the gamble taken by type i with probability $q^i_k \in [0,1]$, where $\sum_{k=1}^{K} q_k = 1$ and $\{1, 2, ...K\}$ is the finite set of commodity bundles[4]. The outcome of the gamble is independent across those who choose the same gamble. Further, a realized bundle induces for each individual of any type a distribution over the number of offspring, which may vary between 0 and B. The distribution is represented by $p_0(x), p_1(x), ...,$ $p_B(x) \geq 0$, where $\sum_{j=1}^{B} p_j(x) = 1$. The outcome is also independent across individuals given consumption. Having consumed the realized bundle (a realization of the gamble chosen), each individual generates offspring (the number of offspring is a realization of another lottery). In short, each type takes a particularly simple gamble over commodities and the result of the gamble determines the probability distribution of the number of its offspring.

Remember that each type is characterized by the choice it makes on the lotteries over commodity bundles. Define a function $\psi : R^n_+ \to R_+$ by $\psi(x) = \sum_{j=1}^{B} j \, p_j(x) \in (0, \infty)$. Then, the value of $\psi(x)$ is the expected number of offspring when an individual consumes the bundle x.

If we interpret $\psi(x)$ as utility function, the type which behave as though it maximizes

$$\sum_{k=1}^{K}\sum_{j=1}^{B} j q^i_k \, p_j(x) = \sum_{k=1}^{K} q^i_k \, \psi(x^i_k) \tag{1}$$

corresponds to expected utility maximizer. In this case, choosing a lottery which maximizes the expected number of offspring translates into expected utility maximizing behavior.

The Expected Survival Rate Interpretation. Another biological foun-

[4]Each bundle is represented by a n-tuple vector.

dation of expected utility theory is given by reading utility as the expected survival rate.[5] The same argument as above establishes the link between the two concepts. Individuals are to choose a lottery over commodity bundles. The realized commodity bundle determines the expected survival rate, which is interpreted as 'utility'. All we need to do is to replace the expected number of offspring given a consumption bundle by the expected survival rate given the bundle. In this case, expected utility theory will be given another vindication if the type which maximizes the expected survival rate becomes dominant. Our finite capacity model adopts this interpretation, while Robson's infinite population model relies on the expected number of offspring interpretation.

3 Infinite Population Model

This section summarizes the results of the infinite population model by Robson. The most important result is that the type which eventually dominates the population depends on the nature of the risk. Having classified risk into two categories (idiosyncratic and aggregate risk), Robson[6] concluded that non-expected utility maximizing behavior will evolve under aggregate risk while expected utility maximizing behavior will be dominating under idiosyncratic risk. We follow the essence of his argument with some examples taken from the paper.

Idiosyncratic Risk. Suppose that each individual chooses a lottery from a set of lotteries over commodity bundles. Attitudes towards risk are characterized by the choice each type makes over these lotteries. As in the previous section, denote the probability that bundle x_k is obtained when lottery i is chosen by q_k^i. The outcome of the chosen lottery generates a probability distribution over the number of the offspring. Denote the probability b that offspring are born when bundle x_k is consumed by p_b^k. We call risk idiosyncratic if both of the two lotteries generate outcomes *independently* across all individuals. In other words, a dice which has one-to-one relation to a consumption bundle is thrown independently for each individual to determine the outcome b. Therefore the number of offspring they leave may be different among those who consume the same bundle x_k, while the expected number of offspring is identical among those who consume the same bundle. This

[5] Karni and Schmeidler?? adopt this interpretation.

case corresponds to the situation in which *each* individual who consumes an identical consumption bundle throws an identical dice. The dice has n faces, and face b turns up with probability p_b. We now try to characterize, through an example, the fittest attitude towards risk when risk is not correlated across individuals. Which lottery should individuals choose if they want to maximize the number of their offspring?

Example 1 *There are two types in the population. Assume that each type chooses a lottery, say lottery* 1 *and* 2. *The outcome of the lotteries (realized consumption) leads to a probability distribution over the number of offspring. Type* 1*'s choice produces* 1 *offspring with probability* $1-p>0$ *and* 3 *offspring with probability* $p>0$, *and these outcomes are independent across individuals. Hence, the mean number of offspring produced by each type 1 individual is* $m_1 = 1+2p$. *The mean* m_1 *is larger than* 2 *when* $p>1/2$. *Type 2 consumption leads to 2 offspring for sure. Hence, its mean* $m_2 = 2$. *Each type is assumed to breed true. Suppose there is only one individual of each type initially and let* $z_T(i)$ *be the population of each type* $i = 1,2$ *at generation* T. *Then,* $z_T(2) = 2^T$ *and* $z_T(1) \to \infty$ *as* $T \to \infty$, *and* $z_T(1)/(m_1)^T \to W$ *as* $T \to \infty$ *for some non-negative random variable* W. *Hence,* $z_T(1)/z_T(2) \to \infty$ *as* $T \to \infty$ *with probability 1. Type 1 dominates the population in the long run*

This example suggests that, if two types are in competition, the type which produces a higher expected number of offspring in dominates the other in the long run. Robson showed that the result survives in an expanded model where a finite number of types are competing and mutation is introduced.

Theorem 2 *The type which maximizes (1) dominates the population with probability one unless the type is extinct.*

Proof. See Robson [6]

If the risk is idiosyncratic, (i.e. the outcome is independent across the individuals within the same group) the type that chooses the lottery which leads to the highest expected number of offspring dominates the population. The discussion in the previous section implies that this result translates into the domination of the type which maximizes expected utility.

Aggregate Shock. What would happen if the independence of outcomes is not satisfied and there is correlation across individuals? We refer to such a case as aggregated risk. In the previous example, the same type of individuals face the identical risk, but the consequence of the risk is independent across the individuals who choose the same lottery. In the case of aggregated risk, the consequence of the shock is correlated across the individuals in the same group. In other words, a dice is thrown *only once* for each group by an individual who is chosen to represent the group and everyone in the group consume the identical bundle determined by the face that turned up.

Example 3 *Suppose that the environment has two states, 1 and 2. Each state occurs independently in each generation* $T = 0, 1, \ldots$ *Type 1 produces 2 offspring in state 1 and 1 offspring in state 2, while type 2 produces 2 or 1 offspring independently with probability* $1/2$ *each. If there is 1 individual of each type initially, the asymptotic behavior of the populations is described by*

$z_T(2)/(3/2)^T \to W$ *as* $T \to \infty$ *for some non-negative random variable* W.
This implies that $(1/T)\ln(z_T(2)) \to \ln(3/2)$ *with probability 1 as* $T \to \infty$

On the other hand, type 1 population growth is described by

$z_T(1) = 2^{n_1(T)}$ *where* $n_1(T)$ *is the number of periods in which state 1 is observed in* T *periods.*

Since $n_1(T)/T \to 1/2$ *with probability 1 as* $T \to \infty$, $(1/T)\ln(z_T(1)) \to \ln(\sqrt{2})$ *with probability 1 as* $T \to \infty$

$$(1/T)\ln(z_T(2)/z_T(1)) \to \ln(3/2\sqrt{2}) > 0 \quad \text{with probability 1 as } T \to \infty$$

$$z_T(2)/z_T(1) \to \infty \quad \text{with probability } 1.$$

Hence, type 2 dominates the population.

Long run performances of the two populations are entirely different while the expected sizes of two populations are identical *at any given date*. This result indicates that the type which has the highest expected number of offspring may be dominated by other types. In particular, the example above suggests that a particular type which maximizes the expected value of the log of the mean will be the candidate for the dominating type under aggregate shock. The following theorem by Robson confirms this intuition.

Theorem 4 *Denote the state realized at period* t *by* $\xi_t \in \{\xi^1, \xi^2, ..., \xi^S\}$ *and define* $m^i(\xi^s)$ *as the number of offspring produced by a type* i *individual in environment* ξ^s. *Then the type which maximizes the following expression dominates the population with probability one unless the type is extinct:*

$$\rho^i = \lim_{T\to\infty} \left[\frac{1}{T} \sum_{t=0}^{T-1} \ln(\ m^i(\xi^s)) \right]$$

Proof. See Robson [6]

This result implies that the type which maximizes average growth rate dominates the population under aggregated shock. Robson also showed that his result survives when mutation is introduced.

Non-Expected Utility. We now ready to introduce Robson' main result that theorem 4 implies that non-expected utility maximizers may dominate the expected utility maximizer using the following argument. So far, we have employed a two-stage lottery to relate biological fitness to utility. First, each type chooses an action which corresponds to a lottery over commodity bundles. Then, the realized commodity bundle generates a probability distribution over the number of offspring. When we dealt with idiosyncratic shock, the independence of the outcome is assumed for both the first and second-stage lotteries. We introduce correlation by dividing the first-stage lottery into two parts. First each type choose an action indexed by i, but this action alone does not specify the probability distribution over commodity bundles. Nature throws a dice to determine the environment indexed by s, which is *common to all individuals*. The combination of the action i and the environment ξ^s generates a probability distribution over commodity bundles. Denote the probability that bundle $x_k^{i,s}$ is attained in environment ξ^s by type i by $q_k^{i,s} \in [0,1]$. The outcome of this lottery is independent across the individuals within the same group, *given the environment*. Theorem 4 implies that the type which maximize the following expression dominates the population. Notice that each type is characterized by q

$$\begin{aligned} & \sum_{s=1}^{S} \pi^s \ln \left[\sum_{k=1}^{K} \sum_{j=1}^{B} j q_k^{i,s} p_j(x_k^{i,s}) \right] \\ = & \sum_{s=1}^{S} \pi^s \ln \left[\sum_{k=1}^{K} q_k^{i,s} \psi(x_k^{i,s}) \right] \end{aligned}$$

Example 5 *All individual are assumed to select one of two gambles. The first gamble yields a wealth level of either w_1 or w_2, where $w_1 > w_2$. Wealth level w_1 occurs with probability p and w_2 with probability $1-p$ independently across individuals. The second gamble also entails w_1 or w_2 but the consequence is the same for all individuals at a given date with a probability of $1/2$ each. Suppose $p = 1/2$ then Jensen's inequality yields that*

$$\ln[(1/2)\psi(w_1) + (1/2)\psi(w_2)]$$
$$> (1/2)\ln(\psi(w_1)) + (1/2)\ln(\psi(w_2))$$

given that ψ is strictly monotonic. Individuals are selected to opt for the first gamble. Since the inequality is strict, it still holds if p is less than $1/2$ but sufficiently close to $1/2$.

This example indicates that evolutionarily induced attitudes to risk need not conform to individual rationality. An attitude which is not consistent with expected utility theory may well evolve when the consequences of any given risk is correlated across individuals.

4 Finite Capacity: Idiosyncratic Risk

In this section, we model the situation where only one individual in the population is eventually affected by any given shock. Suppose the world is inhabited by two types of rabbits and two types of their predators. The rabbit types are Panic and Fear. Predator types are Fox and Wolf. The environment can sustain only N rabbits. In each round, one rabbit is randomly chosen to give a birth. The birth process is asexual and subject to mutation (mutation rate is λ). Over-population is solved by one of the $N+1$ rabbits being killed by a predator.

Each type of rabbit takes one of two evading actions when it recognizes that a predator is coming: Hide or Run. A rabbit's type is characterized by the evading action it takes. A panic rabbit only knows that the predator is a wolf or fox with probability one half each. It panics by randomly choosing one of the evading actions with probability one half each. Fear rabbits, on the other hand, follow "Wait and See" strategy when they recognize a predator is approaching. It can identify the predator's type correctly with a higher accuracy:probability $\alpha > 1/2$, but pays a cost k. (Waiting is costly as it leaves less time to take evading action.).

A predator randomly chooses one of the rabbits as his target. Survival probabilities of the target depends both on the evading action they take and on the predator's type. We assume that the probabilities are represented by the following matrix.

Panic

	Fox	Wolf
Hide	a	b
Run	b	a

where $0 < a < b < 1$

Fear

	Fox	Wolf
Hide	$a-k$	$b-k$
Run	$b-k$	$a-k$

where $0 < k < \min[b-a, a]$

If the rabbit being targeted survives the ordeal, the predator changes its target and repeatedly attacks until a rabbit is killed . Once a predator kills a rabbit, it goes away. Then the surviving rabbits live in peace for a while until one of them gives a birth and causes over-population. The assumptions made allow us to model the evolution of the population mix as a birth-death process. Hence, we can explicitly calculate the stationary distribution of the stochastic process easily. The following proposition characterize the stationary distribution of the process after taking two limits.

Proposition 6 *Of the two types of rabbit, that which maximizes its expected survival rate dominates the population as $\lambda \to 0$ and $N \to \infty$.*

Proof.

Suppose the state space is represented by a straight line with a population in which all rabbits are of the Fear type lies at the right end and a population in which all rabbits are the Panic type lies at the left end. Every other state corresponds to one of the points which equally divide the whole line into $N+1$ segments. In such a case, we can derive the stationary distribution by identifying the probabilities that the system moves one step left or one step right at each state. We use the following notation:

F_f :probability a Fear type dies at the first draw

F_p :probability a Panic type dies at the first draw

G_f :probability that the rabbit who eventually dies is a Fear type

G_p :probability that the rabbit who eventually dies is a Panic type

z^t :Fear rabbit frequency in the whole population at time t

These four probabilities are directly calculated from the assumptions and expressed as follows if the superscript t is omitted.

$$F_f = \frac{z(2-a-b)}{2}, F_p = (1-z)\{\alpha b + (1-\alpha)a + k\}$$

$$G_f = \frac{F_f}{F_f + F_p}, G_p = \frac{F_p}{F_f + F_p}$$

Further, denote the probability that the system moves one step right from z by $R(z)$ and the probability of moving one step right from z by $L(z)$. Then,

$$R(z) = \{z(1-\lambda) + (1-z)\lambda\}G_p$$

$$L(z) = \{z\lambda + (1-z)(1-\lambda)\}G_f$$

The probability mass attached to the state z in the stationary distribution $P(z)$ satisfies the relation below.

$$\frac{P(z)}{P(z+\nu)} = \frac{R(z+\nu)}{L(z)} \text{ where } \nu = \frac{1}{N}$$

If we take the two limits, the ratio converges to a constant.

$$\lim_{\lambda \to 0, N \to \infty} = \frac{R(z+\nu)}{L(z)} = \frac{2\{a + \alpha(b-a) + k\}}{(a+b)}$$

Since the probability mass ratio between any of two neighboring states are constant, the all Fear (all Panic)state has the highest mass if and only if

$$k > (<) = \frac{(a-b)(2\alpha - 1)}{2}.$$

This implies that the process almost always stays in the states in which almost all of the surviving are the type which has a higher expected survival rate. ■

This result should hold when there are more than two types in the population, while we cannot calculate the stationary distribution explicitly in such cases. An intuitive argument can be made to support the conjecture. Suppose that there are many non-expected survival rate maximizing types and one maximizing type initially. Since the birth process is assumed to be neutral to the evolution of the population mix, the evolution is dominated by the survival rate difference. The higher the expected survival rate is, by definition, the individual is more likely survive the ordeal than any of the other types which has a lower expected survival rate in the population. Hence the state is more likely to move from the current state to the neighboring state in which there are one more a higher type and one less one of lower types than to move towards the opposite direction. The system is always more likely to move towards the direction to increase overall fitness of the population. Therefore, the state in which all individuals are expected survival-rate maximizer should have the highest mass in the stationary distribution of the process.

Discussion in Section 2 allows us to interpret this result as follows.

Remark 1 *The type which maximizes expected utility will evolve if the following three conditions are satisfied.*

Condition 1 The population size is sufficiently large.
Condition 2 The mutation rate is sufficiently small.
Condition 3 Only one individual is eventually affected by the risk. (risk is idiosyncratic ex post)

Note that modelling risk as sequential arrivals of shocks allows each type to diversify the risk across its own population. Hence each type is insured in a sense as a group if not as an individual.

5 Finite Capacity: Correlated Risk

Large correlated risk. This section examines the case of correlated risk in a finite population setting. Environment variables such as climate should be modeled as large correlated risk, since the whole population is subject to an identical shock simultaneously. In this paper, we call a shock "large" if the whole population is simultaneously affected by the shock. To analyze the consequence of large correlated risk, we set a capacity limit on the environment in a different way and work on a continuous model. The upper limit of the population is normalized to 1 and the whole population now lies in the continuum $[0, 1]$, rather than being set equal to a finite number N. In each period, the whole population experiences the same type of shock or no shock at all. Each population decreases its size when they experience a shock but grows proportionally up to capacity limit when they do not suffer any shock. We reuse the Fear and Panic story and assume that there are only two types of shock represented by Fox and Wolf as in the previous section to explain the essence of the argument with two simple examples. These assumptions are dropped later in this section when we move on to a general model. The first example considers a case in which there are only two types of rabbits; Panic and Fear. In the second example, we studies a case there are three types of rabbits; Panic-Run, Panic-Hide, and Fear.

Example 4: The Case with Two Types

A large shock corresponds to the situation in which all rabbits in the population face an attack by the predators of a same type. To simplify the calculations, assume that the population of each type shrinks deterministically once the type of shock is revealed. We keep the assumptions made in the previous section. The survival rates of each type are represented by the same matrices and each type of shock occurs with equal probability. Then, if the shock is the appearance of a Fox, the density of Panic shrinks from x_t to $\frac{a+b}{2}x_t$, while the density of Fear shrinks from $(1-x_t)$ to $\{\alpha b+(1-\alpha)a-k\}(1-x_t)$. The Wolf shock has the same effect on the population as the Fox shock. When no shock occurs, the population grows proportionally up to the environment's carrying capacity. We use the following notation:

x_t :the ratio of Panic in the whole population at t

n^f :the number of periods they experience Fox up to t

n^w :the number of periods they experience Wolf up to t

Then the population mix at period t is described as follows.

$$\frac{x_t}{1-x_t}=\frac{\left(\frac{a+b}{2}\right)^{n^f+n^W}x_0}{\{\alpha b+(1-\alpha)a-k\}^{n^f+n^W}(1-x_0)}$$

In the limit, the ratio should diverge to infinity or converge to zero since

$$\begin{aligned}\lim_{t\to\infty}\frac{x_t}{1-x_t} &= \infty \text{ if } \frac{a+b}{2}<\alpha b+(1-\alpha)a-k\\ &= 0 \text{ if } \frac{a+b}{2}>\alpha b+(1-\alpha)a-k\end{aligned}$$

This result suggests that, *in the long run, the type that maximizes the expected survival probability dominates the population.* Intuitively, this result holds, since the mixed strategy followed by the Panic plays the role of insurance for the type. In other words, mixed strategies transform correlated risk into idiosyncratic risk. If a particular type is evolved to play a mixed strategy which maximizes the expected value of the survival rate, the type should dominate even if risk is correlated across the whole population. The next example examines what would happen once we exclude mixed strategies.

Example 5: The Case with Three types

Suppose that there are two types of Panic; Panic Run and Panic Hide. The former chooses Run, whenever it recognizes a shock, and the latter chooses Hide. As the death-rate matrix is symmetric and the two types of shocks occur with the same probability, the two type of Panic has the same survival rate in the expected term. We change the notations to

x_t :the ratio of Panic Hide in the population

y_t :the ratio of Panic Run in the population

$z_t = 1 - x_t - y_t$:the ratio of Fear in the population

Then, the evolution of the population mix is described as follows.

$$
\begin{aligned}
x_{t+1} &= \frac{\pi_{xt}x_t}{\pi_{xt}x_t + \pi_{yt}y_t + \{\alpha b + (1-\alpha)a - k\}z_t} \\
y_{t+1} &= \frac{\pi_{yt}y_t}{\pi_{xt}x_t + \pi_{yt}y_t + \{\alpha b + (1-\alpha)a - k\}z_t}
\end{aligned}
$$

where

$$
\begin{aligned}
\pi_{xt} &= a \text{ and } \pi_{yt} = b \text{ if Fox type shock occur} \\
\pi_{xt} &= b \text{ and } \pi_{yt} = a \text{ if Wolf type shock occur}
\end{aligned}
$$

Taking the limit $t \to \infty$ leads to the following result.

$$
\begin{aligned}
& \lim_{t\to\infty} \frac{x_t}{z_t} \\
= & \lim_{t\to\infty} \frac{a^{n^f} b^{n^w} x_0}{\{\alpha b + (1-\alpha)a + k\}^{n^f + n^W} z_0} \\
= & \lim_{t\to\infty} \left\{ \frac{\sqrt{ab}}{\{\alpha b + (1-\alpha)a + k\}} \right\}^{n^f + n^W} \frac{x_0}{z_0}
\end{aligned}
$$

$$
\begin{aligned}
& \lim_{t\to\infty} \frac{y_t}{z_t} \\
= & \lim_{t\to\infty} \frac{a^{n^w} b^{n^f} y_0}{\{\alpha b + (1-\alpha)a + k\}^{n^f + n^W} z_0} \\
= & \lim_{t\to\infty} \left\{ \frac{\sqrt{ab}}{\{\alpha b + (1-\alpha)a + k\}} \right\}^{n^f + n^W} \frac{y_0}{z_0}
\end{aligned}
$$

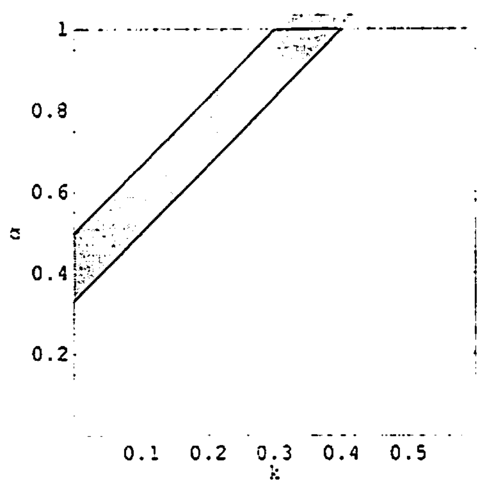

Figure 1:

$$
\begin{aligned}
& \lim_{t\to\infty} \frac{x_t}{z_t} = \lim_{t\to\infty} \frac{y_t}{z_t} \\
= & \ 0 \text{ if } \sqrt{ab} < \{\alpha b + (1-\alpha)a - k\} \\
= & \ \infty \text{ if } \sqrt{ab} > \{\alpha b + (1-\alpha)a - k\}
\end{aligned}
$$

since $n^f + n^W \to \infty$ as $t \to \infty$ and $n^f / n^f + n^W \to 1/2$ almost surely as $n^f + n^W \to \infty$ by the strong law of large numbers.

This result implies that *Non-expected survival probability maximizers dominates when α and k satisfy the following condition* [See Figure 1]

$$
\begin{aligned}
\frac{a+b}{2} &> \alpha b + (1-\alpha)a - k > \sqrt{ab} \\
&\text{or} \\
\frac{\sqrt{ab}-a}{b-a} + \frac{k}{b-a} &< \alpha < \frac{1}{2} + \frac{k}{b-a}
\end{aligned}
$$

This example shows that the type which does not maximize the expected survival rate may dominate the population if we allow only a pure strategy for each type.

Generalization. So far, we have dealt with a rather special setting in the two examples (i.e. There are only two types of shocks and they occur with equal probability). The question now arises; does the result above survive in more general cases? To answer this question, we expand the model as follows. Suppose there are S types of shocks (predators). Each type of shock is indexed by $s \in \{1.2....S\}$. An environment is characterized by a probability distribution over the shocks. This distribution is represented by $(p_1, p_2, ..., p_S)$ where $\sum_{s=1}^{s=S} p_s = 1$.

Suppose there are N types of individuals. Each type of individual (rabbit) is characterized by the action it takes in the environment. Each action (and individual's type) is indexed by $i \in \{1, 2, 3, ...N\}$. Given the type of shock, each type of individual survives the shock with a given probability. Let us denote the survival rate of type i individual when the type of shock is s by $\sigma_{i,s}$. The expected survival rate for type i is $\sum_{s=1}^{s=S} p_s \sigma_{i,s}$. The following proposition asserts that the type which dominates the population will not be the expected survival rate maximizer.

Proposition 7 *The type which maximizes the following expression dominates the population in the long run.*
$\prod_{s=1}^{S} \sigma_{i,s}^{p_s}$ *or* $\sum_{s=1}^{S} p_s \ln \sigma_{i,s}$

Proof. *Suppose that type* 1 *strict maximizer of the above expression. Hence*

$$\sum_{s=1}^{S} p_s \ln \sigma_{1,s} > \sum_{s=1}^{S} p_s \ln \sigma_{i,s} \qquad \text{for any } i \in N\backslash 1$$

Denote the frequency of the type s shock until time t by n^s. Then,

$$\frac{x_{1t}}{x_{it}} = \frac{x_{10}}{x_{i0}} \bullet \frac{\prod_{s=1}^{S} \sigma_{1,s}^{n^s}}{\prod_{s=1}^{S} \sigma_{i,s}^{n^s}} = \left(\frac{\prod_{s=1}^{S} \sigma_{1,s}^{n^s}}{\prod_{s=1}^{S} \sigma_{i,s}^{n^s}} \right)^{\frac{1}{\sum n^s}}$$

$$\lim_{t \to \infty} \frac{x_1}{x_i} = \left(\frac{\prod_{s=1}^{S} \sigma_{1,s}^{p_s}}{\prod_{s=1}^{S} \sigma_{i,s}^{p_s}} \right)^{t} \to \infty$$

Since the actual frequency of the each shock converges almost surely to the probability distribution. Further, as we assumed that N is finite, it follows

$$\frac{x_1}{\sum\limits_{i \neq 1} x_i} \to \infty$$

Hence Type 1 *dominates the population in the compelling sense.* ■

This result shows that the result Robson obtained in his infinite population model survives under a finite capacity setting if the shock affects the whole population. We now see if the result still survives when the shock is correlated but only a certain fraction of the population is affected.**Smaller correlated risks.** Suppose that only a given proportion of the whole population r $(0 < r < 1)$ is affected by the shock (We refer r as the size of correlated shock) The case of large shock we have examined corresponds to the case $r = 1$ in the current setting. Further, we assume that fraction r of *each* population of type is affected by the shock. Now the population ratio of any of two types, say type a and b, evolve as follows.

$$\frac{x_{at}}{x_{bt}} = \frac{x_{a0}}{x_{b0}} \bullet \frac{\prod\limits_{s=1}^{S} (1 - r + r\sigma_{a,s})^{n^s}}{\prod\limits_{s=1}^{S} (1 - r + r\sigma_{b,s})^{n^s}}$$

The same argument as Proposition 7 leads to the conclusion that the type which maximizes the expected values of $\ln(1 - r + r\sigma_{is})$ dominates the population. This implies that the dominant attitude which evolves is not the expected utility maximizer nor the maximizer of log of utility which is being interpreted as the expected survival rate. However, in the limit as $r \to 0$, an equivalent of Proposition 6 holds in this finite capacity model.

Proposition 8 *When the size of the correlated risk becomes vanishing small, the type which dominates the population approaches expected utility maximizer. In the limit of $r \to 0$, the type which maximizes expected utility dominates the population.*

Proof. In the neighborhood of $\ln 1$, $\ln(1-r+r\sigma_{is}) \simeq -r+r\sigma_{is}$. This implies that the dominating type is the one which maximize the expected value of $-r + r\sigma_{is}$. Since this expression is an affine transformation of σ_{is}, the type that maximizes its expected value of utility dominates the population. ■

Intuition behind the result is simple. If only small fraction of the population of any given type is affected by a shock simultaneously, and most of the same type population is intact, the type as a group is insured against correlated risk. In fact, such risk is not correlated from the viewpoint of the group which shares the same preference.

This model with a capacity limit shows that a variety of non-expected utility maximizing behavior may emerge through an evolutionary process. The dominating type depends on the size of the shock and the results Robson obtained in an infinite population setting hold as two extreme cases: the size of shock is very large ($r \to 1$) and the size is very small ($r \to 0$).

6 Discussion

The previous section confirms that the type which maximizes the expected value of the log of utility will evolve when risk is correlated across the whole population even if a capacity limit is imposed on the population. This section discusses what this specific form of risk attitude implies.

Mixed Strategy. Firstly, the result provides the concept of "mixed strategy" with a new evolutionary foundation, although the situation we have so far considered is not game theoretic. Mixed strategy can be justified only if all the strategy in the support has the identical expected payoff in the standard settings. However, when players play a game against nature which is characterized by correlated risk, mixed strategies in which each strategy in the support does not necessarily have the identical expected payoff are given a rational. The intuition behind this somewhat bizarre result is the fact that, as we have already seen, "mixing" serves as insurance for the type.

We reuse the story of Fox and Wolf. Suppose that Fox type shock occurs with probability $p > \frac{1}{2}$, and Wolf type shock occurs with probability $1 - p$. independently across time. Then Panic Run (the type which always to choose Run) has the highest expected survival rate. Any type which is characterized by a mixed strategy has a lower expected survival rate. Hence, if there are only two types of shock, Panic Run and Panic (which independently choose Run and Hide with probability one half each) the former has a higher expected survival rate than the latter. Evolution of the ratio of the two population is described by

$$
\begin{aligned}
& \lim_{t\to\infty} \frac{x_t}{z_t} \\
= & \lim_{t\to\infty} \frac{a^{n^f} b^{n^w} x_0}{\{(a+b)/2\}^{n^f+n^W} z_0} \\
= & \lim_{T\to\infty} \left\{ \frac{a^{n^f/T} b^{n^w/T} x_0}{\{(a+b)/2\}^{(n^f+n^W)/T}} \right\}^T \frac{x_0}{z_0} \\
= & \lim_{T\to\infty} \left\{ \frac{(a^{1-p} \cdot b^p) \cdot 2}{a+b} \right\}^T
\end{aligned}
$$

where x_t : Panic Run population at time t z_t : Panic population at time t

Hence, Panic dominates Panic Run if $(a^{1-p} \cdot b^p) \cdot 2 < a+b$ and vice versa. The inequality is always satisfied as far as p is close to $1/2$. The type which has the higher expected survival rate (Panic Run) is dominated by a mixing type which has the lower expected survival rate. This result indicates that the type evolved to play mixed strategy may dominate the pure strategy type which has the highest expected payoff. This result also offers an explanation to "probability matching" type behavior.

Costly Thinking. Second implication of the main result is that costly thinking is rewarding when risk is correlated. In the Fear and Panic story, there are two factors which allow Fear's domination when the type has the lower expected survival rate. The first factor is a higher accuracy which leads to a higher expected value of the logarithm of utility. The appendix characterizes the parameter values which allow such domination. Another factor is the idiosyncratic nature of the result of thinking. Thinking diversifies individual judgement within the same group and allow each type to behave as though it plays a mixed strategy as a group.. We have argued such "mixing" turns the correlated shock into idiosyncratic shock. Notice that we have ignored the possibility of the types with mixed strategies when we derived the mail result. By concentrating on pure strategies, we intended to model genetically programmed choice behavior by individuals. In other words, Panic represents choices made by individuals by just following his instinct without serious thinking or by simply applying a rule of thumb. Fear, on the other hand, represents a different attitude. When the type have to make a choice, they do not react instinctively but opt for "wait and see" even if taking such strategy involves cost. The fact that Fear can dominate even if it has a lower expected survival rate suggests that the animals which live in large groups are more likely develop thinking habits as the risk they face will be correlated in most cases. Costly thinking is rewarding for the group if not for the individuals which belongs to the group.

Risk Aversion. Another almost direct implication of the main result is that people' attitude towards risk may well be extremely risk averse. If the utility function is concave as normally assumed, the function obtained by taking logarithm of it is even more concave. This point offers an explanation to the seemingly absurdly high degree of risk aversion observed in the household portfolio selection data. Events like market collapse affect everyone in the market simultaneously and such shocks may well be perceived aggregated shock by individuals. Our result is consistent with the observation that people's attitude to such risk is significantly risk averse compared with other risky choices they make.

7 Conclusion

This paper showed that the main results of Robson[6] hold as two extreme cases in a model which imposes a capacity limit on the population size. Non-expected utility maximizing behavior may arise through an evolutionary process if risk is correlated and a significant fraction of the population is affected *ex post*. On the other hand, from an evolutionary viewpoint, the case for the expected utility theory is compelling if risk is idiosyncratic or only a small portion of the population is affected *ex post* by correlated risk. This result justifies mixed strategies in the games against nature even if a pure strategy brings the highest payoff to the players as far as the nature is characterized by correlated shock. Further, the specific model we used suggests that, when it is possible to increase the accuracy of information on the nature of risk by paying a cost, costly thinking behavior may evolve even if it is so costly that thinking decrease the biological fitness at individual level. This implies that the animals living in groups have a better chance to develop "thinking ability."

Appendix

This appendix aims to clarify when costly thinking is rewarding by locating the environmental parameters which is consistent with the case in the Panic-Fear Story. First, we consider the cost of thinking k is constant and find the ranges of the other two parameters (information accuracy parameter α and the frequency of Fox type shock r) which lead to the domination of Fear over Panic when the former has the lower expected survival rate. Then we briefly discuss the case that the value of k varies and α increases as k increases.

Suppose that the Fox type shock occur with probability r and the Wolf type shock occur with probability $1-r$. The cost of waiting (or thinking cost) is k. We assume that k is a constant which satisfies $0 < k < \min[a-b, b]$.

Once the value of k is given, we can calculate the upper and lower limits of r which allow Fear's domination. The upper limit $\overline{r}$ is calculated using the fact that Fear's maximum fitness is attained when they choose right evading action.

$$\begin{aligned} \ln(a-k) &= \overline{r}\ln a + (1-\overline{r})\ln b \\ \overline{r} &= \frac{\ln(a-k) - \ln b}{\ln a - \ln b} \end{aligned}$$

The first and second order derivative are

$$\overline{r}'(k) = -\frac{1}{a-k} < 0, \overline{r}''(k) = -\frac{1}{(a-k)^2} < 0$$

The lower limit $\underline{r}$ is obtained using the fact that the point $(a\underline{r}+b(1-\underline{r}), \underline{r}\ln a + (1-\underline{r})\ln b)$ is on the line represented by

$$y = \frac{\ln(a-k) - \ln(b-k)}{a-b}(x - a + k) + \ln(a-k)$$

hence

$$\underline{r} = \frac{(a-b)(\ln b - \ln(b-k)) - k(\ln(a-k) - \ln(b-k))}{(a-b)(\ln(a-k) - \ln(b-k) - (\ln a - \ln b))}$$

$\underline{r}'(k) > 0$

For a given k and r such that $\overline{r}(k) < r < \underline{r}(k)$ we can find the upper and lower limit of α within which Fear' dominates even if the type has the lower expected survival rate than Panic.

The upper limit $\overline{\alpha}$ needs to be smaller than $1 - r$ for an obvious reason and it also needs to be small enough to guarantee that Fear has the lower expected survival rate. Hence $\overline{\alpha}$ is expressed as below.

$$\overline{\alpha} = \min[1 - r, \frac{k}{a - b}]$$

The lower limit $\underline{\alpha}$ needs to satisfy the following equation to guarantee Fear's domination over Panic.

$$r \ln a + (1 - r) \ln b = (r + \alpha) \ln(a - k) + (1 - r - \alpha) \ln(b - k)$$

Hence

$$\underline{\alpha} = \frac{r (\ln a - \ln(a - k)) - (1 - r) (\ln b - \ln(b - k))}{\ln(a - k) - \ln(b - k)}$$

$$\frac{\partial \alpha}{\partial r} = \text{constant} < 0$$

To summarize the results above, for any value of $0 < k < \min[a - b, b]$, we can find an area in the $\alpha - r$ plane such that Fear dominates Panic when the values of α and r are contained in the area. Further, in the relevant three-dimension space of (k, r, α), we can locate an open set ς in which Fear dominates despite its lower expected survival rate.

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