"Labor Mobility and Dynamic Core-Periphery Patterns: A Two-Region Case"

Masanori Yokoo (横尾 昌紀)　　No.2004

Labor Mobility and Dynamic Core-Periphery Patterns: A Two-Region Case

(preliminary version)

Masanori Yokoo*

Graduate School of Economics, Waseda University

e-mail: yokoo@tkd.att.ne.jp

November 2000

Abstract

Paul Krugman (1991) proposed a geographic model in which a country can endogenously become differentiated into an industrialized "core" and an agricultural "periphery". It is known that the model can give rise to multiple equilibria at which manufacturing production is concentrated in one region or divided between the both regions. To make the basically static two-region model dynamic, we introduce a discrete-time adjustment process that leads the workers who earn lower real wage than the average to migrate to the other region which offers them higher real wage. Numerical simulations show not only that persistent endogenous fluctuations in manufacturing share are possible but also discontinuous changes in manufacturing share can occur without any changes in the underlying system. Furthermore, we numerically demonstrate that the basins of attraction of the concentration steady states can be very complicated for larger transpotaion costs.

Key words: Krugman's model; core-periphery patterns; increasing returns to scale; discrete-time adjustment dynamics; labor mobility; multiple equilibria; bifurcations; chaotic dynamics; transient chaos; discontinuous change; fractal basin of attraction

*The author would like to thank the Japan Society for the Promotion of Science for finacial support.

1 Introduction

One of the most important issues of geographic/spatial/urban economics is to explain how geographic concentration occurs. Using the monopolistic competition model by Dixit and Stiglitz (1977) as a building block, Paul Krugman (1991,1992) proposed a spatial model based on nonconvexity implied by increasing returns. He argued whether a country consisting of two regions becomes divided between a manufacturing "core" and an agricultural "periphery", that is, whether manufacturing is concentrated in only one region or the both regions have some manufacturing. See Fujita *et al* (1999) for generalized and extended versions of the model.

Krugman and his co-workers offered a continuous-time adjustment dynamic model (which can be seen as the replicator dynamics) that gives rise to multiple equilibria. They mainly use the dynamics to examine whether a given core-periphery pattern is sustainable or, in other words, whether an instantaneous equilibrium is stable (i.e. attracting). Seemingly, they implicitly assumed that what matters is to understand in what equilibrium steady state the economy will end up.

Our interest is, however, in possible dynamic patterns (and other nonlinear phenomena) *per se* rather than in local stability of instantaneous equilibria. Our question is then: what kind of dynamic core-periphery patterns are possible in the Krugman's settings? Since it is known that discrete-time dynamics even in one dimension can be very rich in general, we will offer a discrete-time adjustment dynamic model in order to expect to obtain some interesting dynamic results in the minimum setting. The model is *ad hoc* and involves no intertemporal profit/utility maximization. This could be made so, but we don't do that because the model is so complex to analyze that we don't want to complicate it any more for the time being. This will be left for future research.

In exposition of the model, we will use numerical methods because of the model's complexity. We will see below that complicated dynamic core-periphery patterns that cannot arise in the continuous-time setting are possible in our model.

2 A Two-Region Model

2.1 Statement of the Model

In this subsection we will just state the model without detailed explanations. The model we are going to deal with is summarized in the following set of equations. For $t = 0, 1, 2, \cdots$,

$$
\begin{aligned}
Y_{1,t} &= \mu x_t w_{1,t} + \frac{1-\mu}{2}, &(1)\\
Y_{2,t} &= \mu(1-x_t) w_{2,t} + \frac{1-\mu}{2}, &(2)\\
G_{1,t} &= \left[x_t w_{1,t}^{1-\sigma} + (1-x_t)(w_{2,t}T)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, &(3)\\
G_{2,t} &= \left[x_t (w_{1,t}T)^{1-\sigma} + (1-x_t) w_{2,t}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, &(4)\\
w_{1,t} &= \left[Y_{1,t} G_{1,t}^{\sigma-1} + Y_{2,t} G_{2,t}^{\sigma-1} T^{1-\sigma}\right]^{\frac{1}{\sigma}}, &(5)\\
w_{2,t} &= \left[Y_{1,t} G_{1,t}^{\sigma-1} T^{1-\sigma} + Y_{2,t} G_{2,t}^{\sigma-1}\right]^{\frac{1}{\sigma}}, &(6)\\
\omega_{1,t} &= w_{1,t} G_{1,t}^{-\mu}, &(7)\\
\omega_{2,t} &= w_{2,t} G_{2,t}^{-\mu}, &(8)\\
\bar{\omega}_t &= x_t \omega_{1,t} + (1-x_t)\omega_{2,t}, &(9)
\end{aligned}
$$

and

$$
x_{t+1} = \left(1 - g_\beta(\omega_{1,t} - \bar{\omega}_t)\right) x_t + g_\beta(\omega_{2,t} - \bar{\omega}_t)\,(1 - x_t)\,, \qquad (10)
$$

where $g_\beta : \mathbb{R} \to [0,1]$ is a *migration function* given by

$$
g_\beta(z) = \begin{cases} -\tanh(\beta z)\,, & \text{if } z < 0, \\ 0, & \text{if } z \geq 0. \end{cases} \qquad (11)
$$

Notations:

$Y_{i,t}$: income of manufacturing and agricultural workers in total in region i at time t;

μ: constant representing the expenditure share of manufactured goods;

x_t: region 1's share of manufacturing at time t while $1 - x_t$ representing region 2's share;

$w_{i,t}$: nominal wage for manufacturing workers in region i at time t;

$G_{i,t}$: price index for manufactured goods in region i at time t;

σ: elasticity of substitution between any two varieties of manufactured goods;

T: transportation cost for manufactured goods between two regions;

$\omega_{i,t}$: real wage for manufacturing workers in region i at time t;

$\bar{\omega}_t$: average real wage over two regions;

β: intensity parameter (> 0) of reaction to wage difference.

2.2 The Static Part of Model

The static part of the model (i.e. eqs. (1) through (9)) is identical to the model proposed by Krugman (1991) and Fujita *et al* (1999, chapter 4 and 5). We will only give a sketch of the structure of the static part.

Suppose that the economy involves two regions: region 1 and region 2. There are two types of producers: farmers and (manufacturing) workers. Farmers are equally distributed ($(1-\mu)/2$, $\mu \in (0,1)$) and fixed in each region and produce a homogeneous agricultural good A with constant returns to scale. Workers whose total population adds up to μ produce many differentiated manufactured goods $m(i)$, $i \in [0, n]$ with increasing returns: producing a quantity q of any variety of manufactured good in any given region requires labor input l with fixed input F and marginal input requirement c:

$$l = F + cq.$$

Under this technology, the producer of each specific manufactured good will maximize his temporal profit, which is driven to zero by the free entry assumption. Contrary to the farmers, workers are able to move between two regions according to the real wage difference. The agricultural market is perfectly competitive, and transportation of the agricultural goods is costless. On the other hand, the manufacturing market is monopolistically competitive, and each manufactured good is produced in only one region by a single monopolistically competitive producer. It is costly to transport the manufactured goods to the other region: only a fraction $1/T$ ($T > 1$) of the original unit of product actually arrives (Samuelson's "iceberg" form). Every consumer shares the same Cobb-Douglas utility U from the agricultral good A and a composite of manufactured goods C_M,

$$U = U(A, C_M) = A^{1-\mu} C_M^{\mu},$$

where

$$C_M = \left[\int_0^n m(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$$

with $\sigma > 1$ being the elasticity of substitution among any two varieties of manufactured goods. Each consumer maximizes his utility subject to the usual temporal budget constraint. Some exercise together with some normalization of parameters gives eqs. (1) through (9).

2.3 Dynamization of the Static Model

Let us introduce a time-structure into the model. To do this, we assume that in each discrete time period manufacturing workers in a region may move to the other region according to the difference between their real wage and the real average wage, $\omega_{i,t} - \bar{\omega}_t$. Workers can choose to move to the other region or to stay where they are. If the workers are in the region that offers them higher real wage than the average (or, put differently, than the other region's wage), they have no incentive to move, so they will stay. A fraction of workers who are paid lower wage will migrate to the other region, and the rest will stay. The fraction depends on the size of wage difference between the regions: the bigger the difference of wages, the bigger fraction of workers will migrate[1]. This migration process will be formulated by the migration function $g_\beta : \mathbb{R} \to [0,1]$, which is a restatement of eq.(11):

$$g_\beta(\omega_{i,t} - \bar{\omega}_t) = \begin{cases} -\tanh\left(\beta\left(\omega_{i,t} - \bar{\omega}\right)\right), & \text{if } \omega_{i,t} < \bar{\omega}_t, \\ 0, & \text{if } \omega_{i,t} \geq \bar{\omega}_t, \end{cases} \tag{12}$$

where $\beta > 0$ is the parameter of intensity of migration (See Fig 2.1). The larger the parameter β, the more sensitively the workers will react to the wage difference. Note that in the extreme case of $\beta = +\infty$, the migration function (12) becomes a step function given by

$$g_\infty(\omega_{i,t} - \bar{\omega}_t) = \begin{cases} 1, & \text{if } \omega_{i,t} < \bar{\omega}_t, \\ 0, & \text{if } \omega_{i,t} \geq \bar{\omega}_t. \end{cases} \tag{13}$$

— Fig 2.1 —

Now let us define the adjustment dynamics in terms of manufacturing workers' share in region 1, x_t. At the end of period t, the fraction of $g_\beta(\omega_{1,t} - \bar{\omega}_t)$ of workers in region 1 will migrate to region 2, and the fraction of $g_\beta(\omega_{2,t} - \bar{\omega}_t)$ of workers in region 2 will migrate to region 1. Thus at the beginning of period $t+1$, the fraction of manufacturing workers in region 1, x_{t+1}, will amount to:

$$x_{t+1} = \left(1 - g_\beta\left(\omega_{1,t} - \bar{\omega}_t\right)\right) x_t + g_\beta(\omega_{2,t} - \bar{\omega}_t)\left(1 - x_t\right). \tag{14}$$

Expressing ω_i and $\bar{\omega}$ as functions of x by "solving" eqs. (1) to (9), we obtain a map $f : [0,1] \to [0,1]$ with

$$x_{t+1} = f(x_t), \tag{15}$$

[1] This implicitly assumes that there is some unmodeled heterogeneity in the workers.

where

$$f(x) = (1 - g_\beta(\omega_1(x) - \bar{\omega}(x)))\,x + g_\beta(\omega_2(x) - \bar{\omega}(x))\,(1 - x). \qquad (16)$$

It is important to recognize that the static model can have multiple instantaneous equilibria. These equilibria correspond to the fixed points (=steady states) of f. Although we cannot characterize all of the possible equilibria in an explicit form because of our inability to solve the nonlinear simultaneous equations, there are several obvious equilibria given by

$$x^* = 0, \quad x^* = 1, \quad \text{and} \quad x^* = \frac{1}{2}. \qquad (17)$$

At the first two equilibria, manufacturing is concentrated in one of the two regions. At the third equilibrium, manufacturing is equally divided between the two regions. As we will see below, other steady states can arise for some set of parameter values.

3 Dynamic Core-Periphery Patterns

In this section, we will ask, using numerical methods, whether and how the economy becomes divided between a manufacturing core and an agricultural periphery over time. In Krugman's original works (1991,1992) and Fujita *et al* (1999), they used a continuous-time replicator-like adjustment dynamic model[2]

$$\frac{dx}{dt} = \gamma(\omega_1(x) - \bar{\omega}(x))x, \qquad (\gamma > 0)$$

to examine the problem what equilibrium is selected in the long run, in other words, whether an instantaneous equilibrium is sustainable. Unlike their analyses, we are more interested in the dynamic patterns in manufacturing than in the steady states. In what follows, numerical simulations will show that our model is capable of generating persistent fluctuations including periodic and even chaotic dynamics. Furthermore, many other interesting nonlinear phenomeana can be observed.

For our numerical study, let us fix some parameters as follows:

$$\sigma = 5, \quad \mu = 0.5, \quad \text{and} \quad \beta = 200. \qquad (18)$$

[2] A possible discrete-time version of the continuous replicator-like adjustment model is given by

$$x_{t+1} = \frac{\alpha + \omega_1(x_t)}{\alpha + \bar{\omega}(x_t)} x_t, \qquad \alpha > 0.$$

This model is not capable of generating comlicated dynamics.

We will vary the transportation cost parameter T to see how the graph of f changes. Fig 3.1 and Fig 3.2 show that the map $T = 1.01$ and $T = 1.50$ (smaller T) has three fixed points at $x = 0$, $1/2$, and 1. Clearly, the fixed points $x = 0$ and $x = 1$ are stable (i.e., attractors), and the fixed point $x = 1/2$ is unstable (i.e., a repeller). Intervals $(0, 1/2)$ and $(1/2, 1)$ are the basins of attraction of attractors $x = 0$ and 1, respectively. For $T = 1.01$ and $T = 1.50$, there are virtually two possible final states of the economy: manufacturing is concentrated in region 1 or 2 depending on initial conditions. Note that in these cases, every trajectory is either monotone increasing or monotone decreasing.

— Fig 3.1 and Fig 3.2 —

Fig 3.3 depicts the picture for $T = 1.92$. At some T-value near $T = 1.92$, the fixed point $x = 1/2$ undergoes a pitchfork bifurcation[3], gaining stability and generating two other unstable fixed points around it. The fixed points $x = 0$ and 1 are still stable. Thus, in this economy, there are virtually three possible final states: manufacturing can be concentrated in one region or divided between the two regions, which again depends on initial conditions. Note that the basin of attraction of each attractor is an interval.

— Fig 3.3 —

Fig 3.4 for $T = 1.96$. At some T-value near $T = 1.96$, the fixed point $x = 1/2$ undergoes a period-doubling bifurcation. This type of bifurcation cannot occur in the continuous time version of this model. Through this bifurcation, the fixed point $x = 1/2$ loses again its stability and gives birth to a stable periodic orbit of period two in its vicinity. The two fixed points of manufacturing concentration remain stable, so there are at least three possible final states, one of which keeps oscillating over time around the fixed point $x = 1/2$. Thus, in our model, it is possible for the share of manufacturing workers (and other relevant variables) to keep fluctuating between the two regions without any external disturbances.

— Fig 3.4 —

Fig 3.5 shows that nonlinearity of the map f at $T = 2.00$ is strong enough for the manufacturing share to fluctuate around $x = 1/2$ in an erratic or chaotic way. It seems as if the economy would be in the steady state $x = 1/2$, constantly subjected to external random shocks. Fig 3.6 plots the chaotic time series.

[3]For dynamical systems theory and bifurcation theory, see e.g. Guckenheimer and Holmes (1983).

— Fig 3.5 and Fig 3.6 —

As T increases, we can observe other important nonlinear phenomena. First take a look at Fig 3.7. At $T = 2.10$, the maximum and minimum of the humps of the graph of f hit the basins of attraction of manufacturing concentration equilibria $x = 0$ and $x = 1$. The trajectory of almost every initial manufacturing share $x_0 \in [0, 1]$ seems to settle down to either $x = 0$ or $x = 1$ as time goes on. On the interval $(0, 1)$, there seems to be some f-invariant set[4] on which f behaves chaotically. As a result, some trajectories starting near that "chaotic invariant set" which eventually converge to 0 or 1 will fluctuate for a relatively long time; this mechanism will generate so-called *transient chaos*, that is, long-lasting erratic oscillations which eventually cease. Fig 3.8 depicts two transient-chaotic trajectories with different but close initial conditions: one trajectory eventually drops to 0, and another one with a slightly different initial condition converges to 1. As Fig 3.8 indicates, workers move in and out of the two regions in a complicated manner for first many periods, but, *suddenly*, all of them tend to either 0 and 1 depending on initial conditions, and nobody will ever return to the other region.

— Fig 3.7 and Fig 3.8 —

Remarkable is the structure of the basins of attraction for the attractors $x = 0$ and $x = 1$ for $T = 2.10$. Unlike the cases of smaller T, those basins are *no longer intervals* and the boundaries of the two basins look complicated. See Fig 3.9a,b. Fig 3.9b is an enlargement of Fig 3.9a. These basin boundaries are referred to as *fractal basin boundaries*[5]. In the presence of such fractal basin boundaries the final state (i.e., the state to which a trajectory converges) can be sensitive to initial conditions.

— Fig 3.9a,b —

4 Summary and Concluding Remarks

This paper has offered a discrete-time dynamic version of Krugman's geographic model (Krugman 1991) which is based on Dixit and Stiglitz's monopolistic competition model (Dixit and Stiglitz 1977). In Krugman (1992) and Fujita *et al* (1999), a continuous-time dynamic version of Krugman's model was explored to examine the stability of multiple equilibria. On the

[4]Is is "usually" a Cantor set of measure zero

[5]See e.g. Onozaki *et al* (2000) for fractal basin boundaries.

other hand, we have stressed the possible occurrence of complex nonlinear phenomena which may or may not be related to equilibrium selection. In comparison with Krugman and his co-workers' works, our findings are summarized as follows:

a. Given our discrete-time adjustment process, persistent periodic and chaotic endogenous fluctuation in manufacturing share can occur. Such dynamics can not be generated by Krugman's two-region continuous-time model;
b. Even if an equilibrium steady state is unstable (e.g. $x = 1/2$ for $T = 2.00$), manufacturing share may remain in the vicinity of that steady state. In Krugman's two-region model, however, instability of the steady state implies that a trajectory stays away from that steady state;
c. Transient chaos is possible. As a result, seemingly discontinuous change in dynamic patterns can occur *without* any structural change in the underlying system;
d. Unlike Krugman's continuous-time adjustment model, even higher transportation costs (i.e. larger T) may cause manufacturing concentration because of the presence of overshooting adjustment. This seems to be inconsistent with our intuition that at higher transportation costs each region would have some manufacturing;
e. Basin boundaries of multiple attractors can be very complicated, which causes our inability to predict the final state of the economy even if we have highly precise knowledge about initial conditions.

We have numerically demonstrated that our model can generate much richer dynamics than the Krugman's continuous-time model when we modify his model by introducing another type of adjustment process. Our numerical results, however, depend on the *ad hoc* adjustment process as much as those by Krugman do. Basically, our model is intended to illustrate what kind of dynamic core-periphery patterns are possible in the simplest Krugman-type geographic model. If we want to make our results persuasive, we may need a sophisticated dynamization of the static model. An interesting future research topic would be then to replace our *ad hoc* labor adjustment process by a more sophisticated one[6] to check wether the results obtained above can be reproduced for that model.

[6]For other types of adjustment dynamics in the similar context, see e.g. Matsuyama (1992) and Matsuyama and Takahashi (1998).

参考文献

[1] Dixit, A.K. and J.E. Stiglitz, 1977, "Monopolistic Competition and Optimum Product Diversity", *American Economic Review 67*, pp.297–308.

[2] Fujita, M., P. Krugman, and A.J. Venables, 1999, *The Spatial Economy*, MIT Press.

[3] Guckenheimer,J. and P. Holmes, 1983, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer-Verlag.

[4] Krugman, P., 1991, "Increasing Returns and Economic Geography", *Journal of Political Economy 99*, pp.483–499.

[5] Krugman, P., 1992, "A Dynamic Spatial Model", NBER Working Paper No.4219.

[6] Matsuyama, K., 1992, "A Simple Model of Sectoral Adjustment", *Review of Economic Studies 59*, pp.375–388.

[7] Matsuyama, K. and T. Takahashi, 1988, "Self-Defeating Regional Concentration", *Review of Economic Studies 65*, pp.211–234.

[8] Onozaki, T., G. Sieg, and M. Yokoo, 2000, "Stability, Chaos, and Multiple Attractors: Can a Single Heterogeneous Agent Make Differences?", in progress

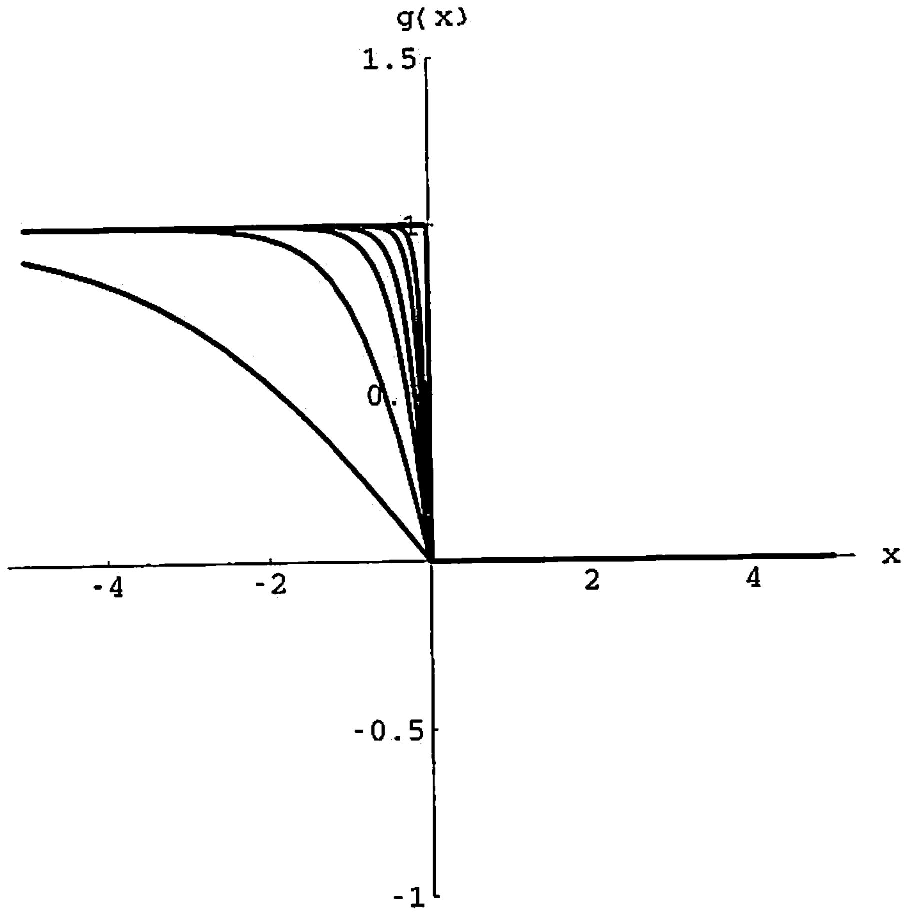

Fig 2.1: Graph of Migration Function g_β

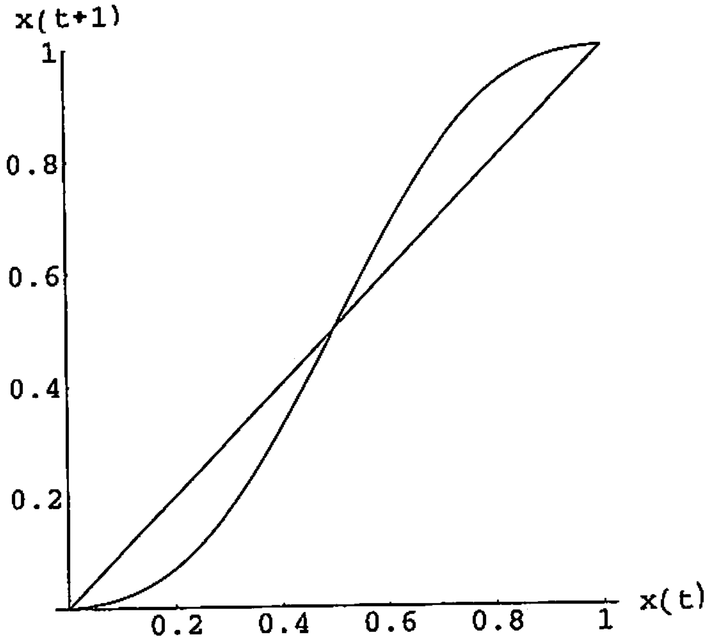

Fig 3.1: Graph of f, $T = 1.01$

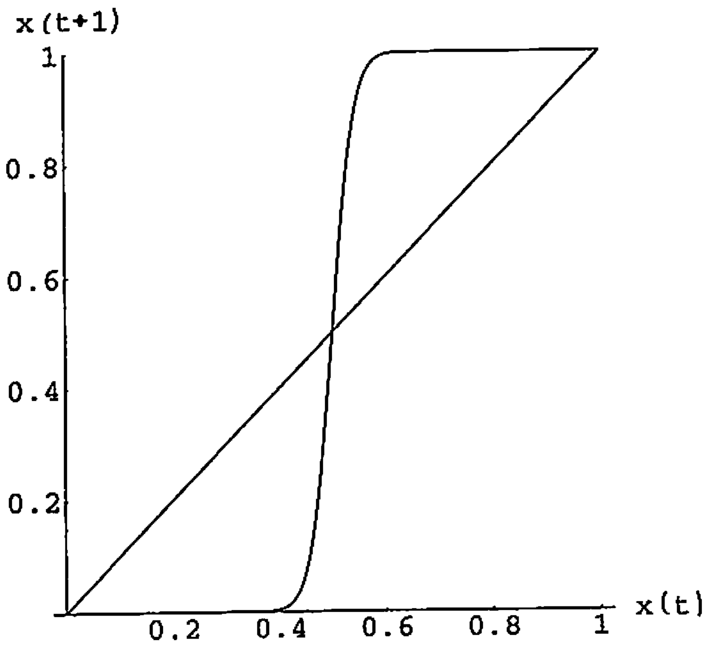

Fig 3.2: Graph of f, $T = 1.50$

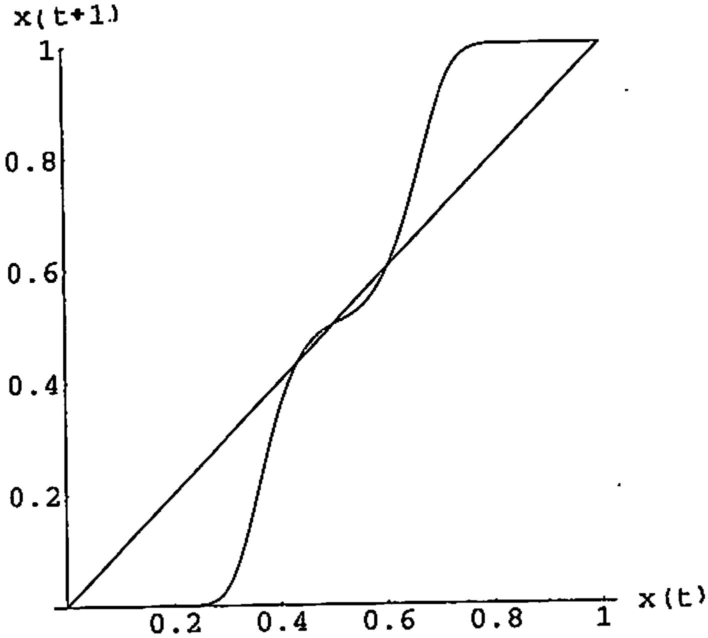

Fig 3.3: Graph of f, $T = 1.92$

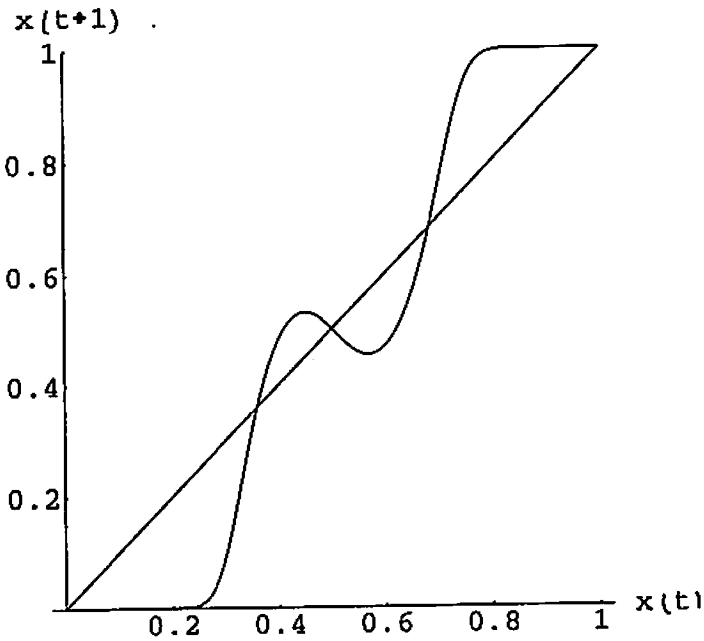

Fig 3.4: Graph of f, $T = 1.96$

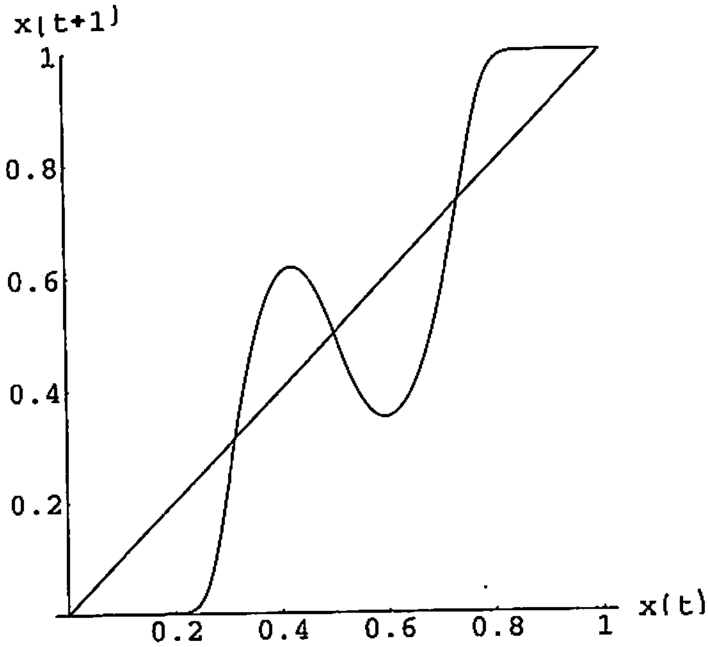

Fig 3.5: Graph of f, $T = 2.00$

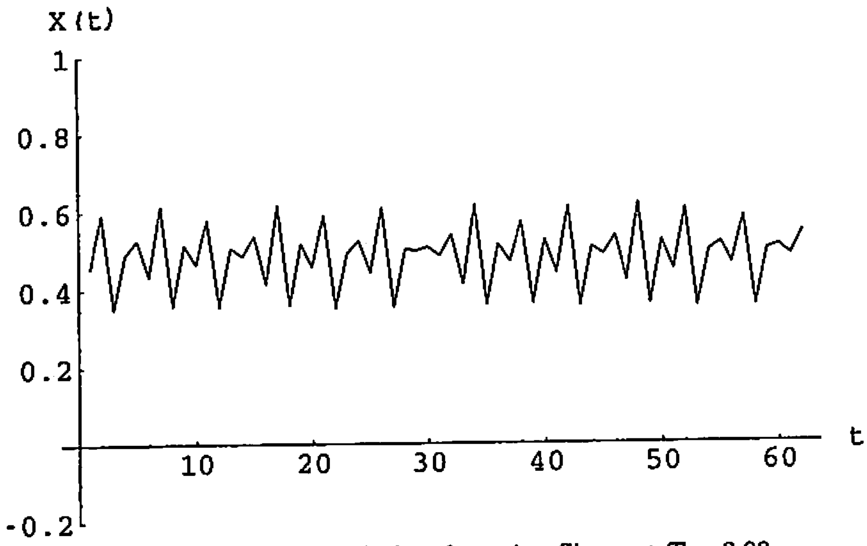

Fig 3.6: Time Series of Manufacturing Share at $T = 2.00$

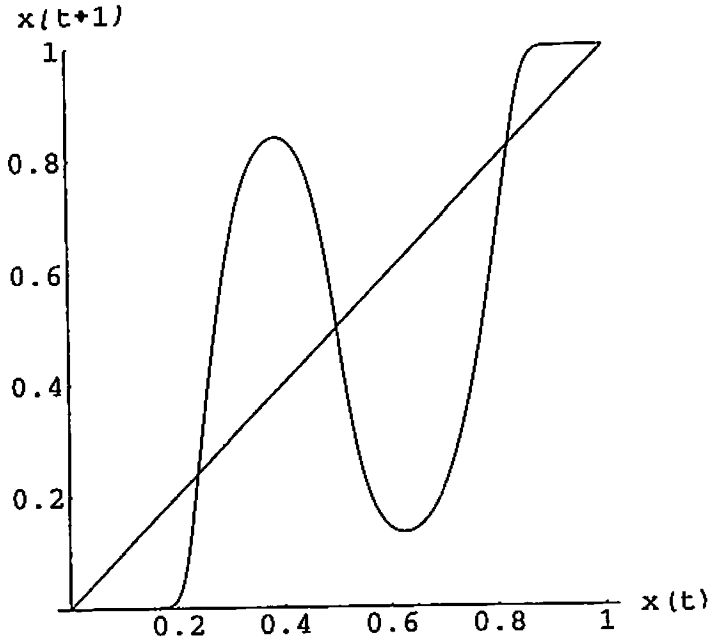

Fig 3.7: Graph of f, $T = 2.10$

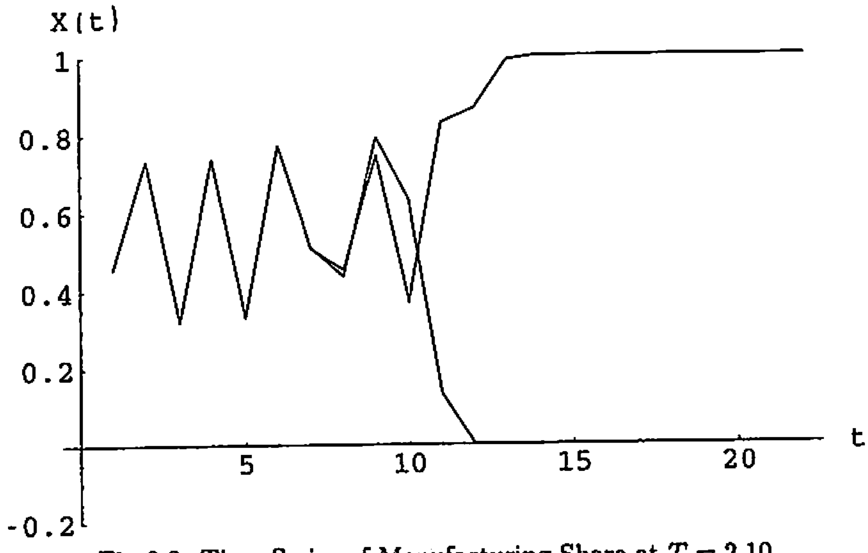

Fig 3.8: Time Series of Manufacturing Share at $T = 2.10$
$x_0 = 0.455551$ and $x_0' = 0.455552$

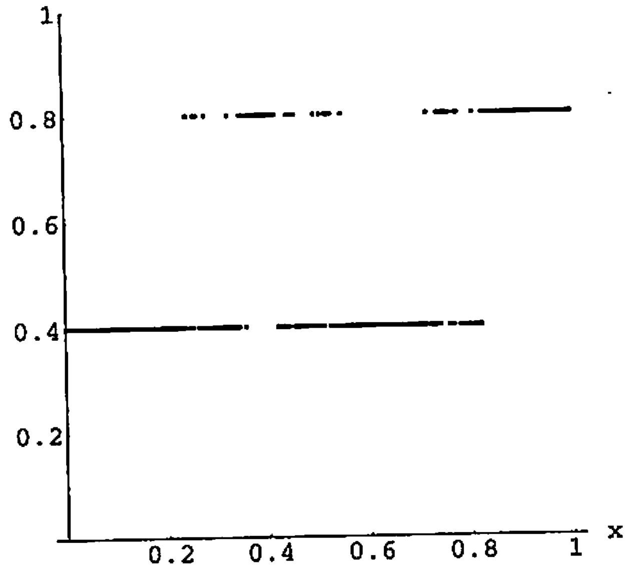

Fig 3.9a: Basins of Attraction $x \in [0, 1]$
Basin for $x = 1$ (on 0.8-line)
Basin for $x = 0$ (on 0.4-line)

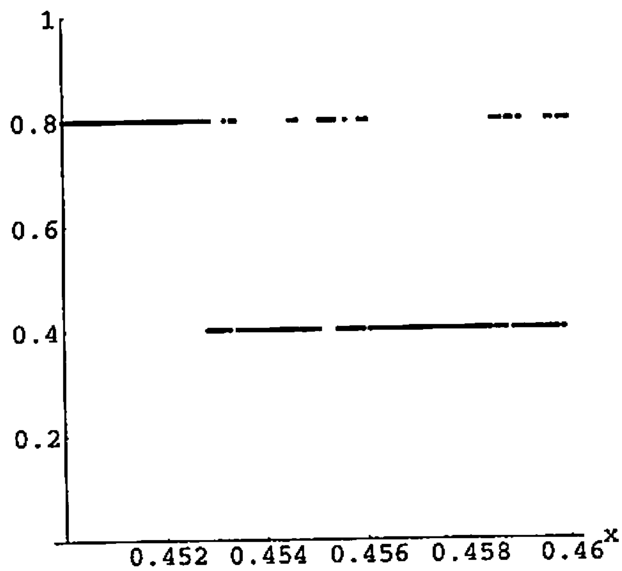

Fig 3.9b: Basins of Attraction $x \in [0.45, 0.46]$
Basin for $x = 1$ (on 0.8-line)
Basin for $x = 0$ (on 0.4-line)

f 6