On the sustainability of a common property resource: An implication from dynamic game theory

Ken-Ichi Akao* and Gerhard Sorger**

* School of Social Sciences, Waseda University

** Department of Economics, University of Vienna

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Abstract

In his influential paper, "The Tragedy of the Commons," Garrett Hardin (1968) concluded that freedom of the commons brings ruin to all. The dynamic game literature verifies that this ecologist's pessimistic view is theoretically supported, but several counter examples have been found. In this study, we show that, under a set of standard assumptions in economics, any resource use, sustainable or unsustainable can be a consequence of a dynamic game involving a common property resource, as long as the path is induced by a Lipschitz continuous dynamical system. This result implies that there are many possibilities for the fate of the commons.

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1 Introduction

Several environmental resources are characterized by common or ill-defined property rights. This may be the result of the physical nature of an environmental resource, such as the global atmosphere, or it may result from an institutional arrangement: for example, fishery resources in the high seas. No matter what the reason, an important consequence is that the resource use cannot be determined by a single agent. The best strategy of each resource user depends on the strategies that the other users take. When the other users are exploiting the resource with maximum efforts, it can be best for each agent to follow this strategy until the resource is devastated. This is the conclusion that Garrett Hardin (1968) reached in his very influential paper, "The Tragedy of the Commons," where the commons is a metaphor for the environmental resource. His core message, which we requote from Dasgupta (1982, Chapter 2), is as follows.

> Picture a pasture open to all. It is to be expected that each herdsman will try to keep as many cattle as possible on the commons...As a rational being, each herdsman seeks to maximize his gain. Explicitly or implicitly, more or less consciously, he asks, 'What is the utility to me of adding one more animal to my herd?'...Adding together the component partial utilities, the rational herdsman concludes that the only sensible course for him to pursue is to add another animal to his herd. And another; and another...But this is the conclusion reached by each and every rational herdsman sharing a commons. Therein is the tragedy. Each man is locked into a system that compels him to increase his herd without limit—in a world that is limited. Ruin is the destination toward which all men rush, each pursuing his own best interest in a society that believes in the freedom of the commons. Freedom of the commons brings ruin to all. (Hardin, 1968, p.1244)

Despite these clear words, however, Hardin's deduction is not always valid. Immediately after the above citation, Dasgupta (1982) wrote that 'it would be difficult to locate another passage of comparable length and fame containing as many errors as the one above.' Dasgupta's point is that while each herdsman may ignore the social cost incurred for an additional animal, an individual herdsman must take into account his private cost, and, if the marginal private cost increases rapidly with the number of his own cattle, the total number of cattle should stabilize before the fate of the pasture becomes ruinous.

Dasgupta's criticism is pertinent. However, if the private cost is negligibly small, does Hardin's tragedy then emerge? The answer depends on whether the resource in question is a common property resource or an open access resource. These types of resources are distinguished by whether the number of resource users are fixed. In the case of a common property resource, the members are fixed, whereas in the case of an open access resource, there are many resource users. As a result, in the case of an open access resource, resource users enter the market or utilize the resource as long as they can earn a positive profit. That is Hardin's story. In contrast, the situation is much more complicated in the case of a common property resource as a result of the strategic interaction among resource users.

In this paper, we show that there are an immense number of possibilities beyond the tragedy of the commons in terms of the outcome for a common property resource. To this end, we consider a symmetric dynamic game with n (≥ 2) players. "Symmetric" means that all players have the same preference and resource harvesting technology. This assumption is made because it makes the game simple. In addition, following the dynamic game literature in economics, we assume that each player determines how much to harvest at each point in time by responding to the current stock level of the resource. In other words, we assume that the strategy of each player is a function that maps the resource stock to the amount of harvest. This type of strategy is called stationary Markovian.[1] Game theory considers a Nash equilibrium as the consequence of a game. A Nash equilibrium is a profile of strategies taken by players where each player maximizes his or her utility as long as the other players keep on using their strategies in the profile. In the present context, a Nash equilibrium is called a Markov perfect Nash equilibrium (MPNE), where "perfect" means subgame perfect; i.e., whenever the game starts and whatever the resource stock is, the profile of strategies always constitutes a Nash equilibrium. If the strategies of an MPNE are the same across players, it is called a symmetric MPNE. We will consider only a symmetric MPNE and suppress "symmetric" hereafter. Note that because the aim of this study is to show that any resource use can occur for a common property resource, these restrictions of symmetry on the game and the equilibrium concept are harmless. That is, if we prove that a thing occurs in a special case, then we can say that it can occur in a general case.

As indicated above, a time path of resource use induced by an MPNE (an MPNE path, for short) can differ from that of the tragedy of the commons even if there is no private cost for harvesting. In the great fish war model by Levhari and Mirman (1980), an MPNE path converges to an equilibrium steady state that is a positive stock level of the resource. That is, the MPNE path is a sustainable path, although the equilibrium steady state is smaller than the optimal steady state that is attained when all players cooperate to maximize their aggregate utilities. Using the same model, Dutta and Sundaram (1993a) obtain an MPNE path that converges to a stock level larger than the optimal steady state, implying that not just over-exploitation but also *under*-exploitation of a common property resource can occur. A similar result is obtained by Hori and Shibata (2010). In the model by Dockner and Nishimura (2005), an MPNE path converges to a positive stock level, whereas a cooperative optimal path converges to the zero stock level. That is, an inefficient but sustainable resource use may even be chosen in a game despite the fact that an unsustainable resource use is optimal.

[1] "Stationary" means time independent and "Markovian" comes from the Markov process in mathematics.

Games are usually characterized by multiple equilibria. These sustainable MPNE paths can coexist with Hardin's path to ruin. In a general class of games, which he referred to as resource games, Sorger (1998) established a sufficient condition under which a sustainable MPNE path coexists with an MPNE path such that the resource is exploited by all players' maximum efforts and is exhausted in a finite time.

In this paper, we show that, under a set of standard assumptions in economics, any resource use, sustainable or unsustainable can be an MPNE path as long as the path is induced by a Lipschitz continuous dynamical system. Compared with previous studies, this study has two features. First, we consider a general class rather than a specific parametric model such as a linear-quadratic model. Second, most of the dynamic game models in economics have only one state variable, but we consider multiple state variables.

The rest of this paper is organized as follows. Section 2 describes the dynamic game model that we are considering. Section 3 shows the results. First, using two existing results by Mitra and Sorger (1999, Theorem 3) and Akao, Mitra and Sorger (2011, Theorem 3.1), we show that for every Lipschitz continuous function h, there is a dynamic game model in which the dynamical system with function h generates an MPNE path (Theorem 3.3). The dynamic game model in Theorem 3.3 uses utility functions with wealth effects; i.e., each player is affected by the resource stock. However, Theorem 3.3 does not impose any restrictions on the wealth effect. If the utility function satisfies the standard assumptions in economics, then the result in Theorem 3.3 is more appealing. For this reason, we show a sufficient condition under which the utility function is concave and increasing in both consumption and the resource stock (Theorem 3.4). Section 4 concludes the paper.

2 Model

2.1 Definitions

In this section, we formally define the class of dynamic games under consideration. Time evolves in discrete periods; i.e., the time variable t takes values in the set of positive integers $\mathbb{N}$. We suppose that there are m ($\in \mathbb{N}$) common property resources in the economy. The state of them at the end of period $t-1$ (or, equivalently, at the beginning of period t) is described by $x_{t-1} \in X \subset \mathbb{R}^m$, where $\mathbb{R}^m$ is the m-dimensional Euclidean space.

The evolution of the common property resources is described by the transition possibility set $\Omega \subseteq X \times X$. A time path of the common property resources $(x_{t-1})_{t=1}^{\infty}$ is feasible if $(x_{t-1}, x_t) \in \Omega$ for all $t \in \mathbb{N}$. The return function $R : \Omega \to \mathbb{R}$ denotes the amount of output that is available for consumption in a period.

The economy is populated by n (≥ 2) players. All players are infinitely lived and identical. We use c_t^i to denote player i's consumption level in period t. Given a feasible path $(x_{t-1})_{t=1}^{\infty}$, the vector of individual consumption levels $(c_t^1, c_t^2, \ldots, c_t^n) \in \mathbb{R}^n$ is feasible in period t if it satisfies $\sum_{i=1}^n c_t^i \leq R(x_{t-1}, x_t)$ for all $t \in \mathbb{N}$. Player $i \in \{1, 2, \ldots, n\}$ seeks to maximize the objective functional

$$\sum_{t=1}^{\infty} \rho^{t-1} u(c_t^i, x_{t-1}),$$

where $\rho \in (0, 1)$ is the discount factor and $u : \mathbb{R} \times X \to \mathbb{R} \cup \{-\infty\}$ is the utility function, which may depend on consumption as well as on the state of the resource stocks.

2.2 Dynamic game

The game under consideration is described by the fundamentals $(n, X, \Omega, R, u, \rho)$. It is a simultaneous-move game with perfect information. Each player i adopts a stationary Markovian strategy $\sigma^i : X \to \mathbb{R}_+$ that determines the individual consumption level in each period t as a function of the state of the system at the beginning of that period, that is $c_t^i = \sigma^i(x_{t-1})$.

A strategy profile $(\sigma^1, \sigma^2, \ldots, \sigma^n)$ is feasible if, for any initial state $x \in X$, there is a feasible path $(x_{t-1})_{t=1}^{\infty}$ that satisfies $x_0 = x$ and $\sum_{i=1}^{n} \sigma^i(x_{t-1}) \leq R(x_{t-1}, x_t)$ for all $t \in \mathbb{N}$.

A strategy profile $(\sigma^1, \sigma^2, \ldots, \sigma^n)$ is an MPNE if it is feasible and if, for each possible initial state $x \in X$ and for each player i, the following dynamic optimization problem:

$$\max_{(c_t^i, x_{t-1}^i)_{t=1}^{\infty}} \left\{ \liminf_{T \to \infty} \sum_{t=1}^{T} \rho^{t-1} u(c_t^i, x_{t-1}^i) \right\} \tag{2.1}$$

$$\begin{aligned} &\text{subject to } (x_{t-1}^i, x_t^i) \in \Omega \text{ for all } t \in \mathbb{N}, \\ &\qquad R(x_{t-1}^i, x_t^i) - \sum_{j \neq i} \sigma^j(x_{t-1}^i) - c_t^i \geq 0 \text{ for all } t \in \mathbb{N}, \\ &\qquad x_0^i = x \end{aligned}$$

has a unique solution $\left(c_t^{i*}, x_{t-1}^{i*}\right)_{t=1}^{\infty}$ that satisfies the following two conditions.

1. $c_t^{i*} = \sigma^i(x_{t-1}^{i*})$ for all $t \in \mathbb{N}$.

2. There exists a feasible path $(x_{t-1})_{t=1}^{\infty}$ with initial state $x_0 = x$ such that $x_t^{i*} = x_t$ holds for all $i \in \{1, 2, \ldots, n\}$ and all $t \in \mathbb{N}$.

The MPNE $(\sigma^1, \sigma^2, \ldots, \sigma^n)$ is called symmetric if there exists a function $\sigma : X \to \mathbb{R}$ such that $\sigma^i = \sigma$ holds for all $i \in \{1, 2, \ldots, n\}$.

2.3 Dynamic optimization problem

A dynamic optimization problem is a special case of a dynamic game in which the number of players n is equal to 1. That is,

$$\max_{(c_t, x_{t-1})_{t=1}^{\infty}} \left\{ \liminf_{T \to \infty} \sum_{t=1}^{T} \rho^{t-1} u(c_t, x_{t-1}) \right\}$$

$$\text{subject to } (x_{t-1}, x_t) \in \Omega \text{ for all } t \in \mathbb{N},$$

$$R(x_{t-1}, x_t) - c_t \geq 0 \text{ for all } t \in \mathbb{N},$$

$$x_0 = x.$$

Our main result utilizes a specific dynamic optimization such that $u(c, x) = c$. Then, the dynamic optimization problem is written as:

$$V(x) = \max_{(x_t)_{t=0}^{\infty}} \sum_{t=1}^{\infty} \rho^{t-1} R(x_{t-1}, x_t) \quad \text{subject to } (x_{t-1}, x_t) \in \Omega \text{ and } x_0 = x \in X. \tag{2.2}$$

The function $V : X \to \mathbb{R}_+ \cup \{\infty\}$ is referred to as the optimal value function, which satisfies the Bellman equation:

$$V(x) = \max\{R(x, y) + \rho V(y) |\ y \in X\}. \tag{2.3}$$

If $h(x)$ is the unique solution to the right-hand side for any $x \in X$, then we call h the optimal policy function.

2.4 Assumption

We assume:

Assumption: $X \subseteq \mathbb{R}^m$ is a compact and convex set with a nonempty interior.

3 Results

Our main result is a corollary of the following two theorems obtained by Mitra and Sorger (1999) and Akao, Mitra and Sorger (2011).

Theorem 3.1 *(Mitra and Sorger, 1999, Theorem 3) Let $h : X \to X$ be a Lipschitz continuous function with Lipschitz constant L. For every $\rho \leq 1/L^2$, there exists a dynamic optimization problem, the optimal policy function of which is given by h. The dynamic optimization problem is characterized by the following properties.*

- *P1: $\Omega \subseteq X \times X$ is a closed and convex set such that the x-section Ω_x is nonempty for all $x \in X$. The set $\cup_{x \in X} \Omega_x$ has a nonempty interior. If $x \leq \bar{x}$, then $\Omega_x \subseteq \Omega_{\bar{x}}$.*
- *P2: $R : \Omega \to \mathbb{R}$ is a continuous and concave function. $R(x, y)$ is nondecreasing in x and nonincreasing in y.*
- *P3: $\rho \in (0, 1)$.*
- *P4: The optimal value function V is α-concave and $(-\beta)$-convex with some $\alpha, \beta \in \mathbb{R}_{++}$.*

Here, the x-section Ω_x is defined by $\Omega_x = \{y \in X | (x, y) \in \Omega\}$. A function $V(x)$ is α-concave if $V(x) + (1/2)\alpha \|x\|^2$ is concave. Similarly, $V(x)$ is $(-\beta)$-convex if $V(x) - (1/2)(-\beta) \|x\|^2$ is convex. By properties P1, P2 and P3, $V(x) < \infty$ for all $x \in X$. As V is α-concave and thus strictly concave, h is a unique optimal policy function to the dynamic optimization problem. This implies that the dynamic optimization problem constructed by Theorem 3.1 satisfies the condition under which Theorem 3.1 in Akao, Mitra and Sorger (2011) holds.

Theorem 3.2 *(Akao, Mitra and Sorger, 2011, Theorem 3.1) Assume that a dynamic optimization problem satisfies Properties P1–P4. Let $h : X \to X$ be the unique policy function. Then, for every $n \geq 2$ there exists a utility function $u : \mathbb{R} \times X \mapsto \mathbb{R}$ that satisfies the following properties.*

- *P5: For all $x \in X$, it holds that $u(c,x)$ is a strictly increasing and concave function of c.*

- *P6: For all $x \in X$, $\partial u(n^{-1}R(x,h(x)),x)/\partial c$ exists and satisfies $\partial u(n^{-1}R(x,h(x)),x)/\partial c = n^{-1}$.*

- *P7: For all $x \in X$, it holds that $u(n^{-1}R(x,h(x)),x) = n^{-1}R(x,h(x))$,*

and the dynamic game $(n, X, \Omega, R, u, \rho)$ has an MPNE path induced by h.

Examples of such a utility function are:

$$u(c,x) = c^{\frac{1}{n}} \left[n^{-1}R(x,h(x))\right]^{1-\frac{1}{n}} \tag{3.1}$$

and

$$u(c,x) = \frac{1}{n}c + \frac{n-1}{n}\left[n^{-1}R(x,h(x))\right]. \tag{3.2}$$

By combining Theorems 3.1 and 3.2, we have the main result, as follows.

Theorem 3.3 *For any Lipschitz continuous function $h : X \to X$ with Lipschitz constant L, there is a dynamic game $(n, X, \Omega, R, u, \rho)$ satisfying the properties P1–P7 in Theorems 3.1 and 3.2 for every $\rho \leq 1/L^2$ and every integer $n \geq 2$, such that an MPNE path of the game is induced by h.*

Theorem 3.3 indicates that not just the tragedy of the commons, but any outcome can occur for a common property resource in theory. However, note that while we impose some restrictions on u with respect to c, the dependence of u on x (the wealth effect) is completely unrestricted. If we have a dynamic game model with u that is increasing and concave jointly in c and x, the result in Theorem 3.3 is more appealing from the viewpoint of economics.

Note that the utility functions

$$u(c,x) = c^{\frac{1}{n}} \left[n^{-1}R(x,h(x))\right]^{1-\frac{1}{n}} \text{ and } u(c,x) = \frac{1}{n}c + \frac{n-1}{n}\left[n^{-1}R(x,h(x))\right].$$

in Theorem 3.2 are concave and increasing if $R(x,h(x))$ is concave and increasing. We show a few examples of $R(x,h(x))$ corresponding to specific policy functions. In the first two examples, $R(x,h(x))$ is concave and increasing, whereas it is not in the last example.

Example 1. Brock–Mirman neoclassical growth model (Brock and Mirman, 1972)

$$R(x,h(x)) = \ln(1-\alpha\rho)x^{\alpha},\ \ \alpha \in (0,1).$$

Example 2. Logistic map policy function (Boldrin and Montrucchio, 1986) when the discount factor satisfies $\rho = \left(3-2\sqrt{2}\right)/16(\approx 0.01072)$,

$$\begin{aligned} R(x,h(x)) = (75/2)\left(2\sqrt{2}+1\right)x - \left(107\sqrt{2}-289/2\right)x^2 - 16\left(4\sqrt{2}-5\right)x^3 \\ - 8\left(113-76\sqrt{2}\right)x^4 + 704\left(3-2\sqrt{2}\right)x^5 - 832\left(3-2\sqrt{2}\right)x^6 \\ + 512\left(3-2\sqrt{2}\right)x^7 - 128\left(3-2\sqrt{2}\right)x^8. \end{aligned}$$

It is easily verified that $R(x,h(x))$ is increasing and concave on X in these examples. The last example fails to meet the requirement of concavity.

Example 3. (Tent map policy function) Nishimura and Yano (1995) demonstrate that, under standard assumptions in economics, even when the discount factor is sufficiently near to one, the optimal policy function may be expansive and unimodal, implying a chaotic equilibrium path.

Let $X = [0, \mu/\beta]$ and $\Omega = \{(x, y) \in X \times X | x \leq \min\{\mu x, \mu/\beta\}\}$, where μ and β are positive numbers and satisfy $\mu/\beta < 1$ and $\rho\mu > 1$. The policy function h is given by:

$$h(x) = \begin{cases} \mu x & \text{if } 0 \leq x \leq 1/\beta, \\ -\frac{\alpha\mu}{\beta-\alpha}x + \frac{\mu}{\beta-\alpha} & \text{otherwise,} \end{cases}$$

where $\alpha \in (\beta/(1+\mu), \beta/\mu)$. The return function $R(x, y)$ is given by:

$$R(x, y) = \begin{cases} x - \mu^{-1}y & \text{if } 0 \leq y \leq -\frac{\alpha\mu}{\beta-\alpha}x + \frac{\mu}{\beta-\alpha}, \\ \frac{1}{\alpha} - \frac{\beta/\alpha}{\mu}y & \text{otherwise.} \end{cases}$$

From these, we have:

$$R(x, h(x)) = \begin{cases} 0 & \text{if } 0 \leq x \leq 1/\beta, \\ \frac{\beta x - 1}{\beta - \alpha} & \text{otherwise.} \end{cases}$$

The return function evaluated at $y = h(x)$ is increasing but not concave because of the corner solutions of $h(x)$ on $[0, 1/\beta]$.

As shown in Example 3, even in a concave model with standard economic assumptions, $u(c, x)$ does not necessarily have the desirable properties, i.e., concavity and monotonicity. However, these properties are ensured if we choose a sufficiently small discount factor.

Theorem 3.4 *Let dynamical system $h : X \to X$ be given. If h is twice continuously differentiable, then the utility function $u(c, x)$ of the dynamic game $(n, X, \Omega, R, u, \rho)$ in Theorem 3.3 can be increasing and concave in both variables by choosing a sufficiently small ρ.*

Proof. As shown in the proof of Theorem 3 in Mitra and Sorger (1999), we can choose the functional form of the value function of the dynamic optimization problem (2.2) as:

$$V(x) = \gamma e_m x - (\alpha/2)||x||^2, \tag{3.3}$$

where α and γ are positive real numbers and $e_m = (1, 1, ..., 1) \in \mathbb{R}^m$. Note that $V(x)$ is strictly concave. Additionally, note that $V(x)$ is strictly increasing if a sufficiently large γ is chosen. As $R(x, h(x))$ is twice continuously differentiable and satisfies:

$$R(x, h(x)) = V(x) - \rho V(h(x)), \tag{3.4}$$

the first and second order derivatives of $R(x, h(x))$ uniformly converge to those of $V(x)$ as $\rho \to 0$. Therefore, by choosing a sufficiently small ρ, $R(x, h(x))$ is increasing and concave. Substitute such an $R(x, h(x))$ into (3.1) or (3.2) into Theorem 3.2 and we have a game with an increasing and concave utility function by Theorem 3.3. ■

4 Concluding remarks

We have shown that any dynamical system generated by a Lipschitz continuous function can be represented as an MPNE path to a dynamic game model that satisfies the standard assumptions in economics. This indicates that any outcome, not just the tragedy of the commons, can occur for a common property resource. This result may not be surprising. We now know that any dynamical system generated by a Lipschitz continuous function can be represented as a solution path to a dynamic optimization problem (Theorem 3 in Mitra and Sorger, 1999). This indicates that an efficient equilibrium path in a competitive economy can take any pattern, from a monotonic convergence to a steady state to a business cycle including chaotic behavior. A game introduces a complication to a competitive economy, which is strategic interaction among agents. Because of this complication, it is natural that we should have more flexibility on the resulting equilibrium path in a dynamic game than a (single agent) dynamic optimization problem.

References

[1] Akao, K., T. Mitra and S. Sorger (2011) "Some rationalizability results for dynamic games," forthcoming in *International Journal of Economic Theory.*

[2] Boldrin, M. and L. Montrucchio (1986) "On the indeterminacy of capital accumulation paths," *Journal of Economic Theory* **40**, 26–39.

[3] Brock, W.A. and L.J. Mirman (1972) "Optimal economic growth and uncertainty: the discounted case," *Journal of Economic Theory* **4**, 479–513.

[4] Dasgupta, P. *The Control of Resources.* Harvard University Press.

[5] Dockner, E.J. and K. Nishimura (2005) "Capital accumulation games with a non-convex production function," *Journal of Economic Behavior and Organization* **57**, 408–420.

[6] Dockner, E.J. and K. Nishimura (1999) "Transboundary pollution in a dynamic game model," *Japanese Economic Review* **50**, 443–456.

[7] Dutta, P. K. and R.K. Sundaram (1993a) "The tragedy of commons?" *Economic Theory* **3**, 413–426.

[8] Dutta, P. K. and R.K. Sundaram (1993b) "How different can strategic models be?" *Journal of Economic Theory* **60**, 42–61.

[9] Hardin, G. (1968) "The tragedy of the commons," *Science* **162**, 1243-1248.

[10] Hori, K. and A. Shibata (2010) "Dynamic game model of endogenous growth with consumption externalities," *Journal of Optimization Theory and Applications* **145**, 93-107.

[11] Levhari, D. and Mirman, L.J., 1980: "The great fish war: an example using a dynamic Cournot-Nash solution," *Bell Journal of Economics* **11**, 322-344.

[12] Mitra, T. and G. Sorger (1999) "Rationalizing policy functions by dynamic optimization," *Econometrica* **67**, 375–392.

[13] Nishimura, K. and M. Yano (1995) "Nonlinear dynamics and chaos in optimal growth: an example," *Econometrica* **63**, 981–1001.

[14] Sorger, G. (1998) "Markov-perfect Nash equilibria in a class of resources games," *Economic Theory* **11**, 78–100.

[15] Stokey, N.L. and R.E. Lucas with E.C. Prescott (1989) *Recursive Methods in Economic Dynamics*. Harvard University Press.