The Harrod Discontinuity and Macroeconomics

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No. 0701

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January 29, 2008

Abstract

The purposes of this paper are threefold. The first is to show that Keynes's *General Theory* can be characterized as a two-sector model with flexible prices and profit-maximizing firms. Such properties are called "Three Features" of the *GT*. The second is to point out that macroeconomics made a drastic change in quality between 1936 and 1939. Such a change is called the Harrod discontinuity. Due to the change, "Three Features" of the *GT* were lost. The third and most important is to propose a general macro model by using "Three Features" of the *GT* and reconsidering Hicks's *IS-LM* model as well. The *simple* macro model proposed here will help to see Keynesian economics and classical economics from a unified point of view.

"You must never judge a master by his disciples."

Friedman (1975)

"It is true that when the two theories are properly understood, and fully worked out, they largely overlap; but they do not overlap all the way, and when they fail to do so, the Keynes theory has the wider coverage."

Hicks (1957)

1 Introduction

The history of macroeconomics is that of dispute. The confrontation between Keynesian economics and classical economics lay at the root of the dispute. Although they have been updated, macroeconomics is still divided into the two. For example, recently Mankiw (2006) made drew a distinction between Keynesians and classics, comparing the former to engineers and the latter scientists. It is very instructive, but how should we understand such a history and the status quo? One way is to think that they can coexist. That is, Keynesian economics is the short-run macroeconomics helpful to the analysis of income determination, business cycles, etc., whereas classical economics is the long-run macroeconomics useful for the analysis of economic growth, income distribution, etc. It traces back to the neoclassical-Keynesian synthesis in the 1950s.

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Indeed it is one way, and still it is alive, but in this paper I am going to take another way to resolve such an unhappy situation as the long-standing division between Keynesian economics and classical economics. That is, I will show that there is a common theoretical base for both of them by perusing Keynes's (1936) *General Theory of Employment, Interest and Money* (also called the *GT* in what follows). It has been generally supposed that Keynesian economics and classical economics are incompatible largely because the former is based on the rigidity of prices whereas the latter on the flexibility of them. But, as far as the *GT* is concerned, that is a delusion. The fact is that prices are assumed to be flexible in the *GT*. Then, what made the economics of the *GT* Keynesian economics of today? By examining the history of macroeconomics, I will find the "discontinuity" of macroeconomics in Harrod who is well known as one of Keynes's best disciples. There were two Harrods between 1936 and 1939! Thus, it will be called the "Harrod discontinuity."

Like air, the influence of the Harrod discontinuity has been so widespread that macroeconomists seem to pay no attention to the fact. First of all, however, we should know it because it is not only what belongs to the past but also what is still dividing macroeconomics into Keynesian economics and classical economics. In my opinion, the two economics had a common framework before the Harrod discontinuity occurred. By understanding properly the *GT* and also Harrod *before* the discontinuity, we are able to see the Keynes theory and the classical theory through one and the same macro model. That is what I am going to do below.

This paper is organized as follows. Section 2 finds that the *GT* can be characterized as a two-sector model with flexible prices and profit-maximizing firms. Such properties are called "Three Features" of the *GT*, which I regard as a good starting point of macroeconomics. Section 3 points out that macroeconomics made a drastic change in quality between 1936 and 1939. Such a change is called the Harrod discontinuity. Thanks to the change represented by the Harrod discontinuity, "Three Features" of the *GT* were lost. Section 4 confirms persistent effects of the Harrod discontinuity on macroeconomics as a whole. And it revisits Hicks's (1937) *IS-LM* model and finds "Three Features" of the *GT* in it. Section 5 discusses Friedman's contribution to the short-run analysis, and thinks highly of his attempt to unify the quantity theory of money and the income-expenditure theory. Section 6 claims that there is a redundant equation in the *IS-LM* model and the classical model Hicks (1937) formulated, while Section 7 addresses the question of what is the raison d'être of the *IS-LM* model. Finally Section 8 proposes a basic macro model for the unification of macroeconomics.

2 "Three Features" of the *General Theory*

I begin by pointing out important but unnoticed features of the *GT*. First, contrary to common belief, the *GT* assumes that prices of *goods* are flexible and that they are so determined as to equate supply and demand as in microeconomics. Although the evidence can be found here and there in the *GT*, Keynes stated it most definitely in the preface to the French edition of the *GT* as follows: "I regard the price level as a whole as being determined in precisely the same way as individual prices; that is to say, under the influence of supply and demand." (Keynes (1973a, p. xxxiv)) Second, the *GT* assumes that firms (or entrepreneurs) maximize their profit,[1] though it is no more than another expression of the first postulate of

[1]Correctly speaking, entrepreneurs determine the level of employment so as to maximize their *expected* profit. See the *GT*, p. 55.

classical economics the *GT* maintained. It implies that output is determined such that the "short-period supply price ... is equal to the *marginal* prime cost," (p. 68), or in modern terms, the price equals with the marginal cost.[2] Therefore, as far as prices of goods are concerned, macroeconomics, which Keynes founded, is very similar to microeconomics or classical economics, which he attacked. Thus prices of goods play no role in making a difference between classical economics and the economics of the *GT*.

It is helpful here to recognize the importance of a "largely forgotten book" (Goodwin (1982, p. vii)), namely, Harrod's (1936) *Trade Cycle* (also called the *TC* in what follows) to see the earliest stage of macroeconomics. The *TC*, which presented a new theory of the trade cycle based on the multiplier theory of the *GT* and the acceleration principle (which Harrod termed the Relation), stressed the flexibility of prices of goods over the trade cycle as follows: "Theory divorced from observation is mere definition or tautology. ... There is wide empirical evidence for the proposition that rising prices are associated with increasing activity and falling prices with declining activity." (p. 39) It can be said, therefore, that macroeconomics was based on the assumption of flexible prices, not sticky or rigid prices, at the very beginning.[3]

The *GT* and the *TC* have much in common. I can pick up two more things. First, both were developing each theory, explicitly or implicitly, under the assumption that the production sector of an economy as a whole is made up of the consumption-goods sector and the investment-goods sector and that each of the two sectors can be described by a production function.[4] As for the *TC*, it is directly connected with the acceleration principle which was depicted as follows: "It has long been a matter of observation that in the upward phase of the trade cycle, activity in the trades producing durable or capital goods increases more rapidly than that in the trades producing concurrently consumable goods, and conversely in the downward phase." (p. 53) As for the *GT*, it is related to the multiplier theory or the principle of effective demand. For example, Keynes (1937, p. 220) explained the conclusion resulting from the principle of effective demand as follows: "Incomes are created partly by entrepreneurs producing for investment and partly by their producing for consumption. The amount that is consumed depends on the amount of income thus made up. Hence the amount of consumption-goods which it will pay entrepreneurs to produce depends on the amount of investment-goods which they are producing. ... [T]here is always a formula ... relating the output of consumption-goods which it pays to produce to the output of investment-goods; and I have given attention to it in my book [i.e., the *GT*] under the name of the *Multiplier*."[5]

Second, and more importantly, both the *GT* and the *TC* tried to make clear the relationship between prices and production of the two sectors. The *TC* asked, "Now he would be a foolish man who would seek to deny that the price fluctuation is intimately connected with the output fluctuation, which is the central phenomenon of the cycle. But how connected?"

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[2]The prime cost is the sum of the factor cost (i.e., the total amount of wage) and the user cost (i.e., the total cost of factors of production other than labor). Thus, "the short-period supply price is the sum of the marginal factor cost and the marginal user cost." (the *GT*, p. 67) However, if output is regarded as that of final goods, the user cost can be dropped in the argument. And that was what the *GT* did implicitly. Compare the equations on pp. 44-45 and those on pp. 283-284. It follows that the short-period supply price equals with the marginal factor cost of labor in the *GT*. See also p. 40 of the *GT*.

[3]I do not know why, but it is very interesting that the *TC* called the *GT* "*General Theory of Unemployment, Interest, and Prices*" (My underline.) and that twice.

[4]The term investment goods is exchangeable for the term capital goods.

[5]See also Lerner (1936, p. 443). But footnote 2 on p. 24 of the *GT* does not seem to be consistent with this interpretation. For the relation with production functions, see the *TC*, p. 27, and the *GT*, pp. 117, 283, 297.

(p. 170) The GT intended to answer such a question by means of the law of price and output for industry r as follows:

> Let the elasticity of the expected price p_{wr} [of a unit of output in terms of the wage-unit W] in response to changes in effective demand D_{wr}, namely $\frac{dp_{wr}}{dD_{wr}}\frac{D_{wr}}{p_{wr}}$, be written e'_{pr}.
>
> Since $O_r \cdot p_{wr} = D_{wr}$, [where O_r is output of industry r,] we have
>
> $$\frac{dO_r}{dD_{wr}} \cdot \frac{D_{wr}}{O_r} + \frac{dp_{wr}}{dD_{wr}} \cdot \frac{D_{wr}}{p_{wr}} = 1$$
>
> $$e'_{pr} + e_{Or} = 1.$$
>
> [Here $e_{Or} = \frac{dO_r}{dD_{wr}} \cdot \frac{D_{wr}}{O_r}$.] That is to say, the sum of the elasticities of price and of output in response to changes in effective demand (measured in terms of wage-units) is equal to unity. Effective demand spends itself, partly in affecting output and partly in affecting price, according to this law. (pp. 284-285)

Only the above law is no more than a truism, so it tells nothing about the specific connection between price and output. However, since the price of industry r is equal to the marginal cost and p_{wr} is the price divided by the wage-unit or the nominal wage rate in a usual expression, it follows that p_{wr} is equal to the inverse of the marginal product of labor in the industry. Hence the change of a truism to a law.[6] The GT does not specify the number of industries, but, given that the GT is based on the above-mentioned assumption, it is appropriate to put $r = 1, 2$.

In sum, the GT has three features that have generally been neglected but I regard as fundamental:

1. Prices of goods are flexible and so adjusted as to make supply and demand equal.
2. The economy has two production sectors, that is, the investment-goods sector and the consumption-goods sector.
3. In each sector, firms maximize their profit with the result that price is equal to marginal cost.

Let us call them "Three Features" of the GT in what follows. As explained above, "Three Features" were shared by the TC to a considerable extent. From the above argument I conclude that the GT as well as the TC can be described as a two-sector model with flexible prices and profit-maximizing firms.[7]

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[6]The TC did not make the relation between the price and the marginal cost as clear as the GT. The TC said, "If my view be accepted, it can legitimately be assumed that broadly ... entrepreneurs do equate marginal revenue to marginal cost in determining the level of current output ... ." (p. 76) It is certain that the TC assumed profit-maximizing firms as in the GT. But it is also certain that there was a tinge of imperfect competition in the TC.

[7]It is well known that Leijonhufvud (1968) interpreted the GT as a two-commodity model with sticky prices. Objecting to the view, Grossman (1972) argued that the GT assumed the instantaneous adjustment of prices, as contrasted with wages, in the short run while Froyen (1976) asserted that a one-commodity interpretation was legitimate.

It should be noticed that "Three Features" are concerned with the goods markets only. The equilibrium of the investment-goods market ($r = 1$) and that of the consumption-goods market ($r = 2$) become respectively as follows:[8]

$$O_1 \cdot p_{w1} = D_{w1}, \tag{1}$$
$$O_2 \cdot p_{w2} = D_{w2}, \tag{2}$$

where p_{w1} and p_{w2} are respectively the price of investment goods and that of consumption goods, both measured in terms of wage-units.[9] An important thing is that p_{w1} and p_{w2} are *expected* prices as stated in the above citation. This implies that expectations play an important role in the determination of output in each sector.

Multiplying both sides of (1) and (2) by the nominal wage rate W leads to

$$p_1 O_1 = D_1, \tag{3}$$
$$p_2 O_2 = D_2, \tag{4}$$

where p_1 and p_2 are the money prices of investment goods and consumption goods, and D_1 and D_2 are the proceeds which firms in the investment-goods sector and the consumption-goods sector *expect* to receive from the employment of labor for production, both measured in terms of money.[10] It would be more convenient to consider the equilibrium of the goods markets using (3) and (4). Remember that firms are assumed to maximize their profit in the *GT*. It means, for example, that p_1 in (3) must be equal to the marginal cost of investment goods at the output level O_1 and therefore that $p_1 O_1$ in (3) represents the "aggregate supply price" of investment goods. This is why D_1 in (3) can be called the *effective* demand for investment goods. The same argument holds in (4), too. Adding each side of (3) and (4) gives a mathematical expression of the principle of effective demand:

$$p_1 O_1 + p_2 O_2 = D_1 + D_2 \ (= D). \tag{5}$$

(5) of course represents the equilibrium of the goods market as a whole. $p_1 O_1 + p_2 O_2$ and $D_1 + D_2$ in it correspond exactly to what the *GT* termed the aggregate supply price Z and the (aggregate) effective demand D.[11]

So far I emphasized the resemblance between the *GT* and the *TC*. But there is a big difference between the two. That is, the former was static with capital stock fixed, whereas the latter sought to be dynamic. The *TC* certainly recognized the limitations of static analysis as follows:

> Static analysis proceeds by asking what rate of flow of goods per unit of time through the exchange process is such that, given tastes, &c., no party to the exchange feels disposed to alter his conduct. This analysis is appropriate to a society which does not accumulate ... .

[8]The *GT* used numbers 1 and 2 to represent respectively the consumption-goods sector and the investment-goods sector. Only as a matter of convenience the order of numbers is reversed in this paper.

[9]The *TC* recognized clearly the difference between the two as follows: "Since, owing to the Relation, the variation in the output of capital goods in boom or slump is greater than that of consumable goods, we may reasonably expect, what in fact we find, that the variation in the price-level of capital goods is greater than the variation in that of consumable goods ... ." (p. 77)

[10]The *GT* also called the proceeds $D_1 + D_2$ the "aggregate demand price." (p. 26)

[11]For aggregate supply price and effective demand, see Chapter 3 of the *GT*.

> But, if accumulation is proceeding, income is growing. The volume of net investment and the rate of interest, that is, the current price which has to be paid for the means of current net investment, are both magnitudes which are essentially related to this process of growth. No static analysis can give a correct account of their determination; yet this is what the traditional analysis of value and distribution has sought to do. Mr. Keynes has perceived that there is something radically faulty in the traditional theory of interest; his perception is related to the objection here stated, since he partly bases his case on a criticism of the traditional presupposition of a constant level of income. But he does not formally set out the proper method of dynamic analysis. (pp. 149-150)

Who can be opposed to the above argument? It was undoubtedly constructive criticism. Needless to say, macroeconomics was founded by Keynes. But macroeconomists must not forget Harrod's contribution that made newborn macroeconomics walk on the right path. Macroeconomics got off to a good start.

3 The Harrod Discontinuity

However, something happened by 1939. Or I cannot help thinking so because I can not suppose that the same person wrote the *TC* and Harrod's (1939) *Essay in Dynamic Theory* at all.[12] Indeed Harrod (1939, p. 14) declared that his dynamic theory consisted in a marriage of the acceleration principle and the multiplier theory and that it was a development and extension of certain arguments advanced in the *TC*. Moreover, it is widely believed that Harrod (1939) extended short-run analysis of the *GT* to long-run analysis. However, in my opinion, all of these are dubious or faulty. I can give three reasons.

First, price(s) disappeared from Harrod (1939).[13] Remember the tremendous importance given to the behavior of prices in the *TC*, where it was also expressed as "far greater guiding light."[14] In Harrod (1939) no role was assigned to prices in the determination of output.

Second, the distinction between consumption goods and investment goods disappeared. It is obvious as Harrod (1939, p. 18) said, "It may be well to emphasise at this point that no distinction is drawn in this theory between capital goods and consumption goods. ... Some trade-cycle theorists concern themselves with a possible lack of balance between these two categories; ... The theory here considered is more fundamental or simple ... ." Who is not confused at this statement? I remember Harrod in the *TC* was certainly one of the trade-cycle theorists who concerned themselves with the lack. And it was directly related to the acceleration principle which constituted the common base of the *TC* and Harrod (1939). Anyway, it can be said, for the moment, that Harrod's (1939) model was a one-good or one-sector model.

Third, the multiplier theory disappeared from Harrod (1939). The *TC* explained the multiplier theory as follows: "The income of any community in a given period is constituted

[12] I know, for a fact, that the Oxford economists' research affected Harrod. But it is not the purpose of this paper to verify its influence on Harrod's thinking.

[13] Correctly speaking, the term price(s) actually appears there *only once* in an unessential context.

[14] This phrase is taken from the following: "But if they imagine that they could have gone through all that maze of deduction and reached this conclusion by the light of pure reason and without the aid of that far greater guiding light, the brute fact that prices do actually rise in booms and fall in slumps, they are most plainly deluding themselves." (the *TC*, p. 41) Then, was Harrod (1939) deluding himself?

by its output. Output in any period consists of the goods consumed in that period plus net investment; if saving is defined as the sum of the incomes of every one ... less what they choose to spend on consumable goods, net investment must be equal to saving. ... The principle that the amount of saving undertaken is accommodated to the amount of net investment through changes in the level of income is called the doctrine of the multiplier. ... The level of activity in the country is determined by the volume of current net investment ... ." (pp. 61-62, 74, 227) It was so clear that no one could not misunderstand it. The point is that causality runs from investment to saving in the multiplier theory. But Harrod (1939) reversed the causality and attributed the reversed causality to the *GT*. This needs some explanation.

Letting x_0 and x_1 stand respectively for output in period 0 and that in period 1 and further s for the (average) propensity to save, Harrod (1939) stated the implication of saving as follows:

$$\begin{aligned} sx_0 &= C_p(x_1 - x_0) \\ \text{[Therefore,]}\ \frac{s}{C_p} &= \frac{x_1 - x_0}{x_0} = G \end{aligned}$$

> G is the rate of increase in total output which actually occurs [in period 1]; C_p is the increment in the stock of capital [in period 0] divided by the increment in total output which actually occurs [in period 1]. (p. 18)

For convenience, let ΔK_1 be defined as the difference between K_1 and K_0, where K_1 and K_0 are respectively the capital stock at the beginning of period 1 (or at the end of period 0 in the same meaning) and that at the beginning of period 0 (or at the end of period -1). Then, from the definition of C_p it follows that $C_p = \Delta K_1/(x_1 - x_0)$. Substituting it into the first equation in the above citation gives

$$sx_0 = \Delta K_1. \tag{6}$$

This is what Harrod (1939) really implied, i.e., the reversed causal relation between saving and investment.

Given the relation, just as Harrod (1939, p. 18) said, the second equation, i.e., $G = s/C_p$, "is a truism, depending on the proposition that actual saving in a period (excess of the income in that period over consumption) is equal to the addition to the capital stock." As is apparent now, (6) is the capital accumulation equation which is familiar in growth theory. It simply means that part of output which was not consumed is added to the existing capital stock. Therefore, it can be said again that Harrod's (1939) model was a one-good macro model.[15] The increase in the capital stock is of course *ex post* investment. Thus (6) implies that saving determines net investment. It follows that it never represents the multiplier theory in which net investment determines saving. Strangely enough, Harrod (1939, p. 17) attributed the idea of (6) to Keynes as follows: "The truism stated above [i.e., $G = s/C_p$] ... gives expression to Mr. Keynes' proposition that saving is necessarily equal to investment—that is, to *ex post* investment. Saving is not necessarily equal to *ex ante* investment ... ."[16]

[15]Ramsey (1928) had already made use of a capital accumulation equation. He also assumed the homogeneity of goods as well as that of labor.

[16]There is another important implication in (6), that is, it means that people never save in the form of money (or cash) and that they always hold their asset only in the form of capital stock. Although it is sure

More important, output can not be determined by (6). As is well known, it was the warranted rate of growth that Harrod (1939) devised for the determination of output. If I am permitted to formulate Harrod's (1939) model in my own way, it becomes as follows:

$$\frac{x_1 - x_0}{x_0} - \frac{x_0 - x_{-1}}{x_{-1}} = \alpha \left(\frac{x_0 - x_{-1}}{x_{-1}} - \frac{s}{C} \right), \ \alpha > 0, \tag{7}$$

where x_{-1}, α, C are respectively output in period -1, the adjustment coefficient, and "the amount of capital required by technological and other conditions." (Harrod (1939, p. 18)) s/C is the famous warranted rate of growth. (7) determines the rate of growth, $(x_1 - x_0)/x_0$, or the output level in period 1, x_1, given the values of x_0 and x_{-1}. According to it, if the actual rate of growth in period 0, $(x_1 - x_0)/x_0$, is greater (less) than the warranted rate of growth, producers accelerates (decelerates) the rate of growth in period 1. Only if the actual rate of growth happens to coincide with the warranted rate, they will be induced to maintain the same rate of growth. (7) may be too simple to describe what Harrod (1939) argued, but I think that it captures the gist of the determination of output. In any case, Harrod (1939) thought of the way to determine output or income which was quite different from that of Keynes as well as his own *TC*.

The above argument reveals that there are crucial differences between the *TC* (or Harrod (1936)) and Harrod (1939). I do not know someone has pointed them out clearly, but they are worthy to be summed up:

1. Harrod (1936) attached great importance to the behavior of prices, whereas Harrod (1939) ignored it completely.

2. Harrod (1936) is based on a two-sector model, whereas Harrod (1939) on a one-sector model.

3. Harrod (1936) relied on the multiplier theory in which investment determines saving, whereas Harrod (1939) used the capital accumulation equation in which saving determines investment.

In 1936, Harrod was close to Keynes. In 1939, there was another Harrod who was away from Keynes. This unbelievable change may be called the "Harrod discontinuity."

4 Changing Macroeconomics

I felt very interested in the "Harrod discontinuity" not only because it occurred inside Keynesian economics but also because it is related to the present state of macroeconomics. This section shows that macroeconomics has been developing directly or indirectly under the influence of the Harrod discontinuity.

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that Harrod disbelieved the quantity theory of money and believed the liquidity preference theory, the *TC* had already suggested what (6) implied as follows: "The capital goods cannot be constructed, unless the wherewithal is surrendered by income receivers for this purpose. ... Nor can income receivers set aside funds for future use, unless new capital goods are concurrently created to the ownership of which they can acquire the titles. ... The *net* saving of income receivers must be equal in value to the new capital goods concurrently being created." (p. 161) See also the *TC*, p. 74.

4.1 Keynesian business cycle theory

The *TC* was so influential that there would have been no Keynesian economists who did not read it at that time. It certainly acted as a catalyst to make business cycle models, also influential, such as Samuelson (1939), Kaldor (1940), and Goodwin (1951), to say nothing of Harrod (1939). But they did not inherit "Three Features" of the *GT*. They underwent the Harrod discontinuity considerably. Generally speaking, Keynesian business cycle models were based on a one-good economy without paying particular attention to the distinction between consumption goods and investment goods despite the fact that they always consisted of consumption function and investment function. The reason why they didn't have to distinguish between consumption goods and investment goods was that they focused on the analysis of *income* which was often denoted by Y as in the *GT*.

By the way, was this Y nominal income or real income? Evidently the *GT* used it as the former, because it said, "Y is the aggregate income. Thus if it is practicable to measure the quantity, O, and the price, P, of current output, we have $Y = OP \ldots$." (p. 209) This is the only mathematical specification in the *GT* of the relation between (nominal gross) income Y and OP that must be the aggregate supply price. It is reasonable to identify $p_1O_1+p_2O_2$ in (5) with this OP.[17] And in the *GT* the proceeds $D_1 + D_2$ in (5) are supposed to be tautologically the same as the aggregate income Y.[18] Thus, $Y = OP$ is equivalent to (5) which in turn can be rewritten as

$$p_1O_1 + p_2O_2 = Y = D_1 + D_2.^{19} \tag{8}$$

It should be remembered, however, that it is simultaneous equations (3) and (4) that matter in order to determine output levels. (8) is not an independent equation because it is no more than the sum of (3) and (4).

Return to Keynesian business cycle models. Focusing on income, they used exclusively the latter half of (8), i.e., the tautological relation $Y = D_1 + D_2$. On the other hand, the aggregate supply price $p_1O_1 + p_2O_2$ on the left-hand side was left out of consideration. By the same token Samuelson (1948, pp. 134-135) explained the heart of Keynesian income analysis as follows:

> By definition, *national income* (at market prices), Y, can initially be set equal to the sum of consumption expenditure, C, and *net investment*, I :
>
> $$Y = C + I$$
>
> If Keynes had stopped with this identity, we should be left with an indeterminate system. In his simplest model of income determination, he added the following two hypotheses: (a) consumption is a function of income, and (b) investment may provisionally be taken, at any one time, as a constant. Mathematically,

[17] In fact, it *is* correct to write $p_1O_1 + p_2O_2 = Y$, because "we cannot aggregate the O_r's, since ΣO_r is not a numerical quantity." (the *GT*, p. 45) But the *GT* sometimes broke the rule as if heterogeneous products could be aggregated and written as O. Incidentally, Keynes (1973b, p. 422-423) called OP effective demand, P being the expected selling price of output O.

[18] See the *GT*, pp. 24, 89-90.

[19] Since (8) is the same as (5), variables in it represent expected values. Then (8) provides a correct expression of effective demand $D(= D_1 + D_2)$ because the *GT* said that "effective demand corresponds to the income the expectation of which has set production moving, not to the actually realised income, and to gross, not net, income." (p. 299) It can be said in this connection that it is not correct to define effective demand as national income rather than GNP or GDP.

these relations may be written

$$C = C(Y) \text{ and } I = \bar{I}$$

When we substitute these into our first identity, we come up with the simplest Keynesian income system:

$$Y = C(Y) + \bar{I} \tag{1}$$

> This is a determinate system, being one equation to determine one unknown variable. ... Equation (1) is crucially important for the history of economic thought. It is the nucleus of the Keynesian reasoning.

In this way the formula $Y = D_1 + D_2$ or $Y = C + I$ in the textbook fashion became the symbol of Keynesian economics in general.[20] Probably that is why Keynesian business cycle models did not need the distinction between consumption goods and investment goods and further they were specified as a one-good model.[21] This neglect of the distinction between consumption goods and investment goods was contrary to the GT (and the TC).

Moreover, in the process of the "homogenization of goods," there happened another change, that is, the "rigidification of price." In his Nobel lecture, Friedman (1976, p. 468) testified to it as follows: "One consequence of the Keynesian revolution of the 1930s was the acceptance of a rigid absolute wage level, and a nearly rigid absolute price level, as a starting point for analyzing short-term economic change." This neglect of the flexibility of prices was contrary to the GT (and the TC), too. After all, it became unimportant to ask if Y represented nominal income or real one.[22] The "homogenization of goods" and the "rigidification of price" became the postulates of Keynesian economics.

And finally, profit-maximizing firms disappeared. This can easily be seen from the formula $Y = C + I$ because Y is determined without the profit-maximizing behavior of firms. In his Nobel lecture, after emphasizing the importance of maximization principles in economics such as the profit maximization of a firm and the utility maximization of a consumer, Samuelson (1970) spoke suddenly in a different tone: "I must not be too imperialistic in making claims for the applicability of maximum principles in theoretical economics. There are plenty of areas in which they simply do not apply. Take for example my early paper dealing with the interaction of the accelerator and the multiplier [i.e., Samuelson (1939)]." (pp. 12-13) He declared, as it were, that there were two Samuelsons, say, microeconomic and macroeconomic.[23] Thus, all of "Three Features" vanished from Keynesian economics. Macroeconomics the GT founded and the TC attempted to improve changed in quality with the Harrod discontinuity.[24]

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[20]Hansen (1949, p. 72) also wrote the income determination equation $Y = I + C(Y)$, but he assumed that these variables were measured in real terms unlike Samuelson above.

[21]In order to express homogeneous goods, Modigliani (1963) and Blanchard (1997) used respectively the terms "*MM*" (read mum), and "shmoo." See Leijonhufvud (1968, p. 132).

[22]See also Leontief (1947, pp. 239-240), Kaldor (1960, p. 3), and Samuelson and Nordhaus (1998, p. 447). Nowadays most macroeconomists, I think, tend to use Y as real income unlike in the GT.

[23]And two Samuelsons appeared together on the stage of a neoclassical synthesis set by Samuelson (1955).

[24]It should be emphasized that I do not argue that these fundamental changes in macroeconomics were responsible solely to Harrod himself. The term is used just because such changes can apparently be seen between Harrod (1936) and Harrod (1939).

4.2 the *IS-LM* model

What about Hicks's (1937) original *IS-LM* model? Did it have "Three Features" that I regard as fundamental in the *GT*? The answer is yes. At the beginning of his paper, Hicks (1937) derived the relationship between the production functions of the two sectors and income as follows:[25]

> Let us begin by assuming that w, the rate of money wages per head, can be taken as given.
>
> Let x, y, be the outputs of investment goods and consumption goods respectively, and N_x, N_y, be the numbers of men employed in producing them. Since the amount of physical equipment specialized to each industry is given, $x = f_x(N_x)$, and $y = f_y(N_y)$, where f_x, f_y, are *given* [production] functions. ...
>
> It is desired to determine N_x and N_y.
>
> First, the price-level of investment goods = their marginal cost = $w(dN_x/dx)$. And the price-level of consumption goods = their marginal cost = $w(dN_y/dy)$.
>
> Income earned in investment trades (value of investment, or simply Investment) = $wx(dN_x/dx)$. Call this I_x.
>
> Income earned in consumption trades = $wy(dN_y/dy)$. [Call this I_y.]
>
> Total income = $[I_x + I_y =]$ $wx(dN_x/dx) + wy(dN_y/dy)$. Call this I.
>
> I_x is therefore a given function of N_x, I [is a given function] of N_x and N_y. Once I and I_x are determined, N_x and N_y can be determined. (p. 148)

How clear it is! It is almost microeconomic. Summarizing the above formulation gives

$$I_x = \frac{wf_x(N_x)}{f'_x(N_x)}, \tag{9}$$

$$I_y = \frac{wf_y(N_y)}{f'_y(N_y)}, \tag{10}$$

$$I = I_x + I_y. \tag{11}$$

$w/f'_x(N_x)$ and $f_x(N_x)$ in (9) correspond respectively to p_1 and O_1 in (3). Similarly, $w/f'_y(N_y)$ and $f_y(N_y)$ in (10) correspond to p_2 and O_2 in (4).[26] (9)-(11) can be unified into

$$\frac{w}{f'_x(N_x)} f_x(N_x) + \frac{w}{f'_y(N_y)} f_y(N_y) = I. \tag{12}$$

(12) is equivalent to the first half of (8), that is, $p_1O_1 + p_2O_2 = Y$ because $I = Y$. It is apparent that in Hicks's formulation above flexible prices, two sectors, and profit maximization are all assumed.[27]

Hicks, moreover, made an attempt to create dynamic theory. In his famous book, *Value and Capital,* Hicks (1939, pp. 3-4) stated as follows: "Mr. Keynes's *General Theory of Employment, Interest, and Money* (1936) appeared at a time when my own work was well under way, but was still incomplete in several respects. Since we were concerned with such similar

[25]The familiar *IS-LM* model will be discussed below in Section 6. The model dealt with in this subsection is concerned exclusively with the second page of Hicks (1937).

[26]$1/f'_x(N_x)$ and $1/f'_y(N_y)$ correspond respectively to p_{w1} in (1) and p_{w2} in (2).

[27]Meade (1936-37), known as one of the members of the Cambridge Circus, presented a surprisingly similar model.

fields, it was inevitable that I should be influenced by Mr. Keynes's work to a very great extent. ... When I began to work on Capital, I had the hope that I should produce an entirely new Dynamic Theory—the theory which many writers had demanded, but which none, at that time, had produced. These hopes have been dashed, for Mr. Keynes has got in first." Although it was the study of the general equilibrium theory, it was strongly affected by the *GT*. Undoubtedly Hicks was going along a line similar to Harrod (1936), though a little belatedly.[28]

But even Hicks incurred the Harrod discontinuity. He was becoming doubtful of what he believed in Hicks (1939), that is, an approach based on the assumption of flexible wages and prices (called the flexprice method by him), and at last he gave it up for lack of relevancy to modern manufacturing industry. It was the fixprice method that Hicks (1965, pp. 74, 77-78) advanced instead: "In Keynesian terms, the Temporary Equilibrium Theory [or the flexprice method] is a Full-Employment theory. ... On the Temporary Equilibrium method, the system is in equilibrium in every single period; and it is by this equilibrium that prices are determined. If we abandon the demand-supply equation, how are prices to be determined? The answer ... is that the new method does not have any way of determining prices. There must be some way by which they are determined, but it is exogenous. The determination of prices is taken right outside the model ... If prices are fixed exogenously, one will naturally begin by assuming them to be constant. The model becomes a Fixprice model." The original *IS-LM* model also degenerated into the textbook *IS-LM* model in which the price level was assumed to be fixed,[29] as if it corresponded with the conversion of its inventor.[30] On the other hand, Patinkin (1956, 1965) and Clower (1965) put forward the dual decision hypothesis that led eventually to energetic researches on macroeconomic disequilibrium theory with rigid prices such as Barro and Grossman (1976), and Negishi (1979).[31]

4.3 Monetarism and new Keynesian economics

However, inflation (i.e., flexible price!) in the 1970s and "monetarist" theory (based on adaptive or rational expectations) threw cold water on bustling Keynesian economics.[32] As is well known, the story went back to the discovery of the negative relation between the rate of change in nominal wage (i.e., flexible nominal wage!) and the rate of unemployment by Phillips (1958). Soon the relation was interpreted as the trade-off between the inflation rate and the unemployment rate, and became widely known as the Phillips curve. Fortunately the Phillips curve was hailed by both Keynesian economists and classical economists. Keynesian economists used it to determine price and/or wage endogenously[33] and to evaluate policy effects. Classical economists used it to attack Keynesian economics because they had already the theory of wage and price in which nominal wage rate was so determined as to equalize the supply of and the demand for labor and price was determined by the quantity of money (the

[28]But why didn't he write that Harrod had got in first? It was Harrod (1936) that pointed out the importance of dynamic theory for the first time, wasn't it?

[29]But the price of bonds continued to be flexible.

[30]The power of the assumption of price rigidity was so strong that Hicks (1980-81) himself threw doubt on his original *IS-LM* model.

[31]For the survey of disequilibrium theory, see Drazen (1980).

[32]See, e.g., Negishi (1979, p. 257).

[33]For example, Phillips (1961) himself made use of the Phillips curve to determine the proportional rate of change of the price level in a Keynesian growth model, though he did not mention the term. See also Samuelson and Solow (1960).

quantity theory of money). History told us that as for the ingenuity of argument classical economists, or in a word Friedman, were a cut above Keynesian economists.

Distinguishing between the actual inflation rate and the inflation rate expected by workers, Friedman (1968) eloquently pointed out the existence of the short-run Phillips curve and the long-run Phillips curve. As long as the two inflation rates differed, there was the negatively sloped short-run Phillips curve, which implied the effectiveness of fiscal and monetary policy. Once they coincided, however, the labor market was in equilibrium due to flexible nominal wage. Hence the vertical long-run Phillips curve and the ineffectiveness of government policy. The rate of unemployment on the long-run Phillips curve was called the natural rate of unemployment by Friedman (1968), while the equilibrium unemployment rate by Phelps (1968). The point was that the ideas of Friedman and Phelps were based on general equilibrium theory with incomplete information. They also assumed that expectations were adjusted gradually or adaptively. Soon the assumption of adaptive expectations was replaced by that of rational expectations according to which, to make a long story short, an economy could be supposed always to be in equilibrium calculated from an economic model under consideration.[34] In due course the two Phillips curves and the interpretation of it through the adaptive or rational expectations hypothesis became one of common property of macroeconomists despite their likes and dislikes.

Under the vastly spread influence of Friedman, many macroeconomists were accustomed to thinking of a macroeconomy in terms of full employment or the natural rate of employment rather than in terms of underemployment which had been the center of macroeconomics. Every intermediate textbook of macroeconomics mentioned Friedman or his natural rate hypothesis without fail. Nevertheless, the assumption of price rigidity continued to stay in macroeconomics. And it resuscitated Keynesian economics as new Keynesian economics by the young generation such as Mankiw and Romer (1991). New Keynesian economics differed from traditional Keynesian economics in that it was able to give the microeconomic foundation to wage and price rigidities. As far as it insisted on wage and price rigidities, however, it was suffering from the Harrod discontinuity. In addition it was affected by monetarism, as Mankiw and Romer (1991, p.3) admitted as follows: "Indeed, since monetarists believe that fluctuations in the money supply have real effects but often leave price rigidities unexplained, much of new Keynesian economics could also be called new monetarist economics." New Keynesian economics was no attempt to resurrect the *GT*.

4.4 Growth Theory

The history of macroeconomics surveyed so far in this section was rather involved. There was another, but intelligible, development in macroeconomics, i.e., the theory of economic growth. Who thought of it for the first time? The conventional answer is that Harrod (1939) and Domar (1946) did. Certainly both resembled each other since both aimed at an extension of static theory of the *GT* to dynamic theory keeping the multiplier theory, though formally, and reached the same conclusion that the equilibrium growth path was unstable. However, Harrod's model was a *trade cycle* model in which full employment were not assumed whereas Domar's model was really a *growth* model in which full employment as well as full utilization

[34]See, e.g., Lucas and Sargent (1979, 1981). But the title of Lucas and Sargent (1979), "After Keynesian Macroeconomics," is rather misleading. Lucas and Sargent (1979) criticized not Keynesian economics itself but Keynesian macroeconom*etrics*. They well recognized the contributions of Keynesian economists who used optimizing microeconomic theory as a guide. See footnote 7 therein.

of productive capacity was assumed definitely.[35] Therefore, or at least in my opinion, it was Domar (1946) that presented the first model of economic growth in a modern fashion.[36]

This field made rapid and theoretically healthy progress because what was wrong with the Harrod-Domar model was made clear soon. First, Hamberg (1952) criticized the Harrod-Domar model for its neglect of labor as a factor of production. The reason was that the growth rate at which production capacity is fully utilized does not necessarily coincide with the growth rate at which full employment is realized. Next, it was Pilvin (1953) that introduced a *production function* of the first degree into the Harrod-Domar model. This was the moment of breakthrough in the growth theory. I cannot think too highly of Pilvin (1953) as well as Hamberg (1952).[37] However, Pilvin (1953) did not find the steady state of economic growth. Then, Tobin (1955), who constructed a monetary growth model, mentioned the existence of (stationary) equilibrium in his model. He almost reached what is now called a neoclassical growth model. But he thought that the equilibrium was hard to attain because of downward inflexibility of prices and, in particular, money wage rates. So he did not make good use of it for analysis. And at last the field of economic growth culminated in Solow's (1956) and Swan's (1956) models. Particularly the former became and still is the universal base of the growth theory partly due to its generality. Recent endogenous growth models such as Lucas (1988) and Romer (1990) were no exceptions.[38]

Now, think of Domar (1946) again. What I want to notice here is that Domar's (1946) model was fully subjected to the Harrod discontinuity, too. That is, it assumed that there was a constant general price level, it adopted a one-good model focusing on income, and it made use of the capital accumulation equation.[39] Thus, the Domar model did not follow the *GT* at all in spite of his respect for Keynes. Then, what made Domar construct such a model? Domar (1947) stated the incentive as follows:

> Keynes's approach is very curious: as a matter of fact, he has two: the familiar short-run analysis, and another one which may be called a long-run one. ... The important point for our purpose is the assumption that the amount of equipment (*i.e.*, capital) in existence is given.
>
> Now, the heart of Keynesian economics is the argument that employment depends on income, which in turn is determined by the current volume of investment (and the propensity to save). But investment (in the net sense) is nothing else but the rate of change of capital. Is it legitimate then first to assume the quantity of capital as given, and then base the argument on its rate of change? (p. 52)

This readily reminds one of Harrod's (1936) criticism of Keynes cited earlier. Apparently both Harrod and Domar looked in the same, and that right, direction. But Domar jumped from the Keynesian world to another world, i.e., the classical world at a stretch. Subsequent growth models were developing into stardom, but in a world which Keynes denied.

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[35] Harrod's warranted rate of growth was intended for the analysis of the trade cycle. The idea was already found in the *TC*, p. 150.

[36] Anyway, they were often put together and were called the Harrod-Domar model.

[37] But Hamberg (1952) and Pilvin (1953) were treated coldly by Domar (1953) and Harrod (1953), respectively.

[38] See Solow (2005, p. 4). In passing, it is an irony that not a few Keynesian economists such as Domar, Tobin, and Solow made a large contribution to the neoclassical growth theory, isn't it?

[39] This generally applies to the growth models above, too.

5 Who succeeded Keynes?

What became of the *GT* after the Harrod discontinuity? Or, to put it in another way, who succeeded Keynes who wrote the *GT*? No one has done it as far as I know. But, though it may sound ironic, Friedman (1956, 1971), known as the most eminent opponent to Keynesian economics, had something in common with Keynes with respect to economic theory. In fact, they were connected through the quantity theory of money despite the fact that Friedman stressed but Keynes negated the validity of it. Friedman (1971) also attempted to go beyond both the (simple) quantity theory of money and Keynesian income-expenditure theory. His attempt was ambitious and stimulating. Unfortunately it was not accepted by macroeconomists.[40] But I think his attempt remains relevant to all macroeconomists and it deserves to be taken up here.

5.1 Quantity Theory of Money

As is well known, Friedman (1956) reformulated the quantity theory of money in the very simple form:

$$Y = v \cdot M, \tag{13}$$

where Y and M are nominal income (in a broad sense) and nominal money supply while v is income velocity which may be a function of several variables such as the nominal rate of return on bonds and the real rate of return on equities. (13) was obtained originally by rewriting a money demand function. It was assumed that M was an exogenous variable while Y was an endogenous variable with the result that a variation in the former caused that in the latter. Friedman's analysis depended crucially on (13).

It is usually believed that Friedman was a classical economist as opposed to a Keynesian economist. But, as I said above, Friedman and Keynes resembled each other, or correctly speaking, the former was affected by the latter to a large extent. I can show four things at once. First, Friedman (1956, p. 14) assumed that the money demand consisted of the transactions motive and the speculative (or asset) motive as Keynes did.[41] It is often said that, for example, money is demanded only due to the transactions motive in the quantity theory of money (or equivalently in classical economics). It does not apply to Friedman. Second, Friedman (1956, 1971) focused on a short-run economy, not a long-run one, as Keynes did. Friedman (1972, pp. 924, 947) stated it as follows: "I introduced the concept of "long-run equilibrium" solely as a preliminary step in sketching the theory of the short-run "adjustment process." This is a straw man if I ever saw one .... [F]or the most part I was concerned with the short run, not the long run ... ." It is often said too that classical economists work with a long-run economy. Again it does not apply to Friedman. Third, Friedman assumed that the supply of money, as said above, was an exogenous variable which was determined by the monetary authority as Keynes did, though this holds in standard macroeconomics.[42]

Fourth, and most importantly, Friedman (1956, 1971) tried to analyze the (short-run) relationship between the supply of money on one hand, and price and output on the other hand as Keynes did. In so doing he utilized (13), on the stability of which he put great

[40] For the details, see Gordon (1974).

[41] See Friedman (1959, p. 349) and Patinkin (1965, pp. 81-82). And it is interesting to point out that Friedman (1956, p.4) had something in common with Hicks (1935, p. 14) who had been approaching the liquidity preference theory before the *GT* as will be explained in Section 7.

[42] See Friedman (1959, p. 330). But see also Friedman (1972, p. 931).

confidence, to determine nominal income and further real income (output). But the problem racked his brain for a long time. In fact, Friedman (1956, p. 15) lamented as follows: "Even under the most favorable conditions, for example, that the demand for money is quite inelastic with respect to the variables in v, equation [(13)] gives at most a theory of money income: it then says that changes in money income mirror changes in the nominal quantity of money. But it tells nothing about how much of any change in Y is reflected in real output and how much in prices." Friedman thought that his analysis was different from that of Keynes. But I think he was wrong. In what follows I will make a survey of Friedman's own theory (the simple monetary theory of nominal income) and elaborate on the reason why he was wrong to think so.

5.2 Friedman's Monetary Theory of Nominal Income

Friedman (1971) set forth a "simple common model" that encompassed both the simple quantity theory and the simple income-expenditure theory. The model was given by six equations:[43]

$$\frac{C}{P} = f\left(\frac{Y}{P},\, i\right), \tag{14}$$

$$\frac{I}{P} = g(i), \tag{15}$$

$$\frac{Y}{P} = \frac{C}{P} + \frac{I}{P}, \tag{16}$$

$$M^D = P \cdot l\left(\frac{Y}{P},\, i\right), \tag{17}$$

$$M^S = M, \tag{18}$$

$$M^D = M^S, \tag{19}$$

where i, M^D, and M^S represent respectively the nominal rate of interest, nominal money demand, and nominal money supply. Other variables have already appeared above. Nominal national income Y can be written as the product of the price level P and real income y, i.e., $Y = Py$.

(14) is a consumption function (Keynes's "marginal propensity to consume") that expresses real consumption C/P as a function of real income $Y/P = y$ and the interest rate i. (15) is an investment function (Keynes's marginal efficiency of investment) expressing real investment I/P as a function of the interest rate i. (16) is the equilibrium condition of the goods market. (17) is the demand function for nominal money balances (Keynes's liquidity preference function). (18) is the supply function of nominal money. The nominal money supply is an exogenous variable taking a value M.[44] (19) is the equilibrium condition of the money market.

Friedman (1971, p. 31) said, "These six equations would be accepted alike by adherents of the quantity theory and of the income-expenditure theory." But the problem is that there are seven unknowns, C, I, Y, i, P, M^D, and M^S, to the six equations. "Some of these variables must be determined by relationships outside this system." According to him, the simple

[43] The nominal rate of interest r in Friedman (1971) is changed here to i.

[44] Friedman (1971) wrote the supply of money as a function of the nominal rate of interest, but he admitted it to be an exogenous variable as pointed out above.

quantity theory assumes that

$$\frac{Y}{P} = y = y_0, \tag{20}$$

that is, real income is determined outside the system. y_0 is the real income corresponding to full employment. Friedman considered y_0 in (20) to be determined by the Walrasian system of general equilibrium. Given (17), the interest rate is determined such that it satisfies (14)-(16) at the same time, or in other words, such that it makes saving and investment equal. Denote this value by i_0. Then, (17)-(19) lead to

$$M = P \cdot l(y_0,\ i_0). \tag{21}$$

(21) is the classical quantity equation, and it now determines P. (20) and (21) imply the classical dichotomy.

On the other hand, the simple income-expenditure theory assumes that

$$P = P_0,$$

that is, the price level is determined outside the system. In this case, the simple common model (14)-(19) becomes the *IS-LM* model. In fact, the *IS* curve can be obtained from (14)-(16) while the *LM* curve from (17)-(19). Y and i are determined at the intersection of the two curves, and therefore other unknowns become determinate. All this is just what is taught in standard textbooks of macroeconomics.

Friedman was satisfied with neither the simple quantity theory nor the simple income-expenditure theory because they are static, they assume a stable price level (hence real and nominal interest rates are the same), they give no explicit role to anticipations about economic magnitudes, and so on. After detailed considerations, he presented the alternative version of the quantity theory referred to as the simple monetary theory of nominal income in the following form:[45]

$$M^D = Y \cdot l(i), \tag{22}$$

$$M^S = M, \tag{18}$$

$$M^D = M^S, \tag{19}$$

$$i = k_0 + \left(\frac{1}{Y}\frac{dY}{dt}\right)^*. \tag{23}$$

It is obvious that formally the above model was obtained by rewriting partly the equilibrium conditions of the money market (17)-(19). The money demand function (17) was changed to (22) under the assumption that the elasticity of the demand for money with respect to real income was unity. (23) represents the determination of the nominal interest rate i. k_0 means the difference between the anticipated real interest rate and the anticipated rate of real growth. The difference is determined outside the system and is assumed to be constant. t being time, $[(1/Y)(dY/dt)]^*$ is the "permanent" or "anticipated" rate of growth of nominal income and it is a predetermined variable. It follows that there are four unknowns to the four equations.[46]

[45] Friedman (1971, pp. 42-43) actually said, "In summary, the key elements of the monetary theory of nominal income are ... borrowed mostly from Irving Fisher and John Maynard Keynes."

[46] Friedman also reformulated the equilibrium of the goods market, but the result disappointed him. So I am not going to touch on it. In fact Friedman (1971, p. 40) said, "I am inclined ... to regard the saving-investment sector as unfinished business, even on the highly abstract general level of this paper." Thus, the four equations above almost constitute his monetary theory of nominal income.

(22), (18), and (19) can be unified into

$$Y(t) = V(i) \cdot M(t), \tag{24}$$

where $V(i) = 1/l(i)$ and it stands for velocity of circulation. The supply of money is allowed to vary over time (and so is nominal income). (24) is much the same as the quantity equation (13). Friedman's simple monetary theory of nominal income are described by two equations (23) and (24). The nominal interest rate is determined by (23) while nominal income by (24). Of course, Friedman (1971) was proud of the superiority of his theory over the other two theories but he also admitted a crucial defect:

> This simple model for analyzing short-term economic fluctuations seems to me more satisfactory than either the simple quantity theory which takes real output as determined outside the system and regards economic fluctuations as a mirror image of changes in the quantity of money or the simple Keynesian income-expenditure theory which takes prices as determined outside the system and regards economic fluctuations as a mirror image of changes in autonomous expenditure. ...
>
> The chief defect that this model shares in common with the other two is that none of the three has anything to say about the factors that determine the proportions in which a change in nominal income will, in the short run, be divided between price change and output change .... The one advantage in this respect of the third approach is that it does not make any assertion about this division as both the others do. It is, as it were, orthogonal to that issue and can therefore be more easily linked to alternative theories about that division. (pp. 43, 45-46)

I can not understand why that his theory does not make any assertion about the division is an advantage, but it is certain that this problem remains unresolved.[47]

5.3 Keynes's Generalized Quantity Theory of Money

Having surveyed Friedman's own theory, I proceed to show his misunderstanding about Keynes, though such a misunderstanding is common to macroeconomists in general. In my opinion, Friedman deemed that Keynes thought of a change in prices and that in the nominal wage rate similarly, or that Keynes assumed that prices changed with the nominal wage rate in the same direction and to a similar degree. But it was Friedman that thought so, not Keynes. As far as I know, Friedman (1975, pp. 14-15) stated this most clearly as follows: "Fisher talked about price changes, Phillips about [nominal] wage changes, but I believe that for our purpose that is not an important distinction. Both Fisher and Phillips took it for granted that wages are a major component of total cost and that prices and wages would tend to move together. So both of them tended to go very readily from rates of wage change to rate of price change and I shall do so as well."

How about Keynes? As Friedman often pointed out, Keynes (1936, p. 276), if provisionally, suggested the rigidity of the nominal wage rate. Friedman adhered to it throughout.

[47] Friedman continued to pay attention to this problem. For example, Friedman (1987, p. 17) restated as follows: "A major unsettled issue is the short-run division of a change in nominal income between output and price. The division has varied widely over space and time and there exists no satisfactory theory that isolates the factors responsible for the variability ... ."

It followed from Friedman's assumption above that Keynes also assumed the price rigidity. Friedman (1971, pp. 18-19) interpreted the link between the price level and the nominal wage rigidity in the GT as follows: "[U]nder conditions when there was no full-employment equilibrium, there was also no equilibrium nominal price level; something had to be brought in from outside to fix the price level; it might as well be institutional wage rigidity. Put differently, flexible nominal wages under such circumstances had no economic function to perform; hence they might as well be made rigid."[48] But, on the contrary, Keynes assumed flexible prices that reacted to the difference between supply and demand, as was fully discussed in Section 2.[49] It was a complete misunderstanding to think that Keynes adopted the assumption of the rigidity of price.

It follows that Friedman was also under the influence of the Harrod discontinuity. Friedman interpreted Keynes in terms of the price rigidity as well as in terms of nominal income Y or a one-good economy. Nevertheless, the two great macroeconomists were working along a similar line on the basis of the quantity theory of money. As said above, Friedman (1971) tried in vain to make it clear how a change in nominal income which was caused by an increase in money supply will be divided between price change and output change in the short run. Keynes (1936) was interested in the same thing and, in my view, he did a much better job than Friedman. Denoting aggregate output, the price level, and the wage-unit (or the nominal wage rate) respectively by O,[50] p, and W, the job is summarized as follows:

> Let us write $MV = D$ where M is the quantity of money, V its income-velocity ... and D the effective demand. If, then, V is constant, prices will change in the same proportion as the quantity of money provided that $e_p \left(= \frac{Ddp}{pdD}\right)$ is unity. ...
>
> Next we can deal with the case where income-velocity is not constant, by introducing yet a further elasticity, namely the elasticity, of effective demand in response to changes in the quantity of money,
>
> $$e_d = \frac{MdD}{DdM}.$$
>
> This gives us
>
> $$\frac{Mdp}{pdM} = e_p \cdot e_d \text{ where } e_p = 1 - e_O(1 - e_w)$$
>
> $$\left[e_p = \frac{Ddp}{pdD},\ e_O = \frac{D_w dO}{OdD_w},\ e_w = \frac{DdW}{WdD}\right];$$
>
> so that
>
> $$\begin{aligned} e &= e_d - (1 - e_w)e_d \cdot e_O \\ &= e_d(1 - e_O + e_O \cdot e_w) \end{aligned}$$
>
> where e without suffix $\left(= \frac{Mdp}{pdM}\right)$ stands for the apex of this pyramid and measures the response of money-prices to changes in the quantity of money.

[48] But it is strange that Friedman (1971, fn 29) admitted that "Keynes distinguished between the price level of products and the wage rate and allowed for a change in the ratio of the one to the other as output changed, even before the point of full employment."

[49] As for a change in the nominal wage rate, the GT said, "The wage-unit [i.e., the nominal wage rate] will tend to rise, before full employment has been reached." (p. 296) See also the GT, pp. 301-302.

[50] See footnote 17 above.

> Since this last expression gives us the proportionate change in prices in response to a change in the quantity of money, it can be regarded as a generalized statement of the Quantity Theory of Money. (the *GT*, pp. 304-305)[51]

It should be noted that the "last expression" is no more than a truism and that it does not represent a causality. But it is very useful to see what the distinction between classical economics and Keynesian economics is. Classical economics or the simple quantity theory in Friedman's terms corresponds to the case in which $e_d = 1$, $e_O = 0$, $e_w = 1$, and $e = 1$. This is what Keynes laughed at. Keynesian economics or the simple income-expenditure theory in Friedman's terms corresponds to the case in which $0 \leq e_d \leq 1$, $e_O = 1$, $e_w = 0$, and $e = 0$. And the generalized quantity theory of money Keynes formulated above corresponds to the case in which $0 \leq e_d, e_O, e_w, e \leq 1$, but "$e$ is, as a rule, less than unity." (the *GT*, p. 306) The "last expression" is no theory, but Keynes also gave it some theoretical foundation as already explained in Section 2. Thus, the simple quantity theory and the simple income-expenditure theory are special cases of the generalized quantity theory of money above. This is, I think, what Friedman (1971) wanted.

Friedman (1972, p. 931) was wrong to say "Keynes, as I have explained, has no theory of the absolute level of prices." But Friedman (1972, p. 933) was probably right to say, "Keynes was a quantity theorist long before he was a Keynesian, and he continued to be one after he became a Keynesian. Many parts of the *General Theory* are a continuation of his earlier interests and beliefs." Keynes certainly preceded Friedman. Conversely Friedman succeeded Keynes.

6 A Redundant Equation in Hicks's *IS-LM* Model

In Subsection 4.2 it was seen that Hicks's (1937) original *IS-LM* model had "Three Features" of the *GT* as shown by (9)-(11). But such evaluation is quite incomplete because it was concerned exclusively with the *second page* of his paper. In this section I continue to discuss Hicks (1937) and learn some lessons from it.

After the formulation summarized by (9)-(11), Hicks presented a classical model and a model of the *GT*. The former is made up of the following three equations

$$M = kI,\ I_x = C(i),\ I_x = S(i,\ I). \tag{25}$$

The first equation is the Cambridge quantity equation with k given. The second equation means that the amount of investment (or demand for capital) I_x depends on the rate of interest i. In the third equation, investment equals saving, and saving S depends on the rate of interest and income. Total money income I is completely determined by the (given) quantity of money M. The rate of interest is determined such that investment and saving coincide and investment I_x is determined by such a rate of interest. Total employment $N_x + N_y$ is therefore determined given (9)-(11).

The latter, i.e., the *IS-LM* model, consists of the following three equations

$$M = L(i,\ I),\ I_x = C(i),\ I_x = S(I). \tag{26}$$

[51] There are some minor errors in the original, so in the citation they have been corrected. For the derivation of the equation $e_p = 1 - e_O(1 - e_w)$, see the *GT*, p. 285.

$L(i, I)$ in the first equation is the liquidity preference which depends on the rate of interest as well as income. The crucial difference from the classical model is that income and the rate of interest are determined together given the quantity of money. And at the same time total employment is determined. It should be emphasized that *the multiplier theory is denied in the IS-LM model* because investment does not determine income but investment and income are determined simultaneously.

It seems that (25) and (26) leave nothing to be desired as modeling of the classical and Keynes theories. Admittedly they are consistent as mathematical models because there are three unknowns, I, I_x, and i, to the three equations. And they are useful to consider the difference between the classical theory and the Keynes theory. That is why they are actually being used today. But when they are seen as economic models, they cannot be said to be consistent. The reason is that there is a redundant equation in both models. I am going to elaborate on this.

First take the *IS-LM* model (26). An important thing to remember is that (9)-(11) as well as (26) constitute the *IS-LM* model. Then, the third equation $I_x = S(I)$ implies

$$\frac{w}{f_y'(N_y)} f_y(N_y) = I_y = I - S(I), \tag{27}$$

because of (10) and (11). (27) represents the equilibrium of the consumption-goods market. Thus it must be the same as (4) except I_y, that is, $[w/f_y'(N_y)]f_y(N_y)$ and $I - S(I)$ in (27) correspond respectively to p_2O_2 (aggregate supply price) and D_2 (effective demand) in (4). Particularly $I - S(I)$ must be the "amount which the community is expected to spend on consumption" (the *GT*, p. 29), or the (expected) expenditure on consumption goods. This interpretation of (27) is such a matter of course that no one needs explanation. That's the point.

If so, the same equation $I_x = S(I)$ should represent the equilibrium of the investment-goods market because it can be written as

$$\frac{w}{f_x'(N_x)} f_x(N_x) = I_x = S(I), \tag{28}$$

due to (9). (28) implies that that part of income which does not go to the purchase of consumption goods goes entirely to the purchase of investment goods. That is, saving $S(I)$ corresponds precisely with the "amount which it [i.e., the community] is expected to devote to new investment." (the *GT*, p. 29) It follows that money paid as income is not held as wealth. In this sense *the liquidity preference theory is also denied in the IS-LM model.*[52] And also (28) is equivalent to (3). $[w/f_x'(N_x)]f_x(N_x)$ and $S(I)$ in (28) correspond respectively to p_1O_1 (aggregate supply price) and D_1 (effective demand) in (3).

In traditional macroeconomics the equality of saving and investment has been paid special attention to. But I think that it is reasonable to regard it just as the equilibrium of the investment-goods market as in (28). If so, saving can be called "investment demand." Indeed this use of the term is not conventional. The term "investment demand" has been used to mean the desire of *firms* to buy investment goods for future returns. But, whoever may buy

[52]Let θ be a positive constant less than one and suppose that $\theta S(I)$ out of saving $S(I)$ is held as money. Then the equilibrium of the investment-goods market becomes $I_x = \theta S(I)$ or $S(I) = I_x/\theta$. Substituting it into (11) yields $(1 - \theta)(I - I_y) = 0$. Therefore $I = I_y$, and $I_x = 0$. These simple calculations mean that the merest propensity to hold money leads to a standstill of economic activity. See also footnote 16.

investment goods produced, it is saving that actually buys them.[53] Anyway, (27) and (28) are symmetric in the sense that one equation can be derived from the other through the definition of income (11). The equilibrium of the investment-goods market and that of the consumption-goods market are attained and broken at once. Of course this is nothing more than Walras' law. Only one of the two markets is necessary to consider the equilibrium of the goods market as a whole. It was to the investment-goods market (or the investment-saving relation) that traditional macroeconomics including Hicks (1937) gave special status.

I have reached a crucial point in this section. In Hicks (1937) there were two I_x's. One is "Income earned in investment trades" (p. 148) in (9) and the other is the "amount of investment (looked at as demand for capital)" (p. 149) in $I_x = C(i)$. It is impossible to understand these two consistently in one model. If I am right above, I_x in $I_x = S(I)$ must represent the aggregate supply price of investment goods while $S(I)$ the effective demand for investment goods. Therefore the second equation $I_x = C(i)$ in (26) is redundant. It has no place to go and the rate of interest has no role to play in (28). The equation $I_x = C(i)$ may be considered to represent the profit-maximizing behavior of firms, but it is already taken into account in the first half of (28), i.e., (9). This is what the *GT* emphasized as follows: "On *every* particular occasion, let it be remembered, an entrepreneur is concerned with decisions as to the scale on which to work a given capital equipment; and when we say that the expectation of an increased demand, *i.e.* a raising of the aggregate demand function, will lead to an increase in aggregate output, we really mean that the firms, which own the capital equipment, will be induced to associate with it a greater aggregate employment of labour." (p. 40) Thus, the *IS-LM* model, which I think to be consistent, should be constituted by (9)-(11), the second equation $I_x = S(I)$, and the first equation $M = L(i, I)$ in (26):

$$\begin{aligned} I_x &= \frac{wf_x(N_x)}{f_x'(N_x)}, \\ I_y &= \frac{wf_y(N_y)}{f_y'(N_y)}, \\ I_x + I_y &= I, \\ I_x &= S(I), \\ M &= L(i, I). \end{aligned}$$

Let us call these the "modified *IS-LM* model" for the time being.

Next consider the classical model (25). But, since (25) and (26) depict essentially the same economy,[54] no more time needs to be spent. The same conclusion is reached by the same reasoning as above: The second equation $I_x = C(i)$ in (25) is redundant, too. Thus, the classical model should consist of (9)-(11), the second equation $I_x = S(i, I)$, and the first equation $M = kI$ in (25):

$$\begin{aligned} I_x &= \frac{wf_x(N_x)}{f_x'(N_x)}, \\ I_y &= \frac{wf_y(N_y)}{f_y'(N_y)}, \\ I_x + I_y &= I, \end{aligned}$$

[53] Sometimes $S(I)$ is called the supply of saving while "investment demand" in the text the demand for saving. See, e.g., Harrod (1939, p. 14).

[54] Hicks never assumed full employment in the classical model.

$$
\begin{aligned}
I_x &= S(i, I), \\
M &= kI.
\end{aligned}
$$

Let us call these the "modified classical model" for the present.[55]

As is obvious, the modified *IS-LM* model and the modified classical model resemble each other. The first three equations are just the same. Only difference lies in how to deal with the rate of interest i. But now in each model there are six unknown variables, I_x, N_x, I_y, N_y, I, and i, to five independent equations. There happened the problem that one unknown is redundant to complete the models.

7 The *IS* Part or the *LM* Part?

Fortunately, as to the modified *IS-LM* model, a way to close it has already been shown in the previous section. That is, you have only to eliminate the rate of interest i from it because the liquidity preference theory does not obtain in it.[56]

This section discusses the liquidity preference theory from another point of view by asking what is the raison d'être of the (original) *IS-LM* model? This question can be made easy to understand by dividing the model into the *IS* part (or the multiplier theory) and the *LM* part (or the liquidity preference theory). As is well known, there are some economists who deem that the raison d'être of the model lies in the latter part because money, as a veil, is usually supposed to have no effect on the output level in classical economics. Hicks (1937, p. 152) himself said, "... the multiplier equation, which performs such queer tricks ... . It is the liquidity preference doctrine which is vital." In fact he had already recognized the importance of money demand other than the transactions motive before the *GT* under the influence of Keynes's *Treatise on Money*. That is, Hicks (1935, p. 5) stated ahead of the *GT* as follows: "The critical question arises when we look for an explanation of the preference for holding money rather than capital goods. For capital goods will ordinarily yield a positive rate of return, which money does not. What has to be explained is the decision to hold assets in the form of barren money, rather than of interest- or profit-yielding securities." This statement reminds us of the *LM* part. Hicks's theory was very similar to Keynes's liquidity preference theory. It was the *LM* part that was important to Hicks.[57] [58]

On the other hand, there are some economists who do not regard the *LM* part as essential to Keynesian economics. For example, in his influential book, Klein (1947, pp. 42-43) wrote, "[W]e need not regard the liquidity-preference theory as an essential element of the modern Keynesian system. ... Keynes later remarked that, as it actually happened, he first conceived

[55]Because investment function $I_x = C(i)$ is a common core of both the classical theory and the Keynes theory, the deletion of it is destructive to the two theories. But it is also constructive because it opens the door to a common theoretical base. It is this that the final section tries to show.

[56]This does not mean to eliminate the rate of interest from a whole system of a macroeconomy. See the next section.

[57]See also Hicks (1967, pp 15-16, 27).

[58]As regards money demand theory as a whole, most macroeconomists seem to continue to rely heavily on the famous three theories, namely, the quantity theory of money, the transactions demand theory by Baumol (1952) and Tobin (1956), and the portfolio demand theory by Tobin (1958) as their theoretical basis. After reviewing such theories and the combination of them, for example, Goldfeld and Sichel (1990, p. 312) concluded as follows: "While we make no pretence at having provided a comprehensive theoretical overview, it nevertheless appears that the bulk of empirical work on money demand has been motivated by one or more of the simple theories we have sketched." In sum the theory of money demand had been completed in the 1950s.

of the savings-investment equation as the determinant of the level of output. This left him without a theory of interest; so he then developed the liquidity-preference theory of interest." According to Klein, it is the *IS* part (multiplier theory) which is vital.

I take sides with Klein and can add one more rationale. That is, money is supposed to be *cash* in the *GT*.[59] Needless to say, the liquidity preference theory claims that people hold part of wealth in the form of money. This money is cash. Thus, is it theoretically reasonable to assume that people hold cash as an asset in the presence of safe interest-bearing assets like bank deposits? The question was already answered by macroeconomists. Immediately after the publication of the *GT*, Viner (1937, pp. 153, 157-159), a classical economist, criticized the liquidity preference theory as follows:

> There is a widely-prevalent aversion to the waste of "dead" cash. ... [I]n what seems to me the most vulnerable part of his analysis, his explanation of the determination of the rate of interest, Keynes assigns to the desire for cash for hoarding purposes a grossly exaggerated importance. ... In Keynes' present attempt the fatal flaw is, to repeat, the exaggerated importance attributed to hoarding. ... The former [i.e., the savers who add each year to their estates] have investment habits, and abhor idle cash as nature abhors a vacuum.[60]

Also in his influential paper Modigliani (1944, pp. 79-80, 85), a Keynesian economist, said as follows:

> [U]nder historically realized conditions, the equilibrium rate of interest may be sufficiently high to make the demand for money to hold so negligible and so scarcely affected by observed changes in the interest rate that this demand can, safely, be neglected. ... Under these conditions, the assumption $M = L(Y)$ [where M, Y, and L represent respectively the quantity of money or cash, money income, and the money demand function] will give a satisfactory approximation to economic reality. ... [T]he property of money is that it is accepted (freely or by force of law) as a medium of exchange ... ."

Indeed people appear actually to hold cash for some reasons other than the transactions motive, but it is, I think, more advisable theoretically to assume that money, i.e., cash, is used only as a means of exchange. It is the *IS* part that is essential to the *GT* and the first four equations of the modified *IS-LM* model constitute that part.[61]

[59] See the *GT*, p. 171. Remember Keynes's source of idea was the Cambridge *cash-balance* quantity theory! Keynesians such as Hansen (1949), Tobin (1958) and Harrod (1969) also regarded money in the *GT* as cash. Hicks (1935, p. 4) thought of "any sort of money" including notes, bank deposits, and metallic coins as money.

[60] The *GT*, in the footnote on p. 167, also used a different broad definition of money which contained even treasury bills. Given that money always appears as cash in the text, it is very confusing. In fact, Viner (1963, p. 258) also criticized it as follows: "Money" was "defined or identified, in the *General Theory*, and later writings, as "cash," as "hoards," as "idle balances," or expressly left to be defined by the reader to his taste."

[61] All in all, current textbooks of macroeconomics deal with the liquidity preference theory favorably. However, an exception is that by Sachs and Larrain (1993, p. 240) which negated it as follows: "The speculative demand for money [discussed by Keynes in the *GT* and Tobin (1958)] may be important only when no safe, liquid asset other than money is available. In most advanced economies, however, the theory no longer applies because of the availability of safe, short-term assets that pay a positive interest rate, but pose no risk of capital losses." Although in favor of the *IS-LM* model itself, Blanchard and Fisher (1989) focus on money not as a store of value but as a medium of exchange, while Romer (1996, 2006) has no chapter on the demand for money. To my surprise, Tobin (1958) does not appear in references of the textbooks by Blanchard and Fisher, and Romer.

8 Conclusion: Toward a General Macro Model

It has been shown above that the *GT* is well represented by a two-sector model with flexible prices and profit-maximizing firms and also that the liquidity preference theory is not an essential part of the *GT*. What about the rate of interest in the modified classical model? As is easily seen from Solow's (1956) neoclassical growth model, the dependence of saving on the rate of interest is not essential to the classical theory. So you may eliminate the rate of interest from the modified classical model, too.

Taking these into consideration, both the modified *IS-LM* model and the modified classical model can be modified again to the same model:

$$\frac{wf_x(N_x)}{f_x'(N_x)} = I_x, \tag{29}$$

$$\frac{wf_y(N_y)}{f_y'(N_y)} = I_y, \tag{30}$$

$$I_x + I_y = I, \tag{31}$$

$$I_x = S(I), \tag{32}$$

$$MV = I, \tag{33}$$

where income velocity of money V is assumed to be given . Let us call these simply the "modified macro model."[62] The first three equations are just the same formally. But I take I_x and I_y as income *paid* in the investment-goods sector and that *paid* in the consumption-sector whereas Hicks (1937) called them "Income *earned* in investment trades" and "Income *earned* in consumption trades," (Both my italics.) respectively. So I in the third equation (31) is aggregate income *paid* in the economy as a whole.

The fourth equation (32) is the same as that of the modified *IS-LM* model. It represents the equilibrium of the investment-goods market. Firms in the investment-sector pay income in the amount of I_x and, at the same period, get back the same amount as saving of households.[63] The fifth equation (33) is similar to that of the modified classical model and (24) in Friedman (1971). But, correctly speaking, it reflects the assumption that money is used only as a medium of exchange. At least formally an increase in money supply M can affect both prices and outputs, as is expected by Keynes's genaralized quantity theory of money. In any case there are five unknowns, N_x, N_y, I_x, I_y, and I, to five equations. The modified macro model is determinate.

Nevertheless there remain two problems to be solved. One is the problem of the determination of the nominal rate of interest i. The other is that of the determination of the nominal wage rate w which has been assumed to be given so far. A general macro model that helps to unify macroeconomics is brought to completion when each of the two variables is placed in an appropriate position. The way I choose is to divide a period into three subperiods, viz., the subperiod of production, that of portfolio selection, and that of plan for production. Within such a framework, the modified macro model above works at the the subperiod of production. And the nomial rate of interest and the nominal wage rate are determined at the subperiod

[62] It resembles in mathematical structure a two-sector general equilibrium model as explained by Dinwiddy and Teal (1988). But they differ in economic implication.

[63] The *GT* called consumption expenditure, which is denoted there by D_1 and here by D_2, "the sum ... which the entrepreneurs can expect to *get back* out of the expenditure of consumers." (p. 30, My italics.)

of portfolio selection and that of plan for production respectively.[64] More concretely, the rate of interest is determined in asset markets as the rate of return on capital, while the nominal wage rate in the labor market, both on the basis of expected prices.[65] Since the description of such a general macro model is not the purpose of this paper, I do not go into details.[66]

Finally I want to point out that the modified macro model can clearly explain the relation between the nominal wage rate and the volume of employment the *GT* put the greatest emphasis on. In fact, the *GT* criticized the classical theory as follow:

> The traditional theory maintains, in short, *that the wage bargains between the entrepreneurs and the workers determine the real wage* ... . [But] [t]here may exist no expedient by which labour as a whole can reduce its *real* wage to a given figure by making revised *money* bargains with the entrepreneurs.
>
> ... For every value of N [i.e., the volume of employment] there is a corresponding marginal productivity of labour in the wage-goods industries; and it is this which determines the real wage. ... The propensity to consume and the rate of new investment determine between them the volume of employment, and the volume of employment is uniquely related to a given level of real wages—not the other way round.
>
> ... There is ... no ground for the belief that a flexible wage policy is capable of maintaining a state of continuous full employment ... .
>
> ... [T]he money-price of wage-goods will depend on the aggregate amount of employment. Therefore we cannot say what aggregate employment will be, until we know the money-price of wage-goods; and we cannot know the money-price of wage-goods until we know the aggregate amount of employment. (pp. 11, 13, 29-30, 267, 276)

In the above citation "wage-goods" and the "wage-goods industries" can be regarded as consumption goods and the consumption-goods sector. Then, since the real wage rate is the nominal wage rate divided by the price of consumption goods, it becomes $f_y'(N_y)$, the marginal produtivity of labor in the consumption-goods sector, in the modified macro model. Moreover, the volume of employment $N_x + N_y$ and the real wage rate $f_y'(N_y)$ are determined simultaneously in the modified macro model. Judging from these observations, it appears that the *GT* was right to argue against a cut in the nominal wage rate. But, as far as this modified macro model is concerned, a reduction of the nominal wage rate will lead to that of the real wage rate and thus that of unemployment. Thus, it should be noted that *the modified macro model is not the GT model itself.*[67]

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[64]Hicks (1937) formulated the classical model (25) and the *IS-LM* model (26) at a point in time. In other words, income and the rate of interest are determined at the same time even if the two do not interact with each other as in the classical case. That is why both I and i appeared in simultaneous equations as unknowns. In such models the subperiod of production and that of portfolio selection are not separated. Indeed it is conventional to assume that production and portfolio selection occur at the same time. But, given that the portfolio selection is the reallocation of the *existing* capital stock among households, it is natural to assume that the portfolio selection takes place *after* investment goods produced have been bought and owned by households. Hence the subperiod of portfolio selection comes *after* that of production.

[65]Remember that all variables in (8) take expected values. See footnote 19.

[66]For the general macro model along the lines of Keynes (1936), Harrod (1936), Friedman (1956, 1971), and Solow (1956), etc., see Sasakura (2006, 2007).

[67]The modified macro model constitutes the basic structure of a general macro model. But it cannot escape the above-mentioned criticism by Harrod and Domar because it belongs to a "static" or "short-run" model. It

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is easy to make it "dynamic" or "long-run" by introducing a usual capital accumulation equation. Recall that Harrod and Domar used the terms static and short-run to refer to the situation in which capital stock remains unchanged while the terms dynamic and long-run to refer to the situation where capital stock is accumulating.

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