Finding Another Linkage between the Short Run and the Long Run in a Macroeconomy Ⅱ

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Abstract

This paper completes the Keynes-Solow model proposed in Sasakura (2006) by adding the government sector and the foreign sector, and investigates a macroeconomy in the short run and in the long run within the general framework. The model used remains *only one*. Main results obtained are concerned with the effectiveness of government fiscal policy consisting of the purchase of both investment and consumption goods. In the short run fiscal policy is effective. In the long-run steady state an increase in government investment leads to that in production and income, whereas an increase in government consumption has the opposite effect. In the golden-rule state fiscal policy is ineffective in changing the level of real GDP due to the consumption maximization by the household sector.

1 Introduction

Microeconomists are happy in that they have the *general* model by Walras. They can concentrate on a *particular* subject without anxiety. They have a common home to return to when at their wits' end. No microeconomist doubts the theoretical foundation of the model. The opposite obtains now in macroeconomics. Macroeconomists have not invented a general model comparable to the Walrasian model yet. They are astray because they have no home to return to with ease. That is why each macroeconomist tends to focus on his/her own particular subject taking no notice of the connection with each other.[1] Thus there is a crucial difference between a microeconomist and a macroeconomist when they work with each one's specific theme.

Indeed there have been attempts to construct a *general* macro model. Noteworthy was, as far as I know, Friedman (1971) who presented a "simple common model" that encompasses both a simplified quantity theory and a simplified income-expenditure theory as special cases. On the basis of the model he proposed a monetary theory of nominal income in which "there is a theoretical link between the short-run model and the long-run model."(p. 45) Although

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[1]For example, Blanchard and Fisher (1989, pp. 26-27), a macroeconomics textbook with the then most "unified" contents, wrote, "the field is now too large and too fragmented. The Keynesian framework embodied in the "neoclassical synthesis," which dominated the field until the mid-1970s, is in theoretical crisis, searching for microfoundations; no new theory has emerged to dominate the field, ... macroeconomics is in crisis... ." Their view holds good even taking into consideration what has been done in macroeconomics since then.

I cannot agree with him on the assumption of price rigidity in the General Theory, it was logically right of him to say, "The rigid price assumption of Keynes is ... entirely a *deus ex machina* with no underpinning in economic theory. Moreover, given that the price level in the long run is determined by the quantity-theory model, there is no theoretical link between the short-run model and the long-run model, no way of connecting the one to the other."(p. 44) But, strangely enough, his theory determines only *nominal* income. It has nothing to say about the factors that determine the proportions in which a change in nominal income will, in the short run, be divided between price change and output change, though such a defect also applies to the other two. After all it was wishful thinking to hope that "almost all economists would accept the framework."(p. 61)[2]

The problem of immeasurable importance to macroeconomists remains unsolved. Nowadays macroeconomists, recognizant of such an old effort or not, seem to be content with the theoretical structure consisting of the *IS-LM* model, the *AD-AS* model, and the Solow model to explain respectively the short run, the medium run, and the long run. But the three-model structure is none other than a historical compromise. In Sasakura (2006), I tried to present an alternative macro model which can analyze basic aspects of a macroeconomy in the short run and in the long run at the same time. The model named the Keynes-Solow model (the *KS* model for short), however, does not contain the government sector and the foreign sector, both of which constitute indispensable parts of an actual macroeconomy. This paper completes the Keynes-Solow model by adding the two sectors, and investigates a macroeconomy in the short run and in the long run within the general framework. But introducing the government sector and the foreign sector in a macro analysis is a cumbersome task. In fact, even the greatest macroeconomists such as Keynes (1936), Solow (1956), and Friedman (1971) dealt with a closed economy without the government sector. In order to extend the *basic KS* model constructed in Sasakura (2006), I follow standard textbooks except for indirect tax. Indirect tax is ignored as such in macroeconomics despite the fact that it accounts for a considerable part of GDP. I treat indirect tax explicitly as well as direct tax, and highlight it as one of the symbols of the extended model. The model is called the *complete* Keynes-Solow model. It is enlarged but only one as the basic *KS* model. It will be shown below that the complete *KS* model is able to analyze basic aspects of a macroeconomy in a fairly general, and therefore realistic, situation.

This paper is organized as follows. The next section provides the outline of the complete *KS* model. The short-run equilibrium state and the long-run equilibrium state are also defined. Sections 3-5 are concerned with the short-run equilibrium state. Sections 3 explains how the investment-goods sector, consumption-goods sector, government sector, and foreign sector behave. Section 4 describes the simultaneous equilibrium of the investment-goods market, consumption-goods market, and bond market, while Section 5 discusses the traditional income-expenditure equation, the quantity theory of money, and the effectiveness of government fiscal policy. Sections 6-9 are concerned with the long-run equilibrium state. Section 6 defines again the long-run equilibrium state using notations of the model. Section 7 characterizes the long-run equilibrium state and, using the results, Section 8 finds the long-run steady state in which macro variables are growing at a constant rate. Section 9 begins by asking what is the golden-rule state in the complete case, and then analyzes the golden-rule

[2]It was his interpretation of Keynes's General Theory that brought his important attempt to a deadlock. For the details of even now stimulating disputes with his critics including Tobin and Patinkin, see (again) Gordon (1974). He could have, in my view, made his theoretical framework more convincing if he had used a production function explicitly, as this paper does.

state which is newly defined as the state in which current *national* consumption is maximized. In order to show the relevancy of the model, an example is presented in Section 10. Section 11 concludes this paper. Appendices discuss Tobin's q theory and the M-M theorem in the presence of indirect tax.

2 Outline of the Complete Keynes-Solow Model

In the complete Keynes-Solow model there are the government sector and the foreign sector in addition to the household sector, the production sector, the central bank, and commercial banks. The government sector imposes both indirect tax and direct tax to buy consumption goods and investment goods. In case of budget deficit government bonds are issued. The macroeconomy considered here exports domestic consumption and investment goods to the foreign sector, and imports foreign consumption and investment goods from there. In case of current account surplus this economy or the household sector buys foreign assets called "foreign bonds" with it. Although the complete model works along the line simlar to the basic model, the government sector and the foreign sector make it complicated. Thus, let us see how this economy proceeds, dividing a period into three subperiods as in the basic case.

At the first subperiod of each period, the (domestic) production sector makes investment goods and consumption goods using labor the household sector supplies[3] and capital stock the household sector, the government sector, and the foreign sector hold. The national economy acquires GNP which constitutes its purchasing power of the period. GNP is the sum of the amount of production of domestic consumption and investment goods, and that of the net receipt of factor income from the foreign sector. GNP net of capital consumption, i.e., NNP is divided between the household sector and the government sector respectively as private disposable income and government disposable income. Each disposable income can be used to purchase consumption and investment goods, domestic and foreign. Domestic consumption and investment goods are bought by the foreign sector, too. Under the assumption that money is not held as wealth, three domestic markets of investment goods, consumption goods, and newly issued government bonds are cleared every period due to correct production plan. At the end of the first subperiod the household sector holds as assets capital stock, government bonds, and foreign bonds, while the government sector has capital stock as well as debts.

The second subperiod is that of portfolio selection. The government sector announces the rates of indirect and direct taxes which are applied at the next period. In the complete KS model there are three kinds of assets, i.e., (domestic) capital stock, government bonds, and foreign bonds. There are four choices for those who hold capital stock. On the one hand, they can hold the capital stock as that of the investment-goods sector or that of the consumption-goods sector. On the other hand, they can hold the capital stock of each sector indirectly as depositors through commercial banks or directly as equity holders. When they hold capital stock as depositors, the nominal rate of return is a fixed rate of interest (or deposit rate). When they hold capital stock as equity holders, they have to expect the rates of return on equities which depend on both how much capital stock exists in each sector and how much the prices and nominal wage rate of the next period are expected to be, which is not known until the third subperiod.[4] It is assumed that the price of investment goods as existing capital stock tends to be so determined as to make the rates of return on equities and the deposit rate

[3]Such labor includes that of employees of the government sector.

[4]For a mathematical description, see (44), (45), and (47) in Section 6.

(or interest rate) equal. It is also assumed that bank deposits, government bonds, and foreign bonds become perfect substitutes as a result of arbitrage, in which the price of government bonds as stock and the nominal exchange rate are adjusted.

The third subperiod is that of plan for production and issuance of government bonds of the next period. The government sector determines the amount of government spending consisting of government investment and government consumption. It also plans to issue newly government bonds when budget deficit is expected to occur. There already exists capital stock in each production sector as a result of portofolio selection during the previous subperiod. Nominal wage rate paid at the next period is determined and the prices and the exchange rate of the next first subperiod are expected.[5] The production sector calculates the profit-maximizing levels of output using all information then available including that of the government sector and the foreign sector. Once the optimal output levels are determinate, the production sector asks the central bank to supply the appropriate amount of money as a medium of exchange.

The first subperiod of the next period comes, and the same processes are repeated over and over again. An economy is said to be in the *short-run equilibium state* if all expectations of the last third subperiod are realized. In this paper too, only the short-run equilibrium state is considered. Thus, the *short run* always means a period in which an economy is in the short-run equilibrium state. It should be noted that markets of domestic goods and government bonds are always cleared whereas labor market is not always. An economy in the short-run equilibrium state is also said to be in the *long-run equilibrium state* if labor market is cleared and the rates of return are equal for all assets. In this paper too, the *long run* always means periods in which an economy is in the long-run equilibrium state. It should be emphasized again that a period in which an economy is in the long-run equilibrium state is only a special case of the short run.

The analysis eventually focuses on the long-run steady state, in which the rate of economic growth is determined by the sum of the growth rate of labor supply and that of technology. In this situation the household sector is able to control the economy by changing the rate of consumption (or equivalently, the rate of saving). It is assumed in the complete KS model that the rate of consumption is so determined as to maximize the sum of current private consumption *and government consumption* each period. For it is the household sector that enjoys government consumption as well as private consumption each period. This view on consumption is quite new to the best of my knowledge. It is found from this extended version of the golden rule that government fiscal policy is ineffective in changing the level of real GDP.

3 Production Sector, Government Sector, and Foreign Sector

3.1 The Investment-Goods Sector

Suppose that an economy is at the third *sub*peiod of period $t-1$. As was explained in the previous section, this is the subperiod of plan for production and issuance of government bonds of period t. First consider the investment-goods sector planning production of period t. Capital stock of the investment-goods sector, K_{1t}, consists of K_{1t}^d and K_{1t}^h. The former is

[5] The nominal exchange rate is determined twice a period, i.e., as part of the terms of trade of the first subperiod and as a result of arbitrage of the second subperiod.

held by depositors through commercial banks, while the latter by equity holders. A subscript 1 represents the investment-goods sector.

The technology of the investment-goods sector at t is given by the Cobb-Douglas production function:

$$Q_{1t} = K_{1t}^{\alpha}(A_t N_{1t})^{1-\alpha},\ K_{1t} = K_{1t}^{d} + K_{1t}^{h},\ 0 < \alpha < 1, \tag{1}$$

$$A_t = (1+g)A_{t-1},\ g > -1, \tag{2}$$

where Q_{1t}, N_{1t}, and A_t are respectively output, labor used, and the effectiveness of labor of the investment-goods sector at t. The effectiveness of labor or "knowledge" is assumed to grow at an exogenous rate g as in (2).

The nominal interest rate, i_t, the price of investment goods as existing capital stock, $\tilde{p}_{1t-1}$, and the rate of indirect tax, μ, have already been determined during the second subperiod of period $t-1$. Thus, after the nominal wage rate, w_t, has been determined, the inventment-goods sector must make production plan under the following budget constraint:

$$\begin{aligned} &p_{1t}^{e}Q_{1t} + p_{1t}^{e}(1-\delta)K_{1t} \\ &= w_t N_{1t} + (1+i_t)\tilde{p}_{1t-1}K_{1t}^{d} + (1+h_{1t}^{e})\tilde{p}_{1t-1}K_{1t}^{h} + \mu p_{1t}^{e}Q_{1t}, \end{aligned} \tag{3}$$

where p_{1t}^{e}, h_{1t}^{e}, and δ are respectively the expected price of investment-goods produced at period t, the expected nominal rate of return on equities, and the capital depreciation rate. A superscript e means an expected value in what follows. $\tilde{p}_{1t-1}K_{1t}^{d}$ is the amount of bank deposits related to K_{1t}^{d}, while $\tilde{p}_{1t-1}K_{1t}^{h}$ is the nominal value of equities related to K_{1t}^{h}.

Rewriting (3) yields

$$p_{1t}^{e}Q_{1t} = w_t N_{1t} + i_t\tilde{p}_{1t-1}K_{1t}^{d} + h_{1t}^{e}\tilde{p}_{1t-1}K_{1t}^{h} + p_{1t}^{e}(\delta - \pi_t^{e})K_{1t} + \mu p_{1t}^{e}Q_{1t}, \tag{4}$$

where $\pi_t^{e} = 1 - (\tilde{p}_{1t-1}/p_{1t}^{e})$. For simplicity let us call π_t^{e} and $\delta - \pi_t^{e}$ respectively the expected inflation rate and the inflation-adjusted depreciation rate. Since the share of capital consumption in GDP is positive, it is assumed that

$$\delta - \pi_t^{e} > 0. \tag{5}$$

The purpose of the investment-goods sector is to maximize h_{1t}^{e} in (4) subject to the production technology (1). From (4), h_{1t}^{e} can be written as

$$h_{1t}^{e} = \frac{(1-\mu)p_{1t}^{e}Q_{1t} - w_t N_{1t} - i_t\tilde{p}_{1t-1}K_{1t}^{d} - p_{1t}^{e}(\delta - \pi_t^{e})K_{1t}}{\tilde{p}_{1t-1}K_{1t}^{h}}. \tag{6}$$

Since the right-hand side of (6) is a function of N_{1t} alone, the investment-goods sector has only to find the level of labor, $N_{1t}^{\mu e}$, which maximizes h_{1t}^{e}. Substituting (1) into (6) and differentiating (6) with respect to N_{1t} yield

$$\frac{dh_{1t}^{e}}{dN_{1t}} = \frac{(1-\mu)p_{1t}^{e}(1-\alpha)A_t^{1-\alpha}N_{1t}^{-\alpha}K_{1t}^{\alpha} - w_t}{\tilde{p}_{1t-1}K_{1t}^{h}}.$$

Then $N_{1t}^{\mu e}$ can easily be obtained by solving $dh_{1t}^{e}/dN_{1t} = 0$ and $d^2h_{1t}^{e}/dN_{1t}^{2} < 0$ as follows:

$$N_{1t}^{\mu e} = \left[(1-\alpha)A_t^{1-\alpha}\frac{(1-\mu)p_{1t}^{e}}{w_t}\right]^{\frac{1}{\alpha}} K_{1t}. \tag{7}$$

A superscript μ, of course, represents a value when indirect tax is imposed at the rate μ. Furthermore, it is used throughout this paper to indicate a value in the complete KS model as distinct from the basic KS model in which the government sector and the foreign sector do not exist. The output of investment-goods which also maximizes h_{1t}^e is calculated as follows:

$$\begin{aligned} Q_{1t}^{\mu e} &= K_{1t}^{\alpha}(A_t N_{1t}^{\mu e})^{1-\alpha} \\ &= \left[(1-\alpha)A_t \frac{(1-\mu)p_{1t}^e}{w_t}\right]^{\frac{1-\alpha}{\alpha}} K_{1t}. \end{aligned} \tag{8}$$

The maximization of h_{1t}^e is equivalent to the usual profit maximization. Let MPL_{1t} be the marginal product of labor at t. Then, since $MPL_{1t} \equiv \partial Q_{1t}/\partial N_{1t}$, the following profit-maximizing condition holds:

$$MPL_{1t} = (1-\alpha)A_t^{1-\alpha}N_{1t}^{-\alpha}K_{1t}^{\alpha} = \frac{w_t}{(1-\mu)p_{1t}^e}, \tag{9}$$

which is equivalent to (7). It is apparent that an increase in the rate of indirect tax depresses the production of investment goods, *ceteris paribus*. The marginal product of capital at t, MPK_{1t}, is

$$MPK_{1t} = \alpha K_{1t}^{\alpha-1}(A_t N_{1t})^{1-\alpha}. \tag{10}$$

When the investment-goods sector expects that investment goods will be sold at the price p_{1t}^e, it is ready to distribute the value added, $p_{1t}^e Q_{1t}^e$, among the factors of production according to (4). Thus nominal factor income in the domestic investment-goods sector, $Y_{1t}^{\mu e}$, is given by

$$\begin{aligned} Y_{1t}^{\mu e} &= w_t N_{1t}^{\mu e} + i_t \tilde{p}_{1t-1} K_{1t}^d + h_{1t}^e \tilde{p}_{1t-1} K_{1t}^h \\ &= (1-\mu)p_{1t}^e Q_{1t}^{\mu e} - p_{1t}^e(\delta - \pi_t^e)K_{1t}. \end{aligned} \tag{11}$$

It is found from (11) that $Y_{1t}^{\mu e}$ is a function of the expected price p_{1t}^e.

3.2 The Consumption-Goods Sector

Next consider the consumption-goods sector planning production of period t. The explanation of the consumption-goods sector proceeds along much the same line as in the investment-goods sector, a subscript 1 being replaced by subscript 2 which in turn represents the consumption-goods sector. Therefore, it suffices to show main features and results in order.

The production function of the consumption-goods sector:

$$Q_{2t} = K_{2t}^{\alpha}(A_t N_{2t})^{1-\alpha},\ K_{2t} = K_{2t}^d + K_{2t}^h,\ 0 < \alpha < 1. \tag{12}$$

The budget constraint on the consumption-goods sector:

$$\begin{aligned} &p_{2t}^e Q_{2t} + p_{1t}^e(1-\delta)K_{2t} \\ &= w_t N_{2t} + (1+i_t)\tilde{p}_{1t-1}K_{2t}^d + (1+h_{2t}^e)\tilde{p}_{1t-1}K_{2t}^h + \mu p_{2t}^e Q_{2t}, \end{aligned}$$

or

$$p_{2t}^e Q_{2t} = w_t N_{2t} + i_t \tilde{p}_{1t-1} K_{2t}^d + h_{2t}^e \tilde{p}_{1t-1} K_{2t}^h + p_{1t}^e(\delta - \pi_t^e)K_{2t} + \mu p_{2t}^e Q_{2t}. \tag{13}$$

The demand for labor in the consumption-goods sector:

$$N_{2t}^{\mu e} = \left[(1-\alpha)A_t^{1-\alpha}\frac{(1-\mu)p_{2t}^e}{w_t}\right]^{\frac{1}{\alpha}} K_{2t}. \tag{14}$$

The planned output of consumption goods for p_{2t}^e:

$$\begin{aligned} Q_{2t}^{\mu e} &= K_{2t}^{\alpha}(A_t N_{2t}^{\mu e})^{1-\alpha} \\ &= \left[(1-\alpha)A_t\frac{(1-\mu)p_{2t}^e}{w_t}\right]^{\frac{1-\alpha}{\alpha}} K_{2t}. \end{aligned} \tag{15}$$

The profit-maximizing condition:

$$MPL_{2t} = (1-\alpha)A_t^{1-\alpha}N_{2t}^{-\alpha}K_{2t}^{\alpha} = \frac{w_t}{(1-\mu)p_{2t}^e}. \tag{16}$$

The marginal product of capital:

$$MPK_{2t} = \alpha K_{2t}^{\alpha-1}(A_t N_{2t})^{1-\alpha}. \tag{17}$$

Nominal factor income distributed in the domestic consumption-goods sector:

$$\begin{aligned} Y_{2t}^{\mu e} &= w_t N_{2t}^{\mu e} + i_t\tilde{p}_{1t-1}K_{2t}^d + h_{2t}^e\tilde{p}_{1t-1}K_{2t}^h \\ &= (1-\mu)p_{2t}^e Q_{2t}^{\mu e} - p_{1t}^e(\delta - \pi_t^e)K_{2t}. \end{aligned} \tag{18}$$

The equilibrium of the domestic consumption-goods market as well as that of the domestic investment-goods market is investigated in Section 4.

3.3 The Government Sector

At the third subperiod of period $t-1$, the government sector decides on the nominal expenditures of investment goods, G_{1t}, and consumption goods, G_{2t}, of the next period. The government expenditures are met in the first place by its expected disposable income

$$IT_t^e + DT_t^e - b_t B_t.$$

Here IT_t^e and DT_t^e are respectively the expected indirect tax and the expected direct tax. B_t and b_t are respectively the total quantity of government bonds already issued and the interest (not the interest *rate*) per unit of bond already promised at the beginning of the second subperiod of period $t-1$. Thus $b_t B_t$ is the total amount of coupon payments at the next period. When budget deficit is expected, new issuance of government bonds is planned according to the following government budget constraint

$$p_{Bt}(B_{t+1}^e - B_t) = G_{1t} + G_{2t} - (IT_t^e + DT_t^e - b_t B_t), \tag{19}$$

where p_{Bt} is the price of newly issued bonds the quantity of which is $B_{t+1}^e - B_t$. The bond price p_{Bt} is set by the government sector. Thus, once the budget deficit represented by the right-hand side of (19) is calculated, the quantity of newly issued bonds is determined.

Since $b_t B_t$ is already fixed, the government sector must calculate IT_t^e and DT_t^e. It is easy. (4) and (13) give the expected indirect tax immediately as follows:

$$IT_t^e = \mu(p_{1t}^e Q_{1t}^{\mu e} + p_{2t}^e Q_{2t}^{\mu e}).$$

Denote the expected nominal national income at t by $Y_t^{\mu e}$. Then, by definition,

$$Y_t^{\mu e} = Y_{1t}^{\mu e} + Y_{2t}^{\mu e} + F_t^e, \tag{20}$$

where F_t^e is the expected net receipt of factor income from the foreign sector in domestic currency. Substituting (11) and (18) into (20) leads to

$$Y_t^{\mu e} = (1-\mu)(p_{1t}^e Q_{1t}^{\mu e} + p_{2t}^e Q_{2t}^{\mu e}) + F_t^e - p_{1t}^e(\delta - \pi_t^e)K_t, \tag{21}$$

where $K_t = K_{1t} + K_{2t}$. Since the rate τ of direct tax has been announced at the second subperiod, the expected direct tax can be calculated as follows:

$$DT_t^e = \tau[(1-\mu)(p_{1t}^e Q_{1t}^{\mu e} + p_{2t}^e Q_{2t}^{\mu e}) + F_t^e - p_{1t}^e(\delta - \pi_t^e)K_t].$$

Remember that it is assumed that government bonds and bank deposits are perfect substitutes as assets. Thus it is understood that the price of government bonds as an asset, $\tilde{p}_{Bt-1}$, has been determined through portfolio selection of the second subperiod of period $t-1$ as follows:

$$\tilde{p}_{Bt-1} = \frac{b_t}{(1-\tau)i_t}.$$

3.4 The Foreign Sector

The foreign sector plans to buy consumption goods and investment goods at the prices p_{1t}^e and p_{2t}^e, respectively. The national economy considered here also plans to buy consumption goods and investment goods produced by the foreign sector at the prices of $e_t^e p_{f1t}^e$ and $e_t^e p_{f2t}^e$. Here e_t^e is the expected nominal exchange rate, while p_{f1t}^e and p_{f2t}^e are respectively the expected prices of foreign investment goods and foreign consumption goods in foreign currency. A subscript f indicates the foreign sector in what follows. A further description of export and import is given in the subsequent sections.

The foreign sector issues foreign bonds the value of which is equal to the current account of the national economy. Since foreign bonds and bank deposits are assumed to be perfect substitutes, the following ought to have held at the end of the second subperiod of period $t-1$:

$$\tilde{e}_{t-1} = \frac{(1-\tau_f)i_{ft}}{(1-\tau)i_t}\, e_t^e,$$

where i_{ft} and τ_f are respectively the nominal interest rate and the rate of direct tax of the foreign sector, and $\tilde{e}_{t-1}$ is the nominal exchange rate that has been determined as a result of portfolio selection. $\tilde{e}_{t-1}$ is to e_t^e what $\tilde{p}_{1t-1}$ is to p_{1t}^e.[6]

4 Domestic Market Equilibrium

Consider how the consumption-goods market, the investment-goods market, and the market of newly issued government bonds reach each equilibrium. First examine the consumption-goods market. In the complete KS model the consumption function is extended to

$$C_t^{\mu e} = cY_{Dt}^{\mu e} + G_{2t} + EX_{2t}^e - IM_{2t}^e,\ 0 < c < 1, \tag{22}$$

[6]Like the price of investment goods, there are two nominal exchange rates during a period. One is applied to transactions at the first subperiod as e_t^e, while the other is used for portfolio selection at the second subperiod as $\tilde{e}_t^e$. No distinction between the two is made in international economics represented by, for example, the Mundell-Fleming model and the overshooting model of Dornbusch (1976).

where

$$Y_{Dt}^{\mu e} = (1-\tau)Y_t^{\mu e} + b_t B_t. \tag{23}$$

$C_t^{\mu e}$ is the expected nominal demand for domestic consumption goods from three sectors. $cY_{Dt}^{\mu e}$ is the expected private consumption demand with $Y_{Dt}^{\mu e}$ and c respectively as the expected private disposable income and the rate of consumption. Furthermore EX_{2t}^e and IM_{2t}^e are respectively the expected nominal export of domestic consumption goods and the expected nominal import of foreign consumption goods. The (private) saving function becomes

$$S_t^{\mu e} = (1-c)Y_{Dt}^{\mu e}, \tag{24}$$

where $S_t^{\mu e}$ is the amount the household sector plans to save out of private disposable income, and $1-c$ is the rate of saving.

The consumption-goods market attains equilibrium when $p_{2t}^e Q_{2t}^{\mu e} = C_t^{\mu e}$. Substituting (22) into this equilibrium condition gives

$$p_{2t}^e Q_{2t}^{\mu e} = cY_{Dt}^{\mu e} + G_{2t} + EX_{2t}^e - IM_{2t}^e. \tag{25}$$

In order to make the model more concrete, let us regard IM_{2t}^e as a linear function of private disposable income:

$$IM_{2t}^e = m_2 Y_{Dt}^{\mu e},\ 0 < m_2 < 1,$$

where m_2 is the marginal propensity to import foreign consumption goods.[7] Substituting (21) into (23) yields

$$Y_{Dt}^{\mu e} = (1-\tau)[(1-\mu)(p_{1t}^e Q_{1t}^{\mu e} + p_{2t}^e Q_{2t}^{\mu e}) + F_t^e - p_{1t}^e(\delta - \pi_t^e)K_t] + b_t B_t. \tag{26}$$

Furthermore, substituting (26) into (25) with the import function specified above leads to

$$p_{2t}^e Q_{2t}^{\mu e} = (c-m_2)\{(1-\tau)[(1-\mu)(p_{1t}^e Q_{1t}^{\mu e} + p_{2t}^e Q_{2t}^{\mu e}) + F_t^e - p_{1t}^e(\delta - \pi_t^e)K_t] + b_t B_t\} + G_{2t} + EX_{2t}^e.$$

Thus the equilibrium amount of production of consumption goods can be obtained as

$$p_{2t}^e Q_{2t}^{\mu e} = \frac{(c-m_2)\{(1-\tau)[(1-\mu)p_{1t}^e Q_{1t}^{\mu e} + F_t^e - p_{1t}^e(\delta - \pi_t^e)K_t] + b_t B_t\} + G_{2t} + EX_{2t}^e}{1-(c-m_2)(1-\tau)(1-\mu)}. \tag{27}$$

And the equilibrium price and output of consumption goods are obtained by substituting (15) into (27) as follows:

$$p_{2t}^e = \left[\frac{w_t}{(1-\mu)(1-\alpha)A_t}\right]^{1-\alpha}\left[\frac{1}{K_{2t}}\right]^{\alpha}$$
$$\times\left\{\frac{(c-m_2)\{(1-\tau)[(1-\mu)p_{1t}^e Q_{1t}^{\mu e} + F_t^e - p_{1t}^e(\delta - \pi_t^e)K_t] + b_t B_t\} + G_{2t} + EX_{2t}^e}{1-(c-m_2)(1-\tau)(1-\mu)}\right\}^{\alpha}, \tag{28}$$

and

$$Q_{2t}^{\mu e} = \left[\frac{(1-\mu)(1-\alpha)A_t}{w_t}\right]^{1-\alpha} K_{2t}^{\alpha}$$

[7]This is just an example of other various specifications of import and export functions. For simplicity, it is assumed above that government does not buy consumption goods from the foreign sector. If it does, an import function may be rewitten, for example, as $IM_{2t}^e = m_2(Y_{Dt}^{\mu e} + G_{2t})$.

$$\times \left\{ \frac{(c-m_2)\{(1-\tau)[(1-\mu)p_{1t}^e Q_{1t}^{\mu e} + F_t^e - p_{1t}^e(\delta - \pi_t^e)K_t] + b_t B_t\} + G_{2t} + EX_{2t}^e}{1-(c-m_2)(1-\tau)(1-\mu)} \right\}^{1-\alpha}. \tag{29}$$

Next consider the investment-goods market and the bond market. The demands for them will come mainly from private saving. Thus it is necessary to look at it carefully. (26) can be rewritten as

$$Y_{Dt}^{\mu e} = p_{1t}^e Q_{1t}^{\mu e} + p_{2t}^e Q_{2t}^{\mu e} + F_t^e - p_{1t}^e(\delta - \pi_t^e)K_t - (IT_t^e + DT_t^e - b_t B_t).$$

Using this, (25) can be expressed as

$$p_{2t}^e Q_{2t}^{\mu e} = \frac{c}{1-c}[p_{1t}^e Q_{1t}^{\mu e} - p_{1t}^e(\delta - \pi_t^e)K_t - (IT_t^e + DT_t^e - b_t B_t - F_t^e)] + \frac{1}{1-c}[G_{2t} + EX_{2t}^e - m_2 Y_{Dt}^{\mu e}].$$

Thus the expected equilibrium private disposable income can be written as

$$Y_{Dt}^{\mu e} = \frac{1}{1-c}[p_{1t}^e Q_{1t}^{\mu e} - p_{1t}^e(\delta - \pi_t^e)K_t - (IT_t^e + DT_t^e - b_t B_t - F_t^e) + G_{2t} + EX_{2t}^e - m_2 Y_{Dt}^{\mu e}]. \tag{30}$$

Finally, from (24) the expected equilibrium saving can be expressed as

$$S_t^{\mu e} = I_t^e + BD_t^e + CA_t^e, \tag{31}$$

$$\begin{aligned} I_t^e &= p_{1t}^e Q_{1t}^{\mu e} - p_{1t}^e(\delta - \pi_t^e)K_t - G_{1t} - EX_{1t}^e + IM_{1t}^e, \\ BD_t^e &= G_{1t} + G_{2t} - (IT_t^e + DT_t^e - b_t B_t), \\ CA_t^e &= EX_{1t}^e + EX_{2t}^e + F_t^e - (IM_{1t}^e + m_2 Y_{Dt}^{\mu e}), \end{aligned}$$

where I_t^e, BD_t^e, and CA_t^e represent the expected nominal private investment, the expected government budget deficit, and the expected current account in domestic currency, respectively. It should be noted that (31) does not mean a causal relation. Correctly speaking, (31) says that the amount of private saving is equal to the sum of the three as a result of the equilibrium of the consumption-goods market with the expected price of investment goods p_{1t}^e as given.

It is necessary that $I_t^e + BD_t^e + CA_t^e > 0$ for positive $Y_{Dt}^{\mu e}$ in (30). In order for the argument made below to be clear, it is convenient to take a "normal" case where $I_t^e \geq 0$ and $BD_t^e \geq 0$. The following lemma is concerned with money hoarding in such a case.

Lemma: Money hoarding implies a domestic economy which is always subordinate to the foreign sector in the sense that private disposable income is determined exclusively by current account.

Proof: See Appendix A.

The case of money hoarding is theoretically possible in the complete KS model,[8] but it is quite unrealistic as is shown in the above lemma. It is assumed, therefore, that the household

[8] "Money" in this context is that of this national economy.

sector uses all money available for store of value, i.e., money is not held as wealth (the no-Pope's-father condition). Under this assumption it is easy to show the following theorem:

Theorem 1: If money is not hoarded, the domestic markets of investment goods and newly issued government bonds reach each equilibrium at the same time and the equilibrium condition is as follows:

$$
\begin{aligned}
\frac{\theta}{1-\theta} &= \frac{I_t^e}{BD_t^e} \\
&= \frac{p_{1t}^e Q_{1t}^{\mu e} - p_{1t}^e(\delta - \pi_t^e)K_t - G_{1t} - EX_{1t}^e + IM_{1t}^e}{G_{1t} + G_{2t} - (IT_t^e + DT_t^e - b_t B_t)},
\end{aligned}
\tag{32}
$$

where $0 \leq \theta \leq 1$ is the ratio of $S_t^{\mu e} - CA_t^e$ that goes to the purchase of domestic and foreign investment goods. The limiting cases $\theta = 0$ and $\theta = 1$ imply respectively zero I_t^e in the numerator and zero BD_t^e in the denominator.

Proof: See Appendix B.

With private saving the household sector can increase assets in the form of investment goods, government bonds, or foreign bonds. Theorem 1 means that the household sector spends the amount equal to I_t^e, BD_t^e, and CA_t^e of the saving on investment goods, government bonds, and foreign bonds, respectively.

It is now found that a new fact has appeared which did not exist in the basic case. That is, the expected equilibrium prices of consumption goods p_{2t}^e and investment goods p_{1t}^e are completely determined by the market equilibrium conditions (27) and (32). In the basic case the expected absolute prices of consumption and investment goods are not determined unless one of the prices are fixed, whereas in the complete case both prices must be determined simultaneously. One of the prices cannot be known before the other. Thus the complete KS model has a flavor of the general equilibrium theory even in the short run.[9] [10]

Figure 1. Indirect Tax and Equilibrium in the Investment-Goods Market.

Figure 2. Indirect Tax and Equilibrium in the Consumption-Goods Market.

Figures 1 and 2 illustrate a simultaneous equilibrium of the investment-goods market and the consumption-goods market. Figure 1 shows the supply curve $Q_{1t}^{\mu S}$ of investment goods when indirect tax is imposed at the rate μ, and the investment-goods demand curve $Q_{1t}^{\mu D}$ with the equilibrium price of *consumption* goods p_{2t}^e shown in Figure 2 as given. The investment-goods market is equilibrated at the intersection of both curves. The supply curve Q_{1t}^S in the absence of indirect tax is also drawn in the figure. As is seen from the figure,

[9]Other specifications can make the situation rather different. For example, if IT_t^e and DT_t^e are lump-sum taxes, as is often assumed in a theoretical analysis, the expected equilibrium price of investment goods is determined independently of the consumption-goods price.

[10]Tobin's q theory and the Modigliani-Miller theorem are applicable in the complete KS model, too, as long as the consideration is confined to the *partial-equilibrium* framework. Appendices G and H deal with the modified version of the q theory and the M-M theorem, respectively, when indirect tax is imposed. And finally their mathematical equivalence is concluded.

indirect tax shifts the supply curve upward. Thus, the less output level Q^{μ}_{1t} corresponds to the same price in the presence of indirect tax. Similarly Figure 2 shows the supply curve $Q^{\mu S}_{2t}$ of consumption goods when indirect tax is imposed at the rate μ, and the consumption-goods demand curve $Q^{\mu D}_{2t}$ with the equilibrium price of *investment* goods p^e_{1t} shown in Figure 1 as given. The equilibrium of the consumption-goods market is represented by the intersection of both curves. The supply curve Q^S_{2t} in the absence of indirect tax is also drawn. It is not at all known whether the equilibrium prices p^e_{1t} and p^e_{2t} in Figures 1 and 2 are higher than the equilibrium prices in the basic case because they depend on various parameters and exogenous variables.[11]

5 The Short-Run Equilibrium State

The arguments developed in the previous two sections can be summarized in terms of a System of National Accounts:

$$\begin{aligned}
GNP^e_t &= p^e_{1t}Q^{\mu e}_{1t} + p^e_{2t}Q^{\mu e}_{2t} + F^e_t \\
&= [Y^{\mu e}_{Dt} + (DT^e_t - b_tB_t)] + IT^e_t + p^e_{1t}(\delta - \pi^e_t)K_t \\
&= Y^{\mu e}_{Dt} + (IT^e_t + DT^e_t - b_tB_t) + p^e_{1t}(\delta - \pi^e_t)K_t \\
&= cY^{\mu e}_{Dt} + S^{\mu e}_t + G_{2t} + [(IT^e_t + DT^e_t - b_tB_t) - G_{2t}] + p^e_{1t}(\delta - \pi^e_t)K_t \\
&= cY^{\mu e}_{Dt} + G_{2t} + \{S^{\mu e}_t + [(IT^e_t + DT^e_t - b_tB_t) - G_{2t}] + p^e_{1t}(\delta - \pi^e_t)K_t\} \\
&= \{cY^{\mu e}_{Dt} + [I^e_t + p^e_{1t}(\delta - \pi^e_t)K_t] + (G_{1t} + G_{2t})\} + CA^e_t. \qquad (33)
\end{aligned}$$

The first line of (33) represents the expected nominal GNP at t, GNP^e_t, which is the sum of the expected nominal GDP at t,

$$GDP^e_t = p^e_{1t}Q^{\mu e}_{1t} + p^e_{2t}Q^{\mu e}_{2t}, \qquad (34)$$

and the expected net receipt of factor income from the foreign sector, F^e_t. The expected nominal NNP at t, can be written as

$$NNP^e_t = p^e_{1t}Q^{\mu e}_{1t} + p^e_{2t}Q^{\mu e}_{2t} + F^e_t - p^e_{1t}(\delta - \pi^e_t)K_t.$$

$Y^{\mu e}_{Dt} + (DT^e_t - b_tB_t)$ in the second line is the expected national income $Y^{\mu e}_t$, which has already appeared in (21). It is very important to recognize that GNP^e_t is the expected *total* purchasing power of this national economy. The ultimate object of macroeconomics is to discover what determines GNP (or, in recent terms, GNI) rather than national income.

The third line says that NNP^e_t is divided among the household sector and the government sector respectively as private disposable income $Y^{\mu e}_{Dt}$ and government disposable income $IT^e_t + DT^e_t - b_tB_t$. The fourth line implies that the household sector divides private disposable income into private consumption cY^e_{Dt} and private saving $S^{\mu e}_t$, while the government sector divides government disposable income into government consumption G_{2t} and government saving $(IT^e_t + DT^e_t - b_tB_t) - G_{2t}$. The fifth line states that GNP^e_t is divided into national consumption $cY^{\mu e}_{Dt} + G_{2t}$ and (gross) national saving $S^{\mu e}_t + [(IT^e_t + DT^e_t - b_tB_t) - G_{2t}] + p^e_{1t}(\delta - \pi^e_t)K_t$. The sixth line is derived using the equilibrium relation (31). $cY^{\mu e}_{Dt} + [I^e_t +$

[11]Supply curve $Q^{\mu S}_{1t}$ and demand curve $Q^{\mu D}_{1t}$ in Figure 1 are derived in Appendix C, while supply curve $Q^{\mu S}_{2t}$ and demand curve $Q^{\mu D}_{2t}$ in Figure 2 are derived in Appendix D.

$p^e_{1t}(\delta - \pi^e_t)K_t] + (G_{1t} + G_{2t})$ in it is the expected absorption. Current account CA^e_t is, as is well known, described as the difference between GNP and absorption.[12]

Now, using the above relations, let us consider the traditional method of income determination in the Keynesian economics. Since $NNP^e_t = GNP^e_t - p^e_{1t}(\delta - \pi^e_t)K_t$, the last line of (33) leads to the so-called equilibrium condition of the goods market:

$$NNP^e_t = cY^{\mu e}_{Dt} + I^e_t + (G_{1t} + G_{2t}) + CA^e_t. \tag{35}$$

Furthermore, from the third line of (33) the private disposable income can be written as

$$Y^{\mu e}_{Dt} = NNP^e_t - (IT^e_t + DT^e_t - b_tB_t). \tag{36}$$

Then, substituting (36) into (35) yields

$$NNP^e_t = \frac{1}{1-c}[I^e_t + (G_{1t} + G_{2t}) + CA^e_t - c(IT^e_t + DT^e_t - b_tB_t)]. \tag{37}$$

This is too famous an expression of the equilibrium income. There are two differences, however. First, (37) draws a distinction between government investment and government consumption. In income determination analysis government expenditure is usually supposed to be government *investment* or "public works."[13] The distinction in (37) is a natural consequence derived from the two-sector model. Second, private investment I^e_t is determined through simultaneous equilibrium of the consumption-goods market and the investment-goods market. (37) does not represent causality.

It should be remembered here that p^e_{1t} and p^e_{2t} in (33) are expected values. It is another problem whether they are realized at the first subperiod of period t. As in the basic case, they are realized just as expected only when the central bank supplies appropriate money as a medium of exchange. Such quantity of money M_t is calculated, e.g., as $[p^e_{1t}Q^{\mu e}_{1t} + p^e_{2t}Q^{\mu e}_{2t} - CA^e_t]/V_t$, where V_t is income velocity of money at t. Then, only if the central bank controls money supply so as to satisfy

$$M_tV_t = p^e_{1t}Q^{\mu e}_{1t} + p^e_{2t}Q^{\mu e}_{2t} - CA^e_t, \tag{38}$$

the economy reaches the short-run equilibrium state where expected or planned values are all realized.

It appears at first sight that (38) implies that the central bank is able to influence the short-run equilibrium state to some extent as in the basic case. But the fact that the expected equilibrium prices p^e_{1t} and p^e_{2t} are determined by (27) and (32) in the complete case limits the role of the central bank. What the central bank can do or must do for the realization of the short-run equilibrium state is only to supply money passively according to (38). If the central

[12]Note that BD^e_t does not appear in (33). Especially from the fifth and sixth lines,

$$S^e_t + [(IT^e_t + DT^e_t - b_tB_t) - G_{2t}] + p^e_{1t}(\delta - \pi^e_t)K_t = (p^e_{1t}Q^{\mu e}_{1t} - EX^e_{1t} + IM^e_{1t}) + CA^e_t.$$

This equality teaches us that national saving leads to either investment goods or foreign bonds. Indeed government bonds are incorporated into portfolio of the household sector as shown in (31), but they are canceled out as an asset of the economy as a whole. However, such a fact does not lessen the significance of government budget deficit in the analysis as is obviously shown in Theorem 1. That is, national saving is not known until government budget deficit is determined endogenously.

[13]See, for example, Keynes (1936, pp. 116). But note that Keynes (1936, pp. 128-129) keeps in mind both government investment and government consumption, using the term "loan expenditure."

bank does not do so, it will lead to the malfunction of the markets. Since this paper focuses on the short-run equilibrium state, the central bank is supposed to do so every period. And then the short-run equilibrium state comes true every period without an auctioneer.

Let a superscript $*$ represent a value realized. Then, in the short-run equilibrium state, $p_{1t}^e = p_{1t}^*$, $p_{2t}^e = p_{2t}^*$, $Q_{1t}^{\mu e} = Q_{1t}^{\mu *}$, $Q_{2t}^{\mu e} = Q_{2t}^{\mu *}$, etc.[14] In the short-run equilibrium state (38) looks like a modified version of the Fisher equation of exchange:

$$M_t V_t = p_{1t}^* Q_{1t}^{\mu *} + p_{2t}^* Q_{2t}^{\mu *} - CA_t^*.$$

However, as has been explained, this is not the equation. The following proposition is right.

Proposition 1: In the complete KS model, the quantitiy theory of money, which claims that the quantity of money determines the price level, does not hold any longer.

In the short-run equilibrium state, (33) and (37) are also rewritten with a superscript e replaced with a superscript $*$:

$$\begin{aligned} GNP_t^* &= p_{1t}^* Q_{1t}^{\mu *} + p_{2t}^* Q_{2t}^{\mu *} + F_t^* \\ &= [Y_{Dt}^{\mu *} + (DT_t^* - b_t B_t)] + IT_t^* + p_{1t}^*(\delta - \pi_t^*)K_t \\ &= Y_{Dt}^{\mu *} + (IT_t^* + DT_t^* - b_t B_t) + p_{1t}^*(\delta - \pi_t^*)K_t \\ &= cY_{Dt}^{\mu *} + S_t^* + G_{2t} + [(IT_t^* + DT_t^* - b_t B_t) - G_{2t}] + p_{1t}^*(\delta - \pi_t^*)K_t \\ &= cY_{Dt}^{\mu *} + G_{2t} + \{S_t^* + [(IT_t^* + DT_t^* - b_t B_t) - G_{2t}] + p_{1t}^*(\delta - \pi_t^*)K_t\} \\ &= \{cY_{Dt}^{\mu *} + [I_t^* + p_{1t}^*(\delta - \pi_t^*)K_t] + (G_{1t} + G_{2t})\} + CA_t^*, \end{aligned} \tag{39}$$

and

$$NNP_t^* = \frac{1}{1-c}[I_t^* + (G_{1t} + G_{2t}) + CA_t^* - c(IT_t^* + DT_t^* - b_t B_t)].$$

In the complete *KS* model the household sector plays two major parts. One is, as in the basic case, the division of disposable income into consumption and saving. Using (24), (25), (30), and (31), this is expressed as follows:

$$\begin{aligned} \frac{cY_{Dt}^{\mu *}}{S_t^{\mu *}} &= \frac{c}{1-c} \\ &= \frac{p_{2t}^* Q_{2t}^{\mu *} - G_{2t} - EX_{2t}^* + IM_{2t}^*}{p_{1t}^* Q_{1t}^{\mu *} - p_{1t}^*(\delta - \pi_t^*)K_t - (IT_t^* + DT_t^* - b_t B_t - F_t^*) + G_{2t} + EX_{2t}^* - IM_{2t}^*}. \end{aligned} \tag{40}$$

In fact (40) is equivalent to (27) except for superscripts $*$ and e. The other is how much to use saving for investment goods or government bonds as is correctly stated in (32). In the short-run equilibrium state this is expressed as follows:

$$\frac{\theta}{1-\theta} = \frac{p_{1t}^* Q_{1t}^{\mu *} - p_{1t}^*(\delta - \pi_t^*)K_t - G_{1t} - EX_{1t}^* + IM_{1t}^*}{G_{1t} + G_{2t} - (IT_t^* + DT_t^* - b_t B_t)}. \tag{41}$$

[14] As to variables regarded as exogenous, the following are simply assumed in the short-run equilibrium state:

$$F_t^e = F_t^*, EX_{1t}^e = EX_{1t}^*, IM_{1t}^e = IM_{1t}^*, EX_{2t}^e = EX_{2t}^*, \text{ and } e_t^e = e_t^*.$$

Therefore, the short-run equilibrium state represented by p_{1t}^* and p_{2t}^* is fully described by (40) and (41). And it is understood that the household sector exerts an influence on it through c and θ. (41) is specific to the complete case. It is found from it that the government sector cannot determine the amount of budget deficit.[15]

G_{1t} and G_{2t} are regarded as policy variables. Then, does an increase in G_{1t} or G_{2t} cause an increase in production and income? Or, to put it in other words, is government fiscal policy effective? Since a rise in prices leads to an increase in production and income, the effectiveness of fiscal policy can be checked by examining the signs of $\partial p_{1t}^*/\partial G_{1t}$, $\partial p_{1t}^*/\partial G_{2t}$, $\partial p_{2t}^*/\partial G_{1t}$, and $\partial p_{2t}^*/\partial G_{2t}$. The following proposition is derived from the fact that these signs are all positive.

Proposition 2: In the short run government fiscal policy is effective.
Proof: See Appendix E.

This proposition itself appears to be the same as what is usually written in textbooks, but there is a difference between them. The latter is supposed to be based on the sluggish adjustment of prices, whereas the former relies merely on the realization of correct production plan by the production sector.[16]

Investment goods produced inside the national economy is added to captial stock of the economy as a whole except the portion exported to the foreign sector. Investment goods imported from the foreign sector also increases the capital stock. Thus the capital accumulatiton equation is written as

$$K_{t+1} = (1-\delta)K_t + Q_{1t}^{\mu *} - \frac{EX_{1t}^*}{p_{1t}^*} + \frac{IM_{1t}^*}{e_t^* p_{f1t}^*}. \tag{42}$$

where e_t^* and p_{f1t}^* are respectively the realized nominal exchange rate and the realized price of foreign investment goods in foreign currency.

6 Definition of the Long-Run Equilibrium State

Having characterized the short-run equilibrium state, let us begin to analyze the long-run macroeconomy. This section defines the long-run equilibrium state in the complete KS model. First, derive the difference between h_{1t}^e and i_t. Rewriting (4) yields

$$(1-\mu)p_{1t}^e Q_{1t}^{\mu e} = w_t N_{1t}^{\mu e} + p_{1t}^e(r_t^e + \delta)K_{1t} + (h_{1t}^e - i_t)\tilde{p}_{1t-1}K_{1t}^h, \tag{43}$$

where $r_t^e = [(1+i_t)\tilde{p}_{1t-1}/p_{1t}^e] - 1$, and r_t^e is the real interest rate. By rearranging (43), the difference between h_{1t}^e and i_t is written as

$$h_{1t}^e - i_t = \frac{p_{1t}^e(r_t^e+\delta)K_{1t}}{\tilde{p}_{1t-1}K_{1t}^h}\left\{\left[\frac{(1-\mu)p_{1t}^e}{\hat{p}_{1t}^e}\right]^{\frac{1}{\alpha}} - 1\right\}, \tag{44}$$

[15] Particularly a balanced budget is possible only if the household sector sets θ at 1, which corresponds to the basic case.

[16] Appendix E also shows that p_{2t}^* is an increasing function of p_{1t}^* in (40) whereas p_{2t}^* is a decreasing function of p_{1t}^* in (41). Therefore, the short-run equilibrium state is unique if it exists.

where

$$\begin{aligned}\hat{p}_{1t}^{e} &= \left[\frac{(1+i_t)\tilde{p}_{1t-1}-(1-\delta)p_{1t}^{e}}{\alpha}\right]^{\alpha}\left[\frac{w_t}{A_t(1-\alpha)}\right]^{1-\alpha} \\ &= p_{1t}^{e}\left[\frac{r_t^e+\delta}{\alpha}\right]^{\alpha}\left[\frac{\frac{w_t}{p_{1t}^{e}}}{A_t(1-\alpha)}\right]^{1-\alpha}.\end{aligned}$$

Figure 3. Indirect Tax and the Expected Normal Supply-Price of Investment Goods.

It should be noted that indirect tax does not change the expected "normal supply-price" $\hat{p}_{1t}^{e}$ of investment goods. But it shifts the point where h_{1t}^{e} is equal to i_t, as shown in Figure 3. It is found from the figure that indirect tax takes equity holders to a disadvantageous position.

From Section 3 the economy is supposed to be at the third subperiod of period $t-1$. *Henceforth,* assume that the economy is always in the short-run equilibrium state, which implies that production plan made each third subperiod are always realized at the first subperiod of the next period. Here the focus of analysis changes from the short-run equilibrium state to the long-run equilibrium state.

In the short-run equilibrium state the difference (44) can be written simply by replacing a superscript e with a superscript $*$:

$$h_{1t}^{*}-i_t=\frac{p_{1t}^{*}(r_t^*+\delta)K_{1t}}{\tilde{p}_{1t-1}K_{1t}^{h}}\left\{\left[\frac{(1-\mu)p_{1t}^{*}}{\hat{p}_{1t}^{*}}\right]^{\frac{1}{\alpha}}-1\right\}, \tag{45}$$

where

$$\begin{aligned}\hat{p}_{1t}^{*} &= \left[\frac{(1+i_t)\tilde{p}_{1t-1}-(1-\delta)p_{1t}^{*}}{\alpha}\right]^{\alpha}\left[\frac{w_t}{A_t(1-\alpha)}\right]^{1-\alpha} \\ &= p_{1t}^{*}\left[\frac{r_t^*+\delta}{\alpha}\right]^{\alpha}\left[\frac{\frac{w_t}{p_{1t}^{*}}}{A_t(1-\alpha)}\right]^{1-\alpha}.\end{aligned} \tag{46}$$

The difference between h_{2t}^{*} and i_t can be calculated using (13) and (28):

$$\begin{aligned} h_{2t}^{*}-i_t &= \frac{p_{2t}^{*}(r_t^*+\delta)K_{2t}}{\tilde{p}_{1t-1}K_{2t}^{h}}\left\{\left[\frac{(1-\mu)p_{2t}^{*}}{\hat{p}_{1t}^{*}}\right]^{\frac{1}{\alpha}}-1\right\} \\ &= \frac{p_{1t}^{*}(r_t^*+\delta)}{\tilde{p}_{1t-1}K_{2t}^{h}}\left\{\frac{c}{1-c}\left[\left(\frac{(1-\mu)p_{1t}^{*}}{\hat{p}_{1t}^{*}}\right)^{\frac{1}{\alpha}}-(\delta-\pi_t^*)\frac{(1-\mu)\alpha}{r_t^*+\delta}\right]K_{1t}-\left[1+(\delta-\pi_t^*)\frac{(1-\mu)\alpha}{r_t^*+\delta}\frac{c}{1-c}\right]K_{2t}\right\} \\ &\quad +\frac{p_{1t}^{*}(r_t^*+\delta)}{\tilde{p}_{1t-1}K_{2t}^{h}}\left\{\frac{G_{2t}+EX_{2t}^{*}-IM_{2t}^{*}}{p_{1t}^{*}}-\frac{IT_t^*+DT_t^*-b_tB_t-F_t^*}{p_{1t}^{*}}c\right\}\frac{(1-\mu)\alpha}{r_t^*+\delta}\frac{1}{1-c}. \end{aligned} \tag{47}$$

The long-run price condition is described as follows:

$$\frac{1}{1-\pi}p_{1t-1}^{**}=\frac{1}{1-\pi}\tilde{p}_{1t-1}^{**}=p_{1t}^{**}, \tag{48}$$

where π is a constant inflation rate.[17] A superscript $**$ represents the long-run equilibrium state in what follows. As for labor market, there is the natural level of employment, N_t, with a constant growth rate n, i.e.,

$$N_t=(1+n)N_{t-1},\ n>-1. \tag{49}$$

[17]Recall that the expected inflation rate was defined as $\pi_t^e=1-(\tilde{p}_{1t-1}/p_{1t}^{e})$ in Section 3.

As a situation peculiar to the complete KS model, the following are assumed:

$$\begin{aligned}
\frac{G_{2t}^{**}}{p_{1t}^{**}} &= \beta_{G_2} K_t^{\mu**},\ \beta_{G_2} \geq 0 \\
\frac{EX_{2t}^{**} - IM_{2t}^{**}}{p_{1t}^{**}} &= \beta_{NX_2} K_t^{\mu**}, \\
\frac{IT_t^{**} + DT_t^{**} - (b_t B_t)^{**}}{p_{1t}^{**}} &= \gamma_T K_t^{\mu**}, \\
\frac{F_t^{**}}{p_{1t}^{**}} &= \gamma_F K_t^{\mu**}, \\
\frac{G_{1t}^{**}}{p_{1t}^{**}} &= \beta_{G_1} K_t^{\mu**},\ \beta_{G_1} \geq 0 \\
\frac{EX_{1t}^{**}}{p_{1t}^{**}} - \frac{IM_{1t}^{**}}{e_t^{**} p_{f1t}^{**}} &= \beta_{NX_1} K_t^{\mu**},\ p_{1t}^{**} = e_t^{**} p_{f1t}^{**},
\end{aligned}$$

where $K_t^{\mu**}$ is capital stock in the long-run equilibrium state and coefficients of $K_t^{\mu**}$ are constants.[18] The above relation between $K_t^{\mu**}$ and other variables is called the *proportionality condition*. It is convenient to write as follows:

$$\beta_2 = \beta_{G_2} + \beta_{NX_2},\ \beta_{NX} = \beta_{NX_1} + \beta_{NX_2},\ \beta = \beta_{G_2} + \beta_{NX},\ \gamma = \gamma_T - \gamma_F.$$

Now let us define the long-run equilibrium state. An economy is in the long-run equilibrium state at t if the following five conditions are all satisfied:

1. The economy is in the short-run equilibrium state.
2. Full employment is realized, i.e., $N_{1t}^{\mu*} + N_{2t}^{\mu*} = N_t$.
3. The rates of return are all equal, i.e., $h_{1t}^* = i_t = h_{2t}^*$.[19]
4. The long-run price condition (48) holds.
5. The proportionality condition holds.

For the sake of convenience, let us call the long-run equilibrium state merely the *long-run state* in what follows.

Conditions 2-4 are commonly used to discuss the long run in macroeconomics, while Condition 5 is new to the extent that it is explicitly stated. The next section examines the long-run state on the basis of the above conditions.

7 The Long-Run State

Taking (45) and the first half of (47) into account, Conditions 3 and 4 imply that

$$\frac{1}{1-\pi}\, p_{1t-1}^{**} = p_{1t}^* = \frac{1}{1-\mu}\, \hat{p}_{1t}^* = p_{2t}^* = p_{1t}^{**}. \tag{50}$$

[18]The last equality $p_{1t}^{**} = e_t^{**} p_{f1t}^{**}$ means that Cassel's theory of purchasing power parity holds in the long-run equilibrium state.

[19]The rates of returns on government bonds and foreign bonds already coincide with the bank rate in the short run due to the adjustment of the bond price and the exchange rate.

It is found from (50) that in the long-run state prices of investment goods and consumption goods coincide and change at the same rate. Thus it is convenient to use only p_{1t}^{**} for them. Moreover, a nominal value divided by p_{1t}^{**} can be interpreted as a real value in a usual sense. Denote, for example, real GDP at t by $Q_t^{\mu**}$. Then, $Q_t^{\mu**} = GDP_t^{**}/p_{1t}^{**}$, and therefore

$$Q_t^{\mu**} = Q_{1t}^{\mu**} + Q_{2t}^{\mu**}, \tag{51}$$

because of (34).

From (50), the labor demand in the investment-goods sector (7) can be written as

$$N_{1t}^{\mu*} = \left[(1-\alpha)A_t^{1-\alpha}\frac{(1-\mu)p_{1t}^{**}}{w_t}\right]^{\frac{1}{\alpha}} K_{1t},$$

and similarly the labor demand in the consumption-goods sector (14) as

$$N_{2t}^{\mu*} = \left[(1-\alpha)A_t^{1-\alpha}\frac{(1-\mu)p_{1t}^{**}}{w_t}\right]^{\frac{1}{\alpha}} K_{2t}.$$

Since $K_{1t} + K_{2t} = K_t$, Condition 2 leads to the following equality:

$$\left[(1-\alpha)A_t^{1-\alpha}\frac{(1-\mu)p_{1t}^{**}}{w_t}\right]^{\frac{1}{\alpha}} K_t = N_t.^{20} \tag{52}$$

(52) gives the long-run-state real wage rate:

$$\frac{w_t^{\mu**}}{p_{1t}^{**}} = (1-\mu)(1-\alpha)A_t\left[\frac{K_t}{A_tN_t}\right]^{\alpha}. \tag{53}$$

$w_t^{\mu**}$ is the long-run-state nominal wage rate, and it is determined on the values of p_{1t}^{**}, K_t, A_t, N_t, α, and μ which are all known at the third subperiod of period $t-1$. Let capital per effective labor in the right-hand side of (53) be designated by k_t^{μ}, and capital per effective labor in the investmet-goods sector and in the consumption-goods sector respectively by k_{1t}^{μ} and k_{2t}^{μ}:

$$k_t^{\mu} = \frac{K_t}{A_tN_t},\ k_{1t}^{\mu} = \frac{K_{1t}}{A_tN_{1t}^{\mu**}},\ \text{and}\ k_{2t}^{\mu} = \frac{K_{2t}}{A_tN_{2t}^{\mu**}},$$

where $N_{1t}^{\mu**} + N_{2t}^{\mu**} = N_t$. Then (53) can be rewritten as

$$\frac{w_t^{\mu**}}{(1-\mu)p_{1t}^{**}} = (1-\alpha)A_t(k_t^{\mu})^{\alpha} = (1-\alpha)A_t(k_{1t}^{\mu})^{\alpha} = (1-\alpha)A_t(k_{2t}^{\mu})^{\alpha}. \tag{54}$$

It is found from (54) that in the long-run state capital per effective labor coinsides in the investment-goods sector and in the consumption-goods sector. Let us call $(1-\alpha)A_t(k_t^{\mu})^{\alpha}$ in (54) the marginal product of labor as a whole, and denote it by MPL_t^{**}. Then, it follows from (9), (16), and (54) that $MPL_t^{**} = MPL_{1t}^{**} = MPL_{2t}^{**}$, and that they are all equal to $w_t^{\mu**}/(1-\mu)p_{1t}^{**}$, not the real wage rate $w_t^{\mu**}/p_{1t}^{**}$.

Condition 3 is satisfied by arbitrage. If $h_{1t}^{e} > (<) i_t$, $\tilde{p}_{1t-1}$ rises (falls).[21] And if $h_{2t}^{e} > (<) i_t$, the ratio of K_{2t} to K_{1t} rises (falls). As a result, $h_{1t}^{*} = i_t = h_{2t}^{*}$ holds. Then it is necessary to

[20]Considering (8), (15), (50), and (52), (51) reduces to the one-sector Cobb-Douglas production function $Q_t^{\mu**} = A_t^{1-\alpha}N_t^{1-\alpha}K_t^{\alpha}$.

[21]See Figure 3. The graph of $\tilde{p}_{1t}^{e}$ shifts upward (downward) when $\tilde{p}_{1t-1}$ rises (falls). When $\tilde{p}_{1t}^{e}$ is adjusted onto the line $\tilde{p}_{1t-1} = (1-\mu)p_{1t}^{e}$, h_{1t}^{e} and i_t coincide.

know what nominal interest and configulation of K_{1t} and K_{2t} are consistent with the long-run state. I will try to find them.

First, consider the interest rate. In the long-run state, the real interest rate as defined and Assumption (5) are respectively simplified as

$$\begin{aligned} r_t^{\mu**} &= \frac{(1+i_t^{\mu**})\tilde{p}_{t-1}^{**}}{p_{1t}^{**}} \\ &= (1+i_t^{\mu**})(1-\pi)-1, \end{aligned} \tag{55}$$

and

$$\delta - \pi > 0, \tag{56}$$

because of Condition 4. And, taking (50) and (54) into account, (46) leads to

$$\frac{r_t^{\mu**}+\delta}{1-\mu} = \alpha(k_t^{\mu})^{\alpha-1} = \alpha(k_{1t}^{\mu})^{\alpha-1} = \alpha(k_{2t}^{\mu})^{\alpha-1}. \tag{57}$$

Let us call $\alpha(k_t^{\mu})^{\alpha-1}$ in (57) the marginal product of capital as a whole, and denote it by MPK_t^{**}. Then, it is found from (10), (17), and (57) that $MPK_t^{**} = MPK_{1t}^{**} = MPK_{2t}^{**}$, and that they are all equal to $(r_t^{\mu**}+\delta)/(1-\mu)$, not the sum $r_t^{\mu**}+\delta$ of the real interest rate and the capital depreciation rate.

More importantly, the first half of (57) means that the level of capital per effective labor as a whole determines the long-run-state real interest rate, which in turn specifies the long-run-state nominal interest rate $i_t^{\mu**}$ through (55) as follows:

$$i_t^{\mu**} = \frac{1}{1-\pi}\,[(1-\mu)\alpha(k_t^{\mu})^{\alpha-1} - (\delta-\pi)]. \tag{58}$$

Once i_t is set at $i_t^{\mu**}$ as in (58) on the values of π, K_t, A_t, N_t, δ, α, and μ which are all known at the second subperiod of period $t-1$, the asset price $\tilde{p}_{1t-1}^{**}$ is so determined as to make h_{1t}^{**} and $i_t^{\mu**}$ equal with the result that the inflation rate takes a value of π. Condition 4 consists of two parts, $[1/(1-\pi)]\ p_{1t-1}^{**} = p_{1t}^{**}$ and $[1/(1-\pi)]\ \tilde{p}_{1t-1}^{**} = p_{1t}^{**}$. It is found from the above argument that it is the nominal interest rate that determines the long-run-state inflation rate as in the latter part. The former part may be realized, e.g., by fiscal policy of the government sector because of its influence on prices in the short run.

Next consider the capital stock. (50) and (56) simplify (47) as

$$\begin{aligned} &\frac{c}{1-c}\left[1-(\delta-\pi)\frac{(1-\mu)\alpha}{r_t^{\mu**}+\delta}\right]K_{1t}^{\mu**} - \left[1+(\delta-\pi)\frac{(1-\mu)\alpha}{r_t^{\mu**}+\delta}\frac{c}{1-c}\right]K_{2t}^{\mu**} \\ &\quad + \left[\frac{G_{2t}^{**}+EX_{2t}^{**}-IM_{2t}^{**}}{p_{1t}^{**}} - \frac{IT_t^{**}+DT_t^{**}-(b_tB_t)^{**}-F_t^{**}}{p_{1t}^{**}}c\right]\frac{(1-\mu)\alpha}{r_t^{\mu**}+\delta}\frac{1}{1-c} = 0. \end{aligned} \tag{59}$$

Substituting $(r_t^{\mu**}+\delta)/(1-\mu) = \alpha(k_t^{\mu})^{\alpha-1}$ and taking Condition 5 into account yield the ratios:

$$\frac{N_{1t}^{\mu**}}{N_t} = \frac{K_{1t}^{\mu**}}{K_t^{\mu**}} = 1-c+[c(\delta+\gamma-\pi)-\beta_2](k_t^{\mu**})^{1-\alpha}, \tag{60}$$

and

$$\frac{N_{2t}^{\mu**}}{N_t} = \frac{K_{2t}^{\mu**}}{K_t^{\mu**}} = c-[c(\delta+\gamma-\pi)-\beta_2](k_t^{\mu**})^{1-\alpha}, \tag{61}$$

where $K_t^{\mu**} = K_{1t}^{\mu**} + K_{2t}^{\mu**}$ and $k_t^{\mu**} = K_t^{\mu**}/A_tN_t$. Capital stock is placed according to (60) and (61) during the second subperiod with the result that $h_{2t}^{**} = i_t^{\mu**}$ holds. $K_{1t}^{\mu**}$ and $K_{2t}^{\mu**}$ are determined on the values of π, β_2, γ, K_t, A_t, N_t, δ, α, and c which are all known at the time. (60) and (61) show that $N_{1t}^{\mu**}$ and $N_{2t}^{\mu**}$ are also determined before the third subperiod of period $t-1$.[22]

8 Analysis of the Long-Run Steady State

The complete KS model in the long-run state is represented by capital per effective labor, $k_t^{\mu**}$. This section analyses the long-run steady state. The method by Solow (1956) is again very helpful to detect it. The equation of capital accumulation in the short run (42) also holds in the long-run state as follows:

$$K_{t+1}^{\mu**} = (1-\delta)K_t^{\mu**} + Q_{1t}^{\mu**} - \frac{EX_{1t}^{**}}{p_{1t}^{**}} + \frac{IM_{1t}^{**}}{e_t^{**}p_{f1t}^{**}}.$$

The proportionality condition simplifies the above equation to

$$K_{t+1}^{\mu**} = (1-\delta-\beta_{NX_1})K_t^{\mu**} + Q_{1t}^{\mu**}. \tag{62}$$

Dividing both sides of (62) by $A_{t+1}N_{t+1}$ and considering (2), (49), and (60) give

$$k_{t+1}^{\mu**} = \frac{1-\delta-\beta+c(\delta+\gamma-\pi)}{(1+g)(1+n)}k_t^{\mu**} + \frac{1-c}{(1+g)(1+n)}(k_t^{\mu**})^{\alpha}.^{23} \tag{63}$$

The economy is said to be in the long-run *steady* state if $k_{t+1}^{\mu**} = k_t^{\mu**}$. Let a subscript S indicate the long-run steady state of the economy in what follows. Furthermore, drop "long-run" in the "long-run steady state" unless it causes confusion. Then the steady-state capital per effective labor can easily be obtained as follows:

$$k_S^{\mu**} = \left[\frac{1-c}{g+n+\pi+\beta-\gamma+(1-c)(\delta+\gamma-\pi)}\right]^{\frac{1}{1-\alpha}}.^{24} \tag{64}$$

It is convenient to assume that

$$g+n+\pi+\beta-\gamma > 0,$$

and

$$\delta+\gamma-\pi > 0.$$

These assumptions make $k_S^{\mu**}$ always positive.[25] Put

$$k_S^{\mu**} = \frac{K_{St}^{\mu**}}{A_tN_t},\ k_{S1}^{\mu**} = \frac{K_{S1t}^{\mu**}}{A_tN_{S1t}^{\mu**}},\ \text{and}\ k_{S2}^{\mu**} = \frac{K_{S2t}^{\mu**}}{A_tN_{S2t}^{\mu**}},$$

[22] It is easy to show that in the long-run state the budget constraints of the two sectors can be unified into $(1-\mu)p_{1t}^{**}Q_t^{\mu**} = w_t^{\mu**}N_t + p_{1t}^{**}(r_t^{\mu**}+\delta)K_t$.

[23] Appendix F shows how to derive (63).

[24] As a matter of convenience $g+n+gn$ is written simply as $g+n$ in what follows. Thus $g+n$ below means $g+n+gn$ mathematically.

[25] They also imply that $g+n+\delta+\beta > 0$. In fact these assumptions are not so powerful, as is seen soon. For example, the assumption that $g+n+\delta+\beta_{NX} > 0$ is also useful. In sum a set of rather cumbersome assumptions are needed to make the argument below economically meaningful.

where

$$K_{St}^{\mu**} = K_{S1t}^{\mu**} + K_{S2t}^{\mu**}, \text{ and } N_t = N_{S1t}^{\mu**} + N_{S2t}^{\mu**}.$$

Then it is assured from the previous analysis that

$$k_S^{\mu**} = k_{S1}^{\mu**} = k_{S2}^{\mu**}.$$

The complete KS model in the steady state is, therefore, characterized by $k_S^{\mu**}$.

As for capital stock,

$$K_{St}^{\mu**} = \left[\frac{1-c}{g+n+\pi+\beta-\gamma+(1-c)(\delta+\gamma-\pi)}\right]^{\frac{1}{1-\alpha}} A_t N_t, \tag{65}$$

$$K_{S1t}^{\mu**} = \frac{(1-c)(g+n+\delta+\beta_{NX_1})}{g+n+\pi+\beta-\gamma+(1-c)(\delta+\gamma-\pi)} K_{St}^{\mu**}, \tag{66}$$

and

$$K_{S2t}^{\mu**} = \frac{c(g+n+\pi+\beta_{NX_1}-\gamma)+\beta_2}{g+n+\pi+\beta-\gamma+(1-c)(\delta+\gamma-\pi)} K_{St}^{\mu**}, \tag{67}$$

because of (60) and (61).

As for output,

$$\begin{aligned} Q_{S1t}^{\mu**} &= A_t N_{S1t}^{\mu**}(k_{S1}^{\mu**})^{\alpha} = A_t N_{S1t}^{\mu**}(k_S^{\mu**})^{\alpha} \\ &= (g+n+\delta+\beta_{NX_1})K_{St}^{\mu**}, \end{aligned} \tag{68}$$

$$\begin{aligned} Q_{S2t}^{\mu**} &= A_t N_{S2t}^{**}(k_{S2}^{**})^{\alpha} = A_t N_{S2t}^{**}(k_S^{**})^{\alpha} \\ &= \frac{c(g+n+\pi+\beta_{NX_1}-\gamma)+\beta_2}{1-c} K_{St}^{\mu**}, \end{aligned} \tag{69}$$

and

$$\begin{aligned} Q_{St}^{\mu**} &= Q_{S1t}^{**} + Q_{S2t}^{**} = A_t N_t (k_S^{**})^{\alpha} \\ &= \left[\frac{1-c}{g+n+\pi+\beta-\gamma+(1-c)(\delta+\gamma-\pi)}\right]^{\frac{\alpha}{1-\alpha}} A_t N_t, \end{aligned} \tag{70}$$

from (1), (12), (60), (61), and (64).

Finally, as for real private disposable income and real saving,

$$\begin{aligned} \frac{Y_{DSt}^{\mu**}}{p_{1t}^{**}} &= Q_{S1t}^{\mu**} + Q_{S2t}^{\mu**} - (\delta-\pi)K_{St}^{\mu**} - \frac{IT_t^{**} + DT_t^{**} - (b_t B_t)^{**} - F_t^{**}}{p_{1t}^{**}} \\ &= Q_{S1t}^{\mu**} + Q_{S2t}^{\mu**} - (\delta-\pi)K_{St}^{\mu**} - \gamma K_{St}^{\mu**} \\ &= \frac{g+n+\pi+\beta-\gamma}{1-c} K_{St}^{\mu**}, \end{aligned} \tag{71}$$

and

$$\begin{aligned} \frac{S_{St}^{**}}{p_{1t}^{**}} &= \frac{(1-c)Y_{DSt}^{\mu**}}{p_{1t}^{**}} \\ &= (g+n+\pi+\beta-\gamma)K_{St}^{\mu**}, \end{aligned} \tag{72}$$

because of (21) and (24).

The market equilibrium conditions (40) and (41) in the short run holds true in the steady state, too. The former is the equilibrium condition of the consumption-goods market, which has already been taken into consideration during the derivation of (47), and it is satisfied by (68), (69), and the proportionality condition. The latter can be simplified by (68) and the proportionality condition as follows:

$$\frac{\theta}{1-\theta} = \frac{g+n+\pi-\beta_{G_1}}{\beta_{G_1}+\beta_{G_2}-\gamma_T}. \tag{73}$$

(73) is the steady-state equilibrium condition of the investment-goods market (and also the bond market). It means that, if the household sector does not change θ, the steady-state net tax revenues $IT_t^{**} + DT_t^{**} - (b_t B_t)^{**}$ must be so determined as to satisfy

$$\gamma_T = \frac{1}{\theta}\beta_{G_1} + \beta_{G_2} - \frac{1-\theta}{\theta}(g+n+\pi),\ 0 < \theta \leq 1.^{26}$$

Then, in the steady state $\gamma(=\gamma_T - \gamma_F)$ can be written as a function of π, β_{G_1}, and β_{G_2}:

$$\gamma = \gamma(\pi, \beta_{G_1}, \beta_{G_2}), \tag{74}$$

where

$$\frac{\partial\gamma}{\partial\pi} = 1 - \frac{1}{\theta} < 0,\ \frac{\partial\gamma}{\beta_{G_1}} = \frac{1}{\theta} > 0,\ \frac{\partial\gamma}{\beta_{G_2}} = 1.$$

Making use of (74), it is easily shown from (65) and (70) that

$$\frac{\partial K_{St}^{\mu**}}{\partial\pi} < 0,\ \frac{\partial K_{St}^{\mu**}}{\partial\beta_{G_2}} < 0,\ \frac{\partial K_{St}^{\mu**}}{\partial\beta_{G_1}} > 0, \tag{75}$$

and

$$\frac{\partial Q_{St}^{\mu**}}{\partial\pi} < 0,\ \frac{\partial Q_{St}^{\mu**}}{\partial\beta_{G_2}} < 0,\ \frac{\partial Q_{St}^{\mu**}}{\partial\beta_{G_1}} > 0. \tag{76}$$

That is, in the long-run steady state with θ constant, economic growth is affected adversely by an increase in π and that in β_{G_2} but favorabley by that in β_{G_1}. These facts are immediately understood by seeing the first term of the right-hand side of (63). A rise in inflation or government consumption tends to decrease capital stock as a whole, while a rise in government investment helps it increase. An increase (a decrease) in capital stock leads to an increase (a decrease) in aggregate output. The steady-state effect of a change in the inflation rate is depressive in the basic case, too. As for government fiscal policy, the following proposition holds.

Proposition 3: In the steady state, government investment has a positive effect whereas government consumption has a negative effect on real economy.

Government fiscal policy as well as money (or the inflation rate) is *not* "neutral" toward the steady-state economy.

[26] As is seen from Theorem 1, the government budget deficit need not follow (73) in the case of $\theta = 0$.

9 Analysis of the Golden-Rule State

In the steady state the household sector is able to adjust the economy by changing the rate of consumption c with other parameters as given.[27] A change in c leads to that in the long-run steady state represented by (65) - (72). As in the basic case, the golden rule is again assumed to govern the consumption behavior of the household sector, i.e., the current real consumption is to be maximized every period. But in the complete case there occurs a rather disputable thing that has to be made clear at once. What is consumption maximized? In the basic case it was undoubtedly consumption goods bought by the household sector itself. Does the same apply to the complete case? I do not know whether such a question has been taken up in macroeconomics. Yes can be an answer, but it is more convincing, I think, to regard nationl consumption, not private consumption, as what is to be maximized by the household sector because government consumption is also enjoyed by the household sector. Indeed the government sector buys consumption goods with money in the form of taxes collected from and national debt to the household sector, but it does not consume them. Thus the long-run steady state where current real national cunsumption is maximized every period is also called the golden-rule state. The golden rule discovered by Oiko Nomos is extended in this way.

Now let us analyze the golden-rule state in the complete KS model. Remembering (25) and taking the proportionality condition into consideration, current real national consumption in the steady state

$$\frac{cY_{DSt}^{\mu**} + G_{S2t}^{**}}{p_{1t}^{**}}$$

is the same as

$$Q_{S2t}^{\mu**} - \beta_{NX_2} K_{St}^{\mu**}.^{28}$$

Thus the golden rule requires the household sector to control the rate of consumption so as to maximize $Q_{S2t}^{\mu**} - \beta_{NX_2} K_{St}^{\mu**}$. It is still simple!

From (65) and (69),

$$Q_{S2t}^{\mu**} - \beta_{NX_2} K_{St}^{\mu**} = \frac{c(g+n+\pi+\beta_{NX_1}-\gamma)+\beta_2}{1-c} K_{St}^{\mu**} - \beta_{NX_2} K_{St}^{\mu**}$$

$$= \frac{c(g+n+\pi+\beta_{NX}-\gamma)+\beta_{G_2}}{1-c}\left[\frac{1-c}{g+n+\pi+\beta-\gamma+(1-c)(\delta+\gamma-\pi)}\right]^{\frac{1}{1-\alpha}} A_t N_t.$$

Let a subscript G represent the golden-rule state as before. Then, the golden-rule-state rate of consumption in the complete KS model can be obtained by solving $d(Q_{S2t}^{\mu**} - \beta_{NX_2} K_{St}^{\mu**})/dc = 0$ and $d^2(Q_{S2t}^{\mu**} - \beta_{NX_2} K_{St}^{\mu**})/dc^2 < 0$:[29]

$$c_G^{\mu} = \frac{(1-\alpha)(g+n+\delta+\beta)-\beta_{G_2}}{g+n+\pi+\beta_{NX}-\gamma+(1-\alpha)(\delta+\gamma-\pi)}, \tag{77}$$

[27]There is another possibility that the household sector changes the ratio, θ, of $S_{St}^{\mu**} - CA_{St}^{**}$ that goes to the purchase of investment goods. It is assumed below that θ is constant.

[28]It is implicitly assumed that $p_{1t}^{**} = e_t^{**} p_{f2t}^{**}$, p_{f2t}^{**} being the steady-state price of foreign consumptoin goods in foreign currency. See also footnote 18.

[29]It is easier to obtain c_G^{μ} from the fact that

$$Q_{S2t}^{\mu**} - \beta_{NX_2} K_{St}^{\mu**} = [(k_S^{\mu**})^{\alpha} - (g+n+\delta+\beta_{NX})\ k_S^{\mu**}\ ] A_t N_t,$$

which is derived from (68) - (70).

where γ is that of (74).[30] If the household sector obeys the golden rule by keeping the rate of consumption at c_G^μ, it proves successful in enjoying the maximum consumption, *all told.* How wise this househould sector is!

When $c = c_G^\mu$, $k_S^{\mu**}$ in (64) is simplified as

$$k_G^{\mu**} = \left[\frac{\alpha}{g+n+\delta+\beta_{NX}}\right]^{\frac{1}{1-\alpha}},$$

where $k_G^{\mu**} = K_{Gt}^{\mu**}/A_tN_t$, a subscript S being replaced by a subscript G. Then the complete KS model in the golden-rule state is characterized by $k_G^{\mu**}$.

As for capital stock,

$$K_{Gt}^{\mu**} = \left[\frac{\alpha}{g+n+\delta+\beta_{NX}}\right]^{\frac{1}{1-\alpha}} A_tN_t, \tag{78}$$

$$K_{G1t}^{\mu**} = \left[\alpha - \frac{\alpha\ \beta_{NX_2}}{g+n+\delta+\beta_{NX}}\right] K_{Gt}^{\mu**}, \tag{79}$$

and

$$K_{G2t}^{\mu**} = \left[(1-\alpha) + \frac{\alpha\ \beta_{NX_2}}{g+n+\delta+\beta_{NX}}\right] K_{Gt}^{\mu**}. \tag{80}$$

As for output,

$$\begin{aligned}
Q_{G1t}^{\mu**} &= A_tN_{G1t}^{\mu**}(k_{G1}^{\mu**})^\alpha = A_tN_{G1t}^{\mu**}(k_G^{\mu**})^\alpha \\
&= (g+n+\delta+\beta_{NX_1})K_{Gt}^{\mu**}, &(81)\\
Q_{G2t}^{\mu**} &= A_tN_{G2t}^{\mu**}(k_{G2}^{\mu**})^\alpha = A_tN_{G2t}^{\mu**}(k_G^{\mu**})^\alpha \\
&= \frac{(1-\alpha)(g+n+\delta+\beta_{NX}) + \alpha\ \beta_{NX_2}}{\alpha} K_{Gt}^{\mu**},\text{[31]} &(82)
\end{aligned}$$

and

$$\begin{aligned}
Q_{Gt}^{\mu**} &= Q_{G1t}^{\mu**} + Q_{G2t}^{\mu**} = A_tN_t(k_G^{\mu**})^\alpha \\
&= \left[\frac{\alpha}{g+n+\delta+\beta_{NX}}\right]^{\frac{\alpha}{1-\alpha}} A_tN_t \\
&= \frac{g+n+\delta+\beta_{NX}}{\alpha} K_{Gt}^{\mu**}, &(83)
\end{aligned}$$

[30] The golde-rule-state rate of saving is calculated as follows:

$$1 - c_G^\mu = \frac{\alpha(g+n+\pi+\beta-\gamma)}{g+n+\pi+\beta_{NX}-\gamma+(1-\alpha)(\delta+\gamma-\pi)}.$$

It is known that the rate of saving increases with population growth and productivity growth. Modigliani (1986) called the positive effects of population and productivity the Neisser effect and the Bentzel effect, respectively, and emphasized that the latter effect was diametrically opposite to the conclusion reached by the permanent income hypothesis of Friedman. The saving rate obtained here, $1 - c_G^\mu$, is an increasing function of both n and g. Thus the two effects are both predicted by the KS model as well as the life cycle hypothesis.

[31] As to the ratio of the investment-goods sector to the consumption-goods sector,

$$\frac{K_{G2t}^{\mu**}}{K_{G1t}^{\mu**}} = \frac{Q_{G2t}^{\mu**}}{Q_{G1t}^{\mu**}} = \frac{1-\alpha}{\alpha} + \frac{\beta_{NX_2}}{\alpha(g+n+\delta+\beta_{NX_1})}.$$

because of (78).[32]

Finally, as for real private disposable income, real private saving, and private consumption,

$$\begin{aligned}
\frac{Y_{DGt}^{\mu**}}{p_{1t}^{**}} &= \frac{g+n+\pi+\beta-\gamma}{1-c_G}K_{Gt}^{\mu**} \\
&= \frac{g+n+\pi+\beta_{NX}-\gamma+(1-\alpha)(\delta+\gamma-\pi)}{\alpha}K_{Gt}^{\mu**} \\
&= \frac{g+n+\delta+\beta_{NX}-\alpha(\delta+\gamma-\pi)}{g+n+\delta+\beta_{NX}}Q_{Gt}^{\mu**}, \qquad (84)
\end{aligned}$$

$$\begin{aligned}
\frac{S_{Gt}^{\mu**}}{p_{1t}^{**}} &= (g+n+\pi+\beta-\gamma)K_{Gt}^{\mu**} \\
&= \frac{g+n+\pi+\beta-\gamma}{g+n+\delta+\beta_{NX}}\alpha Q_{Gt}^{\mu**}, \qquad (85)
\end{aligned}$$

and

$$\begin{aligned}
\frac{c_G Y_{DGt}^{\mu**}}{p_{1t}^{**}} &= \left[\frac{1-\alpha}{\alpha}(g+n+\delta+\beta_{NX})-\beta_{G_2}\right]K_{Gt}^{\mu**} \\
&= \left[\frac{1-\alpha}{\alpha}-\frac{\beta_{G_2}}{g+n+\delta+\beta_{NX}}\right]\alpha Q_{Gt}^{\mu**}. \qquad (86)
\end{aligned}$$

The above results (78)-(85) are much the same as the corresponding results of the basic case. Especially in the case where $\beta_{NX_1}=\beta_{NX_2}=0$, which necessarily holds true in a closed economy, the levels of capital stock and output are just the same! Then, there are three features of the complete KS model to be stressed. The first is the same as in the basic case, i.e., the superneutrality of money. In the golden-rule state, money (or the inflation rate) does not influence real economy represented by (78) - (83).[33] The second is proper to the complete case, i.e., the ineffectiveness of government fiscal policy in the golden-rule state. As is also apparent from (78) - (83), fiscal policy has no influence on real economy represented by them because government policy parameters β_{G_1}, β_{G_2}, and γ do not appear in them. Indeed the government sector can change resource allocations by adjusting those parameters according to (74), but the golden-rule-state level of real economy is not affected by such policy changes.[34]

[32] In terms of the capital-output ratio, (83) becomes

$$\frac{K_{Gt}^{\mu**}}{Q_{Gt}^{\mu**}} = \frac{\alpha}{g+n+\delta+\beta_{NX}}.$$

[33] It should be added, however, that, while private consumption is not influenced by by the inflation rate, real private disposable income (84) and real private saving (85) are affected by it even in the golden-rule state, which shows that the relationship between a short-run and a long-run consumption functions analyzed in Sasakura (2006) remains basically unchanged in the complete case, too.

[34] This argument may remind you of the so-called Ricardian equivalence proposition due to Barro (1974). According to it, a tax reduction compensated with the same amount of newly issued government bonds with government expenditure unchanged has no impact upon consumption with an increase in saving by the amount. Does the complete KS model support the proposition ? The answer is "no and yes." "No" means that it is impossible in the first place to implement such a tax cut in the steady state. As is seen from (73) or (74), the tax ratio γ_T cannot be changed with other parameters unchanged. The tax reduction due to a decrease in γ_T leads to a collapse of the steady state. "Yes" can be considered *if* θ, which is treated as fixed in this paper, moves according to a change in γ_T so as to satisfy (73). Under such circumstances a tax reduction as suggested by the Ricardian equivalence increases the golden-rule-state disposable income (84) and also saving (85), but has no impact on consumption, national (87) or private (86).

Hence the following proposition:

Proposition 4: In the golden-rule state, government fiscal policy is ineffective in changing the level of real GDP.

The third is the stability of the *ratio* of national consumption to GDP. The stability of a variation in consumption level compared with that in income level has long been recognized by macroeconomists since Modigliani and Brumberg (1954) and Friedman (1957) provided the theoretical foundations. The complete *KS* model also shows it. Moreover, it can specify the ratio. Using (82) and (83), the maximized national consumption is calculated as follows:

$$\begin{aligned} Q^{\mu**}_{G2t} - \beta_{NX_2} K^{\mu**}_{Gt} &= \frac{1-\alpha}{\alpha}(g+n+\delta+\beta_{NX})K^{\mu**}_{Gt} \\ &= (1-\alpha)Q^{\mu**}_{Gt}. \end{aligned} \tag{87}$$

Hence the following proposition:

Proposition 5: In the golden-rule state the ratio of national consumption to GDP is $1-\alpha$.

This is a crystal-clear necessary condition for the economy to be in the golden-rule state. Given that α is usually estimated at $\frac{1}{3}$ or so, the ratio above may be around $\frac{2}{3}$.

Figure 4. The Golden-Rule State with Government and Foreign Sectors.

The above-mentioned features come into being because the household sector adjusts the golden-rule-state rate of consumption c^{μ}_{G} to a change in π, β_{G_1}, or β_{G_2}. It is understood from (77) and (74) that c^{μ}_{G} is a decreasing function of π and β_{G_2}, and an increasing function of β_{G_1}. Figure 4 makes this adjustment of c^{μ}_{G} easy to understand.

In the figure is shown the golden-rule state in terms of effective labor.[35] Let us take, as an example, the case of a change in government investment.[36] Consider three values of the ratio of real government investment to capital stock as a whole, $\beta^1_{G_1} < \beta^0_{G_1} < \beta^2_{G_1}$, and the corresponding golden-rule-state rates of consumption, $c^{\mu 1}_{G} < c^{\mu 0}_{G} < c^{\mu 2}_{G}$. When $c = c^{\mu 0}_{G}$, the economy lies on Point A^0, where *national* consumption takes the maximum value A^0B^0. B^0C^0 is the corresponding output of domestic investment goods *plus* real net export of consumption goods. Assume first that the ratio of government investment falls to $\beta^1_{G_1}$. What happens? If the economy diverges from the golden-rule state but remains in the long-run steady state, it shifts leftward, say, to Point A^1 due to (75) and (76). Both national consumption and output of domestic investment goods plus real net export of consumption goods decrease to A^1B^1 and B^1C^1, respectively. Then, what should the household sector do in order to make the maximum national consumption possible again. The answer is very simple: Accumulate capital. To do so the household sector has only to lower the rate of consumption from $c^{\mu 0}_{G}$ to

[35]See also footnote 29.

[36]The case of a changes in government consumption or inflation can be examined in the same fashion if the directions of a change in government investment mentioned below is reversed.

$c_G^{\mu 1}$. Then the economy returns to the original golden-rule state (Point A^0) with a smaller c_G^μ and a larger $1 - c_G^\mu$. This is a transitional process of adjustment to a fall of the government investment ratio β_{G_1}.

Next suppose that the government investment ratio rises from $\beta_{G_1}^0$ to $\beta_{G_1}^2$. Similarly, the economy shifts rightward, say, to Point A^2. National consumption decreases to A^2B^2, while output of domestic investment goods plus real net export of consumption goods increases to B^2C^2. What the household sector should do is to deccumulate capital. This time the household sector has only to raise the rate of consumption from $c_G^{\mu 0}$ to $c_G^{\mu 2}$. Then the economy comes back to the golden-rule state with a larger c_G^μ and a smaller $1-c_G^\mu$. This is a transitional process of adjustment to a rise in the government investment ratio. Changes in the ratio β_{G_1} in both directions never affect the value of a golden-rule-state level $k_G^{\mu **}$, as claimed in Proposition 4.

An alternative way to check the adjustment of c_G^μ lies in the the golden-rule-state version of (40):

$$\begin{aligned}\frac{c_G^\mu Y_{GDt}^{\mu *}}{S_{Gt}^{\mu *}} &= \frac{c_G^\mu}{1-c_G^\mu} \\ &= \frac{Q_{G2t}^{\mu **} - \beta_{G_2}K_{Gt}^{\mu **} - \beta_{NX_2}K_{Gt}^{\mu **}}{Q_{G1t}^{\mu **} - (\delta - \pi)K_{Gt}^{\mu **} - \gamma K_{Gt}^{\mu **} + \beta_{G_2}K_{Gt}^{\mu **} + \beta_{NX_2}K_{Gt}^{\mu **}}. \end{aligned} \tag{88}$$

Take the case of a change in government investment again. A rise in β_1 leads to that in γ of the denominator of (88) due to (74). Since it is already known that $Q_{G1t}^{\mu **}$, $Q_{G2t}^{\mu **}$, and $K_{Gt}^{\mu **}$ are independent of β_{G_1}, the rate of consumption is obliged to rise according to (88). It is an optimal reaction of the household sector to a policy change of the government sector. The dependence of private disposable income and private saving on β_1 in (84) and (85) is the result of this optimal reaction of the household sector.

10 An Example

Pigou (1950, p. 62) once stated, "In a moving world, therefore, Keynes's short-period equilibrium positions are not the positions which are at all likely ever actually to establish themselves. Thus they are on a par with the long-period equilibrium positions, always pursued but never attained, which dominate Marshall's *Principles*. ...Both [Keynes and Marshall] alike deal only with tendencies." The same applies to the short-run equilibrium state and the long-run equilibrium state analyzed in this paper. They are just results derived from simple principles of economics. They are no realities. Someone may regard them as belonging to a sort of a never-never land. Nevertheless, I believe that they are useful to get a good grasp of "tendencies" of an actual macroeconomy. Particularly, I want to show in this section that the golden-rule state[37] can describe a plausible national economy by constructing a numerical example, though I *never* intend it for your country.

As the task of this section I calculate the *golden-rule-state* ratios of various terms to GDP in the following equality:

$$\begin{aligned} GDP_{Gt}^{**} &= p_{1t}^{**}Q_{Gt}^{\mu **} \\ &= c_G^\mu Y_{DGt}^{\mu **} + G_{2t}^{**} + [I_{Gt}^{**} + p_{1t}^{**}(\delta - \pi)K_{Gt}^{\mu **}] + G_{1t}^{**} + NX_{Gt}^{**} \end{aligned}$$

[37]Remember that the golden-rule state is a special case of the steady state whih is a special case of the long-run equilibrium state which is a special case of the short-run equilibrium state.

$$= \quad w_{Gt}^{\mu**}N_t + i_G^{\mu**}\tilde{p}_{1t-1}^{**}K_{Gt}^{\mu**} + p_{1t}^{**}(\delta-\pi)K_{Gt}^{\mu**} + \mu p_{1t}^{**}Q_{Gt}^{\mu**}, \tag{89}$$

where NX_{Gt}^{**} is net export of consumption and investment goods in the golden-rule state, and by definition $NX_{Gt}^{**} = CA_{Gt}^{**} - F_t^{**}$. The first line of (89) follows from (50) and (51). The second line is derived from the last line of (39). The third line can be obtained from (11) and (18), using the condition that $h_{1t}^{**} = i_G^{\mu**} = h_{2t}^{**}$ with $i_G^{\mu**}$ as the golden-rule-state value of the nominal interest rate. The first, second, and third lines can be called respectively GDP from the production side, that from the expenditure side, and that from distribution side.

The SNA relationship as shown in (89) appears in every textbook of macroeconomics, not to mention, but it is explained largely in terms of empirical evidence. Indeed it matters what it is, but it also matters why it is so. Now let us examine the macroeconomy with the following parameters:

Example: $g = 0.015$, $n = 0.001$, $\delta = 0.060$, $\alpha = \frac{1}{3}$, $\pi = 0.020$, $\mu = 0.080$, $\beta_{G_1} = 0.019$, $\beta_{G_2} = 0.023$, $\beta_{NX} = 0.002$, $\gamma_T = 0.040$, and $\gamma_F = 0.002$.

The complete KS model in the golden-rule state is almost characterized by these parameters.[38]

First consider the second line of (89). Private consumption is calculated at once using (86) as $c_G^{\mu}Y_{DGt}^{\mu**} = 0.568p_{1t}^{**}Q_{Gt}^{\mu**}$.[39] From (83), $K_{Gt}^{\mu**} = 4.274p_{1t}^{**}Q_{Gt}^{\mu**}$, or the capital-output ratio is 4.274. Then government consumption: $G_{2t}^{**} = 0.023p_{1t}^{**}K_{Gt}^{\mu**} = 0.098p_{1t}^{**}Q_{Gt}^{\mu**}$, capital consumption: $p_{1t}^{*}(\delta-\pi)K_{Gt}^{\mu**} = 0.040p_{1t}^{**}K_{Gt}^{\mu**} = 0.171p_{1t}^{**}Q_{Gt}^{\mu**}$, government investment: $G_{1t}^{**} = 0.019p_{1t}^{**}K_{Gt}^{\mu**} = 0.081p_{1t}^{**}Q_{Gt}^{\mu**}$, and net export: $NX_{Gt}^{**} = 0.002p_{1t}^{**}K_{Gt}^{\mu**} = 0.009p_{1t}^{**}Q_{Gt}^{\mu**}$. Private investment I_{Gt}^{**} can, of course, be obtained by subtracting the sum of these results from $p_{1t}^{**}Q_{Gt}^{\mu**}$ with the result as $I_{Gt}^{**} = 0.073p_{1t}^{**}Q_{Gt}^{\mu**}$. Hence private gross investment: $I_{Gt}^{**} + p_{1t}^{*}(\delta-\pi)K_{Gt}^{\mu**} = 0.244p_{1t}^{**}Q_{Gt}^{\mu**}$.

Another method of knowing I_{Gt}^{**} is to look how much is used for it from private saving. From (85), private saving can immediately be calculated as $S_{Gt}^{\mu**} = 0.098p_{1t}^{**}Q_{Gt}^{\mu**}$. Furthremore, since current account is the sum of net export and net receipt of factor income from the foreign sector, $CA_{Gt}^{**} = NX_{Gt}^{**} + F_t^{**} = 0.004p_{1t}^{**}K_{Gt}^{\mu**} = 0.017p_{1t}^{**}Q_{Gt}^{\mu**}$. Then the amount that goes to the purchase of investment goods and government bonds becomes $S_{Gt}^{\mu**} - CA_{Gt}^{**} = 0.081p_{1t}^{**}Q_{Gt}^{\mu**}$. From (73), $\theta = 0.895$, i.e., it is found that the household sector uses the ratio 0.895 of the above-mentioned amount for buying investment goods. Therefore, $I_{Gt}^{**} = 0.895(S_{Gt}^{\mu**} - CA_{Gt}^{**}) = 0.073p_{1t}^{**}Q_{Gt}^{\mu**}$, which coincides with the preceding calculation. Incidentally, the budget deficit becomes $BD_{Gt}^{**} = 0.105(S_{Gt}^{\mu**} - CA_{Gt}^{**}) = 0.009p_{1t}^{**}Q_{Gt}^{\mu**}$. The results concerning the second line is summarized in Table 1.

Table 1. GDP from the Expenditure Side: An Example.

Next consider the third line of (89). Since $p_{1t}^{**}(\delta-\pi)K_{Gt}^{\mu**}$ and $\mu p_{1t}^{**}Q_{Gt}^{\mu**}(= 0.080p_{1t}^{**}Q_{Gt}^{\mu**})$ are known, it suffices to calculate those of $w_{Gt}^{\mu**}N_t$ and $i_G^{\mu**}\tilde{p}_{1t-1}^{**}K_{Gt}^{\mu**}$. From (53), labor income can be written as

$$w_{Gt}^{\mu**}N_t = p_{1t}^{**}(1-\mu)(1-\alpha)A_tN_t(k_G^{\mu**})^{\alpha}$$

[38]Moreover, either a value of β_{NX_1} or that of β_{NX_2} is necessary to be able to calculate $Q_{G1t}^{\mu**}$ and $Q_{G2t}^{\mu**}$. See (81) and (82).

[39]All results in this section except Tables 1 and 2 are rounded off to three decimal places.

$$= (1-\mu)(1-\alpha)p_{1t}^{**}Q_{Gt}^{\mu**}.$$[40]

Therefore, $w_{Gt}^{\mu**}N_t = (1-0.080)(1-\frac{1}{3})p_{1t}^{**}Q_{Gt}^{\mu**} = 0.613p_{1t}^{**}Q_{Gt}^{\mu**}$. The golden-rule-state interest rate can be obtained from (58) as follows:

$$i_G^{\mu**} = \frac{1}{1-\pi}\,[(1-\mu)\alpha(k_G^{\mu**})^{\alpha-1} - \delta + \pi]$$
$$= \frac{1}{1-\pi}\,[(1-\mu)(g+n+\delta+\beta_{NX}) - \delta + \pi].$$

Taking the long-run price condition (48) into account, capital income is written as

$$i_G^{\mu**}\tilde{p}_{1t-1}^{**}K_{Gt}^{\mu**} = [(1-\mu)(g+n+\delta+\beta_{NX}) - \delta + \pi]\,p_{1t}^{**}K_{Gt}^{\mu**}.$$

Therefore, $i_G^{\mu**}\tilde{p}_{1t-1}^{**}K_{Gt}^{\mu**} = 0.032p_{1t}^{**}K_{Gt}^{\mu**} = 0.136p_{1t}^{**}Q_{Gt}^{\mu**}$. Incidentally, $i_G^{\mu**} = 0.032$ and $r_G^{\mu**} = 0.012$.[41] The results concerning the third line is summarized in Table 2.

Table 2. GDP from the Distribution Side: An Example.

Finally and most important, the golden-rule-state rate of consumption is calculated from (77) as $c_G^{\mu} = 0.853$. It is no exaggeration to say that in the golden-rule state the macroeconomy is governed by this rate and the household sector that determines it.

11 Conclusion

In reality a macroeconomy is made up of virtually uncountable firms and households interacting with the government and foreign sectors. Although it is conventional to say that a macroeconomy is complex, it certainly is and it will be. That is why an actual macroeconomy should be simplified as a macro model. In this paper I constructed a simple macro model using the wisdom of Keynes and Solow, but tried to make it as realistic as possible by taking the four sectors into consideration.

The main results obtained are as follows:

[40]As is apparent from (53), this relation between labor income and GDP has already held as soon as the economy entered the long-run equilibrium state.

[41]From (57),

$$r_G^{\mu**} + \delta = (1-\mu)\alpha(k_G^{\mu**})^{\alpha-1}$$
$$= (1-\mu)(g+n+\delta+\beta_{NX}).$$

Therefore, $r_G^{\mu**} = (1-\mu)(g+n+\delta+\beta_{NX}) - \delta = 0.012$. The following result is also interesting in relation to footnote 22.

$$p_{1t}^{**}(r_G^{\mu**}+\delta)K_{Gt}^{\mu**} = p_{1t}^{**}(1-\mu)(g+n+\delta+\beta_{NX})\frac{\alpha}{g+n+\delta+\beta_{NX}}Q_{Gt}^{\mu**}$$
$$= (1-\mu)\alpha p_{1t}^{**}Q_{Gt}^{\mu**}.$$

It means that the golden-rule-state value of capital stock can be written formally as the discounted present value of the return on it as follows:

$$\begin{aligned} K_{Gt}^{\mu**} &= \frac{(1-\mu)\alpha Q_{Gt}^{\mu**} - \delta K_{Gt}^{\mu**}}{r_G^{\mu**}} \\ &= \frac{\alpha Q_{Gt}^{\mu**} - \delta K_{Gt}^{\mu**} - \mu\alpha Q_{Gt}^{\mu**}}{1+r_G^{\mu**}} + \frac{\alpha Q_{Gt}^{\mu**} - \delta K_{Gt}^{\mu**} - \mu\alpha Q_{Gt}^{\mu**}}{(1+r_G^{\mu**})^2} + \cdots. \end{aligned}$$

1. The existence of the government sector invalidates the quantity theory of money.
2. In the short run government fiscal policy is effective.
3. In the long-run steady state government investment has a favorable effect on production and income, whereas government consumption has an adverse effect.
4. In the golden-rule state fiscal policy is ineffective.
5. The complete *KS* model predicts that the golden-rule ratio of national consumption to GDP is roughly two thirds.

Result 1 suggests the ineffectiveness of monetary policy of the central bank. Indeed the growth rate of money supply and that of prices are the same in the long-run equilibrium state with a constant income velocity of money, but it is the result of a causal relation from economic activity to money supply. Results 2-4 are concerned with government fiscal policy. The effectiveness of fiscal policy has long been discussed in macroeconomics. The *KS* model is able to give *consistent* answers because it is *only one* throughout. The prediction of Result 5 is simple and can be checked on the spot. It is rather fragile as far as data available are concerned. But data are not necessarily arranged for a particular theory. Thus the correspondence between data and theory needs to be further examined.

Finally I ask you to remember what this paper as well as Sasakura (2006) has been written for. It is because I believe that current macroeconomics textbooks based on the three-model structure is theoretically inconsistent and that another model should be newly constructed. The Keynes-Solow model is my solution.

Appendices

A Proof of Lemma

That part of private saving which amounts to CA_t^e necessarily goes to the purchase of foreign bonds, and the rest of it can be used to buy either investment goods or government bonds. Let $0 \leq \theta \leq 1$ and $0 \leq \theta_B \leq 1$ be the ratio of $S_t^{\mu e} - CA_t^e$ that goes to the purchase of investment goods and government bonds, respectively. The household sector uses the amount equal to IM_{1t}^e to buy foreign investment goods. Thus the nominal demand for domestic investment goods is the sum of $\theta(S_t^{\mu e} - CA_t^e) - IM_{1t}^e$, $p_{1t}^e(\delta - \pi_t^e)K_t$, G_{1t}, and EX_{1t}^e. On the other hand, the nominal supply of domestic investment goods is $p_{1t}^e Q_{1t}^{\mu e}$. Thus the equilibrium of the market is described by

$$p_{1t}^e Q_{1t}^{\mu e} = \theta(S_t^e - CA_t^e) - IM_{1t}^e + p_{1t}^e(\delta - \pi_t^e)K_t + G_{1t} + EX_{1t}^e.$$

Since $S_t^{\mu e} - CA_t^e = I_t^e + BD_t^e$, the above equality leads to $I_t^e = \theta(I_t^e + BD_t^e)$.

The equilibrium of the market of newly issued bonds is described simply by

$$p_{Bt}(B_{t+1}^e - B_t) = \theta_B(S_t^{\mu e} - CA_t^e).$$

Since government bonds are newly issued according to the budget constraint (19), the above relation leads to

$$G_{1t} + G_{2t} - (IT_t^e + DT_t^e - b_t B_t) = \theta_B(S_t^{\mu e} - CA_t^e).$$

Therefore, the bond market equilibrium leads to $BD_t^e = \theta_B(I_t^e + BD_t^e)$. Combining the results of the equilibrium conditions in the investment-goods and bond markets yields

$$(1 - \theta - \theta_B)(I_t^e + BD_t^e) = 0.$$

Money hoarding is equivalent to $0 \le \theta + \theta_B < 1$. In such a case $I_t^e + BD_t^e = 0$, i.e., $S_t^{\mu e} = CA_t^e$. It follows from (30) that

$$Y_{Dt}^{\mu e} = \frac{1}{1-c} CA_t^e.$$

Q.E.D.

B Proof of Theorem 1

From the proof in Appendix A, the equilibrium condition for the investment-goods is $I_t^e = \theta(I_t^e + BD_t^e)$, while for the bond market $BD_t^e = \theta_B(I_t^e + BD_t^e)$. But they are the same if the no-Pope's-father condition holds, i.e., $\theta + \theta_B = 1$. Furthermore, it is obvious that $I_t^e = 0$ for $\theta = 0$ or $\theta_B = 1$ while $BD_t^e = 0$ for $\theta = 1$ or $\theta_B = 0$. Q.E.D.

C Derivation of Supply Curve $Q_{1t}^{\mu S}$ and Demand Curve $Q_{1t}^{\mu D}$

The investment-goods supply curve $Q_{1t}^{\mu S}$ is none other than (8). To express it in a usual way, replace $Q_{1t}^{\mu e}$ and p_{1t}^e in (8) respectively with $Q_{1t}^{\mu S}$ and p_{1t} below. Then,

$$Q_{1t}^{\mu S} = p_{1t}^{\frac{1-\alpha}{\alpha}} \left[\frac{(1-\mu)(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}} K_{1t}.$$

To examine the shape of the graph, differentiate $Q_{1t}^{\mu S}$ w.r.t. p_{1t} once and twice. Then,

$$\frac{dQ_{1t}^{\mu S}}{dp_{1t}} = \frac{1-\alpha}{\alpha} p_{1t}^{\frac{1-2\alpha}{\alpha}} \left[\frac{(1-\mu)(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}} K_{1t} > 0,$$

and

$$\frac{d^2 Q_{1t}^{\mu S}}{dp_{1t}^2} = \frac{1-\alpha}{\alpha}\frac{1-2\alpha}{\alpha} p_{1t}^{\frac{1-3\alpha}{\alpha}} \left[\frac{(1-\mu)(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}} K_{1t} \begin{cases} > 0 & \text{if } 0 < \alpha < \frac{1}{2}, \\ = 0 & \text{if } \alpha = \frac{1}{2}, \\ < 0 & \text{if } \frac{1}{2} < \alpha < 1. \end{cases}$$

The shape of the supply curve in Figure 1 reflects the macro fact that α is around $\frac{1}{3}$.

Next consider the investment-goods demand curve $Q_{1t}^{\mu D}$. As already mentioned in Appendix A, the nominal demand for investment goods is the sum of $\theta(S_t^{\mu e} - CA_t^e) - IM_{1t}^e$, $p_{1t}^e(\delta - \pi_t^e)K_t$, G_{1t}, and EX_{1t}^e. Then,

$$Q_{1t}^{\mu D} = \frac{1}{p_{1t}}[\theta(S_t^{\mu e} - CA_t^e) - IM_{1t}^e + p_{1t}^e(\delta - \pi_t^e)K_t + G_{1t} + EX_{1t}^e],$$

where $S_t^{\mu e}$ and CA_t^e are respectively given in (24) and (31). Substituting (26) into the above equality yields

$$Q_{1t}^{\mu D} = \theta(1-c+m_2)(1-\tau)(1-\mu)Q_{1t}^{\mu S} + \frac{[1-\theta(1-c+m_2)(1-\tau)]\tilde{p}_{1t-1}K_t + D_t}{p_{1t}}$$

$$-[1-\theta(1-c+m_2)(1-\tau)](1-\delta)K_t,$$

where

$$D_t = \theta(1-c+m_2)(1-\tau)(1-\mu)p^e_{2t}Q^{\mu e}_{2t} + \theta(1-c+m_2)b_tB_t + G_{1t}$$
$$-\theta[1-(1-c+m_2)(1-\tau)]F^e_t + (1-\theta)(EX^e_{1t} - IM^e_{1t}) - \theta EX^e_{2t}.$$

This is the investment-goods demand curve with p^e_{2t} as given. It would be reasonable to assume that $[1-\theta(1-c+m_2)(1-\tau)]\tilde{p}_{1t-1}K_t + D_t \geq 0$.

Differentiate $Q^{\mu D}_{1t}$ w.r.t. p_{1t} once and twice. Then,

$$\frac{dQ^{\mu D}_{1t}}{dp_{1t}} = \theta(1-c+m_2)(1-\tau)(1-\mu)\frac{1-\alpha}{\alpha}p_{1t}^{\frac{1-2\alpha}{\alpha}}\left[\frac{(1-\mu)(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}} K_{1t}$$
$$-\frac{[1-\theta(1-c+m_2)(1-\tau)]\tilde{p}_{1t-1}K_t + D_t}{p^2_{1t}},$$

and

$$\frac{d^2Q^{\mu D}_{1t}}{dp^2_{1t}} = \theta(1-c+m_2)(1-\tau)(1-\mu)\frac{1-\alpha}{\alpha}\frac{1-2\alpha}{\alpha}p_{1t}^{\frac{1-3\alpha}{\alpha}}\left[\frac{(1-\mu)(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}} K_{1t}$$
$$+2\frac{[1-\theta(1-c+m_2)(1-\tau)]\tilde{p}_{1t-1}K_t + D_t}{p^3_{1t}} > 0.$$

It follows from these results that demand curve $Q^{\mu D}_{1t}$ is bending forward and that it changes the sign of the slope at $p_{1t} = \bar{p}^{\mu}_{1t}$, where

$$\bar{p}^{\mu}_{1t} = \left[\frac{\alpha}{1-\alpha}\frac{1}{\theta(1-c+m_2)(1-\tau)(1-\mu)}\right]^{\alpha}\left[\frac{w_t}{(1-\mu)(1-\alpha)A_t}\right]^{1-\alpha}$$
$$\times\left\{\frac{[1-\theta(1-c+m_2)(1-\tau)]\tilde{p}_{1t-1}K_t + D_t}{K_{1t}}\right\}^{\alpha}.$$

The demand curve in Figure 1 is pictured under the assumption that $p^e_{1t} < \bar{p}^{\mu}_{1t}$.

D Derivation of Supply Curve $Q^{\mu S}_{2t}$ and Demand Curve $Q^{\mu D}_{2t}$

The consumption-goods supply curve $Q^{\mu S}_{2t}$ is none other than (15). To express it in a usual way, replace $Q^{\mu e}_{2t}$ and p^e_{2t} in (15) respectively with $Q^{\mu S}_{2t}$ and p_{2t}. Then the argument on $Q^{\mu S}_{1t}$ in Appendix C applies to that on $Q^{\mu S}_{2t}$ in the same fashion, a subscript 1 being replaced by a subscript 2. Therefore, let us focus on the consumption-goods demand curve $Q^{\mu D}_{2t}$. The nominal demand for consumption goods is given by the consumption function (22). Then,

$$Q^{\mu D}_{2t} = \frac{1}{p_{2t}}[cY^{\mu e}_{Dt} + G_{2t} + EX_{2t} - m_2Y^{\mu e}_{Dt}].$$

Substituting (26) into the above equality yields

$$Q^{\mu D}_{2t} = (c-m_2)(1-\tau)(1-\mu)Q^{\mu S}_{2t}$$
$$+\frac{(c-m_2)\{(1-\tau)[(1-\mu)p^e_{1t}Q^{\mu e}_{1t} + F^e_t - p^e_{1t}(\delta-\pi^e_t)K_t] + b_tB_t\} + G_{2t} + EX^e_{2t}}{p_{2t}}.$$

This is the consumption-goods demand curve with p_{1t}^e as given.

Differentiate $Q_{2t}^{\mu D}$ w.r.t. p_{2t} once and twice. Then,

$$\frac{dQ_{2t}^{\mu D}}{dp_{2t}} = (c-m_2)(1-\tau)(1-\mu)\frac{1-\alpha}{\alpha}p_{2t}^{\frac{1-2\alpha}{\alpha}}\left[\frac{(1-\mu)(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}}K_{2t}$$

$$-\frac{(c-m_2)\{(1-\tau)[(1-\mu)p_{1t}^e Q_{1t}^{\mu e}+F_t^e-p_{1t}^e(\delta-\pi_t^e)K_t]+b_tB_t\}+G_{2t}+EX_{2t}^e}{p_{2t}^2},$$

and

$$\frac{d^2Q_{2t}^{\mu D}}{dp_{2t}^2} = (c-m_2)(1-\tau)(1-\mu)\frac{1-\alpha}{\alpha}\frac{1-2\alpha}{\alpha}p_{2t}^{\frac{1-3\alpha}{\alpha}}\left[\frac{(1-\mu)(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}}K_{2t}$$

$$+2\frac{(c-m_2)\{(1-\tau)[(1-\mu)p_{1t}^e Q_{1t}^{\mu e}+F_t^e-p_{1t}^e(\delta-\pi_t^e)K_t]+b_tB_t\}+G_{2t}+EX_{2t}^e}{p_{2t}^3} > 0.$$

It follows from these results that demand curve $Q_{2t}^{\mu D}$ is bending forward and that it changes the sign of the slope at $p_{2t} = \bar{p}_{2t}^{\mu}$, where

$$\bar{p}_{2t}^{\mu} = \left[\frac{\alpha}{1-\alpha}\frac{1-(c-m_2)(1-\tau)(1-\mu)}{(c-m_2)(1-\tau)(1-\mu)}\right]^{\alpha}\left[\frac{w_t}{(1-\mu)(1-\alpha)A_t}\right]^{1-\alpha}$$

$$\times\left\{\frac{1}{K_{2t}}\frac{(c-m_2)\{(1-\tau)[(1-\mu)p_{1t}^e Q_{1t}^{\mu e}+F_t^e-p_{1t}^e(\delta-\pi_t^e)K_t]+b_tB_t\}+G_{2t}+EX_{2t}^e}{1-(c-m_2)(1-\tau)(1-\mu)}\right\}^{\alpha}.$$

The position of the demand curve in Figure 2 reflects the assumption that $\alpha < (c-m_2)(1-\tau)(1-\mu)$. This means that $\bar{p}_{2t}^{\mu}$ is smaller than p_{2t}^e in (28).

E Proof of Theorem 2

The equilibrium of the investment-goods market *alone* can be expressed using (41) with the asterisks on the prices deleted as follows:

$$\eta_{111t}\,p_{1t}^{\frac{1}{\alpha}} + \eta_{112t}\,p_{1t} + \eta_{12t}\,p_{2t}^{\frac{1}{\alpha}} = \xi_{1t}, \tag{90}$$

where

$$\begin{aligned}
\eta_{111t} &= [1-\theta(1-\mu)(1-\tau)]\left[\frac{(1-\mu)(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}}K_{1t} > 0,\\
\eta_{112t} &= [1-\theta(1-\tau)](1-\delta)K_t > 0,\\
\eta_{12t} &= \theta[1-(1-\mu)(1-\tau)]\left[\frac{(1-\mu)(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}}K_{2t} > 0,\\
\xi_{1t} &= [1-\theta(1-\tau)]\bar{p}_{1t-1}K_t + G_{1t} + \theta G_{2t} + (1-\theta)(EX_{1t}^* - IM_{1t}^*) - \theta\tau F_t^* + \theta b_t B_t.
\end{aligned}$$

Similarly the equilibrium of the consumption-goods market *alone* can be written using (40) as follows:

$$\eta_{211t}\,p_{1t}^{\frac{1}{\alpha}} + \eta_{212t}\,p_{1t} - \eta_{22t}p_{2t}^{\frac{1}{\alpha}} = \xi_{2t}, \tag{91}$$

where

$$
\begin{aligned}
\eta_{211t} &= (c-m_2)(1-\mu)(1-\tau)\left[\frac{(1-\mu)(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}} K_{1t} > 0, \\
\eta_{212t} &= [(c-m_2)(1-\tau)+c\mu\tau](1-\delta)K_t > 0, \\
\eta_{22t} &= [1-(c-m_2)(1-\mu)(1-\tau)]\left[\frac{(1-\mu)(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}} K_{2t} > 0, \\
\xi_{2t} &= (c-m_2)[(1-\tau)(\tilde{p}_{1t-1}K_t - F_t^*) - b_t B_t] - G_{2t} - EX_{2t}^*.
\end{aligned}
$$

Needless to say, p_{1t}^* and p_{2t}^* in (40) and (41) are the solution of the simultaneous equations (90) and (91). Thus, in the neigborhood of the short-run equilibrium state, the following relation holds:

$$
\begin{pmatrix} \frac{\eta_{111t}}{\alpha}(p_{1t}^*)^{\frac{1-\alpha}{\alpha}} + \eta_{112t} & \frac{\eta_{12t}}{\alpha}(p_{2t}^*)^{\frac{1-\alpha}{\alpha}} \\ \frac{\eta_{211t}}{\alpha}(p_{1t}^*)^{\frac{1-\alpha}{\alpha}} + \eta_{212t} & -\frac{\eta_{22t}}{\alpha}(p_{2t}^*)^{\frac{1-\alpha}{\alpha}} \end{pmatrix} \begin{pmatrix} dp_{1t} \\ dp_{2t} \end{pmatrix} = \begin{pmatrix} d\xi_{1t} \\ d\xi_{2t} \end{pmatrix}.
$$

Denote the determinant of the above coefficient matrix by Det_t, that is,

$$
Det_t = -\left[\frac{\eta_{111t}}{\alpha}(p_{1t}^*)^{\frac{1-\alpha}{\alpha}} + \eta_{112t}\right]\left[\frac{\eta_{22t}}{\alpha}(p_{2t}^*)^{\frac{1-\alpha}{\alpha}}\right] - \left[\frac{\eta_{211t}}{\alpha}(p_{1t}^*)^{\frac{1-\alpha}{\alpha}} + \eta_{212t}\right]\left[\frac{\eta_{12t}}{\alpha}(p_{2t}^*)^{\frac{1-\alpha}{\alpha}}\right] < 0.
$$

Then it is easy to derive the following partial derivatives:

$$
\begin{aligned}
\frac{\partial p_{1t}^*}{\partial G_{1t}} &= -\frac{\eta_{22t}(p_{2t}^*)^{\frac{1-\alpha}{\alpha}}}{\alpha\, Det_t}, \\
\frac{\partial p_{2t}^*}{\partial G_{1t}} &= -\frac{\eta_{211t}(p_{1t}^*)^{\frac{1-\alpha}{\alpha}} + \alpha\,\eta_{212t}}{\alpha\, Det_t}, \\
\frac{\partial p_{1t}^*}{\partial G_{2t}} &= -\theta[1-(c-m_2)](1-\mu)(1-\tau)\frac{(p_{2t}^*)^{\frac{1-\alpha}{\alpha}}}{\alpha\, Det_t}\left[\frac{(1-\mu)(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}} K_{2t}, \\
\frac{\partial p_{2t}^*}{\partial G_{2t}} &= -\frac{(\eta_{111t}+\theta\,\eta_{211t})(p_{1t}^*)^{\frac{1-\alpha}{\alpha}} + \alpha\,(\eta_{112t}+\theta\,\eta_{212t})}{\alpha\, Det_t}.
\end{aligned}
$$

And obviously the signs are all positive. Q.E.D.

F Derivation of Capital Accumulation Equation (63)

$$
\begin{aligned}
k_{t+1}^{\mu**} &= \frac{1-\ \delta-\beta_{NX_1}}{(1+g)(1+n)}k_t^{\mu**} + \frac{A_t N_{1t}^{**}}{A_{t+1}N_{t+1}}(k_{1t}^{\mu**})^\alpha \\
&= \frac{1-\delta-\beta_{NX_1}}{(1+g)(1+n)}k_t^{\mu**} + \frac{1}{(1+g)(1+n)}\frac{N_{1t}^{**}}{N_t}(k_{1t}^{\mu**})^\alpha \\
&= \frac{1-\delta-\beta_{NX_1}}{(1+g)(1+n)}k_t^{\mu**} + \frac{1}{(1+g)(1+n)}\{1-c+[c(\delta+\gamma-\pi)-\beta_2](k_t^{\mu**})^{1-\alpha}\}(k_t^{\mu**})^\alpha \\
&= \frac{1-\delta-\beta_{NX_1}-\beta_2+c(\delta+\gamma-\pi)}{(1+g)(1+n)}k_t^{\mu**} + \frac{1-c}{(1+g)(1+n)}(k_t^{\mu**})^\alpha \\
&= \frac{1-\delta-\beta+c(\delta+\gamma-\pi)}{(1+g)(1+n)}k_t^{\mu**} + \frac{1-c}{(1+g)(1+n)}(k_t^{\mu**})^\alpha.
\end{aligned}
$$

G Indirect Tax and Tobin's q

In the presence of indirect tax Tobin's (average and also marginal) q is modified as $[(1-\mu)p^e_{1t}/\hat{p}^e_{1t}]^{1/\alpha}$. Here is a proof. Multiplying each side of (8) by $(1-\mu)p^e_{1t}$ yields planned amount of production of investment goods

$$\begin{aligned}(1-\mu)p^e_{1t}Q^{\mu e}_{1t} &= (1-\mu)p^e_{1t}\left[(1-\alpha)A_t\frac{(1-\mu)p^e_{1t}}{w_t}\right]^{\frac{1-\alpha}{\alpha}}K_{1t}\\ &= \left[\frac{(1-\mu)p^e_{1t}}{\hat{p}^e_{1t}}\right]^{\frac{1}{\alpha}}\frac{r^e_t+\delta}{\alpha}p^e_{1t}K_{1t}. \qquad (92)\end{aligned}$$

Therefore,

$$\left[\frac{(1-\mu)p^e_{1t}}{\hat{p}^e_{1t}}\right]^{\frac{1}{\alpha}} = \frac{(1-\mu)p^e_{1t}\frac{\alpha Q^{\mu e}_{1t}}{r^e_t+\delta}}{p^e_{1t}K_{1t}}. \qquad (93)$$

The denominator of the right-hand side of (93) represents the value of existing capital stock evaluated at the expected price p^e_{1t} of investment goods as flow at the first subperiod of period t, and p^e_{1t} is the replacement cost of capital stock. $(1-\mu)p^e_{1t}\alpha Q^{\mu e}_{1t}$ is the expected gross return on existing capital stock because

$$\begin{aligned}(1-\mu)p^e_{1t}Q^{\mu e}_{1t} - w_tN^{\mu e}_{1t} &= (1-\mu)p^e_{1t}Q^{\mu e}_{1t} - (1-\mu)p^e_{1t}(1-\alpha)Q^{\mu e}_{1t}\\ &= (1-\mu)p^e_{1t}\alpha Q^{\mu e}_{1t},\end{aligned}$$

due to (9). Since

$$(1-\mu)p^e_{1t}\frac{\alpha Q^{\mu e}_{1t}}{r^e_t+\delta} \approx \frac{(1+\pi^e_t)(1-\mu)p^e_{1t}\alpha Q^{\mu e}_{1t}}{1+i_t+\delta} + \frac{(1+\pi^e_t)^2(1-\mu)p^e_{1t}\alpha Q^{\mu e}_{1t}}{(1+i_t+\delta)^2} + \cdots,$$

the numerator of the right-hand of (93) may be thought of as the discounted present value of the gross return on capital, or the value of capital stock. Thus, $[(1-\mu)p^e_{1t}/\hat{p}^e_{1t}]^{1/\alpha}$ corresponds to what Tobin (1969, p. 21) called q which is "the value of capital relative to its replacement cost." (See footnote 41 for the value of capital stock in the golden-rule state.)

Apparently the right-hand side of (93) represents Tobin's average q. But it is also marginal q because

$$\begin{aligned}\frac{d\left[(1-\mu)p^e_{1t}\frac{\alpha Q^{\mu e}_{1t}}{r^e_t+\delta}\right]}{d[p^e_{1t}K_{1t}]} &= \frac{\frac{d[(1-\mu)\alpha Q^{\mu e}_{1t}]}{dK_{1t}}}{r^e_t+\delta}\\ &= \left[\frac{(1-\mu)p^e_{1t}}{\hat{p}^e_{1t}}\right]^{\frac{1}{\alpha}},\end{aligned}$$

due to (92). Therefore, Tobin's q, average and marginal, is $[(1-\mu)p^e_{1t}/\hat{p}^e_{1t}]^{1/\alpha}$.

There are two qs in the complete KS model, too. They may be called q^μ_{1t} and q^μ_{2t}, where

$$q^\mu_{1t} = \left[\frac{(1-\mu)p^e_{1t}}{\hat{p}^e_{1t}}\right]^{\frac{1}{\alpha}},$$

and

$$q^\mu_{2t} = \left[\frac{(1-\mu)p^e_{2t}}{\hat{p}^e_{1t}}\right]^{\frac{1}{\alpha}} = \frac{(1-\mu)p^e_{2t}\frac{\alpha Q^{\mu e}_{2t}}{r^e_t+\delta}}{p^e_{1t}K_{2t}}.$$

q^{μ}_{1t} corresponds to the original q, while q^{μ}_{2t} is q of the consumption-goods sector in the complete case. q^{μ}_{1t} and q^{μ}_{2t} appeared respectively in (45) and the first half of (47) where e is replaced with $*$.

H Indirect Tax and the *M-M* Theorem

The *M-M* theorem holds true, if modified, even in the presence of indirect tax. Furthermore, it can be shown, using the complete *KS* model, that the *M-M* theorem and Tobin's q theory remain mathematically equivalent under such circumstances, too. As explained in Sasakura (2006), Modigliani and Miller (1958) focused on a group of firms or an industry which is characterized by ρ_k with k as a class of the group. ρ_k is the expected rate of return on equities in the absence of debt-financing, where all of profit earned belongs to equity holders. They proved three propositions concerning the cost of capital. So let us proceed in order again.

First take the investment-goods sector as an industry examined here and let k be 1. (In the case of the consumption-goods sector $k = 2$.) Then, in the complete *KS* model,

$$\rho^{\mu}_{1t} \equiv \frac{(1-\mu)p^{e}_{1t}Q^{\mu e}_{1t} - w_t N^{\mu e}_{1t} - p^{e}_{1t}(\delta - \pi^{e}_{t})K_{1t}}{\tilde{p}_{1t-1}K_{1t}} \tag{94}$$

$$= \frac{p^{e}_{1t}}{\tilde{p}_{1t-1}}\left\{\left[\frac{(1-\mu)p^{e}_{1t}}{\hat{p}^{e}_{1t}}\right]^{\frac{1}{\alpha}}(r^{e}_{t}+\delta) - (\delta - \pi^{e}_{t})\right\}, \tag{95}$$

because of (7) and (8). ρ^{μ}_{1t} in (95) corresponds to ρ_k in the original *M-M* theorem when indirect tax is imposed at the rate μ.

Let $\bar{X}^{\mu}_{1t}$ stand for the expected return on the assets owned by the investment-goods sector. Denote by D^{μ}_{1t} the market value of the debts of the sector; by S^{μ}_{1t} the market value of its equities: and by $V^{\mu}_{1t} \equiv S^{\mu}_{1t} + D^{\mu}_{1t}$ the market value of the sector. In terms of the complete *KS* model, $\bar{X}^{\mu}_{1t} = (1-\mu)p^{e}_{1t}Q^{\mu e}_{1t} - w_t N^{\mu e}_{1t} - p^{e}_{1t}(\delta - \pi^{e}_{t})K_{1t}$, $D^{\mu}_{1t} = \tilde{p}_{1t-1}K^{d}_{1t}$, and $S^{\mu}_{1t} = \tilde{p}_{1t-1}K^{h}_{1t}$, where $K^{h}_{1t} + K^{d}_{1t} = K_{1t}$. Then, the budget constraint on the investment-goods sector (4) can be written as:

$$V^{\mu}_{1t} \equiv S^{\mu}_{1t} + D^{\mu}_{1t} = \frac{\bar{X}^{\mu}_{1t}}{\rho^{\mu}_{1t}}. \tag{96}$$

(96) corresponds to Proposition I of Modigliani and Miller (1958, p. 268). The average cost of capital of the investment-goods sector is defined as the ratio of the expected return to the market value. Then, (96) can also be expressed as:

$$\frac{\bar{X}^{\mu}_{1t}}{V^{\mu}_{1t}} = \rho^{\mu}_{1t}.$$

That is, the average cost of capital of the investment-goods sector is completely independent of its capital structure and is equal to the capitalization rate ρ^{μ}_{1t} of a pure equity stream of the sector.

Next consider the relationship among h^{e}_{1t}, i_t, and ρ^{μ}_{1t}. From (43),

$$h^{e}_{1t} - i_t = \frac{(1-\mu)p^{e}_{1t}Q^{\mu e}_{1t} - w_t N^{\mu e}_{1t} - p^{e}_{1t}(\delta - \pi^{e}_{t})K_{1t} - p^{e}_{1t}(r^{e}_{t} + \pi^{e}_{t})K_{1t}}{\tilde{p}_{1t-1}K_{1t}}\frac{\tilde{p}_{1t-1}K_{1t}}{\tilde{p}_{1t-1}K^{h}_{1t}}. \tag{97}$$

Substituting (94) into (97) and remembering the definitions of D^{μ}_{1t} and S^{μ}_{1t} lead to

$$h^{e}_{1t} = \rho^{\mu}_{1t} + (\rho^{\mu}_{1t} - i_t)\frac{D^{\mu}_{1t}}{S^{\mu}_{1t}}.$$

That is, the expected rate of return on equities h^{e}_{1t} is equal to the capitalization rate ρ^{μ}_{1t} for a pure equity stream, plus a premium related to financial risk equal to the debt-to-equity ratio times the spread between ρ^{μ}_{1t} and i_t. This result corresponds to Proposition II of Modigliani and Miller (1958, p. 271).

Proposition III of Modigliani and Miller (1958, p. 288) can be rephrased in terms of the complete *KS* model as follows: If the investment-goods sector is acitng in the best interest of the equity holders, the marginal cost of capital (or equivalently the rate of return on the investment) should be equal to the average cost of capital, which is in turn equal to the capitalization rate ρ^{μ}_{1t} for an unlevered stream in the sector. The marginal cost of capital may be defined as $d\bar{X}^{\mu}_{1t}/dV^{\mu}_{1t}$. Then,

$$\begin{aligned}
\frac{d\bar{X}^{\mu}_{1t}}{dV^{\mu}_{1t}} &= \frac{d[(1-\mu)p^{e}_{1t}Q^{\mu e}_{1t} - w_t N^{\mu e}_{1t} - p^{e}_{1t}(\delta - \pi^{e}_{t})K_{1t}]}{d[\tilde{p}_{1t-1}K_{1t}]} \\
&= \frac{p^{e}_{1t}}{\tilde{p}_{1t-1}}\frac{d[(1-\mu)\alpha Q^{\mu e}_{1t} - (\delta - \pi^{e}_{t})K_{1t}]}{dK_{1t}} \\
&= \frac{p^{e}_{1t}}{\tilde{p}_{1t-1}}\left\{\left[\frac{(1-\mu)p^{e}_{1t}}{\hat{p}^{e}_{1t}}\right]^{\frac{1}{\alpha}}(r^{e}_{t} + \delta) - (\delta - \pi^{e}_{t})\right\} = \rho^{\mu}_{1t}.
\end{aligned}$$

As is obvious, $d\bar{X}^{\mu}_{1t}/dV^{\mu}_{1t}$ is also the rate of return on the investment. Remember that the investment-goods sector maximizes h^{e}_{1t} in (4). Therefore, Proposition III also obtains in the complete *KS* model. Similar arguments applies to the consumption-goods sector where $k = 2$.

Now the relationship between the modified *M-M* theorem and modified Tobin's q theory can be made clear. From the previous appendix $[(1-\mu)p^{e}_{1t}/\hat{p}^{e}_{1t}]^{1/\alpha}$ and $[(1-\mu)p^{e}_{2t}/\hat{p}^{e}_{1t}]^{1/\alpha}$ are two qs, q^{μ}_{1t} and q^{μ}_{2t}, respectively. Taking (95) into account, the following simple relations hold:

$$q^{\mu}_{1t} \geq (<) 1 \Leftrightarrow \rho^{\mu}_{1t} \geq (<) i_t,$$

and

$$q^{\mu}_{2t} \geq (<) 1 \Leftrightarrow \rho^{\mu}_{2t} \geq (<) i_t.$$

That is, the modified *M-M* theorem and modified Tobin's q theory are mathematically equivalent.

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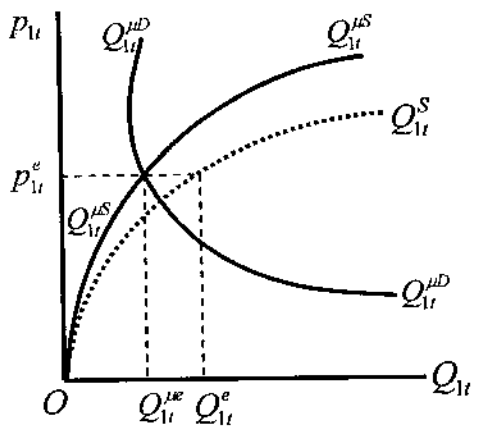

Figure 1. Indirect Tax and Equilibrium in the Investment-Goods Market.

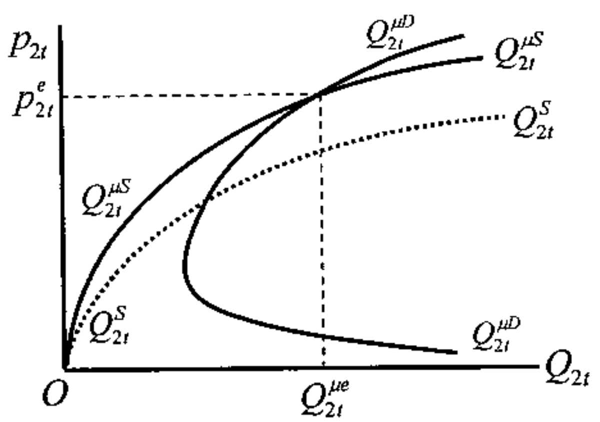

Figure 2. Indirect Tax and Equilibrium in the Consumption-Goods Market.

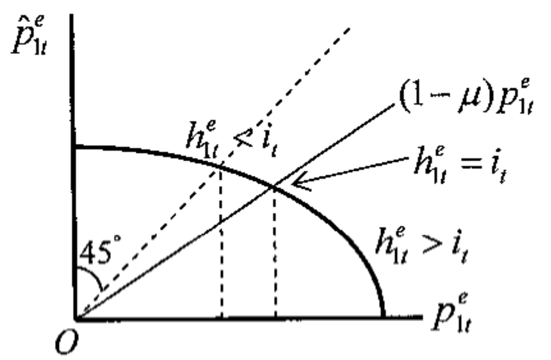

Figure 3. Indirect Tax and the Expected Normal Supply-Price of Investment Goods.

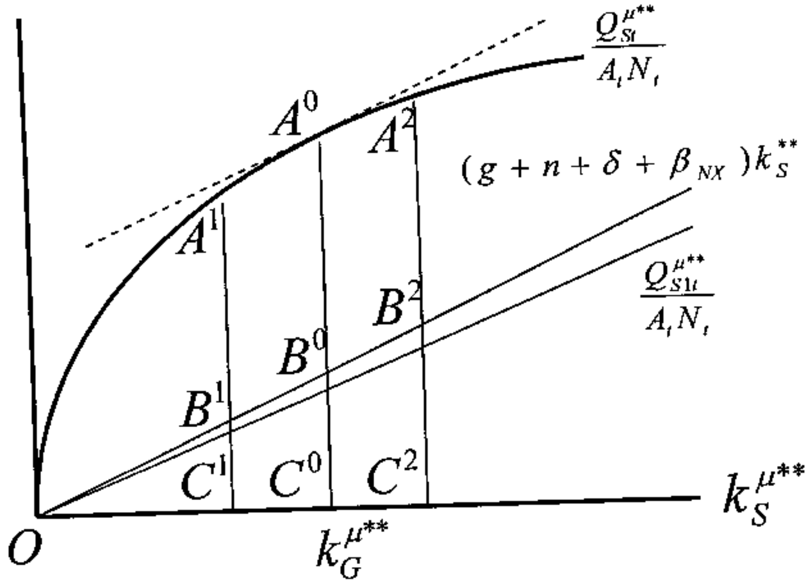

Figure 4. The Golden-Rule State with Government and Foreign Sectors.

GDP^{**}_{Gt}	$c^{\mu}_G Y^{\mu**}_{DGt}$	G^{**}_{2t}	$I^{**}_{Gt} + p^{**}_{1t}(\delta - \pi)K^{\mu**}_{Gt}$	G^{**}_{1t}	NX^{**}_{Gt}
100 (%)	57	10	24	8	1

Table 1. GDP from the Expenditure Side: An Example.

GDP^{**}_{Gt}	$w^{\mu**}_{Gt} N_t$	$i^{\mu**}_G \widetilde{p}^{**}_{1t-1} K^{\mu**}_{Gt}$	$p^{**}_{1t}(\delta - \pi)K^{\mu**}_{Gt}$	$\mu\ p^{**}_{1t} Q^{\mu**}_{Gt}$
100 (%)	61	14	17	8

Table 2. GDP from the Distribution Side: An Example.