Finding Another Linkage between the Short Run and the Long Run in a Macroeconomy

by

Kazuyuki Sasakura
Waseda University

No. 0601

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July 19, 2006

Abstract

This paper presents a model which can explain basic aspects of a macroeconomy both in the short run and in the long run. It is based simply on such principles as profit maximization of firms and utility maximization of households. The way the macro model is constructed is much the same as in microeconomics. No new concepts are introduced. The model used is *only one*. Nevertheless, it provides a new insight into the theories of consumption and investment. As an example, the once-famous consumption function controversy is reconsidered with this model, and the relationship between a short-run and a long-run consumption functions is made clear from quite a new point of view.

1 Introduction

I am a teacher of macroeconomics. A problem has been bothering me since I began teaching macroeconomics in class. The problem is that there is no model in macroeconomics which can analyze basic aspects of a macroeconomy in the short run and in the long run at the same time. The purpose of this paper is to present a basic model which can do so.

Recent macroeconomics textbooks adopt the theoretical structure consisting of three basic models, i.e., the *IS-LM* model, the *AD-AS* model, and the Solow model to explain the short run, the medium run, and the long run, respectively. I know that macroeconomics has remarkable progress since Keynes (1936) and that the three-model structure is an ingeneous workmanship by great macroeconomists. In fact, when the renowned textbook reached its golden birthday, Samuelson and Nordhause (1998, p. 372) wrote, "One of the major breakthroughs of twentieth-century economics has been the development of macroeconomics." I myself am also one of users of "standard" textbooks.[1] Nonetheless, a question has been sticking to me: Why are three models necessary for one economy?

As is well known, the *IS-LM* model and neo-classical growth models such as the Solow model have quite different backgrounds. And the original idea of the *AS* curve in the *AD-AS* model is based mainly on the theory of monetarists. Is it, therefore, natural to think that the three-model structure as a whole is theoretically inconsistent and that macroeconomics has

*School of Political Science and Economics, Waseda University, Japan. E-mail: sasakura@waseda.jp

[1]Intermediate textbooks I have read are Dornbusch and Fischer (1994), Mankiw (1994), Sachs and Larrain (1993), Blanchard (1997) and so on and so forth. Advanced textbooks every macroeconomist knows are Blanchard and Fischer (1989), and Romer (1996).

not been laid on a sound foundation yet? If so, (and I do believe so, which is the very motive of this paper,) another model should be newly constructed for all macroeconomists.[2]

I do not at all, however, intend to destroy all of them. What can be regarded as useful should be used. My main proposals for a new macro-model are as follows:

1. For the *IS-LM* model, the *IS* part is used, while the *LM* part is abandoned.

2. The *AD-AS* model is abandoned.

3. The Solow model is basically accepted.

4. The production sector is divided into two industries, i.e., the investment-goods sector and the consumption-goods sector.

Proposal 1 implies that money is demanded only as a medium of exchange as in the Solow model.[3] Proposal 4 leads to a two-sector model. It seems to be a *long-run* growth model as Proposal 3 appears to suggest, but it is not necessarily.[4] The model constructed on the basis of the above proposals is *onley one*. The two-sector model is applicable to the short run, the medium run, and the long run without modification. The dynamics of an economy is always described as a series of the short-run market equilibria represented by the *IS* part in which full employment is not always guaranteed. Full employment is realized in the long run, but the long-run state is theoretically a special case of the short-run equilibrium state. The medium run is regarded only as a transitional process from the short-run equilibrium state to the long-run equilibrium state, which is what Proposal 2 suggests.[5]

The model, which shall be explained in detail below, is both tractable and trustable in the sense that it is composed of only a few equations and that it can give quite new and consistent answers to important problems in macroeconomics. For example, it sheds new light on the interpretation of the relationship between a short-run and a long-run consumption functions which is nowadays thought to be completely solved, and also that of the effects of inflation on

[2]Solow (1988, p. 310) stated, "The problem of combining long-run and short-run macroeconomics has still not been solved." Solow (1997, 2000) accepts something like the *IS-LM* model for a short-run analysis and his own growth model for describing a lon-run economy. And he thinks the fix-price approach or the imperfect-competition approach as useful to construct a medium-run macro-model. The model presented in this paper is quite different from what he proposes. In order to know what other leading macroeconomists regarded as a core of macroeconomics, see also Taylor (1997), Eichenbaum (1997), Blinder (1997), and Blanchard (1997a).

[3]There have been different views on money among macroeconomists. Needless to say, Keynes (1936) himself emphasized the role of money (or, correctly speaking, cash) as a means of store of value in the short run. His proponents such as Tobin (1965), Mundell (1971), and recently Ono (2001) attached importance to the influence of money on a macroeconomy even in the long run or in a dynamic setting. On the other hand, old and new Keynesians such as Klein (1947) and Romer (2000), and neo-classical economists such as Viner (1937) cast doubt on the relevancy of Keynes's liquidity preference theory (or the *LM* curve).

[4]The earliest studies on two-sector growth models include Shinkai (1960), Meade (1961), and Uzawa (1961-62, 1963). Particularly under the stimulus of interesting features of Uzawa's neo-classical model immeadiately followed further investigations including Solow (1961-62), Inada (1963), and Takayama (1963). Since then neo-classical two-sector models have been thoroughly examined and extended by Foley and Sidrauski (1971), Boldrin and Montrucchio (1986), and recently Benhabib et al. (2002), to name only a few. There are also studies on Keynesian two-sector models such as Mackay and Waud (1975), Benavie (1976), and Chakrabarti (1979). However, no one made an attempt to analyze a macroeconomy both in the short run and in the long run using *a* two-sector model.

[5]There are the pros and cons of the use of the *AD-AS* model at least as a teaching tool. For example, Blnachard and Fischer (1989) and Mankiw (1998) are for it, while Barro (1994) and most writers in Rao (1998) against it.

economic growth which seem to perplex macroeconomists. Such paradoxical problems can be resolved only by considering an economy as a whole, not by focusing on a particular aspect.

This paper is organized as follows. The next section provides the outline of the model. I think it is necessary because such a model does not exist as far as I know. The short-run equilibrium state and the long-run equilibrium state are also defined in the section. Sections 3-5 are concerned with the short-run equilibrium state. Sections 3 explains how the investment-goods sector and the consumption-goods sector behave, while Section 4 describes the euilibrium in the investment-goods market and the consumption-goods market. The short-run equilibrium state is not completely understood until the roles of the central bank and the household sector are discussed in Sections 5. Sections 6-9 are concerned with the long-run equilibrium state. Section 6 defines again the long-run equilibrium state using notations of the model. Section 7 characterizes the long-run equilibrium state and, using the results, Section 8 finds the long-run steady state in which macro variables are growing at a constant rate. Section 9 analyzes the golden-rule state, a special case of the long-run steady state, in which current consumption is maximized. In this paper it is the golden-rule state which is considered useful for analyzing an actual macroeconomy, though it is not thought much of in modern macroeconomics. In order to show the relevancy of the model, the consumption function controversy is reconsidered in Section 10. Section 11 concludes this paper. In appendices Tobin's q theory and the Modigliani-Miller theorem are considered through the model and their equivalence is shown.

As the above proposals suggest, the model presented is based largely on the pioneering work of Keynes (1936) and Solow (1956). Thus, it is appropriate to call it the Keynes-Solow model (the KS model for short) throughout the paper.

2 Outline of the Keynes-Solow Model

This paper deals with a basic case in which a macroeconomy is made up of the household sector, the production sector, the central bank, and commercial banks.[6] The production sector consists of the investment-goods sector and the consumption-goods sector. The KS model is a discrete-time model and, correctly speaking, each period is divided into three subperiods.

At the first subperiod of each period, the production sector makes investment goods and consumption goods using labor the household sector supplies and capital stock the household sector holds. Labor is supposed to be homogenenous, while capital stock malleable. The household sector receives income in the form of money from the production sector and buys goods of the two types. Under the assumption that money is not held as wealth,[7] the household sector uses all of income received and thus all of goods are sold out, that is, both the investment-goods market and the consumption-goods market are cleared every period.[8] At

[6]In fact, only the existence of the household sector is essential. It is possible to think that the household sector plays all roles of others.

[7]As will be shown in the lemma in Section 4, this assumption is not a mere one but an indispensable one to the KS model.

[8]As will be discussed in Section 5, the market equilibrium is attained neither through the Walrasian price adjustment process nor through the Marshallian (or so-called Keynesian) quantity adjustment process. It is assumed to be realized by correct production plan by each sector, or to put it in a modern way, rational expectations, which are "essentially the same as the predictions of the relevant economic theory," as Muth (1961, p. 316) proposed.

the end of the first subperiod the household sector holds capital stock available for production of the next period.

The second subperiod is that of portfolio selection. In this basic case there is only one kind of wealth (or asset in the same meaning), i.e., real capital. Households have basically four choices as asset holders. On one hand, they can hold capital stock as that of the investment-goods sector or that of the consumption-goods sector. On the other hand, they can hold the capital stock of each sector directly as equity holders or indirectly as depositors through commercial banks. When they hold capital stock as depositors, the nominal rate of return is a fixed rate of interest which is determined, for example, by negotiations between commercial banks and the production sector. In the KS model commercial banks are merely institutions that hold capital stock, which bears interest at the fixed rate, on behalf of households as depositors. All interest income earned belongs to depositors.[9] Households as equity holders have to expect the rates of return on equities which depend on both how much capital stock exists in each sector and how much the prices and nominal wage rate of the next period are expected to be, which is not known untill the third subperiod.[10] The price of asset is that of investment goods as existing capital stock, and it is unique in this case. It is assumed that the asset price tends to be so determined as to make all rates of return equal. In sum, price of investment goods is determined *twice* during a period, as that of output produded (or flow) at the first subperiod and as that of asset (or stock) at this second subperiod.

The third subperiod is that of plan for production of the next period. There already exists capital stock in each sector as a result of portofolio selection during the previous subperiod. Nominal wage rate paid at the next period is determined, for example, by negotiations between the production sector and the household sector, and prices of investment goods and consumption goods at the next period are expected by the production sector. Once expected prices are fixed, the production sector can calculate profit-maximaizing output (and also the corresponding demand for labor) using the existing capital stock, the fixed nominal wage rate, and the expected prices. Hence a certain amount of money as a medium of exchange which realizes the calculated optimal production. The central bank is a unique institution that can supply money. The expected prices and the planned production are realized if the central bank promises the production sector that it will issue the same amount of money as the production sector requires.[11] If the central bank announces that it will issue less money than the production sector desires, expected prices and planned production are adjusted downward according to recalculation.[12]

The first subperiod of the next period comes, and the same processes are repeated again

[9] "Bank deposits" can be replaced by "corporate bonds." The point is that housholds have a means of store of value which makes the nominal rate of return certain.

[10] For a mathematical description, see (31), (32) and (34) in Section 6.

[11] Time-inconsistency is excluded.

[12] What if the central bank offers to issue more money than the production sector needs? According to the quantity theory of money, expected prices and planned production should be adjusted *upward* accordingly. It is interesting to point out that Adam Smith (1776, p. 323) argued for the "reverse" quantity theory of money, in which "The quantity of money, therefore, which can be annually employed in any country, must be determined by the value of the consumable goods annually circulated within it. ... The quantity of money, on the contrary, must in every country naturally increase as the value of the annual produce increases." It may be comparable to the relationship between the number of books students demand to borrow and that which a university library holds (and can supply). I take a compromise between the two, as will be discussed in Section 5. For "reverse causation," see also King and Plosser (1984) who, using a real business cycle model, found empirically that the expansion of *inside* money (bank deposits) followed that of output, while changes in *outside* money (currency or high-powered money) and real activity resulted in inflation.

and again. An economy is said to be *in the short-run equilibium state* if expectations of prices and the corresponding production plan are realized. In this paper only the short-run equilibrium state is analyzed. Thus, *the short run* always means a period in which an economy is in the short-run equilibrium state. Note that goods markets are always cleared whereas labor market is not always. An economy in the short-run equilibrium state is also said to be *in the long-run equilibrium state* if labor market is cleared and the interest rate is equal to the rates of returns on equities. In this paper *the long run* always means periods in which an economy is in the long-run equilibrium state. The Solow model works in the long run, not to mention. It should be emphasized that a period in which an economy is in the long-run equilibrium state is only a special case of the short run.[13] This is why two models are not needed for one economy.

As is well known, the rate of economic growth is determined in the long-run steady state by the sum of the growth rate of labor supply and that of technology, which is called the natural growth rate. It holds in the Keynes-Solow model, too. In this situation the household sector alone can control the economy in the sense that it can change the ratio of consumpton goods produced to investment goods produced through the rate of consumption (or the rate of saving in familiar terms). It is assumed in the *KS* model that the rate of consumption is so determined as to maximize current consumption the household sector enjoys each period. This means that a long-run macroeconomy is not in the modified golden-rule state but in the "truc" golden-rule state. Under the assumption that an actual economy is approximated by the golden-rule state, the once-disputed relationship between a short-run and a long-run consumption functions can be reinterpreted.

3 The Production Sector

3.1 The Investment-Goods Sector

Suppose that an economy is at the third *sub*peiod of period $t-1$. As was explained in the previous section, this is the subperiod of plan for production of period t. First consider the investment-goods sector planning production of period t. Capital stock of the investment-goods sector, K_{1t}, consists of K_{1t}^d and K_{1t}^h. The former is held by households as depositors, while the latter as equity holders. A subscript 1 represents the investment-goods sector.

The technology of the investment-goods sector at t is given by the Cobb-Douglas production function:

$$Q_{1t} = K_{1t}^{\alpha}(A_t N_{1t})^{1-\alpha},\ K_{1t} = K_{1t}^d + K_{1t}^h,\ 0 < \alpha < 1, \tag{1}$$

$$A_t = (1+g)A_{t-1},\ g > -1, \tag{2}$$

where Q_{1t}, N_{1t}, and A_t are respectively output, labor used, and the effectiveness of labor of the investment-goods sector at t. The effectiveness of labor or "knowledge" is assumed to grow at an exogenous rate g as in (2).[14]

The nominal interest rate, i_t, and the asset price of the investment goods, $\tilde{p}_{1t-1}$, have already been determined during the second subperiod of period $t-1$. Thus, after the nominal

[13]These definitions of the short run and the long run can also be applied in their own right to the argument on the Phillips curve pioneered by Friedman (1968) and Phelps (1968).

[14]I know well that this assumption dissatisfies endogenous growth theorists.

wage rate, w_t, has been determined, the inventment-goods sector must make production plan under the following budget constraint:[15]

$$p^e_{1t}Q_{1t} + p^e_{1t}(1-\delta)K_{1t} = w_tN_{1t} + (1+i_t)\tilde{p}_{1t-1}K^d_{1t} + (1+h^e_{1t})\tilde{p}_{1t-1}K^h_{1t}, \tag{3}$$

where p^e_{1t}, h^e_{1t}, and δ are respectively the expected price of investment-goods produced at period t, the expected nominal rate of return on equities, and the capital depreciation rate which is assumed as usual to be a positive constant.[16] A superscript e means an expected value in what follows. $\tilde{p}_{1t-1}K^d_{1t}$ is the amount of bank deposits related to K^d_{1t}, while $\tilde{p}_{1t-1}K^h_{1t}$ is the nominal value of equities related to K^h_{1t}.

Rewriting (3) yields

$$p^e_{1t}Q_{1t} = w_tN_{1t} + i_t\tilde{p}_{1t-1}K^d_{1t} + h^e_{1t}\tilde{p}_{1t-1}K^h_{1t} + p^e_{1t}(\delta - \pi^e_t)K_{1t}, \tag{4}$$

where $\pi^e_t = 1 - (\tilde{p}_{1t-1}/p^e_{1t})$. π^e_t is approximately equal to $(p^e_{1t} - \tilde{p}_{1t-1})/\tilde{p}_{1t-1}$, when it is not far from zero. For simplicity let us call π^e_t the expected inflation rate in what follows. Then $\delta - \pi^e_t$ can be called the "inflation-adjusted depreciation rate."[17] Taking into account the usual observation that the share of capital consumption in GDP is positive, it is assumed that

$$\delta - \pi^e_t > 0. \tag{5}$$

The purpose of the investment-goods sector is to maximize h^e_{1t} in (4) subject to the production technology (1). From (4), h^e_{1t} can be written as

$$h^e_{1t} = \frac{p^e_{1t}Q_{1t} - w_tN_{1t} - i_t\tilde{p}_{1t-1}K^d_{1t} - p^e_{1t}(\delta - \pi^e_t)K_{1t}}{\tilde{p}_{1t-1}K^h_{1t}}. \tag{6}$$

Since the right-hand side of (6) is a function of N_{1t} alone, the investment-goods sector has only to find the level of labor, N^e_{1t}, which maximizes h^e_{1t}. Substituting (1) into (6) and differentiating (6) with respect to N_{1t} yield

$$\frac{dh^e_{1t}}{dN_{1t}} = \frac{p^e_{1t}(1-\alpha)A_t^{1-\alpha}N_{1t}^{-\alpha}K^\alpha_{1t} - w_t}{\tilde{p}_{1t-1}K^h_{1t}}.$$

Then N^e_{1t} can easily be obtained by solving $dh^e_{1t}/dN_{1t} = 0$ and $d^2h^e_{1t}/dN^2_{1t} < 0$ as follows:

$$N^e_{1t} = \left[(1-\alpha)A_t^{1-\alpha}\frac{p^e_{1t}}{w_t}\right]^{\frac{1}{\alpha}} K_{1t}. \tag{7}$$

And the output of investment-goods which also maximizes h^e_{1t} is calculated as follows:[18]

$$\begin{aligned} Q^e_{1t} &= K^\alpha_{1t}(A_tN^e_{1t})^{1-\alpha} \\ &= \left[(1-\alpha)A_t\frac{p^e_{1t}}{w_t}\right]^{\frac{1-\alpha}{\alpha}} K_{1t}. \end{aligned} \tag{8}$$

[15] It should be noted that this constraint is a nominal one, not a real one as a resource constraint.

[16] As said in the previous section, investment goods of, say, period $t-1$ have two prices, i.e., that of investment goods as flow (or equivalently, output price), and that of investment goods as stock (or asset price). The latter is distinguished by a superimposed tilde as in $\tilde{p}_{1t-1}$.

[17] Allow me to use the term "inflation rate" for $(p^e_{1t} - \tilde{p}_{1t-1})/\tilde{p}_{1t-1}$ since I don't think of a proper name for it. It is usual to define the inflation rate as the rate of change of a weighted average of prices of investment goods and consumption goods. In fact this problem disappears in the long run because all prices are assumed to change at the same rate.

[18] It is assumed that the investment-goods sector is always on the labor demand curve.

The maximization of h^e_{1t} looks like the short-run profit maximization in microeconomics.[19] Let MPL_{1t} be the marginal product of labor at t. Then, since $MPL_{1t} \equiv \partial Q_{1t}/\partial N_{1t}$, the familiar-looking profit-maximizing condition holds:

$$MPL_{1t} = (1-\alpha)A_t^{1-\alpha}N_{1t}^{-\alpha}K_{1t}^{\alpha} = \frac{w_t}{p^e_{1t}}, \tag{9}$$

which is equivalent to (7). It should be noticed, however, that the right-hand side is *not* the real wage rate in a usual sense. The marginal product of capital at t, MPK_{1t}, is

$$MPK_{1t} = \alpha K_{1t}^{\alpha-1}(A_t N_{1t})^{1-\alpha}. \tag{10}$$

When the investment-goods sector expects that investment goods will be sold at the price p^e_{1t}, it is ready to distribute the value added, $p^e_{1t}Q^e_{1t}$, among the factors of production according to (4). Hence nominal income in the investment-goods sector Y^e_{1t}:

$$\begin{aligned} Y^e_{1t} &= w_t N^e_{1t} + i_t \tilde{p}_{1t-1}K^d_{1t} + h^e_{1t}\tilde{p}_{1t-1}K^h_{1t} \\ &= p^e_{1t}Q^e_{1t} - p^e_{1t}(\delta - \pi^e_t)K_{1t}. \end{aligned} \tag{11}$$

(11) means that the magnitude of Y^e_{1t} depends crucially on the *expected* price p^e_{1t}. This is one of the remarkable characteristics of the KS model.

3.2 The Consumption-Goods Sector

Next consider the consumption-goods sector planning production of period t.[20] The explanation of the consumption-goods sector proceeds along much the same line as in the investment-goods sector, a subsript 1 being replaced by subscript 2 which in turn represents the consumption-goods sector. Therefore, it suffices to show main features and results in order.

The production function of the consumption-goods sector:

$$Q_{2t} = K_{2t}^{\alpha}(A_t N_{2t})^{1-\alpha},\ K_{2t} = K^d_{2t} + K^h_{2t},\ 0 < \alpha < 1. \tag{12}$$

The budget constraint on the consumption-goods sector:

$$p^e_{2t}Q_{2t} + p^e_{1t}(1-\delta)K_{2t} = w_t N_{2t} + (1+i_t)\tilde{p}_{1t-1}K^d_{2t} + (1+h^e_{2t})\tilde{p}_{1t-1}K^h_{2t},$$

or

$$p^e_{2t}Q_{2t} = w_t N_{2t} + i_t\tilde{p}_{1t-1}K^d_{2t} + h^e_{2t}\tilde{p}_{1t-1}K^h_{2t} + p^e_{1t}(\delta - \pi^e_t)K_{2t}. \tag{13}$$

The demand for labor in the consumption-goods sector:

$$N^e_{2t} = \left[(1-\alpha)A_t^{1-\alpha}\frac{p^e_{2t}}{w_t}\right]^{\frac{1}{\alpha}} K_{2t}. \tag{14}$$

[19] But what is profit in macroeconomics? Strange to say, macroeconomics textbooks do not define it clearly. In order to discuss it, it is advisable to wait until the real interest rate appears. See footnote 32 below for the definitions of profit.

[20] Remember that the economy is at the third subperiod of period $t-1$. Period t has not come yet.

The planned output of consumption goods for p_{2t}^e:

$$\begin{aligned} Q_{2t}^e &= K_{2t}^\alpha (A_t N_{2t}^e)^{1-\alpha} \\ &= \left[(1-\alpha)A_t \frac{p_{2t}^e}{w_t}\right]^{\frac{1-\alpha}{\alpha}} K_{2t}. \end{aligned} \tag{15}$$

The profit-maximizing condition:

$$MPL_{2t} = (1-\alpha)A_t^{1-\alpha} N_{2t}^{-\alpha} K_{2t}^\alpha = \frac{w_t}{p_{2t}^e}. \tag{16}$$

The marginal product of capital:

$$MPK_{2t} = \alpha K_{2t}^{\alpha-1}(A_t N_{2t})^{1-\alpha}. \tag{17}$$

Nominal income distributed in the consumption-goods sector:

$$\begin{aligned} Y_{2t}^e &= w_t N_{2t}^e + i_t \tilde{p}_{1t-1} K_{2t}^d + h_{2t}^e \tilde{p}_{1t-1} K_{2t}^h \\ &= p_{2t}^e Q_{2t}^e - p_{1t}^e(\delta - \pi_t^e) K_{2t}. \end{aligned} \tag{18}$$

(2) and (5) are assumed in the consumption-goods sector, too. The consumption-goods sector resembles the investment-goods sector in formal structure, but there is a difference in the budget constraints. The budget constraint on the investment-goods sector (4) has one expected price, p_{1t}^e, while the budget constraint on the consumption-goods sector (13) has two expected prices, p_{1t}^e and p_{2t}^e. The relationship between the two and also that between the two sectors are found out in the next section.[21]

4 Market Equilibrium

Consider how the investment-goods market and the consumption-goods market reach equilibrium. It is the investment-goods sector and the consumption-goods sector that decide how much should be so supplied as to maximize the rates of return on equities each period. The demand for goods as a whole is national income itself. Nominal national income at t, Y_t^e, is the sum of Y_{1t}^e and Y_{2t}^e. From (11) and (18),

$$Y_t^e = p_{1t}^e Q_{1t}^e + p_{2t}^e Q_{2t}^e - p_{1t}^e(\delta - \pi_t^e) K_t, \tag{19}$$

where $K_t = K_{1t} + K_{2t}$. It follows from (19) that it is also the investment-goods sector and the consumption-goods sector that decide how much income is paid to the household sector.

What can the household sector do in the KS model? The household sector receives national income in return for labor and capital stock. All that the household sector can do is to use it for consumption or, for saving, not to use it.[22] The decision is described by two alternative ways. One is the consumption function:

$$C_t^e = cY_t^e,\ 0 < c < 1, \tag{20}$$

[21] Production functions (1) and (12) satisfy three assumptions made by Meade (1961), namely, the assumption of perfect malleability of machinery, that of perfect substitutability in production between capital goods and consumption goods, and that of depreciation by evaporation. Kurz (1963) investigated a two-sector neo-classical growth model when the two sectors have the Cobb-Douglas production functions with different exponents.

[22] In this basic case national income equals disposable income of the household sector.

where C_t^e is the planned nominal demand for consumption goods, and c is called the rate of consumption in what follows.[23] The other is the saving function:

$$S_t^e = (1-c)Y_t^e, \tag{21}$$

where S_t^e is the amount the household sector plans to save, and $1-c$ is of course the rate of saving. Although the Keynesian school stressed (20) and the neo-classical school laid weight on (21), the two functions are on an equal footing in the KS model.

Output levels of investment goods and consumption goods are determined when supply and demand coincide in each market. The equilibrium in the consumption-goods market is described as follows:

$$p_{2t}^e Q_{2t}^e = C_t^e. \tag{22}$$

Sustituting (20) and then (19) into (22) gives the equilibrium amount of production of consumption goods:

$$p_{2t}^e Q_{2t}^e = \frac{c}{1-c}[p_{1t}^e Q_{1t}^e - p_{1t}^e(\delta - \pi_t^e)K_t], \tag{23}$$

and also the equilibrium national income:

$$Y_t^e = \frac{1}{1-c}[p_{1t}^e Q_{1t}^e - p_{1t}^e(\delta - \pi_t^e)K_t], \tag{24}$$

where $p_{1t}^e Q_{1t}^e - p_{1t}^e(\delta - \pi_t^e)K_t$ is nominal *net* investment.[24]

The equilibrium price and output of consumption goods can be obtained by substituting (15) into (23) as follows:

$$p_{2t}^e = \left[\frac{w_t}{(1-\alpha)A_t}\right]^{1-\alpha}\left[\frac{1}{K_{2t}}\right]^{\alpha}\left\{\frac{c}{1-c}[p_{1t}^e Q_{1t}^e - p_{1t}^e(\delta - \pi_t^e)K_t]\right\}^{\alpha}, \tag{25}$$

and

$$Q_{2t}^e = \left[\frac{(1-\alpha)A_t}{w_t}\right]^{1-\alpha} K_{2t}^{\alpha}\left\{\frac{c}{1-c}[p_{1t}^e Q_{1t} - p_{1t}^e(\delta - \pi_t^e)K_t]\right\}^{1-\alpha}. \tag{26}$$

How about the investment-goods market? To answer it, the following lemma is needed.

Lemma: Money hoarding implies shutdown.

Proof: See Appendix A.

[23] c corresponds to the *average* propensity to consume in a usual sense, but it is different from the *marginal* propensity to consume. This is fully discussed in Section 10.

[24] Substituting (22) into (19) yields

$$Y_t^e = C_t^e + [p_{1t}^e Q_{1t}^e - p_{1t}^e(\delta - \pi_t^e)K_t].$$

This equation and the consumption function (20) constitute what Samuelson(1948, p. 135) called the "nucleus of the Keynesian reasoning."

The KS model cannot deal with the case of money hoarding, where the economy is not sustained. It is assumed, therefore, that money is not held as wealth.[25] It means that S_t^c in (21) is all spent for investment goods. Under the assumption it is straightforward to show the following theorem:

Theorem 1: If money is not hoarded, the investment-goods market always reaches equilibrium with positive price and output.

Proof: See Appendix B.

Furthermore, from (8), (24), (25), (26), and Theorem 1 follows the proposition which appeals to common sense of "ordinary people:"[26]

Proposition 1: In the short run an increase in prices leads to an increase in production and income in both nominal and real terms.

Figure 1. Equilibrium in the Investment-Goods Market.

Figure 2. Equilibrium in the Consumption-Goods Market.

The formal argument above can easily be understood by a familiar method using a supply curve and a demand curve. Figures 1 and 2 represent respectively the investment-goods market and the consumption-goods market. The strictly concave curves with upward slope in those figures are the supply curves. In Figure 1 once expected price p_{1t}^e is fixed, the planned output Q_{1t}^e is known through the supply curve Q_{1t}^S. Information about the demand for investment goods is not necessary due to Theorem 1. In Figure 2 the consumption-goods demand curve is needed to discover expected price p_{2t}^e and planned output Q_{2t}^e in addition to the supply curve Q_{2t}^S. It is derived from the consumption function which in turn depends on output of investment goods through national income. The unfamiliar forward bending curve in Figure 2 is the demand curve Q_{2t}^D. Both p_{2t}^e and Q_{2t}^e are determined in the intersection of two curves Q_{2t}^S and Q_{2t}^D. In passing, as far as I know, no supply curve or demand curve with such shape as in Figures 1 and 2 has not been drawn in economics.[27]

[25]This may be called the no Pope's father condition. See Keynes (1936, p. 221). Keynes argued that *high* propensity to hoard depresses economy. The above lemma says that even *low* propensity to hoard collapses economy. In his article approved by Keynes, Lerner (1936, p. 443) wrote as follows: "The total income of society (Y) is made up of the income earned in making cosumption goods (C) and the income earned in making investment goods (I). Y = C + I. Now C, which stands for income earned in making consumption goods, must also stand for the amount spent on buying consumption goods, since these two are in fact the same thing. (Similarly I stands also for the amount of *money spent on investment goods.*)" (Italics added by me.) This statement is also a proof of the lemma, though against their will.

[26]Proposition 1 is related with the famous Tobin's q theory of investment. The q theory has been studied in a long-run neo-classical environment, but in my opinion it should be understood within a short-run partial-equilibrium framework. This is discussed in Appendix E. Furthermore, in Appendix F (the last appendix) the Modigliani-Miller theorem, which is also well-known in investment theory, is restated within the same framework as Appendix E, and it is concluded that the q theory and the M-M theorem are theoretically equivalent. You are rather recommended to read Appendixes E and F after the conclusion (Section 11) of this paper in order to be able to know the relationship between the short run and the long run.

[27]Supply curves Q_{1t}^S and Q_{2t}^S, and demand curve Q_{2t}^D are derived in Appendix C.

5 Roles of the Central Bank and the Household Sector

As was shown in the previous section, main features of the short-run macroeconomy can be grasped by seeing the levels of p^e_{1t}, Q^e_{1t}, p^e_{2t}, Q^e_{2t}, etc. with capital stock as given. But p^e_{1t} is the most important because all other variables are functions of p^e_{1t}. They responds to any change in p^e_{1t}. In other words the economy is dominated by p^e_{1t}.

There is, however, an obstacle to realization of p^e_{1t}. For p^e_{1t}, the value added in the economy as a whole is calculated as $p^e_{1t}Q^e_{1t} + p^e_{2t}Q^e_{2t}$ according to (8), (25), and (26). But whether p^e_{1t} is realized is another problem.[28] Transactions represented by $p^e_{1t}Q^e_{1t} + p^e_{2t}Q^e_{2t}$ is possible only if an appropriate amount of a medium of exchange, i.e., money, is supplied by the central bank. Such an amount M_t is, for example, $(p^e_{1t}Q^e_{1t} + p^e_{2t}Q^e_{2t})/V_t$ with V_t as income velocity of money at t. Hence

$$M_tV_t = p^e_{1t}Q^e_{1t} + p^e_{2t}Q^e_{2t}. \tag{27}$$

(27) reminds us of the traditional quantity theory of money. But it is assumed in the KS model that in general the causal relationship between prices and money supply is opposite. M_t determines p^e_{1t} and p^e_{2t} in the quantity theory of money, whereas p^e_{1t} and p^e_{2t} determines M_t in the KS model.

It is necessary to explain correctly. If the central bank promises the production sector that it will supply just the same amount of money as the production sector desires, then the original production plan comes true at the first subperiod of period t. Let a superscript $*$ designate a value realized, i.e., that *in the short-run equilibrium state*. Then $p^e_{1t} = p^*_{1t}$. And therefore, $Q^e_{1t} = Q^*_{1t}$, $p^e_{2t} = p^*_{2t}$, $Q^e_{2t} = Q^*_{2t}$, etc. The central bank may reject the request of the production sector. If the central bank announces that it will issue less money than the production sector requires, p^e_{1t} has to be recalculated. But the modified production plan due to the downward revision of expected prices comes true at t, too. What about the case where the central bank is going to issue more money than the production sector wants? Although there is no theoretical reason why the prodution sector declines such an offer, a pessimistic production sector may actually do so. As a result, the central bank is obliged to supply money passively according to the demand of the production sector. In this case, too, the original production plan comes true at t. In sum, money supply is determined by the "short-side principle."[29]

The price mechanism explained above means that the KS model needs no fictitious auctioneer in a Walrasian sense. The productin sector is assumed to be able to know the short-run equilibrium state using all information available including the quantity of money the central bank is scheduled to supply. Therefore, produntion plan is always realized, and (27) can be written as

$$M_tV_t = p^*_{1t}Q^*_{1t} + p^*_{2t}Q^*_{2t}.$$

As a result, the Fisher equation of exchange formally holds even in the short run. The KS model needs no time-consuming tâtonnement process. But, as was stated above, the causal relation depends upon circumstances. Anyway the short-run market equilibrium is accomplished *not by the flexibility of prices*, but by the correctness of production plan by each

[28] Remember once again that the economy is still at the third subpeiod of period $t-1$. Period t has not come yet.

[29] This principle implies that the central bank can check inflation but cannot stop deflantion. Thus, Friedman (1966, p. 24) is right in saying, "Since inflation results from unduly rapid monetary expansion, the government is responsible for any inflation that occurs," but it is not the case with deflation.

sector based on expected (and realized) supply of money. I believe that this is a practical view and that a macroeconomy actually works in such a way.

In the short run the household sector is implicitly supposed to appear in negotiatins for nominal wage rate as said in Section 2. It plays another role in production plan. It is implied in the consumption function (20) (or the saving function (21)). It goes without saying that the consumption function describes the behavior of the household sector, but the consumption-goods sector cannot make production plan without it. It is obvious from (25) and (26). Conversely, the consumption function makes no sense unless it is used by the consumption-goods sector. To put it in another way, it looks as if the consumption-goods sector made production plan in cooperation with the household sector. The role played by the household sector in production plan can be expressed as

$$\begin{aligned}\frac{C_t^*}{S_t^*} &= \frac{c}{1-c} \\ &= \frac{p_{2t}^* Q_{2t}^*}{p_{1t}^* Q_{1t}^* - p_{1t}^*(\delta - \pi_t^*) K_t}. \end{aligned} \tag{28}$$

Again it is convenient to classify two cases to understand correctly what (28) means. When money is so supplied as to satisfy the need of the production sector, output level of investment goods determines that of consumption goods through (28). This case holds in the traditional Keynesian economics which teaches that, say, an increase in investment gives rise to a multiplier times as much as that in income.[30] On the other hand, when money supply falls short of the need of the production sector, "rationing" occurs. The investment-goods sector can not produce as much as it likes, and it is obliged to reduce output according to (28).[31] In this case the household sector has influence on output level of investment goods, too. Money certainly matters. In both cases the household sector affects the ratio of Q_{2t}^* to Q_{1t}^*, and capital is accumulated each period according to

$$K_{t+1} = (1-\delta)K_t + Q_{1t}^*. \tag{29}$$

6 Definition of the Long-Run Equilibrium State

Since the short-run equilibrium state has been characterized, this section begins a consideration of the long-run equilibrium state. As said in Section 2, an economy in the short-run equilibrium state is also said to be in the long-run equilibirum state if labor market is cleared and the interest rate (or deposit rate) equals the rates of returns on equities.

To analyze the long-run economy, it is necessary to define the long-run equilibrium state using notations of the KS model. First, derive the difference between h_{1t}^e and i_t. Rewriting (4) yields

$$p_{1t}^e Q_{1t}^e = w_t N_{1t}^e + p_{1t}^e (r_t^e + \delta) K_{1t} + (h_{1t}^e - i_t)\tilde{p}_{1t-1} K_{1t}^h, \tag{30}$$

where $r_t^e = [(1+i_t)\tilde{p}_{1t-1}/p_{1t}^e] - 1$. r_t^e is the real interest rate, which is approximately equal to $i_t - \pi_t^e$ when the nominal interest rate i_t and the inflation rate π_t^e are not far from zero.[32]

[30] See also (24).

[31] See also (27).

[32] As was suggested in footnote 19, the definition of profit is ambiguous in macroeconomics. I doubt if macroeconomists have the definition of profit in common. In microeconomics profit is always defined as the difference between total revenue and total cost. And total cost is the sum of variable cost and fixed cost. But

By rearranging (30), the difference between h^e_{1t} and i_t is written as

$$h^e_{1t} - i_t = \frac{p^e_{1t}(r^e_t + \delta)K_{1t}}{\tilde{p}_{1t-1}K^h_{1t}} \left[\left(\frac{p^e_{1t}}{\hat{p}^e_{1t}} \right)^{\frac{1}{\alpha}} - 1 \right], \tag{31}$$

where

$$\begin{aligned} \hat{p}^e_{1t} &= \left[\frac{(1+i_t)\tilde{p}_{1t-1} - (1-\delta)p^e_{1t}}{\alpha} \right]^{\alpha} \left[\frac{w_t}{A_t(1-\alpha)} \right]^{1-\alpha} \\ &= p^e_{1t} \left[\frac{r^e_t + \delta}{\alpha} \right]^{\alpha} \left[\frac{\frac{w_t}{p^e_{1t}}}{A_t(1-\alpha)} \right]^{1-\alpha} . \end{aligned}$$

Figure 3. The Expected Normal Supply-Price of Investment Goods.

$\hat{p}^e_{1t}$ may be called the expected "normal supply-price" of investment goods.[33] It is pictured in Figure 3 as a function of p^e_{1t}. The graph is a strictly concave curve with downward slope. p^e_{1t} and $\hat{p}^e_{1t}$ coincide on the intersection of the graph and the 45° line. When p^e_{1t} exceeds (falls short of) $\hat{p}^e_{1t}$, $h^e_{1t} > (<) i_t$. This means that the higher the expected price of investment goods becomes, the more profitable equities grow. $\hat{p}^e_{1t}$ is also a function of $\tilde{p}_{1t-1}$, i_t, w_t, etc. The graph shifts according as these parameters change.

From Section 3 the economy is assumed to be at the third subperiod of period $t-1$. *From now on,* suppose that the economy is always in the short-run equilibrium state, which means that production plan made each third subperiod are always realized at the first subperiod of the next period. The focus of analysis shifts from the short-run equilibrium state to the long-run equilibrium state.

In the short-run equilibrium state the difference (31) can be written simply by replacing a superscript e with a superscript $*$:

$$h^*_{1t} - i_t = \frac{p^*_{1t}(r^*_t + \delta)K_{1t}}{\tilde{p}_{1t-1}K^h_{1t}} \left[\left(\frac{p^*_{1t}}{\hat{p}^*_{1t}} \right)^{\frac{1}{\alpha}} - 1 \right], \tag{32}$$

where

$$\begin{aligned} \hat{p}^*_{1t} &= \left[\frac{(1+i_t)\tilde{p}_{1t-1} - (1-\delta)p^*_{1t}}{\alpha} \right]^{\alpha} \left[\frac{w_t}{A_t(1-\alpha)} \right]^{1-\alpha} \\ &= p^*_{1t} \left[\frac{r^*_t + \delta}{\alpha} \right]^{\alpha} \left[\frac{\frac{w_t}{p^*_{1t}}}{A_t(1-\alpha)} \right]^{1-\alpha} . \end{aligned} \tag{33}$$

even in the light of this definition the profit of, say, the investment-goods sector can be interpreted twofold. One is $p^e_{1t}Q_{1t} - w_tN_{1t} - [i_t\tilde{p}_{1t-1}K^d_{1t} + p^e_{1t}(\delta - \pi^e_t)K_{1t}](= h^e_{1t}\tilde{p}_{1t-1}K^h_{1t})$, while the other is $p^e_{1t}Q^e_{1t} - w_tN_{1t} - p^e_{1t}(r^e_t + \delta)K_{1t}(= (h^e_{1t} - i_t)\tilde{p}_{1t-1}K^h_{1t})$. In both cases the total revenue and the variable cost are respectively $p^e_{1t}Q^e_{1t}$ and w_tN_{1t}. The difference is the fixed cost. It is $i_t\tilde{p}_{1t-1}K^d_{1t} + p^e_{1t}(\delta - \pi^e_t)K_{1t}$ in the former case while $p^e_{1t}(r^e_t + \delta)K_{1t}$ in the latter case. The former case is more faithful to the microeconomics definition, but the latter is often to be seen and more convenient because a usual microeconomic analysis can directly be applied. When it comes to the rate of profit, the suitable definition is $p^e_{1t}Q^e_{1t} - w_tN_{1t} - p^e_{1t}\delta K_{1t}$ where $p^e_{1t}\delta K_{1t}$ is the "true" capital consumption, not the inflation-adjusted capital consumption. According to it, the rate of profit is defined as $(p^e_{1t}Q^e_{1t} - w_tN_{1t} - p^e_{1t}\delta K_{1t})/p^e_{1t}K_{1t}$. Fortunately the maximization of any "profit" mentioned above leads to the first-order condition (9). The same argument holds in the consumption-goods sector.

[33] See Keynes (1936, p. 228).

The derivation of the difference between h_{2t}^* and i_t in the short-run equilibrium state is a little bit complicated, but it can be obtained using (13) and (25) :

$$h_{2t}^* - i_t = \frac{p_{2t}^*(r_t^* + \delta)K_{2t}}{\tilde{p}_{1t-1}K_{2t}^h}\left[\left(\frac{p_{2t}^*}{\hat{p}_{1t}^*}\right)^{\frac{1}{\alpha}} - 1\right]$$

$$= \frac{p_{1t}^*(r_t^* + \delta)}{\tilde{p}_{1t-1}K_{2t}^h}\left\{\frac{c}{1-c}\left[\left(\frac{p_{1t}^*}{\hat{p}_{1t}^*}\right)^{\frac{1}{\alpha}} - (\delta - \pi_t^*)\frac{\alpha}{r_t^* + \delta}\right]K_{1t} - \left[1 + \frac{c}{1-c}(\delta - \pi_t^*)\frac{\alpha}{r_t^* + \delta}\right]K_{2t}\right\}. \tag{34}$$

Next consider the following price trend:

$$\frac{1}{1-\pi}\, p_{1t-1}^{**} = \frac{1}{1-\pi}\, \tilde{p}_{1t-1}^{**} = p_{1t}^{**}, \tag{35}$$

where π is a constant value of the inflation rate.[34] A superscript $**$ represents the long-run equilibrium state in what follows. (35) means that the rate of change of the price of investment goods as flow is equal to the inflation rate. Let us call such a situation as (35) the long-run price condition. This condition leads to the equality of the price of investment goods produced and the asset price during the same period.

Lastly, it is assumed, as usual in modern macroeconomics, that there is the natural level of employment, N_t, where

$$N_t = (1+n)N_{t-1},\ n > -1. \tag{36}$$

Now the long-run equilibrium state can be defined. An economy is in the long-run equiliturium state at t if the four conditions below are all satisfied:

1. The economy is in the short-run equilibrium state.

2. Full employment is realized, i.e., $N_{1t}^* + N_{2t}^* = N_t$.

3. The rates of return are all equal, i.e., $h_{1t}^* = i_t = h_{2t}^*$.

4. The long-run price conditon (35) holds.

For simplicity let us call the long-run equilibrium state just *the long-run state* in what follows.

Condition 1 says that the long-run state is a special case of the short-run equilibrium state where goods markets are cleared. A period in which an economy is in the long-run state is necessarily a period in which the economy is in the short-run equilibrium state. Never forget the previous short-run analysis!

Condition 2 means that labor market is also cleared in the long-run state, not to mention. Condition 3 implies that it is indifferent whether households hold asset as depositors or equity holders in the long-run state. From Condition 4 there is no distinction between the output price of investment goods and the asset price. I think that Conditions 2-4 are usually taken for granted to define the long run in macroeconomics. In fact the three conditions all stand and fall together. The next section explains how they are satisfied, and characterizes the long-run state.

[34]Remember that the expected inflatin rate was defined as $\pi_t^e = 1 - (\tilde{p}_{1t-1}/p_{1t}^e)$ in Section 3.

7 The Long-Run State

Taking (32) and the first half of (34) into cosideration, Conditions 3 and 4 imply that

$$\frac{1}{1-\pi}\,p_{1t-1}^{**} = p_{1t}^{*} = \hat{p}_{1t}^{*} = p_{2t}^{*} = p_{1t}^{**}. \tag{37}$$

Hence the following theorem concernig output prices:

Theorem 2: In the long-run state prices of investment goods and consumption goods coincide and change at the same rate.

In the short-run equilibrium state it is necessary to distinguish the two prices, but it is not in the long-run state. Therefore, it is convenient in what follows to write both prices only as p_{1t}^{**}, in which case a nominal value divided by p_{1t}^{**} can be interpreted as a "real" value in a usual sense.

Theorem 2 makes it possible to describe the two-sector KS model in the long-run state as if it were a one-sector model like the Solow model. Let Q_t^{**} be defined as the long-run-state total amount of production divided by p_{1t}^{**}. Then, real GDP is expressed simply as

$$Q_t^{**} = Q_{1t}^{**} + Q_{2t}^{**}. \tag{38}$$

But it should be emphasized that there is a crucial difference between a two-sector model and a one-sector model: The latter divides output into consumption goods and investment goods *after* production is finished, whereas the former distinguishes the two goods *from beginning to end.* Which one do you like? I, for one, don't like to eat a machine.

From (37), the demand for labor in the investment-goods sector (7) can be written as

$$N_{1t}^{*} = \left[(1-\alpha)A_t^{1-\alpha}\frac{p_{1t}^{**}}{w_t}\right]^{\frac{1}{\alpha}} K_{1t},$$

and similarly that in the consumption-goods sector (14) as

$$N_{2t}^{*} = \left[(1-\alpha)A_t^{1-\alpha}\frac{p_{1t}^{**}}{w_t}\right]^{\frac{1}{\alpha}} K_{2t}.$$

Since $K_{1t} + K_{2t} = K_t$, Condition 2 leads to the following equality:

$$\left[(1-\alpha)A_t^{1-\alpha}\frac{p_{1t}^{**}}{w_t}\right]^{\frac{1}{\alpha}} K_t = N_t.^{35} \tag{39}$$

[35] Considering (8), (15), (37), and (39), (38) can also be written as

$$Q_t^{**} = A_t^{1-\alpha}N_t^{1-\alpha}K_t^{\alpha}.$$

This may be regarded as the original Cobb-Douglas production function. $A_t^{1-\alpha}$ corresponds to what Cobb and Douglas (1928, p. 155) called a "catch-all." I am afraid that macroeconomists forgot his presidential address, in which Douglas (1948, pp. 20-21) stated, "The fact that on the basis of fairly wide studies there is an appreciable degree of uniformity, and that the sum of the exponents approximates unity, fairly clearly suggests that there are *laws of production* which can be approximated by inductive studies and that we are at least approaching them." (My italics.) It was Solow (1957) that reconfirmed the law. I don't know that the law was broken since then. Thus, I don't know the reason why other production function than this beautiful one is used in macro analysis. Or, to say the least of it, the Cobb-Douglas production function has no doubt the highest priority.

(39) gives the long-run equilibrium real wage rate:

$$\frac{w_t^{**}}{p_{1t}^{**}} = (1-\alpha)A_t\left[\frac{K_t}{A_tN_t}\right]^{\alpha}. \tag{40}$$

w_t^{**} is the long-run-state nominal wage rate, and it is determined on the values of p_{1t}^{**}, K_t, A_t, N_t, and α which are all known at the third subperiod of period $t-1$.[36] Let capital per effective labor in the right-hand side of (40) be designated by k_t, and that in the investmet-goods sector and in the consumption-goods sector respectively by k_{1t} and k_{2t}:

$$k_t = \frac{K_t}{A_tN_t},\ k_{1t} = \frac{K_{1t}}{A_tN_{1t}^{**}},$$

and

$$k_{2t} = \frac{K_{2t}}{A_tN_{2t}^{**}},$$

where $N_{1t}^{**} + N_{2t}^{**} = N_t$. Then (40) can be rewritten as

$$\frac{w_t^{**}}{p_{1t}^{**}} = (1-\alpha)A_tk_t^{\alpha} = (1-\alpha)A_tk_{1t}^{\alpha} = (1-\alpha)A_tk_{2t}^{\alpha}. \tag{41}$$

Therefore the following theorem holds:

Theorem 3: In the long-run state capital per effective labor coinsides in the investment-goods sector and in the consumption-goods sector.

Let us call $(1-\alpha)A_tk_t^{\alpha}$ in (41) the marginal product of labor as a whole, and denote it by MPL_t^{**}. Then, it follows from (9), (16), and (41) that $MPL_t^{**} = MPL_{1t}^{**} = MPL_{2t}^{**}$, and that they are all equal to the real wage rate w_t^{**}/p_{1t}^{**}.

Condition 3 holds as a result of arbitrage. It is not so clear how the arbitrage takes place, but it is reasonable to think that the asset price, $\tilde{p}_{1t-1}$, and capital stock in each sector, K_{1t} and K_{2t}, are adjusted as follows. If $h_{1t}^e > (<) i_t$, $\tilde{p}_{1t-1}$ rises (falls).[37] And if $h_{2t}^e > (<) i_t$, the ratio of K_{2t} to K_{1t} rises (falls). As a result, $h_{1t}^* = i_t = h_{2t}^*$ holds.[38] I will elaborate on this.

In the long-run state, the real interest rate as defined and Assumption (5) are respectively simplifies as

$$\begin{aligned} r_t^{**} &= \frac{(1+i_t^{**})\tilde{p}_{t-1}^{**}}{p_{1t}^{**}} \\ &= (1+i_t^{**})(1-\pi) - 1, \end{aligned} \tag{42}$$

and

$$\delta - \pi > 0, \tag{43}$$

[36] It happens that labor market is cleared even in the short run, but it requires more information like K_{1t}, K_{2t}, p_{1t}^e, and p_{2t}^e.

[37] See Figure 3. The graph of $\hat{p}_{1t}^e$ shifts upward (downward) when $\tilde{p}_{1t-1}$ rises (falls). When $\hat{p}_{1t}^e$ coincides with p_{1t}^e on the 45° line, $h_{1t}^e = i_t$ holds.

[38] An important point is that the asset price must always be so determined as to satisfy the budget constraints. In this respect so-called asset bubble can be directly caused only by a sharp rise in the expected price of investment goods *as flow*, not as stock, of the next period.

because of Condition 4. And, taking (37) and (41) into consideration, (33) leads to

$$r_t^{**} + \delta = \alpha k_t^{\alpha-1} = \alpha k_{1t}^{\alpha-1} = \alpha k_{2t}^{\alpha-1}. \tag{44}$$

Call $\alpha k_t^{\alpha-1}$ in (44) the marginal product of capital as a whole, and denote it by MPK_t^{**}. Then, it is found from (10), (17), and (44) that $MPK_t^{**} = MPK_{1t}^{**} = MPK_{2t}^{**}$, and that they are all equal to the sum of the real interest rate and the capital depreciation rate.

More important, the first half of (44) means that the level of capital per effective labor as a whole determines the long-run-state real interest rate, which in turn specifies the long-run-state nominal interest rate i_t^{**} through (42) as follows:

$$i_t^{**} = \frac{1}{1-\pi}\,[\alpha k_t^{\alpha-1} - (\delta - \pi)]. \tag{45}$$

i_t^{**} is approximately equal to the difference between the marginal product of capital as a whole and the inflation-adjusted depreciation rate when the inflation rate is not far from zero. Once i_t is set at i_t^{**} as in (45) on the values of π, K_t, A_t, N_t, δ, and α which are all known at the second subperiod of period $t-1$, the asset price $\tilde{p}_{1t}^{**}$ is so determined as to make h_{1t}^{**} and i_t^{**} equal with the result that the inflation rate takes a value of π. Condition 4 consists of two parts, $[1/(1-\pi)]\ p_{1t-1}^{**} = p_{1t}^{**}$ and $[1/(1-\pi)]\ \tilde{p}_{1t-1}^{**} = p_{1t}^{**}$. It is found from the above argument that it is the nominal interest rate that determines the long-run-state inflation rate as in the latter part.[39] The former part may come true, e.g., by means of monetary policy of the central bank.

(37) and (43) simplify (34) as

$$\frac{c}{1-c}\left[1 - (\delta - \pi)\frac{\alpha}{r_t^{**}+\delta}\right] K_{1t}^{**} - \left[1 + \frac{c}{1-c}(\delta-\pi)\frac{\alpha}{r_t^{**}+\delta}\right] K_{2t}^{**} = 0. \tag{46}$$

Then, substituting $r_t^{**} + \delta = \alpha k_t^{\alpha-1}$ in (44) into (46) and some calculations yield the ratios:

$$\frac{N_{1t}^{**}}{N_t} = \frac{K_{1t}^{**}}{K_t^{**}} = (1-c) + c(\delta-\pi)(k_t^{**})^{1-\alpha}, \tag{47}$$

and

$$\frac{N_{2t}^{**}}{N_t} = \frac{K_{2t}^{**}}{K_t^{**}} = c - c(\delta-\pi)(k_t^{**})^{1-\alpha}. \tag{48}$$

The rightmost-hand sides of (47) and (48) include two terms. The former is the sum of the rate of saving $1-c$ and the term related to the inflation-adjusted depreciation rate $\delta - \pi$, while the latter is the difference between the rate of consumption c and the same term related to the inflation-adjusted depreciation rate. This inflation-adjusted depreciation rate plays a very important role in the analysis below.

Capital stock in each sector is adjusted during the second subperiod according to (47) and (48) with the result that $h_{2t}^{**} = i_t^{**}$ holds. K_{1t}^{**} and K_{2t}^{**} are determined on the values of π, K_t, A_t, N_t, δ, α and c which are all known at the time. (47) and (48) show that N_{1t}^{**} and N_{2t}^{**} are also determined before the third subperiod of period $t-1$.[40] It turns out that the long-run state is a kind of the Nash equilibrium.

[39]It has been claimed in the name of the Fisher effect that the nominal interest rate is determined as the sum of the real interest rate and the inflation rate in the long run. I argue for the opposite, i.e., the claim that the inflation rate is determined as the sum of the real interest rate and the nominal interest rate in the long run.

[40]It is easy to show that in the long-run state the budget constraints of the two sectors can be unified into

8 Analysis of the Long-Run Steady State

The KS model in the long-run state is represented by capital per effective labor, k_t^{**}, as in usual growth models. The problem is what value k_t^{**} takes in this two-sector model. The answer is, however, just simple because the familiar method to analyze the long-run state which was developed by Solow (1956) can be used without reservation.[41]

The equation of capital accumulation in the short run (29) also holds in the long-run state as follows:

$$K_{t+1}^{**} = (1-\delta)K_t^{**} + Q_{1t}^{**}, \tag{49}$$

where $K_t^{**} = K_{1t}^{**} + K_{2t}^{**}$. Dividing both sides of (49) by $A_{t+1}N_{t+1}$ gives

$$k_{t+1}^{**} = \frac{(1-\delta)+c(\delta-\pi)}{(1+g)(1+n)}k_t^{**} + \frac{1-c}{(1+g)(1+n)}(k_t^{**})^{\alpha}, \tag{50}$$

because of (2), (36), and (47).[42] The long-run-state capital accumulation equation (50) is much the same as that of Solow (1956). A difference is the term $c(\delta-\pi)$, which comes from the budget constraints of the two sectors (4) and (13).

The economy is said to be in the long-run *steady* state when $k_{t+1}^{**} = k_t^{**}$, and the analysis focuses on the state. Let a subcript S represent the long-run steady state of the economy in what follows. Furthermore, let us drop "long-run" in the "long-run steady state" unless it involves ambiguity. Then it is easy to obtain the steady-state capital per effective labor:

$$k_S^{**} = \left[\frac{1-c}{g+n+\pi+(1-c)(\delta-\pi)}\right]^{\frac{1}{1-\alpha}}.^{43} \tag{51}$$

Here is a crucial assumption for the steady-state analysis:

$$g+n+\pi > 0, \tag{52}$$

which roughly asserts that the sum of the natural growth rate and the inflation rate should be positive.[44] Assumptions (43) and (52) make k_S^{**} always positive. They also imply that $g+n+\delta > 0$.

the following equation:

$$p_{1t}^{**}Q_t^{**} = w_t^{**}N_t + p_{1t}^{**}(r_t^{**}+\delta)K_t,$$

where $p_{1t}^{**}(r_t^{**}+\delta)$ corresponds to what Jorgenson (1963, p. 249) called the user cost of capital. On the basis of (44), someone may say that Condition 3 means the equality of the capital demand by firms with existing capital through the adjustment of the real interest rate, as is often argued. But it doesn't. In the KS model it is households that demand capital (as a means of store of value). Firms are merely institutions that produce goods using existing capital for profit maximization. Condition 3 is the result of arbitrage as said in the text.

[41] There may be someone who somehow feels that Solow's neoclassical growth model is a mere textbook one or that a surge of new growth theory rendered it old-fashioned. But it is a sheer misunderstanding. The status quo is as follows: We have reached "the 50th anniversary of the neoclassical model of growth; astonishingly, it is still alive and well. There is not really any competing model. In the broad sense in which I use the term, the "endogenous growth" models of Romer and Lucas and their successors are entirely neoclassical. So the *basic* model has survived for 50 years" (Solow, 2005, p. 4, the italics in the original.) Macroeconomists have not gotten a more robust growth model than the Solow model.

[42] Appendix D shows how to derive (50).

[43] For conveniencc' sake $g+n+gn$ is written simply as $g+n$ in what follows. Thus $g+n$ such as that in the denominator of (51) must be read as $g+n+gn$.

[44] If Condition (52) does not hold, an economy itself ceases to exist as is seen soon.

Theorem 3 assures that

$$k_S^{**} = k_{S1}^{**} = k_{S2}^{**},$$

where

$$k_S^{**} = \frac{K_{St}^{**}}{A_t N_t},\ k_{S1}^{**} = \frac{K_{S1t}^{**}}{A_t N_{S1t}^{**}},\ k_{S2}^{**} = \frac{K_{S2t}^{**}}{A_t N_{S2t}^{**}},\ K_{St}^{**} = K_{S1t}^{**} + K_{S2t}^{**},$$

and

$$N_t = N_{S1t}^{**} + N_{S2t}^{**}.$$

The KS model in the steady state is, therefore, completely characterized by k_S^{**}.

As for capital stock,

$$K_{St}^{**} = \left[\frac{1-c}{g+n+\pi+(1-c)(\delta-\pi)}\right]^{\frac{1}{1-\alpha}} A_t N_t, \tag{53}$$

$$K_{S1t}^{**} = \frac{(1-c)(g+n+\delta)}{g+n+\pi+(1-c)(\delta-\pi)} K_{St}^{**}, \tag{54}$$

and

$$K_{S2t}^{**} = \frac{c(g+n+\pi)}{g+n+\pi+(1-c)(\delta-\pi)} K_{St}^{**}, \tag{55}$$

because of (47) and (48).

As for output,

$$\begin{aligned} Q_{S1t}^{**} &= A_t N_{S1t}^{**}(k_{S1}^{**})^{\alpha} = A_t N_{1t}^{**}(k_S^{**})^{\alpha} \\ &= (g+n+\delta)\left[\frac{1-c}{g+n+\pi+(1-c)(\delta-\pi)}\right]^{\frac{1}{1-\alpha}} A_t N_t, \end{aligned} \tag{56}$$

$$\begin{aligned} Q_{S2t}^{**} &= A_t N_{2t}^{**}(k_{S2}^{**})^{\alpha} = A_t N_{2t}^{**}(k_S^{**})^{\alpha} \\ &= (g+n+\pi)\frac{c}{1-c}\left[\frac{1-c}{g+n+\pi+(1-c)(\delta-\pi)}\right]^{\frac{1}{1-\alpha}} A_t N_t, \end{aligned} \tag{57}$$

and

$$\begin{aligned} Q_{St}^{**} &= Q_{S1t}^{**} + Q_{S2t}^{**} = A_t N_t (k_S^{**})^{\alpha} \\ &= \left[\frac{1-c}{g+n+\pi+(1-c)(\delta-\pi)}\right]^{\frac{\alpha}{1-\alpha}} A_t N_t, \end{aligned} \tag{58}$$

from (1), (12), (47), (48), and (51).[45]

Finally, as for nationl income and saving in real terms,

$$\begin{aligned} \frac{Y_{St}^{**}}{p_{1t}^{**}} &= Q_{S1t}^{**} + Q_{S2t}^{**} - (\delta-\pi)K_{St}^{**} \\ &= \frac{g+n+\pi}{1-c}\left[\frac{1-c}{g+n+\pi+(1-c)(\delta-\pi)}\right]^{\frac{1}{1-\alpha}} A_t N_t, \end{aligned} \tag{59}$$

[45] (53) and (58) yield $(1-c)Q_{St}^{**}/K_{St}^{**} = g+n+\pi+(1-c)(\delta-\pi)$. This is one of "fundamental growth equations" Hahn and Matthews (1964, p. 824) enumerated in their survey of the theory of economic growth, if $g = \pi = 0$.

and

$$
\begin{aligned}
\frac{S_{St}^{**}}{p_{1t}^{**}} &= \frac{(1-c)Y_{St}^{**}}{p_{1t}^{**}} \\
&= (g+n+\pi)\left[\frac{1-c}{g+n+\pi+(1-c)(\delta-\pi)}\right]^{\frac{1}{1-\alpha}} A_t N_t, \qquad (60)
\end{aligned}
$$

because of (19) and (21).

Note that these macro variables are all influenced by the inflation rate π even in the long-run steady state unlike in usual growth models. Particularly it is easily shown from (53) and (58) that

$$\frac{\partial K_{St}^{**}}{\partial \pi} < 0, \qquad (61)$$

and

$$\frac{\partial Q_{St}^{**}}{\partial \pi} < 0. \qquad (62)$$

I think that these are very interesting facts which have not been established. Therefore, these results are worthy to be written down as the following proposition: [46]

Proposition 2: In the long-run steady state economic growth is adversely affected by inflation.

Proposition 2 appears to contradict Proposition 1 because an increase in prices has a favorable influence on an economy in the latter whereas the opposite is claimed in the former. Why? It is because an increase in prices has direct influence on production with capital stock as given in the latter, while capital stock and labor in each sector are adjusted to the inflation rate according to (47) and (48) in the former. In the long-run steady state the production sector retains $(\delta-\pi)K_{St}^{**}$ in real terms for capital depreciation each period. This constitutes the demand for investment goods. Therefore, the higher the inflation becomes, the lower the demand for investment goods, *ceteris paribus*. This long-run-state effect appears in (47). And captital accumulation is decelerated due to the effect in the first term of the right-hand side of (50).

In other words it is the household sector that is responsible. The result obtained in the short-run equilibrium state (28) still holds in the long-run steady state in the following form:

$$
\begin{aligned}
\frac{C_{St}^{**}}{S_{St}^{**}} &= \frac{c}{1-c} \\
&= \frac{Q_{S2t}^{**}}{Q_{S1t}^{**}-(\delta-\pi)K_{St}^{**}}. \qquad (63)
\end{aligned}
$$

Intuitively speaking, since the ratio of consumption to saving is fixed *by the bousehold sector*, an increase in π crowds out a part of Q_{S1t}^{**} from the denominator of (63), which in turn causes

[46] Moreover it is obvious that $\partial K_{S1t}^{**}/\partial\pi < 0$, $\partial Q_{S1t}^{**}/\partial\pi < 0$, $\partial(K_{S1t}^{**}/K_{St}^{**})/\partial\pi < 0$, and $\partial(K_{S2t}^{**}/K_{St}^{**})/\partial\pi > 0$, but the signs of $\partial K_{S2t}^{**}/\partial\pi$ and $\partial Q_{S2t}^{**}/\partial\pi$ are not determinate.

K^{**}_{St} to fall because output of investment goods is the source of capital stock itself. In due course Q^{**}_{St} is reduced.[47]

Nevertheless Proposition 2 certainly breaks the law of superneutrality of money which is now recognized by most macroeconomits as true in the long run. Why? It is because the analysis is not yet completed.

9 Analysis of the Golden-Rule State

Consider the long-run steady state represented by (53) - (60). It is interesting to note that it is the household sector that is in a position to "control" the economy. If g, n, π, δ, and α are supposed to be given, only the rate of consumption c is variable. And it is the household sector that can change it. Then, what is the optimal rate of consumption for the household sector?

The rate that comes into my mind naturally is that which maximizes the *current* real consumption *every* period. The long-run steady state where current cunsumption is maximized is called the *golden-rule state* among macroeconomists. Needless to say, the golden rule was discovered by Oiko in Phelps (1961). The golden rule focuses simply on current consumption. What a nice idea! I remember that the Solovians were satisfies with that simple rule. However, recent textbooks of macroeconomics as well as academic researches are generally based on the rate which realizes the modified golden-rule state, in which an infinite-lived household maximizes a sum of discounted utilities in the infinite horizon subject to a resource constraint. Such an idea goes back to Ramsey (1928). One of the reasons why a sum of discounted utilities was recommended in his one-sector model is that the rate of saving is too high (or equivalently, the rate of consumption is too low) in terms of reality if utilities are not discounted.[48] The golden-rule state is regarded as an "undiscounted" case which is, according to Ramsey (1928, p. 543), "ethically indefensible."

For example, in the case of the Cobb-Douglas production function, the golden-rule-state rate of consumption is calculated at $1-\alpha$, or the golden-rule-state rate of saving at α, as in Phelps (1961). On the other hand, it is well known that an actual value of α is around $\frac{1}{3}$. Therefore, the golden-rule-state rate of consumption (saving) turns out to be about $\frac{2}{3}$ ($\frac{1}{3}$). But this result does not fit the macro fact that an actual value of c is usually over 0.8 on average. Too low rate of consumption and too high rate of saving made the golden rule unrealistic. That is, I think, why the golden rule has been ignored in macroeconomics. Indeed the modified golden rule may make the rate of consumption a realistic value, but what if the golden rule can do it, too? If so, (and that is shown below,) there is not any reason, according to Occam's razor, why the simple "true" golden rule is not used for analysis.[49]

In the case considered in this paper the maximization of current consumption in the steady state is equivalent to that of output of consumption goods Q^{**}_{S2t} in (57). Let a subscript G represent the golden-rule state. Then, the golden-rule-state rate of consumption can be

[47] Based on a statistical analysis of roughly a hundred countries since 1965, Barro (1997) obtained the result that higher inflation leads to a lower rate of economic growth. But no theoretical grounds are provided. Proposition 2 may serve as a clue.

[48] See Ramsey (1928, pp. 548-549).

[49] The golden rule may be rather for "rich" countries if Harrod (1969, p. 200) is right to say, "Opinions differ about how important a part ... preference for present over future utilities, called by Pigou 'lack of telescopic faculty,' plays in the individual's saving schedule. I would suppose it to play an unimportant part, except in the case of very poor, and thereby improvident, societies."

obtained by solving by $dQ^{**}_{S2t}/dc = 0$ and $d^2Q^{**}_{S2t}/dc^2 < 0$:[50]

$$c_G = \frac{(1-\alpha)(g+n+\delta)}{g+n+\pi+(1-\alpha)(\delta-\pi)}. \tag{64}$$

If the household sector chooses the rate of consumption c_G following the golden rule, it can always enjoy the maximum consumption the production technologies available make possible.[51] What a wonderful world!

Put, e.g., $g = 0.01$, $n = 0.005$, $\delta = 0.06$, $\alpha = \frac{1}{3}$, and $\pi = 0.01$. Then c_G is something like 0.86, which is plausible enough. I do not think that this example alone convinces macroeconomists, mainly because this paper deals only with a case without the government sector or the foreign sector. But it can be said even in this basic case that c_G is more than $1-\alpha$ under Assumptions (43) and (52) since

$$c_G = (1-\alpha)\left[1+\frac{\alpha(\delta-\pi)}{g+n+\pi+(1-\alpha)(\delta-\pi)}\right] > 1-\alpha.$$

Similarly the golden-rule-state rate of saving is calculated at

$$1-c_G = \frac{\alpha(g+n+\pi)}{g+n+\pi+(1-\alpha)(\delta-\pi)}, \tag{65}$$

which is of course less than α.

When $c = c_G$, k^{**}_S in (51) is simplified as

$$k^{**}_G = \left[\frac{\alpha}{g+n+\delta}\right]^{\frac{1}{1-\alpha}},$$

where $k^{**}_G = K^{**}_{Gt}/A_tN_t$, a subsript S being replaced by a subscript G.[52] Then the KS model in the golden-rule state is completely characterized by k^{**}_G.

As for capital stock,

$$K^{**}_{Gt} = \left[\frac{\alpha}{g+n+\delta}\right]^{\frac{1}{1-\alpha}} A_tN_t, \tag{66}$$

$$K^{**}_{G1t} = \alpha K^{**}_{Gt}, \tag{67}$$

and

$$K^{**}_{G2t} = (1-\alpha)K^{**}_{Gt}. \tag{68}$$

As for output,

$$\begin{aligned} Q^{**}_{G1t} &= A_tN^{**}_{G1t}(k^{**}_{G1})^{\alpha} = A_tN^{**}_{G1t}(k^{**}_{G})^{\alpha} \\ &= (g+n+\delta)\left[\frac{\alpha}{g+n+\delta}\right]^{\frac{1}{1-\alpha}} A_tN_t, \end{aligned} \tag{69}$$

[50] In fact it is easier to get c_G from the fact that $Q^{**}_{S2t} = [(k^{**}_S)^{\alpha} - (g+n+\delta)k^{**}_S]A_tN_t$, which is derived immediately from (56) - (58).

[51] There were also economists who, on the contrary, paid attention to the "optimum propensity to consume" which maximizes production of investment goods. For details, see Lange (1938).

[52] Note that the golden-rule state is a special case of the steady state which is a special case of the long-run equilibrium state which is a special case of the short-run equilibrium state. The KS model, basic building blocks of which are (1), (4), (12), (13), (19), and (20), is only one throughout.

$$
\begin{aligned}
Q^{**}_{G2t} &= A_t N^{**}_{G2t}(k^{**}_{G2})^{\alpha} = A_t N^{**}_{G2t}(k^{**}_{G})^{\alpha} \\
&= \frac{1-\alpha}{\alpha}(g+n+\delta)\left[\frac{\alpha}{g+n+\delta}\right]^{\frac{1}{1-\alpha}} A_t N_t, \qquad (70)
\end{aligned}
$$

and

$$
\begin{aligned}
Q^{**}_{Gt} &= Q^{**}_{G1t} + Q^{**}_{G2t} = A_t N_t (k^{**}_{G})^{\alpha} \\
&= \left[\frac{\alpha}{g+n+\delta}\right]^{\frac{\alpha}{1-\alpha}} A_t N_t. \qquad (71)
\end{aligned}
$$

Finally, as for national income and saving in real terms,

$$
\begin{aligned}
\frac{Y^{**}_{Gt}}{p^{**}_{1t}} &= Q^{**}_{G1t} + Q^{**}_{G2t} - (\delta - \pi)K^{**}_{Gt} \\
&= \frac{g+n+\pi+(1-\alpha)(\delta-\pi)}{\alpha}\left[\frac{\alpha}{g+n+\delta}\right]^{\frac{1}{1-\alpha}} A_t N_t, \qquad (72)
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{S^{**}_{Gt}}{p^{**}_{1t}} &= (1-c_G)Y^{**}_{Gt} \\
&= (g+n+\pi)\left[\frac{\alpha}{g+n+\delta}\right]^{\frac{1}{1-\alpha}} A_t N_t. \qquad (73)
\end{aligned}
$$

In the long-run steady state macro variables are generally influenced by the inflation rate π as was seen from (53) - (60) in the previous section, while in the golden-rule state levels of capital stock and output are independent of it as is seen from (66) - (71). The superneutrality of money obtains. Hence the following proposition:

Proposition 3: In the golden-rule state money (or the inflation rate) does not influence real economy.

This is precisely what is called the neo-classical world. The law of superneutrality of money is kept due to the consumption-maximizing behavior of the household sector. Putting Propositions 1-3 together gives us a consistent understanding of the rather paradoxical relationship between prices and real economy.[53]

[53] It is also important to point out the following facts in the golen-rule state: As to the ratio of the investment-goods sector to the consumption-goods sector,

$$\frac{K^{**}_{G1t}}{K^{**}_{G2t}} = \frac{Q^{**}_{G1t}}{Q^{**}_{G2t}} = \frac{\alpha}{1-\alpha},$$

and as to the capital-output ratio as a whole,

$$\frac{K^{**}_{Gt}}{Q^{**}_{Gt}} = \frac{\alpha}{g+n+\delta}.$$

The latter result can also be written as $K^{**}_{Gt} = [\alpha/(g+n+\delta)]Q^{**}_{Gt}$. This may be the relationship between capital stock and output from which the acceleration principle and the capital stock adjustment principle have been derived. Particularly the value of the coefficient of Q^{**}_{Gt} is around 4.4, using the example given in the text.

Proposition 3 is easy to understand by seeing the golden-rule-state version of (63):

$$\frac{C_{Gt}^{**}}{S_{Gt}^{**}} = \frac{c_G}{1-c_G} = \frac{Q_{G2t}^{**}}{Q_{G1t}^{**}-(\delta-\pi)K_{Gt}^{**}}. \tag{74}$$

Intuitively speaking again, an expansion of the denominator of (74) due to an increase in π causes the household sector to lower the rate of consumption because maximized output of comsumption goods in the numerator is unchanged to π. It is the rate of consumption that is adjusted to inflation in the golden-rule state while it was output in the steady state as was seen in (63).

There remains to be considered the relationship appearing in (72) and (73). The next section discusses it in connection with an old controversy in macroeconomics.

10 Consumption Function Controversy Revisited

10.1 The Permanent Income Hypothesis

"Once upon a time," there was a consumption function controversy among macroeconomists. It began when Kuznets (1942) found out the fact of and asked the reason for the secular stability in division of national income (or net national product) between consumption and investment, or the secular constancy of the rate of saving. The discovery of a "long-run" consumption function led to the re-examination of a "short-run" consumption function according to which the rate of saving should rise with income and in fact it had done so. Some new hypotheses appeared, trying to explain why a long-run consumption function is steeper than a short-run consumption function. Time has passed, and it is the permanent imcome hypothesis by Friedman (1957) that survived most influentially.[54]

According to it, income can be divided into two components, permanent and transitory. Households tend to spend a certain fraction of permanent (or expected) income. Variation in income as whole is caused by that in transitory (or unexpected) income. The two components of income are not correlated, and the increase (decrease) in transitory income leads to the increase (decrease) in saving because consumption depends on permanent income which is assumed to be stable. As a result, when transitory income is positive (negative), the rate of consumption becomes lower (higher) than a long-run average. Hence the crossing of a short-run consumption function with a steeper long-run consumption function. Now the paradox is thought to be completely solved.[55]

But the *KS* model sheds new light on this problem. Remember that in the *KS* model the economy is assumed to be always in the short-run equilibrium state. This means that unexpected income like transitory income never happens. The amount of national income paid

[54]For example, Deaton (1992), a critical survey of the modern consumption theories, is for the most part related with the permanent income hypothesis. For the consumption function controversy, see Ackley (1961, chap 10).

[55]The terms "permanent" and "transitory" components were originally used by Friedman and Kuzunets (1945) in their study of incomes of professions such as physicians, dentists, lawyers, and certified public accountants. It is interesting to note that Friedman (1957) focused on consumption, whereas Kuznets (1952) placed emphasis on the saving process, to explain essentially the same thing, the secular stability of the rate of consumption or saving.

at period t is determined at the third subperiod of period $t-1$ in the process of production plan. The production plan is made on the basis of all information then available including the rate of consumption c of the household sector. Period t comes, and national income is paid as is planned. And income paid is divided into two parts, the purchasing power for consumption goods and that for investment goods as is expected at the previous period. The household sector does not change the rate of consumtpion, and all expectations are realized.

The permanent income hypothesis pays attention to the reaction of the househould sector to an unexpected variation in income. In other words the household sector is permitted to change the rate of consumption *after* period t has come. It seems that the hypothesis is based on a one-sector model and it may be assumed that the rate of consumption is easy to change because it is equivalent to a change in the division of a single-type good between consumption and investment. If corn is in a good harvest, store the surplus. In the light of the two-sector KS model, however, the permanent income hypothesis is tantamount to the claim that an unexpected variation in income comes from unplanned production of investment goods since production of consumption goods is assumed to be realized as planned. Indeed unxpected shocks to an economy appear to play a temporary part in the short-run consumption behavior, it may not be convincing to claim that unexpected shocks continued to generate pretty regular pattern of the short-run consumption behavior for decades. Nowadays it is widely agreed among macroeconomists that economic agents form expectations rationally using all infomation available and they do not repeat systematic failures. Thus it will be more convincing if the paradox of consumption functions is made clear on the assumption of nonexistence of unexpected factors. The KS model can do this.

10.2 The True Golden-Rule Hypothesis

Let us think of the golden-rule-state rate of consumption c_G as the slope of a long-run consumption function. This means that the long-run consumption behavior is the result of the optimal behavior of the household sector. This is a natural starting point. But it is also the end of argument. That is, in order to explain the consumption puzzle the KS model assumes that the economy is always in the golden-rule state. It should be stressed at once that I do not argue that an actual economy always grows presicely on the golden-rule-state path. It is too apparent that the golden-rule state is an ideal one and that an economy diverges from it or even from the steady state. I just say that an economy tends to be in the neighborhood of the golden-rule state, so it is covenient and useful to analyze the economy as if it were exactly in the golden-rule state. If such principle is accepted, the slope of a short-run consumption function is also analysed using c_G. The upshot is that the distinction between a short-run and a long-run consumption functions itself must be made obsolete.

As is seen from (64), c_G is calculated on the basis of such information as g, n, δ, α and π. Therefore, c_G, the slope of *the* consumption function, varies according as these parameters change. The slope of a "long-run" consumption function is the average value of c_G. Which parameter, then, dominates a "short-run" change in c_G? The most plausible is the inflation rate π, which can vary during a comaratively short period. Thus, let us focus on the relationship between c_G and π with other paramets as fixed.[56]

(64) tells us that in the golden-rule state the rate of consumption is a decreasing function

[56]RBC theorists shall not miss g.

of the inflation rate. In other words,

Proposition 4: Inflation (deflation) implies the lower (higher) rate of cosumption and the higher (lower) rate of saving.

One may be under the impression that inflation (deflation) causes the incerase (decrease) in real consumption *level*. It does *not*. The permanent income hypothesis was quite right concerning the stability of consumption level. As is obvious in (70), real consumption level is not at all affected by the inflation rate. It is real national income that varies with the inflation rate. (72) shows that inflation leads to an increase in real national income. Since real consumption is independent of the inflation rate, the ratio of consumption to national income, i.e., the rate of consumption, decreases as the inflation rate rises.[57]

With regard to the rate of saving, both real saving (73) and real national income increase with the inflation rate. But (64) and (65) teach us that the rate of increase in saving is faster than that in national income.[58]

Figure 4. The Golden-Rule State.

The argument above is made clear graphically. In Figure 4 is shown the golden-rule state in terms of effective labor. Consider three values of the inflation rate, $\pi^1 > \pi^0 > \pi^2$, and the corresponding golden-rule-state rates of consumption, $c_G^1 < c_G^0 < c_G^2$. When $c = c_G^0$, the economy lies on Point A^0, where consumption takes the maximum value A^0B^0. B^0C^0 is the corresponding output of investment goods. Assume that the inflation rate rises to π^1. What happens? If the economy diverges from the golden-rule state but remains in the long-run steady state, it shifts leftward, say, to Point A^1 due to (61) and (62). Both consumption and output of investment goods decrease to A^1B^1 and B^1C^1, respectively. Then, what should the household sector do in order to make the maximum consumption possible again. The answer is very simple: Accumulate capital. To do so the household sector has only to lower the rate of consumption from c_G^0 to c_G^1. Then the economy returns to the original golden-rule state (Point A^0) with a smaller c_G and a larger $1 - c_G$. This is a transitional process of adjustment to a rise in the inflation rate.

Next suppose that the inflation rate falls from π^0 to π^2. π^2 may be negative, i.e., a case of deflation. Similarly, the economy shifts rightward, say, to Point A^2. Consumption decreases to A^2B^2 while output of investment goods increases to B^2C^2. What the household sector should do is to deccumulate capital. This time the household sector has only to raise the rate of consumption from c_G^0 to c_G^2. Then the economy comes back to the golden-rule state with

[57]On the basis of the life cycle-permanent income hypothesis, Hall (1978) established empirically the famous "random walk hypothesis." According to it, future consumption is unrelated to current income, and only current consumption has the predictive power with respect to future consumption. From the viewpoint of the *KS* model, his result reflects the stability of consumption trend as shown in (70) and the variability of income in response to inflation as shown in (72). He also found that changes in stock prices have a measurable value in predicting changes in consumption. This result can be explained by Proposition 1 to some extent.

[58]How about labor share? Labor share here is the ratio of real labor income to real national income. Golden-rule-state labor income is calculated at $(1-\alpha)Q_{Gt}^{**}$ using (9) and (16). It is constant irrespective of the inflation rate. Then it is conjectured that inflation (deflation) gives rise to lower (higher) labor share.

a larger c_G and a smaller $1 - c_G$. This is a transitional process of adjustment to a fall of the inflation rate including deflation.[59] [60]

Finally let us make clear what has been called the "marginal propensity to consume" for a long time, using the above graphical example again. Take two periods. One is period t with the inflation rate π^2, and the other is the next period $t+1$ with the inflation rate π^1. And assume that $g + n > 0$. Then this economy is charaterized by a positive growth rate and an accelerating inflation rate which were usually typical of prosperity. Let c_G^{12} be

$$\frac{\left(\frac{C_{Gt+1}^{**}}{p_{1t+1}^{**}}\right) - \left(\frac{C_{Gt}^{**}}{p_{1t}^{**}}\right)}{\left(\frac{Y_{Gt+1}^{**}}{p_{1t+1}^{**}}\right) - \left(\frac{Y_{Gt}^{**}}{p_{1t}^{**}}\right)}.$$

c_G^{12} is the ratio of an increase in real consumption to that in real income. It is my opinion that c_G^{12} can be identified as the marginal propensity to consume which of course Keynes (1936) invented and became one of the symbols of Keynesian economics. c_G^{12} is also the slope of an observed "short-run" consumption function.

Simple calculations show that

$$c_G^{12} = \frac{(1-\alpha)(g+n+\delta)}{g+n+\pi^2+(1-\alpha)(\delta-\pi^2)+\alpha(\pi^1-\pi^2)\frac{(1+g)(1+n)}{g+n}}, \tag{75}$$

because of (22), (70), and (72). Hence the proposition concerning the "marginal propensity to consume:"

Proposition 5: The "marginal propensity to consume" is positive when the economy is growing and the inflation rate is accelerating.

Figure 5. The "Short-Run" and the "Long-Run" Consumption Functions.

Since $\pi^1 > \pi^0 > \pi^2$, it is found that $c_G^0 > c_G^{12}$. Figure 5 shows this situation.[61] If c_G^0 is regarded as the average value of c_G, i.e., the slope of an observed "long-run" consumption

[59] A comment on deflation can be made, though it is not the subject of this paper. Deflation has been a rare phenomenon after World War II. Thus, unlike inflation, it was not a main theme in macroeconomics. In this sense deflation of Japan since mid-1990's is a new challenge to macroeconomits. Here is an numerical example the *KS* model gives:

Put, $g = 0.015$, $n = 0$, $\delta = 0.06$, $\alpha = \frac{1}{3}$, and $\pi = -0.01$. Then c_G is around o.97, and $1 - c_G$ is around 0.03. This example seems to represent very recent experience of Japan.

[60] The origin O is the limiting case as c_G tends to 1, while the intersection of curves Q_{St}^{**}/A_tN_t and Q_{S1t}^{**}/A_tN_t, which is not shown in Figure 4, is that as c_G appoaches 0. The former case worries me. It was assumed in (52) that $g+n+\pi > 0$. Then, $c_G \to 1$ as $g+n+\pi \to 0$. Deflatin is surely a serious problem. When deflation is considered, a value of $g+n$ needs to be taken into consideration without fail to examine whether Condition (52) is satisfied. This is one of the reasons why the growth rate of the effectiveness of labor and that of labor supply are included in the model throughout this paper.

[61] The reader is urged to compare this figure with Figure 13 in Friedman (1957, p. 117). See also Figure 4 in Duesenberry (1949, p. 114).

function, the following proposition has been established:[62]

Proposition 6: In the golden-rule state the slope of a "long-run" consumption function is steeper than that of a "short-run" consumption function.

Proposition 6 means a settlement of the consumption function controversy by the KS model.

The permanent income hypothesis explains the paradox of consumption functions exclusively within consumption behavior. Saving is regarded only as a "residual."[63] And investment plays no part despite the literally long-run data analysis. On the contrary, the KS model solves the puzzle within the unified structure of consumption on one hand and, saving and investment on the other hand. It should be remembered that a macroeconomy is an organism like a human body, and therefore a one-sided analysis may be misleading.

11 Conclusion

In this paper I presented a model which can explain basic aspects of macroeconomy both in the short run and in the long run. It also works as a model which can analyze both business cycles and economic growth. Looking back, it is obvious that it is based simply on such principles as profit maximization of firms and utility maximization of households. The way the macro model is constructed is much the same as in microeconomics.[64] No new concepts were introduced. It consists of ideas of great economists, a few of which I may have failed to mention. The model used is *only one.* Nevertheless it provided, I believe, a new insight into the theories of consumption and investment. For example, it has been shown that the consumption function controversy has not ended yet.

Having studied the relationship between the short run and the long run in macroeconomics in one and the same model, I am ready to make two remarks on how to see the short run and the long run. Firstly, it is important to recognize that even the short run involves dynamic decisions. The short run is often defined as a situation in which firms can make production plan with capital stock as given. The short run in such a usual sense may correspond to two subperiods in the KS model, viz., the third subperiod of, say, period $t-1$ where capital stock K_{1t} and K_{2t} are already fixed, and the first subperiod of period t where production plan is realized. In fact Sections 3-5 gave an analysis in such a traditional framework. Indeed capital stock K_{1t} and K_{2t} are taken as given at the third subperiod, but they are also the results of arbitrage at the second subperiod. And in the process of the arbitrage, K_{1t} and K_{2t} are adjusted on the basis of the expected values of nominal wage rate and expected price

[62] Let us make a numerical example using the same parameter values as in the previous section: $g = 0.01$, $n = 0.005$, $\delta = 0.06$, and $\alpha = \frac{1}{3}$. Set $\pi^1 = 0.02$, $\pi^0 = 0.01$, and $\pi^2 = 0.00$. 0.01 was used in the previous section as a value for π. Then approximately $c_G^1 = 0.81$, $c_G^0 = 0.86$, and $c_G^2 = 0.91$. c_G^{12} turns out, in this case, to be about 0.1. Certainly $c_G^{12} > c_G^0$. But this value may be too low as compared with an example often cited in textbooks like 0.75. Nevertheless, it should be added that the marginal propensity to consume out of *current* income is fairly lower than is generally recognized. For example, Friedman and Becker (1957) estimated it at 0.29, while Blanchard (1997b, p. 71) at 0.17.

[63] See Friedman (1957, p. 28). This view coincides with that of Keynes (1936, pp. 64, 210).

[64] I am a teacher of microeconomics, too.

of investment goods which are determined at the third subperiod.[65] Similarly, capital stock which is assumed to be given in a usual short-run analysis can be regarded as endogeneous variables in the relevant model. It is not correct to consider it literally given from outside the model.[66]

Seconodly, in my opinion, a macro model which lacks the *short-run foundation* is not attractive even if it has the long-run *microeconomic foundation*. It must always be prepared to explain what happens if a macroeconomy diverges from the long-run equilibrium state. But it is often more appropriate to analyze an actual macroeconomy within the *long-run* framework because, as all macroeconomists will admit, a macroeconomy is a truely dynamic phenomenon.[67] For example, it has been shown that the marginal propensity to consume can be reinterpreted from such a point of view. It usually appears around Chapter 3 of macroeconomics textbooks as a short-run concept. However, if I am right, it should be explained in a dynamic environment, i.e., it should be taken up nearly in the last chapter.

This paper dealt with a basic case in which a macroeconomy is made up of the production sector, the household sector, the central bank, and commercial banks. The *KS* model is not completed untill both of the government sector and the foreign sector are introduced in it. In the complete *KS* model further new results are expected.

Appendices

A Proof of Lemma in Section 4

Let $0 \leq \theta_t \leq 1$ be the ratio of saving that goes to the purchase of investment goods at t. Then $\theta_t S_t^e$ constitutes the nominal demand for investment goods, while $(1-\theta_t)S_t^e$ is hoarded. On the other hand the production sector withholds the amount $p_{1t}^e(\delta - \pi_t^e)K_t$ for capital depreciation. Thus, the nominal demand for investment goods is the sum of $\theta_t S_t^e$ and $p_{1t}^e(\delta - \pi_t^e)K_t$. The equilibrium of the investment-goods market is described by

$$p_{1t}^e Q_{1t}^e = \theta_t S_t^e + p_{1t}^e(\delta - \pi_t^e)K_t.$$

Taking (21) and (24) into account, the equilibrium condition can be written as

$$(1-\theta_t)[p_{1t}^e Q_{1t}^e - p_{1t}^e(\delta - \pi_t^e)K_t] = 0.$$

When $0 \leq \theta_t < 1$, money is hoarded. In that case $p_{1t}^e Q_{1t}^e - p_{1t}^e(\delta - \pi_t^e)K_t = 0$. It follows from (24) that the equilibrium national income vanishes, and therefore production is stopped. Q.E.D.

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[65] In other words, K_{1t} and K_{2t} are adjusted on information that are not fixed until the third subperiod. Thus the adjustment of asset market is more difficult than that of labor market.

[66] The short-run approach suggested in the text can be called the profit maximization after portfolio selection. I think that it explains why a linear homogeneous production function like the Cobb-Douglas can be used in a macro analysis. Mathematically it is well known that a two-variable function homogeneous of degree one can not be maximized with respect to the two variables, but it seems to me that the important fact is usually ignored especilly in a neo-classical analysis. Then it may not be meaningless to stress that profit caluculated from a linear homogeneous function is *not* maximized with respect to labor *and* capital. It is correct to say that the profit is maximized with respect to labor *after* capital is adjusted through portfolio selection. See (7) and (14) again.

[67] But then again time should not be taken too long. Who can convince ordinary people of an infinite-time-horizon model in the strict sense of the term?

B Proof of Theorem 1

For an arbitrary value of p^e_{1t}, the amount of production of investment goods is $p^e_{1t}Q_{1t}$, while the nominal demand for investment goods is the sum of S^e_t and $p^e_{1t}(\delta - \pi^e_t)K_t$. But, from (21) and (24), this sum is always equal to $p^e_{1t}Q^e_{1t}$. Whatever prices are expected, investment goods produced are always sold out. Q.E.D.

C Derivation of Supply Curves Q^S_{1t} and Q^S_{2t}, and Demand Curve Q^D_{2t}

The consumption-goods supply curve Q^S_{2t} is none other than (15). To express it in a usual way, replace Q^e_{2t} and p^e_{2t} in (15) respectively with Q^S_{2t} and p_{2t} below. Then,

$$Q^S_{2t} = p_{2t}^{\frac{1-\alpha}{\alpha}} \left[\frac{(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}} K_{2t}.$$

To examine the shape of the graph, differentiate Q^S_{2t} w.r.t. p_{2t} once and twice. Then,

$$\frac{dQ^S_{2t}}{dp_{2t}} = \frac{1-\alpha}{\alpha} p_{2t}^{\frac{1-2\alpha}{\alpha}} \left[\frac{(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}} K_{2t} > 0,$$

and

$$\frac{d^2Q^S_{2t}}{dp^2_{2t}} = \frac{1-\alpha}{\alpha}\frac{1-2\alpha}{\alpha} p_{2t}^{\frac{1-3\alpha}{\alpha}} \left[\frac{(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}} K_{2t} \begin{cases} > 0 & \text{if } 0 < \alpha < \frac{1}{2}, \\ = 0 & \text{if } \alpha = \frac{1}{2}, \\ < 0 & \text{if } \frac{1}{2} < \alpha < 1. \end{cases}$$

The shape of the supply curve in Figure 2 reflects the macro fact that α is around $\frac{1}{3}$. The above argument on Q^S_{2t} applies to that on Q^S_{1t} in the same fashion.

Next consider the consumption-goods demand curve Q^D_{2t}. Needless to say, the demand for comsumption goods is represented by the consumption function (20). Substituting (19) into (20) gives

$$\begin{aligned} C^e_t &= cY^e_t \\ &= c[p^e_{1t}Q^e_{1t} + p^e_{2t}Q^e_{2t} - p^e_{1t}(\delta - \pi^e_t)K_t] \\ &= c[p_{2t}Q^S_{2t} + p^e_{1t}Q^e_{1t} - p^e_{1t}(\delta - \pi^e_t)K_t]. \end{aligned}$$

But it is measured in nominal terms. The *real* demand for consumption goods is obtained simply by deviding it by price:

$$\begin{aligned} Q^D_{2t} &= \frac{C^e_t}{p_{2t}} \\ &= cQ^S_{2t} + \frac{c[p^e_{1t}Q^e_{1t} - p^e_{1t}(\delta - \pi^e_t)K_t]}{p_{2t}}. \end{aligned}$$

This is the demand for consumption goods in a usual way. Keep the above-mentioned macro fact in mind and differentiate Q^D_{2t} w.r.t. p_{2t} once and twice. Then,

$$\frac{dQ^D_{2t}}{dp_{2t}} = c\frac{1-\alpha}{\alpha} p_{2t}^{\frac{1-2\alpha}{\alpha}} \left[\frac{(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}} K_{2t} - \frac{c[p^e_{1t}Q^e_{1t} - p^e_{1t}(\delta - \pi^e_t)K_t]}{p^2_{2t}},$$

and

$$\frac{d^2Q_{2t}^D}{dp_{2t}^2} = c\frac{1-\alpha}{\alpha}\frac{1-2\alpha}{\alpha}p_{2t}^{\frac{1-3\alpha}{\alpha}}\left[\frac{(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}}K_{2t} + \frac{2c[p_{1t}^eQ_{1t}^e - p_{1t}^e(\delta-\pi_t^e)K_t]}{p_{2t}^3} > 0.$$

It follows from these results that demand curve Q_{2t}^D is bending forward and that it changes the sign of the slope at $p_{2t} = \bar{p}_{2t}$, where

$$\bar{p}_{2t} = \left[\frac{\alpha}{1-\alpha}\frac{1-c}{c}\right]^{\alpha}\left[\frac{w_t}{(1-\alpha)A_t}\right]^{1-\alpha}\left[\frac{1}{K_{2t}}\right]^{\alpha}\left\{\frac{c}{1-c}[p_{1t}^eQ_{1t}^e - p_{1t}^e(\delta-\pi_t^e)K_t]\right\}^{\alpha}.$$

The position of the demand curve in Figure 2 reflects another macro fact that $\alpha < c$. This means that $\bar{p}_{2t}$, not shown in the figure, is smaller than p_{2t}^e in (25).

D Derivation of Capital Accumulation Equation (50)

$$\begin{aligned}
k_{t+1}^{**} &= \frac{1-\delta}{(1+g)(1+n)}k_t^{**} + \frac{A_tN_{1t}^{**}}{A_{t+1}N_{t+1}}(k_{1t}^{**})^{\alpha} \\
&= \frac{1-\delta}{(1+g)(1+n)}k_t^{**} + \frac{1}{(1+g)(1+n)}\frac{N_{1t}^{**}}{N_t}(k_{1t}^{**})^{\alpha} \\
&= \frac{1-\delta}{(1+g)(1+n)}k_t^{**} + \frac{1}{(1+g)(1+n)}[(1-c)+c(\delta-\pi)(k_t^{**})^{1-\alpha}](k_t^{**})^{\alpha} \\
&= \frac{(1-\delta)+c(\delta-\pi)}{(1+g)(1+n)}k_t^{**} + \frac{1-c}{(1+g)(1+n)}(k_t^{**})^{\alpha}.
\end{aligned}$$

E On Tobin's q

Tobin's q theory has been a very stimulating theme in macroeconomics as well as the paradox of a short-run and a long-run consumtpion functions. It was first proposed by Tobin (1969), and researchers such as Yoshikawa (1980) and Hayashi (1982) strengthened the theoretical ground with the help of the concept of adjustment costs introduced by Uzawa (1969). In general, analyses of the q theory are rather neo-classical long-run ones and they are very sophisticated as compared with simplicity of the original idea of Tobin. But an answer the KS model gives is very simple: Tobin's (average and also marginal) q is $(p_{1t}^e/\hat{p}_{1t}^e)^{1/\alpha}$. The short run will do. No adjustment costs need to be relied on.

Here is a proof. Multiplying each side of (8) by p_{1t}^e yields planned amount of production of investment goods

$$\begin{aligned}
p_{1t}^eQ_{1t}^e &= p_{1t}^e\left[(1-\alpha)A_t\frac{p_{1t}^e}{w_t}\right]^{\frac{1-\alpha}{\alpha}}K_{1t} \\
&= \left(\frac{p_{1t}^e}{\hat{p}_{1t}^e}\right)^{\frac{1}{\alpha}}\frac{r_t^e+\delta}{\alpha}p_{1t}^eK_{1t}.
\end{aligned} \tag{76}$$

Therefore,

$$\left(\frac{p_{1t}^e}{\hat{p}_{1t}^e}\right)^{\frac{1}{\alpha}} = \frac{p_{1t}^e\frac{\alpha Q_{1t}^e}{r_t^e+\delta}}{p_{1t}^eK_{1t}}. \tag{77}$$

The denominator of the right-hand side of (77) represents the value of existing capital stock evaluated at the expected price p_{1t}^e of investment goods as flow at the first subperiod of period t, and p_{1t}^e is the replacement cost of capital stock. $p_{1t}^e \alpha Q_{1t}^e$ is the expected gross return on existing capital stock because

$$\begin{aligned} p_{1t}^e Q_{1t}^e - w_t N_{1t}^e &= p_{1t}^e Q_{1t}^e - p_{1t}^e (1-\alpha) Q_{1t}^e \\ &= p_{1t}^e \alpha Q_{1t}^e, \end{aligned}$$

due to (9). Since

$$p_{1t}^e \frac{\alpha Q_{1t}^e}{r_t^e + \delta} \approx \frac{(1+\pi_t^e) p_{1t}^e \alpha Q_{1t}^e}{1 + i_t + \delta} + \frac{(1+\pi_t^e)^2 p_{1t}^e \alpha Q_{1t}^e}{(1 + i_t + \delta)^2} + \frac{(1+\pi_t^e)^3 p_{1t}^e \alpha Q_{1t}^e}{(1 + i_t + \delta)^3} + \cdots,$$

the numerator of the right-hand of (77) may be thought of as the discounted present value of the gross return on capital, or the value of capital stock, though some qualifications are required. Thus, $(p_{1t}^e / \hat{p}_{1t}^e)^{1/\alpha}$, in my view, can be considered what Tobin (1969, p. 21) called q which is "the value of capital relative to its replacement cost."

Obviously the right-hand side of (77) represents Tobin's average q. But it is also marginal q because

$$\begin{aligned} \frac{d\left(p_{1t}^e \frac{\alpha Q_{1t}^e}{r_t^e + \delta}\right)}{d(p_{1t}^e K_{1t})} &= \frac{\frac{d(\alpha Q_{1t}^e)}{dK_{1t}}}{r_t^e + \delta} \\ &= \left(\frac{p_{1t}^e}{\hat{p}_{1t}^e}\right)^{\frac{1}{\alpha}}, \end{aligned}$$

due to (76). Therefore, Tobin's q, average and marginal, is $(p_{1t}^e / \hat{p}_{1t}^e)^{1/\alpha}$.

$(p_{1t}^e / \hat{p}_{1t}^e)^{1/\alpha}$ is an increasing function of p_{1t}^e. (See Figure 3.) When the investment-goods sector expects the price of investment goods to rise faster (less fast) than the expected normal supply-price $\hat{p}_{1t}^e$, it tends to accelerate (decelerate) production, *ceteris paribus*. In other words, $q >$ ($<$)1 implies the acceleration (deceleration) of production. When $p_{1t}^e = \hat{p}_{1t}^e$, i.e., $q = 1$, the investment-goods sector will not be tempted to alter the rate of growth of production. Someone may think of the effect of nominal interest rate on production of investment goods. But, as is obvious from (8), nominal interest rate has no influence on production of investment goods. Indeed production of investment goods is superficially affected by a change of real interest rate, but it is the expected price of investment goods that has direct influence on production of investment goods.

Tobin's q represented by $(p_{1t}^e / \hat{p}_{1t}^e)^{1/\alpha}$ is defined both in the short run and in the long run. But, as is obvious, the q theory comes into its own in the short run or in the *non*-neo-classical environment where $q \neq 1$ in general. In the long-run state or in the neo-classical environment q always equals unity. (See (37).) In fact Tobin (1969, p. 23) wrote, "$\ldots q = 1$. This may be regarded as a condition of equilibrium in the long run." In such a situation the relation between price and production is quite different. Recall Propositions 2 and 3.

The KS model gives further insight into the original Tobin's q. There are two qs in fact. They may be called q_1 and q_2, where

$$q_1 = \left(\frac{p_{1t}^e}{\hat{p}_{1t}^e}\right)^{\frac{1}{\alpha}},$$

and

$$q_2 = \left(\frac{p_{2t}^e}{\hat{p}_{1t}^e}\right)^{\frac{1}{\alpha}} = \frac{p_{2t}^e \frac{\alpha Q_{2t}^e}{r_t^e+\delta}}{p_{1t}^e K_{2t}}.$$

q_1 is the original q, while q_2 is that of the consumption-goods sector. q_1 and q_2 appeared respectively in (32) and the first half of (34) where e is replaced with $*$. The above argument on q (or q_1) similarly applies to that on q_2, i.e., $q_2 > (<)1$ implies the acceleration (deceleration) of production of consumption goods. Thus, q theory is applicalbe not only to investment goods but also to consumption goods. However, it is p_{1t}^e that counts, because p_{2t}^e is an increasing function of p_{1t}^e. (See (25).) An increase in p_{1t}^e leads to an increase in both q_1 and q_2, which in turn causes an increase of production in both the investment-goods sector and the consumption-goods sector. This is the interpretation of Proposition 1 by Tobin's q.

F On the *M-M* Theorem

To me, including related literature such as Stiglitz (1969), the *M-M* theorem has been difficult and thus mysterious except for an impression that it was a declaration of triumph of economic theory over contemporary doctrines on corporate investment policy. To be honest, I have not dwelt on it as an essential part of macroeconomics. However, having constructed the *KS* model, I have noticed that the *M-M* theorem and the *KS* model are closely connected, though at a macroeconomic level, and as a collorary that so are the *M-M* theorem and Tobin's q theory. In this final appendix I will show these relationships.

Modigliani and Miller (1958) concentrated on a group of firms or an industry which is characterized by ρ_k with k as a class of the group. ρ_k is the expected rate of return on equities in the absence of debt-financing, where all of profit earned belongs to equity holders. They showed three propositions concernig the cost of capital. So let us proceed in order.

First take the investment-goods sector as an industry examined here and let k be 1. (In the case of the consumption-goods sector $k = 2$.) Then, in the *KS* model,

$$\rho_{1t} \equiv \frac{p_{1t}^e Q_{1t}^e - w_t N_{1t}^e - p_{1t}^e(\delta - \pi_t^e)K_{1t}}{\tilde{p}_{1t-1}K_{1t}} \tag{78}$$

$$= \frac{p_{1t}^e}{\tilde{p}_{1t-1}}\left[\left(\frac{p_{1t}^e}{\hat{p}_{1t}^e}\right)^{\frac{1}{\alpha}}(r_t^e + \delta) - (\delta - \pi_t^e)\right], \tag{79}$$

because of (7) and (8). In the *M-M* theorem ρ_k appears only as "a constant," while the *KS* model can specify ρ_{1t} as in (79).

Let $\bar{X}_{1t}$ stand for the expected return on the assets owned by the investment-goods sector. Denote by D_{1t} the market value of the debts of the sector; by S_{1t} the market value of its equities: and by $V_{1t} \equiv S_{1t} + D_{1t}$ the market value of the sector. In terms of the *KS* model, $\bar{X}_{1t} = p_{1t}^e Q_{1t}^e - w_t N_{1t}^e - p_{1t}^e(\delta - \pi_t^e)K_{1t}$, $D_{1t} = \tilde{p}_{1t-1}K_{1t}^d$, and $S_{1t} = \tilde{p}_{1t-1}K_{1t}^h$, where $K_{1t}^h + K_{1t}^d = K_{1t}$. Then, the budget constraint on the investment-goods sector (4) can be written as:

$$V_{1t} \equiv S_{1t} + D_{1t} = \frac{\bar{X}_{1t}}{\rho_{1t}}. \tag{80}$$

(80) is a macro version of Proposition I of Modigliani and Miller (1958, p. 268). But (80) is only a budget constraint, while their proposition states that the market value of any firm in class k is independent of its capital structure as a result of arbitrage. The average cost of

capital of the investment-goods sector is defined as the ratio of the expected return to the market value. Then, (80) can also be expressed as:

$$\frac{\bar{X}_{1t}}{V_{1t}} = \rho_{1t}.$$

That is, the average cost of capital of the investment-goods sector is completely independent of its capital structure and is equal to the capitalization rate ρ_{1t} of a pure equity stream of the sector.

Next consider the relationship among h_{1t}^e, i_t, and ρ_{1t}. From (30),

$$h_{1t}^e - i_t = \frac{p_{1t}^e Q_{1t}^e - w_t N_{1t}^e - p_{1t}^e(\delta - \pi_t^e)K_{1t} - p_{1t}^e(r_t^e + \pi_t^e)K_{1t}}{\tilde{p}_{1t-1}K_{1t}} \frac{\tilde{p}_{1t-1}K_{1t}}{\tilde{p}_{1t-1}K_{1t}^h}. \tag{81}$$

Substituting (78) into (81) and remembering the definitions of D_{1t} and S_{1t} lead to

$$h_{1t}^e = \rho_{1t} + (\rho_{1t} - i_t)\frac{D_{1t}}{S_{1t}}.$$

That is, the expected rate of return on equities h_{1t}^e is equal to the capitalization rate ρ_{1t} for a pure equity stream, plus a premium related to financial risk equal to the debt-to-equity ratio times the spread between ρ_{1t} and i_t. This result corresponds to Proposition II of Modigliani and Miller (1958, p. 271).

Proposition III of Modigliani and Miller (1958, p. 288) can be rephrased in terms of the *KS* model as follows: If the investment-goods sector is acitng in the best interest of the equity holders, the marginal cost of capital (or equivalently, the rate of return on the investment) should be equal to the average cost of capital, which is in turn equal to the capitalization rate ρ_{1t} for an unlevered stream in the sector. The marginal cost of capital may be defined as $d\bar{X}_{1t}/dV_{1t}$, though they did not define it explicitly. Then,

$$\begin{aligned}
\frac{d\bar{X}_{1t}}{dV_{1t}} &= \frac{d[p_{1t}^e Q_{1t}^e - w_t N_{1t}^e - p_{1t}^e(\delta - \pi_t^e)K_{1t}]}{d[\tilde{p}_{1t-1}K_{1t}]} \\
&= \frac{p_{1t}^e}{\tilde{p}_{1t-1}} \frac{d[\alpha Q_{1t}^e - (\delta - \pi_t^e)K_{1t}]}{dK_{1t}} \\
&= \frac{p_{1t}^e}{\tilde{p}_{1t-1}} \left[\left(\frac{p_{1t}^e}{\tilde{p}_{1t}^e}\right)^{\frac{1}{\alpha}} (r_t^e + \delta) - (\delta - \pi_t^e)\right] = \rho_{1t}.
\end{aligned}$$

As is apparent, $d\bar{X}_{1t}/dV_{1t}$ is also the rate of return on the investment. Remember that the investment-goods sector maximizes h_{1t}^e in (4). Therefore, Proposition III also obtains in the *KS* model. Similar arguments applies to the consumption-goods sector.

Like the q theory, the *M-M* theorem also has its raison d'être in the short run, where $h_{1t}^e \neq \rho_{1t}$ and $\rho_{1t} \neq i_t$ in general. In the long run $h_{1t}^e = \rho_{1t} = i_t$ holds, and it degenerates into tautology. Modigliani and Miller (1958, p. 264) rightly recognized it, saying, "the approach is essentially a partial-equilibrium one focusing on the firm and "industry." Accordingly, the "prices" of certain income streams will be treated as constant and given from outside the model, just as in the standard Marshallian analysis of the firm and industry the prices of all inputs and of all other products are taken as given."

Now the relationship between the *M-M* theorem and Tobin's q theory can be made clear. From the previous appendix $(p^e_{1t}/\hat{p}^e_{1t})^{1/\alpha}$ and $(p^e_{2t}/\hat{p}^e_{1t})^{1/\alpha}$ are two qs, q_1 and q_2, respectively. Taking (79) into account, the following simple relations hold:

$$q_1 \geq (<)1 \Leftrightarrow \rho_{1t} \geq (<) i_t,$$

and

$$q_2 \geq (<)1 \Leftrightarrow \rho_{2t} \geq (<) i_t.$$

That is, the *M-M* theorem and Tobin's q theory are mathmatically equivalent. Both of them are a short-run partial-equilibrium approach and deal with a production sector that maximizes the rate of return on equities. A difference from an economic point of view lies in how to see investment behavior. The q theory sees it through a production function while the *M-M* theorem through a budget constraint.

References

[1] Ackley, Gardner, 1961, *Macroeconomic Theory,* New York: Macmillan.

[2] Barro, Robert J., 1994, "The Aggregate-Supply/Aggregate-Demand Model," *Eastern Economic Journal,* 20, 1-6.

[3] Barro, Robert J., 1997, *Determinants of Economic Growth: A Cross-Country Empirical Study,* Cambridge, Massachusetts: The MIT Press.

[4] Benavie, Arthur, 1976, "Monetary and Fiscal Policy in a Two-Sector Keynesian Model," *Journal of Money, Credit, and Banking,* 8, 63-84.

[5] Benhabib, Jess, Kazuo Nishimura, and Alain Venditti, 2002, "Indeterminacy and Cycles in Two-Sector Discrete-Time Model," *Economic Theory,* 20, 217-235.

[6] Blanchard, Oliver, 1997a, "Is There a Core of Usable Macroeconomics?" *American Economic Review,* 87, 244-246.

[7] Blanchard, Olivier J., 1997b, *Macroeconomics,* Upper Saddle River, New Jersey: Prentice-Hall.

[8] Blanchard, Olivier J., and Stanley Fischer, 1989, *Lectures on Macroeconomics,* Cambridge, Massachusetts: The MIT Press.

[9] Blinder, Alan S., 1997, "Is There a Core of Practical Macroeconomics That We Should All Believe?" *American Economic Review,* 87, 240-243.

[10] Boldrin, Michele, and Luigi Montrucchio, 1986, "On the Indeterminacy of Capital Accumulation Paths," *Journal of Economic Theory,* 40, 26-39.

[11] Chakrabarti, Santi K., 1979, *The Two-Sector General Theory Model,* London: Macmillan.

[12] Cobb, Charles W., and Paul H. Douglas, 1928, "A Theory of Production," *American Economic Review,* 18, Supplement, 139-165.

[13] Deaton, Angus, 1992, *Understanding Consumption,* Oxford: Oxford University Press.

[14] Duesenberry, James S., 1949, *Income, Saving and the Theory of Consumer Behavior,* Cambridge, Massachusetts: Harvard University Press.

[15] Dornbusch, Rudiger, and Stanley Fischer, 1994, *Macroeconomics, 6th Edition,* New York: McGraw-Hill.

[16] Douglas, Paul H., 1948, "Are There Laws of Production?" *American Economic Review,* 38, 1-41.

[17] Eichenbaum, Martin, 1997, "Some Thoughts on Practical Stabilization Policy," *American Economic Review,* 87, 236-239.

[18] Foley, Duncan K., and Miguel Sidrauski, 1971, *Monetary and Fiscal Policy in a Growing Economy,* London: Macmillan.

[19] Friedman, Milton, 1957, *A Theory of the Consumption Function,* Princeton: Princeton University Press.

[20] Friedman, Milton, 1966, "What Price Guideposts?" in George P. Shultz and Robert Z. Aliber, eds., *Guidelines, Informal Controls, and the Market Place,* Chicago: University of Chicago Press, 17-39.

[21] Friedman, Milton, 1968, "The Role of Monetary Policy," *American Economic Review,* 63, 1-17.

[22] Friedman, Milton, and Gary S. Becker, 1957, "A Statistical Illusion in Judging Keynesian Models," *Journal of Political Economy,* 65, 64-75.

[23] Friedman, Milton, and Simon Kuznets, 1945, *Income from Independent Professional Practice,* New York, National Bureau of Economic Research.

[24] Hall, Robert E., 1978, "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy,* 86, 971-987.

[25] Harrod, Roy F., 1969, *Money,* London: Macmillan.

[26] Hayashi, Fumio, 1982, "Tobin's Marginal q and Average q: A Neoclassical Interpretation," *Econometrica,* 50, 213-224.

[27] Inada, Ken-Ichi, 1963, "On a Two-Sector Model of Economic Growth: Comments and a Generalization," *Review of Economic Studies,* 30, 119-127.

[28] Jorgenson, Dale W., 1963, "Capital Theory and Investment Behavior," *American Economic Review,* 53, 247-259.

[29] Keynes, John M., 1936, *The General Theory of Employment, Interest and Money,* London: Macmillan.

[30] King, Robert G., and Charles I. Plosser, 1984, "Money, Credit, and Prices in a Real Business Cycles," *American Economic Review,* 74, 363-380.

[31] Klein, Lawrence R., 1949, *The Keynesian Revolution,* London: Macmillan.

[32] Kurz, Mordecai, 1963, "A Two-Sector Extension of Swan's Model of Economic Growth: The Case of No Technical Change," *International Economic Review, 4,* 68-79.

[33] Kuznets, Simon, 1942, *Uses of National Income in Peace and War,* New York: National Bureau of Economic Research.

[34] Kuznets, Simon, 1952, " Proportion of Capital Formation to National Product," *American Economic Review,* 42, 507-526.

[35] Lange, Oskar, 1938, "The Rate of Interest and the Optimum Propensity to Consume," *Economica,* 5, 12-32.

[36] Lerner, Abba P., 1936, "Mr. Keynes' "General Theory of Employment, Interest and Money"," *International Labour Review,* 34, 435-454.

[37] Mackay, Robert J., and Roger N. Waud, 1975, "A Re-Examination of Keynesian Monetary and Fiscal Orthodoxy in a Two-Sector Keynesian Paradigm," *Canadian Journal of Economics,* 8, 548-573.

[38] Mankiw, N. Gregory, 1994, *Macroeconomics, 2nd Edition,* New York: Worth Publishers.

[39] Mankiw, N. Gregory, 1998, "Teaching the Principles of Economics," *Eastern Economic Journal,* 24, 519-524.

[40] Meade, James E., 1961, *A Neo-Classical Theory of Economic Growth,* London: George Allen & Unwin.

[41] Modigliani, Franco, and Merton H. Miller, 1958, "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review,* 48, 261-297.

[42] Mundell, Robert A., 1971, *Monetary Theory: Inflation, Interest and Growth in the World Economy,* Pacific Palisades, California: Goodyear Publishing Company.

[43] Muth, John F., 1961, "Rational Expectations and the Theory of Price Movements," *Econometrica,* 29, 315-335.

[44] Ono, Yoshiyasu, 2001, "A Reinterpretation of Chapter 17 of Keynes's General Theory: Effective Demand Shortage Under Dynamic Optimization," *International Economic Review,* 42, 207-236.

[45] Phelps, Edmund, 1961, "The Golden Rule of Accumulation: A Fable for Growthmen," *American Economic Review,* 51, 638-643.

[46] Phelps, Edmund S., 1968, "Money-Wage Dynamics and Labor-Market Equilibrium," *Journal of Political Economy,* 76, 678-711.

[47] Ramsey, Frank P., 1928, "A Mathematical Theory of Saving," *Economic Journal,* 38, 543-559.

[48] Rao, B. Bhaskara, ed., 1998, *Aggregate Demand and Supply: A Critique of Orthodox Macroeconomic Modelling,* London: Macmillan.

[49] Romer, David, 1996, *Advanced Macroeconomics*, New York: McGraw-Hill.

[50] Romer, David, 2000, "Keynesian Macroeconomics without the LM Curve," *Journal of Economic Perspective,* 14, 149-169.

[51] Sachs, Jeffrey D., and Felipe Larrain B., 1993, *Macroeconomics in the Global Economy,* Englewood Cliffs, New Jersey: Prentice-Hall.

[52] Samuelson, Paul A., 1948, "The Simple Mathematics of Income Determination," in Lloyd A. Metzler, *et al., Income, Employment and Public Policy,* New York: W. W. Norton, 133-155.

[53] Samuelson, Paul A., and William D. Nordhaus, 1998, *Economics, 16th Edition,* New York: McGraw-Hill.

[54] Shinkai, Yoichi, 1960, "On Equilibrium Growth of Capital and Labor," *International Economic Review,* 1, 107-111.

[55] Smith, Adam, 1776, *An Inquiry into the Nature and Causes of the Wealth of Nations,* Edwin Cannan, ed., 1937, New York: The Modern Library.

[56] Solow, Robert M., 1956, "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics,* 70, 65-94.

[57] Solow, Robert M., 1957, "Technical Change and the Aggregate Production Function," *Review of Economics and Statistics,* 39, 312-320.

[58] Solow, Robert M., 1961-62, "Note on Uzawa's Two-Sector Model of Economic Growth," *Review of Economic Studies,* 29, 48-50.

[59] Solow, Robert M., 1988, "Growth Theory and After," *American Economic Review,* 78, 307-317.

[60] Solow, Robert M., 1997, "Is There a Core of Usable Macroeconomics That We Should All Believe In?" *American Economic Review,* 87, 230-232.

[61] Solow, Robert M., 2000, "Toward a Macroeconomics of the Medium Run," *Journal of Economic Perspective,* 14, 151-158.

[62] Solow, Robert M., 2005, "Reflections on Growth Theory," in Philippe Aghion and Steven N. Durlauf, eds., Handbook of Economic Growth, 1A, Amsterdam: Elsevier B. V., 3-10.

[63] Stiglitz, Joseph E., 1969, "A Re-Examination of the Modigliani-Miller Therem," *American Economic Review,* 59, 784-793.

[64] Takayama, Akira, 1963, "On a Two-Sector Model of Economic Growth - A Comparative Statics Analysis -," *Review of Economic Studies,* 30, 95-104.

[65] Taylor, John B., 1997, "A Core of Practical Macroeconomics," *American Economic Review,* 87, 233-235.

[66] Tobin, James, 1965, "Money and Economic Growth," *Econometrica,* 33, 671-684.

[67] Tobin, James, 1969, "A General Equilibrium Approach to Monetary Theory," *Journal of Money, Credit, and Banking*, 1, 15-29.

[68] Uzawa, Hirofumi, 1961-62, "On a Two-Sector Model of Economic Growth," *Review of Economic Studies*, 29, 40-47.

[69] Uzawa, Hirofumi, 1963, "On a Two-Sector Model of Economic Growth II," *Review of Economic Studies*, 30, 105-118.

[70] Uzawa, Hirofumi, 1969, "Time Preference and the Penrose Effect in a Two-Class Model of Economic Growth," *Journal of Political Economy*, 77, 628-652.

[71] Viner, Jacob, 1937, "Mr. Keynes on the Causes of Unemployment," *Quarterly Journal of Economics*, 51, 611-619.

[72] Yoshikawa, Hiroshi, 1980, "On the 'q' Theory of Investment," *American Economic Review*, 70, 739-743.

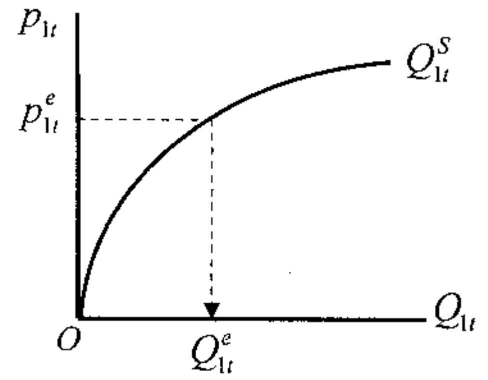

Figure 1. Equilibrium in the Investment-Goods Market

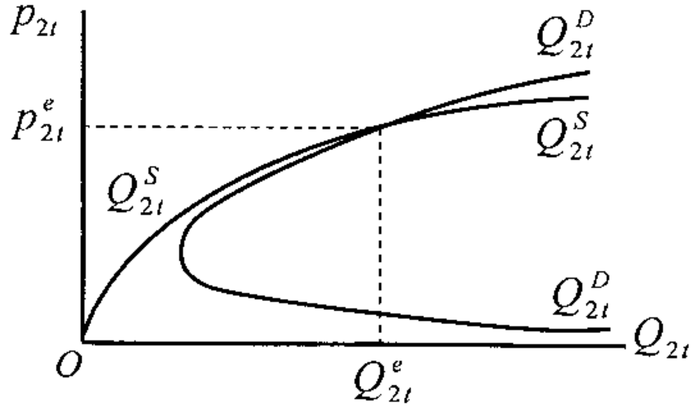

Figure 2. Equilibrium in the Consumption-Goods Market

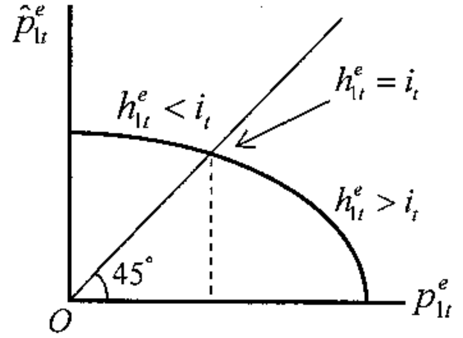

Figure 3. The Expected Normal Supply-Price of Investment Goods.

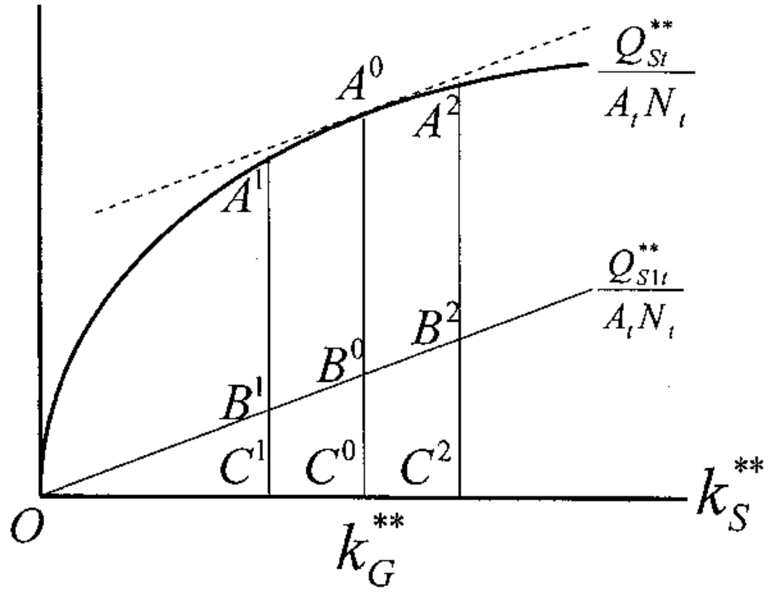

Figure 4. The Golden-Rule State.

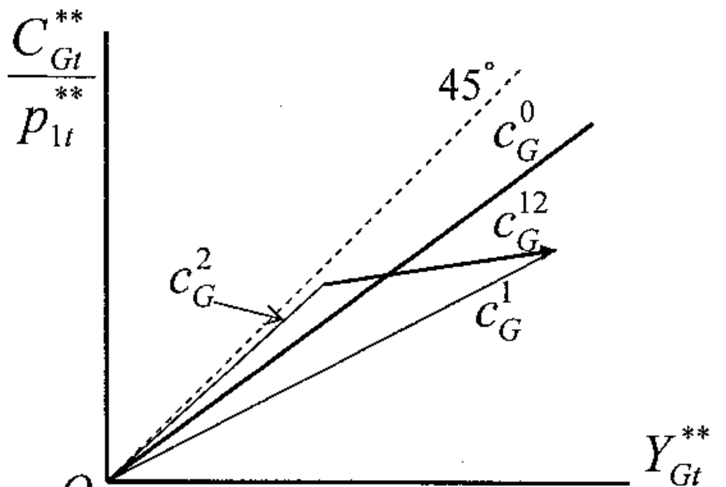

Figure 5.
The “Short-Run” and the “Long-Run” Consumption Functions.