Searching for an "Optimal" Waste Management Option and Evaluating Its Economic Cost: The Waste Input-Output Model and Its Application to the Japanese Economy

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1 Introduction

A necessary condition for achievement of sustainable waste management requires the minimization of environmental impact. However, it is almost impossible to find out an "optimal" waste management option, or an environmental impact-minimizing waste management option, from among a quite large number of alternative feasible options by trial and error without using a systematic analytical method which fully takes into account that there are highly complicated inter-sectoral relationships with regard to the flow of goods and the flow of waste associated with it. It is thus desirable that one can systematically obtain an "optimal" waste management option with regard to a given objective function from among a given set of feasible alternatives. Even if we know what an "optimal" waste management option is, its effects cannot be materialized unless the introduction of it is also economically affordable. It is thus desirable that both environmental impact and economic cost of a waste management option are evaluated in a consistent way.

The purpose of this paper is to propose a new methodology for searching for an "optimal" waste management option and evaluating its economic cost; the option is found optimal from the viewpoint of life cycle assessment (LCA). The methodology is based on a joint use of the two extensions of the waste input-output (WIO) model (Nakamura, 1999; Nakamura and Kondo, 2002a). One is the WIO linear programming (WIO-LP) model (Nakamura and Kondo, 2002c; Kondo and Nakamura, 2004b) that searches for an "optimal" waste management option with regard to a given objective function from among a given set of feasible alternatives. The other is the WIO price model (Nakamura and Kondo, 2002b) that evaluates the cost and price of sectoral outputs including waste management services under full consideration of the interdependence between the flow of goods and the associated waste stream.

The applicability of the proposed methodology has been illustrated by a case study for Japanese waste management options of both municipal and industrial origins. As alternative waste management options we consider regional concentration of incineration combined with different incineration options with regard to energy recovery and treatment of residue, gasification of organic waste, alternative recycling technologies of end-of-life appliances, and alternative ways of recycling recovered materials.

The structure of this paper is as follows. The first part of Section 2 gives a brief summary of the WIO model (the WIO quantity model) to readers who are unfamiliar with it. We the proceed in the second part of Section 2 to the description of the joint use of the WIO price model and the WIO-LP model, which is the main topic of this paper. Section 3 then shows empirical results obtained by applying the Japanese WIO models to a search for an "optimal" waste management option and an evaluation of its economic cost. Concluding remarks in section 4 close the paper.

2 The Waste Input-Output Models

2.1 The Quantity Model

We briefly review the WIO quantity model (Nakamura, 1999; Nakamura and Kondo, 2002a) in this section. Let there be n^{I} goods- and service-producing sectors (henceforth "goods sector"), n^{II} waste treatment sectors, and n^{W} waste types, and define $n := n^{\mathrm{I}} + n^{\mathrm{II}}$. In order to keep notations simple, we define the sets of natural numbers referring to each of these sectors and waste types by $N^{\mathrm{I}} := \{1,\ldots,n^{\mathrm{I}}\}$, $N^{\mathrm{II}} := \{n^{\mathrm{I}}+1,\ldots,n^{\mathrm{I}}+n^{\mathrm{II}}\} = \{n^{\mathrm{I}}+1,\ldots,n\}$, $N := N^{\mathrm{I}} \cup N^{\mathrm{II}}$, and $N^{\mathrm{W}} := \{1,\ldots,n^{\mathrm{W}}\}$. We then denote, for sector j, its output by x_j, the input from sector i by X_{ij}, the generation of waste k by $W^{\oplus}_{kj}$ and the input of waste k by $W^{\ominus}_{kj}$ ($i, j \in N$, $k \in N^{\mathrm{W}}$). For a waste treatment sector, its "output" is measured by the amount of waste that it treated. Similarly, we denote the final demand for good i by $X_{i\mathrm{F}}$, the generation of waste k from the final demand sector by $W^{\oplus}_{k\mathrm{F}}$, and the input of waste k into the final demand sector by $W^{\ominus}_{k\mathrm{F}}$ ($i \in N$, $k \in N^{\mathrm{W}}$). We denote by $W_{kj} := W^{\oplus}_{kj} - W^{\ominus}_{kj}$ the net generation of waste k from sector j. When $W_{kj} > 0$, sector j generates a greater amount of waste k than it uses as input, and creates a positive demand for waste treatment. On the other hand, when $W_{kj} < 0$, sector j reduces the amount of waste k that has to be treated as waste. The sum of W_{kj}'s for all j, w_k, then gives the total amount of waste k that undergoes waste treatment.

Let $a_{ij} := X_{ij}/x_j$ be the conventional input coefficient, $g^{\oplus}_{kj} := W^{\oplus}_{kj}/x_j$ be the waste generation coefficient, $g^{\ominus}_{kj} := W^{\ominus}_{kj}/x_j$ be the waste input coefficient, and $g_{kj} := g^{\oplus}_{kj} - g^{\ominus}_{kj}$ be the net waste generation coefficient ($k \in N^{\mathrm{W}}$, $j \in N$). Let S denote an $n^{\mathrm{II}} \times n^{\mathrm{W}}$ non-negative matrix whose (i,k)-component s_{ik} represents the share of waste k that is treated by treatment method i; the equality $\sum_{i=1}^{n^{\mathrm{II}}} s_{ik} = 1$ holds by definition for every $k \in N^{\mathrm{W}}$. We call S an 'allocation matrix' because it represents an allocation pattern of wastes to treatment methods. We then have the following equations for the output of goods, the net generation of waste, and the amount of waste treatment:

$$x_i = \sum_{j \in N^{\mathrm{I}}} a_{ij} x_j + \sum_{j \in N^{\mathrm{II}}} a_{ij} x_j + X_{i\mathrm{F}} \quad (i \in N^{\mathrm{I}}), \tag{1}$$

$$w_k = \sum_{j \in N^{\mathrm{I}}} g_{kj} x_j + \sum_{j \in N^{\mathrm{II}}} g_{kj} x_j + W_{k\mathrm{F}} \quad (k \in N^{\mathrm{W}}), \tag{2}$$

$$x_i = \sum_{k \in N^{\mathrm{W}}} s_{ik} w_k \quad (i \in N^{\mathrm{II}}), \tag{3}$$

or in an obvious matrix notations,

$$x_{\mathrm{I}} = A_{\mathrm{I,I}} x_{\mathrm{I}} + A_{\mathrm{I,II}} x_{\mathrm{II}} + X_{\mathrm{I,F}}, \tag{4}$$

$$w = G_{\cdot,\mathrm{I}} x_{\mathrm{I}} + G_{\cdot,\mathrm{II}} x_{\mathrm{II}} + W_{\cdot,\mathrm{F}}, \tag{5}$$

$$x_{\mathrm{II}} = S w. \tag{6}$$

Inserting (5) into (6), we have

$$x_{\mathrm{II}} = S G_{\cdot,\mathrm{I}} x_{\mathrm{I}} + S G_{\cdot,\mathrm{II}} x_{\mathrm{II}} + S W_{\cdot,\mathrm{F}}. \tag{7}$$

In order to obtain an IO model that can analyze issues of waste management and recycling, Nakamura (1999) and Nakamura and Kondo (2002a) developed the waste input-output (WIO) model. In terms of WIO, the solution of the above system of equations (4) and (7) is given by

$$\begin{bmatrix} x_{\mathrm{I}} \\ x_{\mathrm{II}} \end{bmatrix} = \left(I_n - \begin{bmatrix} A_{\mathrm{I,I}} & A_{\mathrm{I,II}} \\ S G_{\cdot,\mathrm{I}} & S G_{\cdot,\mathrm{II}} \end{bmatrix} \right)^{-1} \begin{bmatrix} X_{\mathrm{I,F}} \\ S W_{\cdot,\mathrm{F}} \end{bmatrix}, \tag{8}$$

where I_n is an identity matrix of order n. The environmental IO (EIO) model of Leontief (1970), Leontief and Ford (1972) and Duchin (1990) corresponds to a special case of (8) where S is an identity matrix of order n^{II}. The condition $S = I_{n^{\mathrm{II}}}$ implies that there exists for each pollutant one and only one abatement method that treats no other pollutant but that pollutant. This condition hardly holds in the reality of waste management because, in general, there is no one-to-one correspondence between a waste and its treatment method. For instance, the following cases cannot be dealt with if the unrealistic condition $S = I_{n^{\mathrm{II}}}$ is imposed: (i) the number n^{W} of waste types is, in general, much larger than the number n^{II} of treatment methods, that is, the allocation matrix S is not square; (ii) garbage can be composted, gasified, incinerated, and/or landfilled, that is, a column of the allocation matrix can have more than one non-zero components; (iii) any solid waste can be landfilled (MacDonald, 1996), that is, a row of the allocation matrix can have more than one non-zero components (see Nakamura and Kondo (2002a) for further details of the WIO quantity model).

Let there be n^{E} types of environmental loads under consideration. Let us denote by $R_{\cdot,\mathrm{I}}$ for an $n^{\mathrm{E}} \times n^{\mathrm{I}}$ matrix of the emissions from a unit of output in goods sectors and $R_{\cdot,\mathrm{II}}$ for an $n^{\mathrm{E}} \times n^{\mathrm{II}}$ matrix of the emissions from a unit of output in waste treatment sectors. The vector of total emissions e is then given by

$$\begin{aligned} e &= R_{\cdot,\mathrm{I}} x_{\mathrm{I}} + R_{\cdot,\mathrm{II}} x_{\mathrm{II}} + E_{\cdot,\mathrm{F}} \\ &= \begin{bmatrix} R_{\cdot,\mathrm{I}} & R_{\cdot,\mathrm{II}} \end{bmatrix} \left(I - \begin{bmatrix} A_{\mathrm{I,I}} & A_{\mathrm{I,II}} \\ S G_{\cdot,\mathrm{I}} & S G_{\cdot,\mathrm{II}} \end{bmatrix} \right)^{-1} \begin{bmatrix} X_{\mathrm{I,F}} \\ S W_{\cdot,\mathrm{F}} \end{bmatrix} + E_{\cdot,\mathrm{F}}, \end{aligned} \tag{9}$$

where $E_{\cdot,\mathrm{F}}$ refers to the direct emission from the final demand sector. Using (9), one can carry out an LCA study by evaluating the emission e associated with a given scenario which consists of goods-producing technologies $\langle A_{\mathrm{I,I}}, G_{\cdot,\mathrm{I}}, R_{\cdot,\mathrm{I}} \rangle$, waste treatment technologies $\langle A_{\mathrm{I,II}}, G_{\cdot,\mathrm{II}}, R_{\cdot,\mathrm{II}} \rangle$, consumer's life style $\langle X_{\mathrm{I,F}}, W_{\cdot,\mathrm{F}}, E_{\cdot,\mathrm{F}} \rangle$, and an allocation pattern of wastes to treatment methods $\langle S \rangle$.

2.2 The WIO Price Model

We now turn to the aspect of cost and price of the WIO model, the WIO price model (Nakamura and Kondo, 2002b). Let p_j be the price of output of sector j ($j \in N$), p^{W}_k be the price of waste k ($k \in N^{\mathrm{W}}$), V_j be the cost for primary factors of production that includes depreciations as well as taxes less subsidies, and $U_{kj} \geq 0$ be the quantity of waste k ($k \in N^{\mathrm{W}}$) that was used as input in sector j ($j \in N \cup \{\mathrm{F}\}$). This explicit consideration of the transaction of waste materials distinguishes the definition of costs in the WIO from that of the conventional IOA. Note that the price p^{W}_k of waste is not necessarily positive. Based on its sign, three cases can be distinguished: the waste is valuable when $p^{\mathrm{W}}_k > 0$; it has no value but can be accepted by other sectors as input with no charge when $p^{\mathrm{W}}_k = 0$; and it has no value and its acceptance needs a positive charge when $p^{\mathrm{W}}_k < 0$. Henceforth, we call U_{kj} "sale of waste" regardless of whether the price of waste k is positive, zero, or negative.

In the input-output account system we have the identity that equates the value of output to the total cost. Considering the trade of waste, this identity can be given for sector j ($j \in N$) by

$$p_j x_j = \underbrace{\sum_{i \in N^{\mathrm{I}}} p_i a_{ij} x_j}_{\text{(a)}} + \underbrace{\sum_{l \in N^{\mathrm{II}}} p_l \sum_{k \in N^{\mathrm{W}}} s_{lk}(g^{\oplus}_{kj} x_j - U_{kj})}_{\text{(b)}} + \underbrace{\sum_{k \in N^{\mathrm{W}}} p^{\mathrm{W}}_k g^{\ominus}_{kj} x_j}_{\text{(c)}} - \underbrace{\sum_{k \in N^{\mathrm{W}}} p^{\mathrm{W}}_k U_{kj}}_{\text{(d)}} + \underbrace{v_j x_j}_{\text{(e)}}, \quad (10)$$

where $v_j := V_j / x_j$ refers to the cost for primary factors of production per unit of output. The cost can be decomposed into five parts: (a) the cost for the input of goods, (b) the cost for waste treatment, (c) the cost for the input of waste materials, (d) the revenue from the sale of waste materials, and (e) the cost for the input of primary factors. The terms (b), (c), and (d) are unique to the WIO price model. When there is no recycling, $U_{kj} = 0$ holds for all $k \in N^{\mathrm{W}}$ and $j \in N$, and the terms (c) and (d) vanish, while the term (b) reduces to the treatment cost of wastes generated in the sector. The term (b) indicates that the amount of waste for treatment is reduced by the amount of U_{kj}.

While the term U_{kj} plays an essential role in the cost equation (10), it does not occur in the system of equations for the WIO quantity model (8). It is thus necessary to establish the relationship between U_{kj} and the elements occurring in (8). Let denote by U_{kji} the amount of waste k generated by sector j that was used as input in sector i ($i \in N$, $j \in N \cup \{\mathrm{F}\}$, $k \in N^{\mathrm{W}}$). Suppose that $W^{\ominus}_{kF} = 0$, for the sake of simplicity; the household does not "directly" participate in recycling in the sense that it does not directly use waste, while they would indirectly participate in recycling by purchasing goods made of recovered waste or produced by using waste heat. Suppose also that the classification of waste is so detailed that the user (recycler) of a given type of waste is indifferent to its origin; the portion of waste k used in sector i that originates from sector j would then be proportional to the share of sector j in the total generation of that waste, i.e., $U_{kji} = W^{\ominus}_{ki}(W^{\oplus}_{kj} / W^{\oplus}_k)$. It then follows

$$U_{kj} = \sum_{i \in N} U_{kji} = \sum_{i \in N} W^{\ominus}_{ki}(W^{\oplus}_{kj} / W^{\oplus}_k) = W^{\oplus}_{kj}(W^{\ominus}_k / W^{\oplus}_k) = g^{\oplus}_{kj} x_j (W^{\ominus}_k / W^{\oplus}_k), \quad (11)$$

where the first and third equalities hold by definition and the assumption that $W^{\ominus}_{kF} = 0$. Letting r_k denote the rate of recycling of waste k with $r_k := W^{\ominus}_k / W^{\oplus}_k$, we obtain

$$U_{kj} = g^{\oplus}_{kj} x_j r_k. \quad (12)$$

Insertion of (12) into (10), division of both the sides by x_j, and rearranging terms yield the following expression of the price equation:

$$p_j = \sum_{i \in N^{\mathrm{I}}} p_i a_{ij} + \sum_{l \in N^{\mathrm{II}}} p_l \sum_{k \in N^{\mathrm{W}}} s_{lk}(1 - r_k) g^{\oplus}_{kj} + \sum_{k \in N^{\mathrm{W}}} p^{\mathrm{W}}_k (g^{\ominus}_{kj} - r_k g^{\oplus}_{kj}) + v_j. \tag{13}$$

Using obvious matrix notations, (13) can be rewritten as

$$\begin{aligned} \begin{bmatrix} p_{\mathrm{I}} & p_{\mathrm{II}} \end{bmatrix} = \begin{bmatrix} p_{\mathrm{I}} & p_{\mathrm{II}} \end{bmatrix} & \begin{bmatrix} A_{\mathrm{I,I}} & A_{\mathrm{I,II}} \\ S(I_{n^{\mathrm{W}}} - D) G^{\oplus}_{\cdot,\mathrm{I}} & S(I_{n^{\mathrm{W}}} - D) G^{\oplus}_{\cdot,\mathrm{II}} \end{bmatrix} \\ & + p^{\mathrm{W}} \begin{bmatrix} G^{\ominus}_{\cdot,\mathrm{I}} - D G^{\oplus}_{\cdot,\mathrm{I}} & G^{\ominus}_{\cdot,\mathrm{II}} - D G^{\oplus}_{\cdot,\mathrm{II}} \end{bmatrix} + \begin{bmatrix} v_{\mathrm{I}} & v_{\mathrm{II}} \end{bmatrix}, \end{aligned} \tag{14}$$

or in a more compact way as

$$p = p \begin{bmatrix} A_{\mathrm{I},\cdot} \\ S(I_{n^{\mathrm{W}}} - D) G^{\oplus} \end{bmatrix} + p^{\mathrm{W}} (G^{\ominus} - D G^{\oplus}) + v, \tag{15}$$

where $p = (p_{\mathrm{I}}, p_{\mathrm{II}}) = (p_1, \ldots, p_n)$, $v = (v_{\mathrm{I}}, v_{\mathrm{II}}) = (v_1, \ldots, v_n)$, $p^{\mathrm{W}} = (p^{\mathrm{W}}_1, \ldots, p^{\mathrm{W}}_{n^{\mathrm{W}}})$, D is a diagonal matrix whose (k,k)-component is r_k, $A_{\mathrm{I},\cdot} = (A_{\mathrm{I,I}}, A_{\mathrm{I,II}})$, $G^{\oplus} = (G^{\oplus}_{\cdot,\mathrm{I}}, G^{\oplus}_{\cdot,\mathrm{II}})$, and $G^{\ominus} = (G^{\ominus}_{\cdot,\mathrm{I}}, G^{\ominus}_{\cdot,\mathrm{II}})$. The solution to the WIO price model (15) can then be given by

$$p = \left\{ p^{\mathrm{W}} (G^{\ominus} - D G^{\oplus}) + v \right\} \left(I_n - \begin{bmatrix} A_{\mathrm{I},\cdot} \\ S(I_{n^{\mathrm{W}}} - D) G^{\oplus} \end{bmatrix} \right)^{-1}. \tag{16}$$

Recall that, using the WIO quantity model (9), we can perform an LCA study by evaluating the emission e associated with a given scenario which consists of technologies, life style, and an allocation pattern of wastes to treatment methods. Using the price model (16), we can also evaluate the aspect of price and cost of a given scenario.

2.3 The WIO Linear Programming Model

We have so far considered the technological coefficient matrices and the allocation matrix to be constant. It has been considered, in other words, that there exists only one technology for each goods and a given allocation pattern of wastes to treatment methods is unchanged. In this section, we take account of the possibility that there exist alternative goods-producing technologies and an allocation pattern can be changed. In order to consider this possibility, Nakamura and Kondo (2002c) and Kondo and Nakamura (2004b) developed the waste input-output linear programming (WIO-LP) model that is a decision analytic extention of the WIO quantity model based on the method of linear programming (LP). Application of LP to process LCA has been considered by Azapagic and Clift (1995, 1998) while the WIO-LP model is an application of LP to the WIO quantity model for hybrid LCA.

We first consider the selection of goods-producing technologies. Recall that, in Section 2.1, there were n^{I} goods and n^{I} goods sectors, that is, the possibility of selection of technologies were excluded. Now let there be n^{I} goods and m^{I} technologies for producing them such that $n^{\mathrm{I}} \leq m^{\mathrm{I}}$. Note that the input coefficient matrix $A_{\mathrm{I,I}}$ is of $n^{\mathrm{I}} \times m^{\mathrm{I}}$ and is not square in general. The balance equation (4) of the flow of goods is rewritten as

$$Jx_{\mathrm{I}} = A_{\mathrm{I,I}}x_{\mathrm{I}} + A_{\mathrm{I,II}}x_{\mathrm{II}} + X_{\mathrm{I,F}}, \tag{17}$$

where J is an $n^{\mathrm{I}} \times m^{\mathrm{I}}$ matrix of zeros and unities, and its (i,j)-component J_{ij} is defined as

$$J_{ij} = \begin{cases} 1 & \text{technology } j \text{ produces goods } i, \\ 0 & \text{otherwise.} \end{cases} \tag{18}$$

It is not necessary to introduce another notation like m^{II} so as to consider the possibility of selection of waste treatment technologies because the WIO model can contain an arbitrary number of waste treatment sectors regardless of their activity levels. Thus, the only thing to do here is to replace a constant allocation matrix S with a variable one.

The basic form of the WIO-LP model is defined as a minimization problem:

$$\begin{aligned} &\text{minimize} && u(R_{\cdot,\mathrm{I}}x_{\mathrm{I}} + R_{\cdot,\mathrm{II}}x_{\mathrm{II}} + E_{\cdot,\mathrm{F}}) && (19)\\ &\text{subject to} && Jx_{\mathrm{I}} = A_{\mathrm{I,I}}x_{\mathrm{I}} + A_{\mathrm{I,II}}x_{\mathrm{II}} + X_{\mathrm{I,F}}, && (20)\\ &&& w = G_{\cdot,\mathrm{I}}x_{\mathrm{I}} + G_{\cdot,\mathrm{II}}x_{\mathrm{II}} + W_{\cdot,\mathrm{F}}, \quad x_{\mathrm{II}} = Sw, && (21)\\ &&& \iota_{n^{\mathrm{II}}}^{\mathsf{T}} S = \iota_{n^{\mathrm{W}}}^{\mathsf{T}}, && (22)\\ &&& x_{\mathrm{I}} \geq 0_{n^{\mathrm{I}}},\, x_{\mathrm{II}} \geq 0_{n^{\mathrm{II}}},\, S \geq O_{n^{\mathrm{II}},n^{\mathrm{W}}}, && (23)\\ &\text{with respect to} && x_{\mathrm{I}},\, x_{\mathrm{II}},\, w,\, S, && (24) \end{aligned}$$

where ι_n is an $n \times 1$ vector of unities, 0_n is an $n \times 1$ vector of zeros, $O_{m,n}$ is an $m \times n$ matrix of zeros, and the superscript T refers to the transpose of a matrix or vector. The objective function (19) may be a specific emission or an integrated indicator of various emissions, depending upon a given $1 \times n^{\mathrm{E}}$ vector u of weights.

The minimization problem above is not an LP due to the presence of a nonlinear constraint, $x_{\mathrm{II}} = Sw$, in (21), so that the name 'WIO-LP' might seem inappropriate. To rewrite the above nonlinear problem as an LP, we replace the constraints (21) and (22) with

$$Z^{\mathsf{T}} \iota_{n^{\mathrm{II}}} = G_{\cdot,\mathrm{I}}x_{\mathrm{I}} + G_{\cdot,\mathrm{II}}x_{\mathrm{II}} + W_{\cdot,\mathrm{F}}, \quad x_{\mathrm{II}} = Z\iota_{n^{\mathrm{W}}}, \tag{25}$$

and the non-negativity condition $Z \geq O_{n^{\mathrm{II}},n^{\mathrm{W}}}$. To summarize, the basic form of the WIO-LP model is redefined as an LP:

$$\begin{aligned} &\text{minimize} && u(R_{\cdot,\mathrm{I}}x_{\mathrm{I}} + R_{\cdot,\mathrm{II}}x_{\mathrm{II}} + E_{\cdot,\mathrm{F}}) && (26)\\ &\text{subject to} && Jx_{\mathrm{I}} = A_{\mathrm{I,I}}x_{\mathrm{I}} + A_{\mathrm{I,II}}x_{\mathrm{II}} + X_{\mathrm{I,F}}, && (27)\\ &&& Z^{\mathsf{T}} \iota_{n^{\mathrm{II}}} = G_{\cdot,\mathrm{I}}x_{\mathrm{I}} + G_{\cdot,\mathrm{II}}x_{\mathrm{II}} + W_{\cdot,\mathrm{F}}, \quad x_{\mathrm{II}} = Z\iota_{n^{\mathrm{W}}}, && (28)\\ &&& x_{\mathrm{I}} \geq 0_{n^{\mathrm{I}}},\, x_{\mathrm{II}} \geq 0_{n^{\mathrm{II}}},\, Z \geq O_{n^{\mathrm{II}},n^{\mathrm{W}}}, && (29) \end{aligned}$$

with respect to $\quad x_{\mathrm{I}}, x_{\mathrm{II}}, Z.$ (30)

One may take into consideration additional constraints such as the capacity of a recycling sector, that of a treatment sector, and environmental regulations. Solving the above LP, we can systematically obtain an "optimal" waste management option from among a given set of feasible alternatives.

2.4 Joint Use of the WIO-LP Model and the WIO Price Model

We now turn to explaining how to evaluate the aspect of price and cost of the WIO-LP model. Our task here is to obtain the allocation matrix and technological coefficient matrices of goods sectors that correspond to a given optimal solution to the WIO-LP model. Let denote by $(\bar{x}_{\mathrm{I}}, \bar{x}_{\mathrm{II}}, \bar{Z})$ an optimal solution to the WIO-LP model. The allocation matrix $\bar{S}$ corresponding to the solution is easily obtained as

$$\bar{S} = \bar{Z}\{\mathrm{diag}(\bar{Z}^{\mathrm{T}}\iota_{n^{\mathrm{II}}})\}^{-1} \tag{31}$$

by normalizing each column of $\bar{Z}$ so that each column sum is equal to unity.

Turning to the coefficient matrices of goods sectors, define an $m^{\mathrm{I}} \times n^{\mathrm{I}}$ matrix C as

$$C = \mathrm{diag}(\bar{x}_{\mathrm{I}})J^{\mathrm{T}}\{\mathrm{diag}(J\bar{x}_{\mathrm{I}})\}^{-1}.$$

Suppose for a while that $n^{\mathrm{I}} = 2$, $m^{\mathrm{I}} = 5$, and

$$J = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix},$$

i.e., technologies 1 and 2 are for producing goods 1 and technologies 3, 4, and 5 are for producing goods 2. In this case, C can be written as

$$\begin{aligned} C &= \mathrm{diag}(\bar{x}_{\mathrm{I}})J^{\mathrm{T}}\{\mathrm{diag}(J\bar{x}_{\mathrm{I}})\}^{-1} \\ &= \begin{bmatrix} \bar{x}_{11} & 0 \\ \bar{x}_{12} & 0 \\ 0 & \bar{x}_{13} \\ 0 & \bar{x}_{14} \\ 0 & \bar{x}_{15} \end{bmatrix} \begin{bmatrix} \bar{x}_{11} + \bar{x}_{12} & 0 \\ 0 & \bar{x}_{13} + \bar{x}_{14} + \bar{x}_{15} \end{bmatrix}^{-1} = \begin{bmatrix} \bar{x}_{11}/q_{12} & 0 \\ \bar{x}_{12}/q_{12} & 0 \\ 0 & \bar{x}_{13}/q_{345} \\ 0 & \bar{x}_{14}/q_{345} \\ 0 & \bar{x}_{15}/q_{345} \end{bmatrix}, \end{aligned}$$

where $q_{12} = \bar{x}_{11} + \bar{x}_{12}$ and $q_{345} = \bar{x}_{13} + \bar{x}_{14} + \bar{x}_{15}$. Note that each column sum of C is equal to unity; the (j,i)-component of C represents the share of goods i that is produced by technology j. Thus, the technological coefficient matrices $(\bar{A}_{\mathrm{I,I}}, \bar{G}_{\cdot,\mathrm{I}}, \bar{R}_{\cdot,\mathrm{I}}, \bar{v}_{\mathrm{I}})$ of goods sectors are defined as

$$\bar{A}_{\mathrm{I,I}} = A_{\mathrm{I,I}}C, \quad \bar{G}_{\cdot,\mathrm{I}} = G_{\cdot,\mathrm{I}}C, \quad \bar{R}_{\cdot,\mathrm{I}} = R_{\cdot,\mathrm{I}}C, \quad \bar{v}_{\mathrm{I}} = v_{\mathrm{I}}C. \tag{32}$$

Right-multiplying a technological coefficient matrix of goods sectors, which has m^{I} rows, by C yields a matrix of n^{I} "average" technologies, where the "average" is taken so as to reflect the market shares obtained as an optimal solution $\bar{x}_{\mathrm{I}}$. Going back to the above case where $n^{\mathrm{I}} = 2$ and $m^{\mathrm{I}} = 5$, we can see how the "average" is taken:

$$
\begin{aligned}
\bar{A}_{\mathrm{I,I}} &= A_{\mathrm{I,I}} C \\
&= \begin{bmatrix} A_{\mathrm{I},1} & A_{\mathrm{I},2} & A_{\mathrm{I},3} & A_{\mathrm{I},4} & A_{\mathrm{I},5} \end{bmatrix} \begin{bmatrix} \bar{x}_{\mathrm{I}1}/q_{12} & 0 \\ \bar{x}_{\mathrm{I}2}/q_{12} & 0 \\ 0 & \bar{x}_{\mathrm{I}3}/q_{345} \\ 0 & \bar{x}_{\mathrm{I}4}/q_{345} \\ 0 & \bar{x}_{\mathrm{I}5}/q_{345} \end{bmatrix} \\
&= \begin{bmatrix} \dfrac{\bar{x}_{\mathrm{I}1}}{q_{12}} A_{\mathrm{I},1} + \dfrac{\bar{x}_{\mathrm{I}2}}{q_{12}} A_{\mathrm{I},2} & \dfrac{\bar{x}_{\mathrm{I}3}}{q_{345}} A_{\mathrm{I},3} + \dfrac{\bar{x}_{\mathrm{I}4}}{q_{345}} A_{\mathrm{I},4} + \dfrac{\bar{x}_{\mathrm{I}5}}{q_{345}} A_{\mathrm{I},5} \end{bmatrix}.
\end{aligned}
$$

The solution to the WIO price model corresponding to a given optimal solution $(\bar{x}_{\mathrm{I}}, \bar{x}_{\mathrm{II}}, \bar{Z})$ can thus be obtained as

$$
\begin{aligned}
\begin{bmatrix} p_{\mathrm{I}} & p_{\mathrm{II}} \end{bmatrix} = &\left(p_{\mathrm{W}} \begin{bmatrix} \bar{G}^{\ominus}_{\cdot,\mathrm{I}} - \bar{D}\bar{G}^{\oplus}_{\cdot,\mathrm{I}} & G^{\ominus}_{\cdot,\mathrm{II}} - \bar{D}G^{\oplus}_{\cdot,\mathrm{II}} \end{bmatrix} + \begin{bmatrix} \bar{v}_{\mathrm{I}} & v_{\mathrm{II}} \end{bmatrix} \right) \\
&\times \begin{bmatrix} I_{n^{\mathrm{I}}} - \bar{A}_{\mathrm{I}} & -A_{\mathrm{II}} \\ -\bar{S}(I_{n^{\mathrm{w}}} - \bar{D})\bar{G}^{\oplus}_{\cdot,\mathrm{I}} & I_{n^{\mathrm{II}}} - \bar{S}(I_{n^{\mathrm{w}}} - \bar{D})G^{\oplus}_{\cdot,\mathrm{II}} \end{bmatrix}^{-1},
\end{aligned}
\tag{33}
$$

where the matrix $\bar{D}$ of rate of recycling has a bar on top of it because it depends upon the optimal solution $(\bar{x}_{\mathrm{I}}, \bar{x}_{\mathrm{II}}, \bar{Z})$ to the WIO-LP model.

After obtaining an "optimal" waste management option expressed as an optimal solution to the WIO-LP model, we can evaluate the aspect of price and cost of the option using (33). In other words, once a waste management option is found environmentally sound, we can evaluate how economically affordable the option is.

3 Empirical Analyses

We applied the WIO-LP model and the WIO price model to the Japanese WIO table of 1995 (Nakamura, 2003), which has 80 goods sectors, 4 waste treatment sectors (incineration, landfill, composting, and shredding), and 42 types of waste. We deal with import and export in a standard manner although we have so far ignored them for the sake of simplicity. To be concrete, we consider only the domestic products in the balance equation of the flow of goods using the so-called "$(I-(I-M)A)^{-1}$-type" of inverse matrix. Therefore, we account for the environmental loads emitted inside Japan in our empirical analyses.

As for environmental emissions, we consider the two loads. One is the consumption of landfill site in volume. The other is the CO_2 emission originating from burning fossil fuel and limestone. It also includes the global warming potential over 100 years (GWP100) CO_2-equivalent value of methane (CH_4) originating from biomass fermenting at landfill sites and that of chlorofluorocarbons (CFC's) from end-of-life refrigerators and air conditioners. The main reason why we consider quite a small number of environmental loads only is the lack of appropriate data. However, the two environmental loads are appealing in their own rights. A landfill site is one of the scarce resources at least in Japan. It can, in addition, be regarded as an environmental impact if reclaiming of a closed landfill site is technologically excluded in the foreseeable future. The CO_2 emission can be regarded as a proxy, or a first approximation, of a greenhouse gas impact, where we assume an input-output structure same as the Japanese structure prevails in relevant foreign countries.

Kondo and Nakamura (2004b) considered various alternative goods-producing and waste management technologies, and allocation patters of waste to treatment, and obtained the results summarized in Table 1. The considered technologies, besides ones given in the Japanese WIO table of 1995, are as follows: (H) Intensive disassembling and shredding of end-of-life electric home appliances with high yield rates (AEHA, 1999; Kondo and Nakamura 2004a); (I) Regional concentration of incineration (Nakamura and Kondo, 2002a); (G) Gasification of kitchen garbage with power generation (JAFIC, 2002); (B) Injection of waste plastics into blast furnaces (Sanou et al., 2000); (R) Substitution of virgin materials (iron, copper, aluminum, and silica stone) with recycled materials (iron scraps, copper scraps, aluminum scraps, and glass cullet, respectively) recovered from end-of-life electric home appliances (Yoshida et al., 2000; Kondo and Nakamura 2004a). Kondo and Nakamura (2004b) also employed several sorts of additional constraints in order for unrealistic solutions to be ruled out.

The symbols, A, C1, L1, C2, and L2, in the first row labeled 'Case' in Table 1 indicate the possibility of selection of alternative options and the objective to be minimized. Case A corresponds to the current status of Japan in 1995. In Cases C1 and L1, the allocation pattern of wastes to treatment methods can be optimally selected while any additional recycling technologies cannot be chosen; this is the case with $m^I = n^I$. In Cases C2 and L2, both the allocation pattern and additional recycling technologies can be optimally selected. The objective function to be minimized is the CO_2 emission in Cases C1 and C2, and the consumption of landfill site in Cases L1 and L2.

The middle of Table 1 shows which technologies are chosen as components of an optimal solution. It is found that alternative technologies selected at optimality depend upon the possibility of selection of alternative options and the objective to be optimized. In Cases C1 and C2 where the CO_2 emission is minimized, garbage is not gasified but incinerated with the regional concentration and waste plastics are not incinerated but injected into blast furnace or landfilled. In Cases L1 and L2 where the consumption of landfill site is minimized, on the other hand, reducing the bulk of wastes is of great importance, so that garbage is gasified or incinerated with the regional concentration and waste plastics are injected into blast furnace or incinerated with the regional concentration. In particular, waste plastics are injected into blast furnace at optimality in all the cases up to the capacity. The optimal treatment method for the remaining waste plastics varies across cases: on the one hand, the incineration with regional concentration is chosen when the consumption of landfill site is minimized, on the other hand, landfill is chosen when the CO_2 emission is minimized.

In minimizing CO_2 emission, it decreases by 5.0% in Case C1 and by 5.3% in Case C2; optimizing allocation pattern of wastes to treatment sectors is quite effective in the reduction of CO_2 emission. However, in minimizing CO_2 emission, the consumption of landfill site increases by 9.2%

Table 1: Main results of Applying the WIO-LP and price models to Japanese economy

Case		A	C1	L1	C2	L2
1. Alternative waste treatment technologies						
(L) landfilling		yes	yes	yes	yes	yes
(S) old shredding		yes	yes	yes	yes	yes
(H) new shredding		no	yes	yes	yes	yes
(I) regional concentration		no	yes	yes	yes	yes
(G) gasification		no	yes	yes	yes	yes
2. Alternative recycling technologies						
(B) blast furnace		no	no	no	yes	yes
(R) recovered materials		no	no	no	yes	yes
Objective to minimize[a]			CO_2	LS	CO_2	LS
Alternative technologies chosen at optimality[b]						
garbage			I	G/I	I	G/I
waste plastics			L	I	B/L	B/I
TV sets			L	H	S	H
washing machine			L	H	H	H
refrigerator, air conditioner			H	H	H	H
metal scraps			L	L	R	R
glass cullet			L	L	R/L	R/L
Environmental load emission[c]						
CO_2		0.00	-4.97	-4.06	-5.33	-4.45
landfill consumption		0.00	9.22	-26.02	5.22	-27.95
Activity level, cost, and price of waste treatment methods[c]						
activity level:	incineration	0.00	-5.57	-13.86	-5.63	-14.95
	landfill	0.00	2.38	-9.74	-1.01	-12.85
	all the treatment methods	0.00	-1.94	-0.39	-2.83	-2.58
price:	incineration	0.00	-2.10	3.58	-2.19	4.51
	landfill	0.00	2.46	-2.92	1.92	-1.77
	all the treatment methods	0.00	-5.56	8.71	-3.97	10.23
cost:	incineration	0.00	-7.55	-10.77	-7.70	-11.11
	landfill	0.00	4.90	-12.38	0.89	-14.39
	all the treatment methods	0.00	-7.39	8.29	-6.68	7.38

Source: Kondo and Nakamura (2004b).

[a] 'LS' stands for 'consumption of landfill site'.

[b] Chosen technologies are indicated with symbols in the upper part of the table. However, two symbols connected by a slash, say 'G/I', indicate that a type of waste is gasified up to the capacity of the gasification, and the remaining part of that waste is treated by regionally concentrated incineration.

[c] Rate of change relative to the current status (A) in percentage.

in Case C1 and by 5.2% in Case C2. It is found that there exists a trade-off relationship between our two objective functions, CO_2 emission and the consumption of landfill site. That is, the one of the two objectives increases when the other is minimized and reduced. A trade-off relationship between the two objectives is also observed in Cases L1 and L2 where the consumption of landfill site is minimized, although both emissions decrease in these cases.

Turning to the aspects of cost and price of optimally chosen waste management options, in Case C1, the total cost of landfill increases by 4.9% due to the increase of both the activity level and price of landfill. In minimizing CO_2 emission, waste plastics not recycled are landfilled, some of which newly come to be landfilled as a result of optimization. This change of the allocation increases the activity level of landfill and worsens the efficiency of transportation because waste plastic is bulky. The increase of price due to a decline in the efficiency of transportation is also observed as follows: the price of incineration increases in Cases L1 and L2, and the price of landfill increases in Case C2. The costs of both incineration and landfill fall by 11.1% and 14.4%, respectively, in Case L2, although the price of incineration increases by 4.5%. However, the total cost of all the treatments increases by 7.4% because the average price increases by 10.2%. This is partly due to the introduction of high-tech treatment methods such as intensive disassembling and shredding with high yield rates.

4 Concluding Remarks

This paper has proposed a joint use of the WIO-LP model (Nakamura and Kondo, 2002c; Kondo and Nakamura, 2004b) and the WIO price model (Nakamura and Kondo, 2002b). The WIO-LP model is an decision analytic extension of the basic WIO model (Nakamura and Kondo, 2002a) and it can be used to systematically search for an "optimal" waste management option from among a given set of feasible alternatives. After obtaining an environmental impact-minimizing waste management option expressed as an optimal solution to the WIO-LP model, we can evaluate the aspect of price and cost of the option using the WIO price model. In other words, once a waste management option is found environmentally sound, we can evaluate its economic affordability. Empirical application of the proposed methodology to the Japanese economy of 1995 revealed that there exists a trade-off relationship between CO_2 emission and the consumption of landfill site and that the efficiency of transportation crucially affects the price of waste treatment.

References

AEHA [Association for Electric Home Appliances] (1999) Development of the Integrated Recycling System of Waste Electric Home Appliances: Progress Report 1998 (in Japanese with English summary).

Azapagic, A. and R. Clift (1995) Life-cycle assessment and linear-programming: Environmental optimization of product system, *Computers and Chemical Engineering*, 19, Supplement, pp. S229–S234.

Azapagic, A. and R. Clift (1998) Linear programming as a tool in life cycle assessment, *International Journal of Life Cycle Assessment*, 3, 6, pp. 305–316.

Duchin, F. (1990) The conversion of biological materials and waste to useful products, *Structural Change and Economic Dynamics*, 1, 2, pp. 243–262.

JAFIC [Japan Food Industry Center] (2002) The 2001 Report of National Undertaking on Development of Resource-Cycle Food Industry: Zero-emission Promotion Committee of Western Kanagawa (in Japanese).

Kondo, Y. and S. Nakamura (2004a) Evaluating Alternative Life-Cycle Strategies for Electrical Appliances by the Waste Input-Output Model, *International Journal of Life Cycle Assessment*, 9, 4, pp. 236–246.

Kondo, Y. and S. Nakamura (2004b) Waste Input-Output Linear Programming Model with Its Application to Eco-Efficiency Analysis, submitted to *Economic Systems Research*; Working Paper Series, No. 0404, Institute for Research in Contemporary Political and Economic Affairs, Waseda University.

Leontief, W. W. (1970) Environmental repercussions and the economic structure: An input-output approach, *Review of Economics and Statistics*, 52, 3, pp. 262–271.

Leontief, W. W. and D. Ford (1972) Air pollution and the economic structure: Empirical results of input-output computations, in: A. Brody and A. P. Carter (eds) *Input-Output Techniques* (Amsterdam, North-Holland), pp. 9–30.

MacDonald, M. L. (1996) Solid waste management models: A state of the art review. *Journal of Solid Waste Technology and Management* 23(2): 73–83.

Nakamura, S. (1999) Input-Output Analysis of Waste Cycles, *First International Symposium on Environmentally Conscious Design and Inverse Manufacturing, Proceedings* (Los Alamitos, IEEE Computer Society), pp. 475–480.

Nakamura, S. (2003) The Waste Input-Output Table for Japan 1995, Version 31e, (Tokyo, Japan, School of Political Science and Economics, Waseda University). Available at `http://www.f.waseda.jp/nakashin/` (Accessed on October 2003).

Nakamura, S. and Y. Kondo (2002a) Input-output analysis of waste management, *Journal of Industrial Ecology*, 6, 1, pp. 39–63.

Nakamura, S. and Y. Kondo (2002b) Environmental Impact and Economic Cost of Waste Treatment: Analysis by the Waste IO Model, presented at 14th International Conference on Input-Output Techniques, October 10–15, 2002, Montreal, Canada; Working Paper Series, No. 0204, Institute for Research in Contemporary Political and Economic Affairs, Waseda University, July 2002; French translation is also available: "Impact environnemental et coût économique du traitement des déchets: Une analyse d'entrée-sortie," in L. M. Cloutier and C. Debresson eds. *Changement Climatique, Flux Technologiques, Financiers et Commerciaux: Nouvelles directions d'analyse d'entrée-sortie*, Presses de l'Université du Québec, 2004, pp. 91–103.

Nakamura, S. and Y. Kondo (2002c) Waste input-output linear programming analysis of waste management and recycling, The 12th SETAC Europe LCA Case Studies Symposium (SETAC Europe/ISIE Joint Meeting), Barcelona, Spain, December 2–4, 2002.

M. Sanou, A. Fujita, C. Kosan, A. Yamada, K. Fujisaki (2000) Environmental Assessment for Treatment of Used Office Appliances, in: Proceedings of the Fourth International Conference on EcoBalance, Tsukuba, Japan, pp. 581–584.

T. Yoshida, Y. Otsuka, K. Ueno, and A. Sunami (2000) Life cycle assessment on recycling of electrical home appliances: Prediction and results through operation, in: Proceedings of the Fourth International Conference on EcoBalance, Tsukuba, Japan, pp. 551–554.