FDI Subsidies When Plant Locations and Sizes Are Both Endogenous under Capital Constraints

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Abstract

This paper exploits a simple three-stage reciprocal market game model of an international Cournot duopoly consisting of home and foreign multinational firms to examine FDI subsidy policies of home and foreign governments. It considers firms' endogenous choices of plant (FDI) locations and sizes under capital constraints. The governments decide the FDI subsidies in the first stage. Then, the firms choose endogenously the plant locations and sizes under capital constraints in the second stage and the output-export levels in the third stage. This paper finds that governments can use the FDI subsidies as tools for implementing strategic policies, and that the FDI subsidies do not have drastic changes on firms' plant locations.
1. Introduction

Recently, the number of studies on international duopoly (or oligopoly) has been increasing. One of their main purposes is to investigate whether governments can use certain economic policies, such as tariffs, export subsidies and so on, as the basis for economic strategies to deprive foreign countries of their economic welfare when a few big firms compete in imperfectly competitive international markets.

Many papers, initiated by Brander and Krugman (1983) and Brander and Spencer (1984 and 1985), have established various types of imperfectly competitive trade model in which only uninational firms engaging in no overseas production compete against each other for international market shares by exporting goods and have shown that trade policies, such as tariffs and export subsidies, are generally effective as economic strategies. By contrast, Janeba (1998) and Ishii (2001) have exploited duopoly models where multinational firms with subsidiaries in foreign countries compete with each other for market shares by overseas production as well as exports and have demonstrated that tariffs remain effective but export subsidies lose their effectiveness.

Though the proposition presented by Janeba (1998) and Ishii (2001) is useful and of interest, both their models adopt the implicit assumption that plant sizes of multinational firms are exogenously given as constant. However, when firms act as multinational firms in the present world they determine endogenously both plant sizes and locations. Furthermore, they do not consider policies to control plant sizes and locations of multinational firms. Therefore, the proposition shown by Janeba (1998) and Ishii (2001) is restrictive from a political, as well as a descriptive, point of view.

As the number of imperfectly competitive international markets consisting of a few big multinational firms that have subsidiary plants in multiple countries by foreign direct
investment (henceforth, FDI) has increased, so has the possibility that governments adopt policies for controlling firms’ FDI (henceforth, FDI policies) with strategic economic aims. Indeed, multinational firms provide their products for foreign countries not only by exporting from the parent plants in their home countries, but also by supplying from their subsidiary plants in foreign countries. Therefore, it is of great significant and utility to establish an international duopoly model considering firms’ FDI or plant choices and to investigate the effects of FDI policies on firms’ FDI sizes and optimal levels of FDI policies.

Although their purpose is different from that of this paper, Markusen et al. (1993 and 1995) have presented a model in which firms endogenously determine their plant locations and have shown that governmental policies (environmental taxes in their model) cause drastic changes in firms’ plant locations. Then, in order to highlight their result they have assumed that firms can easily move all production plants anywhere in the world (that is, firms are ‘footloose’), that firms’ plant sizes are exogenously determined as constant, and that firms are free from capital constraints. However, these assumptions are not plausible in all international industries.

In some real-world international industries, firms that already have production plants in their countries of origin (henceforth, parent plants) do not move their parent plant across countries as easily as they do their plants in other countries (henceforth, subsidiary plants), that is, they are not ‘footloose’. It is also true that when firms intend to construct newly production plants by using FDI, they can choose their plant sizes or FDI sizes as well as their locations endogenously. Furthermore, firms cannot expand their plant sizes or FDI sizes, because the available amount of capital for FDI is bounded by credit constraints and/or technological constraints. Thus, firms have capital constraints with an upper limit. In this paper, we first establish a more generalized international duopoly model that considers these
aspects. Then, we analyze the effectiveness and optimal levels of FDI subsidies, as strategic EDI policies.¹

In order to investigate the issues mentioned above, this paper will expand the model exploited by Markusen et al. to include endogenous choices of plant size and location by duopolistic multinational firms under capital constraints and optimal FDI subsidy decisions by governments.² Hence, though the framework of the present model seems to be similar to that of the Markusen et al. model, there exists an essential difference between these models.

The Markusen et al. model is a two-stage game model, since they have implicitly assumed that plant locations and outputs are simultaneously determined in the same stage, the second. However, since plant constructions take considerable time in the actual world, it is quite plausible to regard plant decisions and output-export choices as being made in different stages, which introduces one more stage into the Markusen et al. two-stage model.³ Therefore, the present model becomes a three-stage game model. In the first stage the governments determine the FDI subsidies. Then, the firms decide their plant locations and sizes under capital constraints in the second stage and choose the optimal outputs and exports in the third stage.

For solving these three-stage decision problems, we use backward induction. Thus, we find, among other things, that FDI subsidies are also effective as strategic political instruments and that they have generally positive effects on firms' FDI sizes. Furthermore, we also show that the optimal FDI subsidies are not always positive and that FDI subsidies do not have such drastic effects on the firms' plant choices as indicated by Markusen, et al. (1993 and 1995).

The rest of this paper is organized as follows. Section 2 establishes a generalized model of an international Cournot duopoly, as was explained above. Section 3 analyzes the firms'
output-export choices. Section 4 examines the firms’ FDI decisions. Section 5 discusses the optimal FDI subsidies. Section 6 presents concluding remarks.

2. Assumptions and Basic Model

This section will establish a very simple generalized model of an international Cournot duopoly consisting of a home firm and a foreign firm. The present model supposes an international Cournot duopoly, in which, both firms have already parent plants of fixed sizes, \( A \) and \( A' \), that produce homogenous goods in their original countries, and intend to expand the parent plants by additional domestic investment (henceforth, ADI), \( K \) and \( K' \), and/or to build new subsidiary plants producing the same goods by FDI, \( k \) and \( k' \), in their rival countries under capital constraints, respectively. (Henceforth, the notations with asterisks, *, express the foreign variables that correspond to the home variables).

The above reflects the fact that many multinational firms in the actual economy first establish parent plants and later set up subsidiaries, after engaging in exports for several years. Therefore, firms’ parent and subsidiary plant sizes are both endogenously determined by choosing ADI and FDI under capital constraints, respectively. When firms choose zero FDI, they are still uninalional firms, but when they decide to build their subsidiary plants by choosing positive FDI, they become multinational firms. Moreover, it is clear that if \( A = 0 \) and \( A' = 0 \) hold, the present model reduces to the model of Mulkusen et al. and hence includes it as a special case.

Demand functions

Suppose that the home and foreign markets are segregated from each other. While the home and foreign firms supply goods produced by their parent plants to both countries, they sell goods produced by their subsidiary plants only in the rival country where the subsidiary
plants are constructed, respectively. Then, both the home and foreign firms have two routes to supply their goods to the rival country: exports and overseas production. The assumption that the subsidiaries do not export their products back to the parent countries may seem to be strict at first sight, but it is not so in a homogenous good model, with respect to both the real-world situation and the theoretical standpoint.\(^4\)

Thus, the home and foreign inverse demand (twice differentiable) functions are given respectively by \( p(Z) = p(X + T^* + Y^*) \) with \( p'(Z) < 0 \) and \( p'(Z^*) = p'(X^* + T + Y) \) with \( p'(Z^*) < 0 \), where \( p \) and \( p^* \) are respectively the home and foreign prices, \( X \) and \( T \) (\( X^* \) and \( T^* \)) are respectively domestic sale and export of the home (foreign) parent plant, and \( Y \) (\( Y^* \)) is output (= sale) of the home (foreign) subsidiary plant.

**Capital constraints**

We consider capital constraints. One important and oft-observed factor in big multinational firms facing such capital constraints is technological in nature. When capital equipment demanded by the firms in question has a particular technical specification and/or is produced by a few small firms that can not adopt mass-production systems, big multinational firms might have difficulty in obtaining sufficient capital equipment to carry out their ADI and FDI plans. Furthermore, even big multinational firms cannot always gather enough funds to achieve their plant choices. In such cases the home and foreign firms face capital constraints, respectively. This paper analyzes such a case where firms' capital constrains are both bounded:

\[
    K + k = \bar{K} \quad \text{and} \quad K^* + k^* = \bar{K}^*,
\]

where \( \bar{K} \) and \( \bar{K}^* \) are respectively the upper limits of (real) capital that are available for
firms' ADI and FDI.

Cost functions

When firms plan to be multinationals they consider three different categories of costs: plant construction, production and export. Therefore, it is necessary to examine these cost functions in some detail.

**Plant expansion and construction costs.** Since the home and foreign firms respectively plan to expand parent plants by ADI and to construct subsidiary plants by FDI, they must incur expansion costs, $V$ and $V^*$, for parent plants and construction costs, $v$ and $v^*$, for their subsidiary plants. It is supposed that these costs are all expressed by twice differentiable and strictly increasing-convex functions of their ADI and FDI levels, respectively. Then, while firms' parent plant expansion cost functions are respectively given by $V(K)$ (with $V'(K) > 0$ and $V''(K) > 0$ for $K > 0$) and $V^*(K^*)$ (with $V^{**}(K^*) > 0$ and $V^{***}(K^*) > 0$ for $K^* > 0$), their subsidiary plant construction cost functions are respectively given by $v(k)$ (with $v'(k) > 0$ and $v''(k) > 0$ for $k > 0$) and $v^*(k^*)$ (with $v^{**}(k^*) > 0$ and $v^{***}(k^*) > 0$ for $k^* > 0$). It is clear that, while these plant expansion and construction costs are fixed in the third stage when the firms choose their optimal output-export levels, they are variable in the second stage when the firms decide their optimal ADI and FDI levels.

**Production costs.** While positive ADI levels expand sizes of firms' parent plants, positive FDI levels determine sizes of firms' subsidiary plants. Hence, all unit production costs, $C$ and $C^*$, of the firms' parent plants and unit production costs, $c$ and $c^*$, of the firms' subsidiary plants are regarded as decreasing with their ADI, $K$ and $K^*$, and their FDI, $k$ and $k^*$, respectively (due to scale merits, for example). Thus, it is assumed that all unit production costs of these plants, $C$, $C^*$, $c$ and $c^*$ are given by (twice differentiable) functions such as
C(K) with C'(K) < 0, C''(K) with C''(K) < 0, c'(k) < 0 and c''(k) < 0, C(K), C'(K), c(k) and c''(k) are also regarded as strictly convex, i.e., C'(K) > 0, K''(K) > 0, c'(k) > 0 and c''(k) > 0, since the scale merit decreases with plant size.

Furthermore, it is assumed that all unit production costs, C, C', c and c', of firms' parent and subsidiary plants are independent of their outputs in the third stage and that C \neq C', c \neq c', C \neq c, C' \neq c', C \neq c' and c \neq C' hold, due, for example, to differences between the factor prices in the two countries and differences between production technologies in production facilities.⁵

*Export Costs:* Export costs of the home and foreign firms consist of transportation, sales and office transaction costs. In order to increase exports firms have to send their goods to more distant areas in their rivals' counties. Therefore, export costs of the home and foreign firms are respectively expressed as (twice differentiable) export cost functions, G(T) and G'(T'), whose marginal costs are strictly positive, i.e., G'(T) > 0 and G''(T') > 0, for the entire positive range of exports and increase with exports, i.e., G'(T) > 0 and G''(T') > 0, for some of, but not the entire, positive range of exports.⁶

In this paper, since marginal production costs of plants are all independent of their outputs, exports and overseas production are perfectly substitutive for each other when export costs are also constant. If the marginal export costs are larger than the marginal production costs of the subsidiary plant, then the firm chooses only overseas production and does not export goods, and *vice versa*. Therefore, the variable marginal export costs, as is considered in this paper, are necessary to ensure that the multinationals engage in both exports and overseas production in addition to domestic production.
Political instruments

Various instruments may be used to control firms' FDI flows (= negative ADI flows), but the present model concentrates only on FDI subsidies (= negative ADI subsidies), because FDI subsidies are generally regarded as more useful and appropriate than any other political instrument for controlling the firms' ADI or FDI flows. Of course, though it is not so difficult to incorporate explicitly some other policy instruments, such as tariffs, export subsidies and so on, into the model, the effects of these policies on FDIs have already received much attention. It is assumed that the home and foreign governments provide, respectively, per-unit FDI subsidies, $s$ and $s^*$, for out-flow FDIs and per-unit FDI subsidies, $s_k$ and $s_k^*$, for in-flow FDIs. Of course, negative FDI subsidies imply FDI taxes.

However, as is clear from their definitions, the home (foreign) firm's FDI is regarded as the out-flow FDI by the home (foreign) country but as the in-flow FDI by the foreign (home) country. So, let $s$, $s_k$, $s^*$ and $s_k^*$ be, respectively, the home out-flow FDI subsidy, the home in-flow FDI subsidy, the foreign out-flow FDI subsidy, and the foreign in-flow FDI subsidy. Since the home and foreign governments choose their FDI subsidies so as to maximize their own welfare in the first stage and keep them constant thereafter, all the FDI subsidies, $s$, $s^*$, $s_k$ and $s_k^*$, are parameters for the home and foreign firms in the second and third stages.

Firms' profits

Under the assumptions and features explained in the previous subsections, profits of the home and foreign firms, $\Pi$ and $\Pi^*$, are defined respectively as

$$\Pi = \{p(Z)X + p^*(Z^*)T - C\left(\bar{K} - k\right)(X + T) - V\left(\bar{K} - k\right) - G(T)\}$$

$$+ \{p^*(Z^*)Y - c(k)Y - v(k)\} + (s + s_k^*)k,$$  \hspace{1cm} (1)

10
\[ \Pi^* = (p^*(Z^*)X^* + p(Z)Y^* - C^*(\tilde{K}^* - k^*) (X^* + T^*) - V^*(\tilde{k}^* - k^*) - G^*(T^*)) \]
\[ + (p(Z)Y^* - c^*(k^*)Y^* - v^*(k^*)) + (s^* + s_k)k^*, \] (2)

where \( Z = X + T^* + Y^* \) and \( Z^* = X^* + T + Y \) are total sales (= consumption) in the home and foreign countries, respectively. In (1) (and (2)), the first and second terms braced by \{ \} are profits of the home (foreign) parent and subsidiary plant, respectively, and the last term is revenue from the governments' FDI subsidies.

The home and foreign firms in a Cournot industry act independently to maximize their own profits defined by (1) and (2), respectively, and the home and foreign governments determine their respective FDI subsidies so as to maximize their own welfare, as defined in section 5. This paper will solve these problems from the third stage equilibrium to the first stage equilibrium by backward induction.

3. Optimal Output-Export Choices in the Third Stage

In the third stage, both the home and foreign firms face output-export decisions. The home (foreign) firms' control variables are output \( X \) (\( X^* \)), exports \( T \) (\( T^* \)) of the parent plant and output \( Y \) (\( Y^* \)) of the subsidiary plant, respectively. Since the firms belonging to a Cournot industry act non-cooperatively, the home firm chooses \( X \), \( T \) and \( Y \) so as to maximize its profit defined as (1), given the foreign firm's output-export levels and all other exogenous variables in the third stage, and the foreign firm decides \( X^* \), \( T^* \) and \( Y^* \) which maximize its profit defined by (2), given the home firm's output-export levels and all other exogenous variables in the third stage.
Profit Maximization Conditions

Concentrating on a case of inner solutions since the case of corner solutions is trivial, the first-order conditions of the home and foreign firms for maximizing their profits are respectively given by:

\[ p(Z) + p'(Z)X - C\left(\bar{K} - k\right) = 0, \]  
\[ p'(Z^*) + p''(Z^*)(T + Y) - G'(T) - C\left(\bar{K} - k\right) = 0, \]  
\[ p'(Z^*) + p''(Z^*)(T + Y) - c(k) = 0, \]  

and

\[ p'(Z^*) + p''(Z^*)X^* - C\left(\bar{K}^* - k^*\right) = 0, \]  
\[ p(Z) + p'(Z)(T^* + Y^*) - G''(T^*) - C\left(\bar{K}^* - k^*\right) = 0, \]  
\[ p(Z) + p'(Z)(T^* + Y^*) - c'(k^*) = 0. \]

Furthermore, since the second-order conditions can be satisfied under the demand and cost conditions introduced in the previous section, we assume that they hold in the neighborhood of the equilibrium.

In the first-order conditions, while (3), (7) and (8) are the reaction functions among home parent domestic output, foreign parent export and foreign subsidiary output, (4), (5) and (6) are the reaction functions among home parent export, home subsidiary output and foreign parent output. Whether these reaction functions are respectively depicted on a plane as downward or upward-sloping curves depends on whether the goods produced by the firms are strategically substitutive for, or complementary to, each other. However, it is more reasonable to regard their products as being strategically substitutive for each other when they are
homogenous. Then, as is well known, the demand functions present the following conditions:

\[ p'(Z) + p''(Z)X < 0, \quad p''(Z') + p''(Z')(T + Y) < 0, \quad p'(Z') + p''(Z')X' < 0 \text{ and } p'(Z) + p'(Z')(T'* + Y') < 0. \]

Hence, the reaction curves of the home and foreign firms are all downward-sloping.

**Industry Equilibrium**

The Cournot-Nash industry equilibrium in the third stage is given by a vector of \((X, T, Y, X^*, T^* \text{ and } Y^*)\) that simultaneously satisfies the equation system consisting of (3)-(8). However, it is easily shown that while the equilibrium levels of \(X, T^*\) and \(Y^*\) are derived by solving (3), (7) and (8), the equilibrium levels of \(X^*, T\) and \(Y\) are obtained by solving (4), (5) and (6). Thus, this kind of separation theorem, the demand functions, and the cost functions mentioned above combine to show that the industry equilibrium in the third stage is locally stable (see the Routh theorem).

It is easily shown, from (3)-(8), that \(T\) and \(Y\) (\(T^*\) and \(Y^*\)) in the present model become positive simultaneously at the industry equilibrium. This is why, though unit production costs are assumed to be constant, the marginal export costs are regarded as variable. If it is assumed that not only marginal production costs but also marginal export costs are constant, it is impossible for \(T\) and \(Y\) (\(T^*\) and \(Y^*\)) to become simultaneously positive at the industry equilibrium. In such a case, at least one of \(T\) and \(Y\) (\(T^*\) and \(Y^*\)) always becomes zero, and firms' exports and FDI become always perfectly substitutive for each other. However, this seems to be contrary to observed fact. Therefore, it seems to be very restrictive to assume simultaneously constant marginal production costs and constant marginal export costs when analyzing the optimal FDI and output-export choices of the multinational firms.

As is described in the previous section, though firms' FDI levels, \(k\) and \(k^*\), are
parameters in the third stage, they are firms' control variables in the second stage. Therefore, it is interesting to examine the effects of changes in $k$ and $k^*$ on firms' output-export choices because they provide useful information about relations between firms' decisions in the second and third stages.

Taking the total differential of each equation of (3)-(8) and considering features of the demand functions and the cost functions mentioned above, one obtains the effects of a change in the firms' FDI levels, $k$ and $k^*$, on the industry equilibrium, $X$, $T$, $Y$, $X^*$, $T^*$ and $Y^*$ (see Appendix 3):

$$
\frac{\partial T}{\partial k} < 0, \quad \frac{\partial Y}{\partial k} > 0, \quad \frac{\partial X^*}{\partial k} < 0, \quad \frac{\partial T^*}{\partial k} = 0, \quad \frac{\partial Y^*}{\partial k} > 0, \quad \frac{\partial X^*}{\partial k} < 0,
$$

(9-i)

and

$$
\frac{\partial T^*}{\partial k^*} < 0, \quad \frac{\partial Y^*}{\partial k^*} > 0, \quad \frac{\partial X^*}{\partial k^*} < 0, \quad \frac{\partial T}{\partial k^*} = 0, \quad \frac{\partial Y}{\partial k^*} > 0, \quad \frac{\partial X^*}{\partial k^*} < 0.
$$

(9-ii)

Then, (9-i) and (9-ii) present:

Proposition 1: A rise in the home (foreign) firm's FDI increases outputs of both the home and foreign subsidiary plants, but reduces all exports of the home (foreign) parent plant and outputs of the home and foreign parent plants, and vice versa. However, it has no effect on exports of the foreign (home) parent plant.

Furthermore, from (9) one gets the following relations (see also Appendix 3):

$$
\frac{\partial Z^*}{\partial k} > 0, \quad \frac{\partial Z^*}{\partial k^*} < 0, \quad \frac{\partial Z^*}{\partial k^*} > 0, \quad \frac{\partial Z^*}{\partial k^*} < 0,
$$

(10-i)

and

$$
\frac{\partial (Y + T^* + X^*)}{\partial k} > 0, \quad \frac{\partial (Y^* + T + X)}{\partial k} < 0, \quad \frac{\partial (Y^* + T^* + X^*)}{\partial k^*} < 0, \quad \frac{\partial (Y^* + T + X)}{\partial k^*} > 0.
$$

(10-ii)

While $\frac{\partial Z^*}{\partial k} > 0$ and $\frac{\partial Z^*}{\partial k^*} > 0$ entail that a raise in the FDI expands the market size (= consumption) of the host country, $\frac{\partial Z}{\partial k} < 0$ and $\frac{\partial Z^*}{\partial k^*} < 0$ demonstrate that an increase in the
FDI reduces the market size (= consumption) of the parent country. On the other hand, \( \frac{\partial Y}{\partial k} + \frac{\partial T^*}{\partial k} + \frac{\partial X^*}{\partial k} > 0 \) and \( \frac{\partial Y^*}{\partial k} + \frac{\partial T^*}{\partial k} + \frac{\partial X^*}{\partial k} > 0 \) mean that a rise in the FDI increases the output level (= GDP) of the host country, but \( \frac{\partial Y}{\partial k} + \frac{\partial T}{\partial k} + \frac{\partial X}{\partial k} < 0 \) and \( \frac{\partial Y^*}{\partial k} + \frac{\partial T^*}{\partial k} + \frac{\partial X^*}{\partial k} < 0 \) indicate that an increase in the FDI results in a reduction of the output level (= GDP) of the parent country.

When taken together, Proposition 1 and these results 1 show that if the governments can manage the firms' FDI levels by changing FDI subsidies they can use their FDI subsidies strategically to control firms' market shares and revenues, respectively. Therefore, it is essential for the governments to obtain definite information about the effects of changes in FDI subsidies on the firms' FDI decisions. In the next section, we will investigate whether the governments can control appropriately the firms' FDI levels by changing their FDI subsidies.

4. Optimal FDI Levels in the Second Stage

In the second stage, the Cournot home and foreign firms non-cooperatively choose their FDI levels, \( k \) and \( k^* \), so as to maximize their own profits, given all of the FDI subsidies, the rival's FDI level, the firms' optimal output-export levels, and the parameters included in (1) and (2), respectively. Hence, considering that the conditions of (3)-(8) will always hold in industry equilibrium of the third stage, the first-order conditions for maximizing the home and foreign firms' profits in the second stage are respectively given by:

\[
p'(Z)X \frac{\partial Y}{\partial k} + p''(Z')X \frac{\partial X}{\partial k} (T + Y) + C'\left(\frac{K}{k} - k\right)(X + T) + V'\left(\frac{K}{k} - k\right)
\]

\[= c'(k)Y - v'(k) + (s + s^*_k) = 0, \quad (11)\]

and
\[ p^*(Z^*)X^* \frac{\partial Y}{\partial k^*} + p'(Z) \frac{\partial X}{\partial k^*}(T^* + Y^*) + C''(k^* - k^*) (X^* - T^*) + V''(k^* - k^*) \\
- c'(k^*)T^* - v'(k^*) + (s^* + s_0) = 0. \tag{12} \]

The second-order conditions of the home and foreign firms, \( \Pi_{kk} \) < 0 and \( \Pi_{kk^*}^* \) < 0, are assumed to hold (henceforth, \( \Pi_y \) and \( \Pi_y^* \) denote the second-order derivatives of \( \Pi \) and \( \Pi^* \) with respect to \( i \) and \( j \), \( i, j = k, k^* \), respectively).\(^{10}\)

The equations (11) and (12) are respectively the reaction functions of the home and foreign firms for FDI decisions. Therefore, the Cournot-Nash industry equilibrium in the second stage is given by \( k \) and \( k^* \) satisfying (11) and (12) simultaneously. When FDIs of the home and foreign firms are strategically substitutive (complementary) for each other, the reaction functions in FDI decisions are both depicted as downward (upward) sloping curves. Since these two firms engage in FDIs producing homogenous goods, it is more plausible to assume that FDIs of these two firms are strategically substitutive for each other as is similar to their goods. Then, \( \Pi_{kk^*} < 0 \) and \( \Pi_{kk^*}^* < 0 \) hold.

Furthermore, the firms' marginal profits, \( \frac{\partial \Pi}{\partial k} \) and \( \frac{\partial \Pi^*}{\partial k^*} \), with respect to its own FDIs are generally regarded as being more sensitive to a change in its own FDI than to a change in its rival's FDI, respectively. Therefore, it is plausible to assume that \( | \Pi_{kk} | > | \Pi_{kk^*} | \) and \( | \Pi_{kk^*}^* | > | \Pi_{kk^*}^* | \) hold. Then, these relationships and the second-order conditions combine to ensure that the locally stability conditions of the industry equilibrium in the second stage are satisfied: \( \Pi_{kk} < 0 \), \( \Pi_{kk^*}^* < 0 \) and \( 0 < \Pi_{kk} \Pi_{kk^*}^\prime - \Pi_{kk}^\prime \Pi_{kk^*} \) (see the Routh theorem).

As is obvious from (11) and (12), both the industry equilibrium FDI levels, \( k \) and \( k^* \),
depend on out-flow and in-flow FDI subsidies, $s$, $s^*$, $s_k$ and $s_k^*$, which are determined by the home and foreign governments in the first stage. However, it is easily shown, from (11) and (12), that $s$ and $s_k^*$ ($s^*$ and $s_k$) have the same effects on the industry equilibrium. Then, it is impossible to discern the effects of changes in $s$ and $s_k^*$, and of changes in $s^*$ and $s_k$, on the equilibrium. Therefore, let us examine these effects more carefully in the rest of this section.

Taking the total differential of (11) and (12), and considering the second-order conditions, the negative slope conditions of firms' FDI reaction curves and the stability conditions of the industry equilibrium, one can obtain the effects of changes in $s$, $s_k^*$, $s^*$ and $s_k$ on the industry equilibrium, respectively:

$$
\frac{\partial k^*}{\partial s} = \frac{\partial k^*}{\partial s_k^*} = \frac{-\Pi_k^*}{\Omega} > 0, \quad \frac{\partial k}{\partial s} = \frac{\partial k}{\partial s_k} = \frac{-\Pi_k^*}{\Omega} > 0,
$$

(13)

and

$$
\frac{\partial k^*}{\partial s^*} = \frac{\partial k^*}{\partial s_k^*} = \frac{-\Pi_k^{s^*}}{\Omega} > 0, \quad \frac{\partial k^*}{\partial s^*} = \frac{\partial k^*}{\partial s_k^*} = \frac{-\Pi_k^{s^*}}{\Omega} > 0,
$$

(14)

where $\Omega = \Pi_k^{s^*} \Pi_k^{s_k^*} - \Pi_k^{s^*} \Pi_k^{s_k^*}$. Therefore, considering $|\Pi_k| > |\Pi_k^{s^*}|$ and $|\Pi_k^{s_k^*}| > |\Pi_k^*|$, (13) and (14) are summarized together with as the following proposition:

Proposition 2: A rise in any FDI subsidy increases both home and foreign FDIs, and vice versa. However, a change in the out-flow (in-flow) FDI subsidy has a larger effect on out-flow (in-flow) FDI than on in-flow (out-flow) FDI, measured by the absolute value.

It is quite natural that when the home government raises the out-flow (in-flow) FDI
subsidy the home (foreign) firm increases its FDI and thus the foreign (home) firm also increases its FDI (ADI). Under the capital constraint, since a rise in the home FDI implies the equal reduce of the home ADI, it induces an increase in the foreign FDI and a decrease in the foreign ADI. Therefore, Proposition 2 also coincides with our intuition.

This proposition demonstrates that both the home and foreign firms alter their FDI levels smoothly when the governments change the FDI subsidies marginally. Therefore, it is shown that a small change in FDI subsidy does not cause drastic changes in firms’ plant locations as is indicated by Markusen et al. (1993 and 1995) when firms’ plant sizes, as well as plant locations, are determined endogenously. On the contrary, there exists a possibility that such drastic changes in plant locations are not induced by small changes in FDI subsidies even if firms have fixed costs.

This proposition also entails that both the home and foreign governments can control the rival firm’s FDI (ADI) by changing appropriately one or both of their out-flow and in-flow FDI subsidies. Therefore, the governments can use FDI subsidies as strategic policies that manage the economic welfare by controlling market sizes and/or GDPs in the rivals’ countries, respectively.

5. Optimal FDI Subsidies in the First Stage

At first glance, since a rise (reduction) in FDI level increases (decreases) consumption and domestic output of the host country (see proposition 1) and both changes in out-flow and in-flow FDI subsidies have positive effects on both the in-flow and out-flow FDIs (see Proposition 2), it seems to be generally true that while the in-flow FDI subsidy is positive, the out-flow FDI subsidy is negative, in the home and foreign countries. However, this is not always the case. This section discusses such issues.
In the first stage, the home and foreign governments non-cooperatively and simultaneously decide their FDI subsidies, \((s, s_k)\) and \((s^*, s_k^*)\), that maximize their own economic welfare. Adopting the same notation and functions as used in the previous sections, we define the home social welfare, \(W\), as the following:

\[
W = \left\{ \int_0^f p(\theta)d\theta - p(Z)Z \right\} + \Pi - sk - s_k k^*,
\]

(15)

where the first term, \(\left\{ \int_0^f p(\theta)d\theta - p(Z)Z \right\}\), is the home consumer's surplus, the second term, \(\Pi\), is the home firm's profit, the third term, \(sk\), is the home subsidy payment for out-flow FDI, and the last term, \(s_k k^*\), is the home subsidy payment for in-flow FDI. Thus, the home government chooses the home out-flow and in-flow subsidies, \(s\) and \(s_k\), so as to maximize the home social welfare defined by (15). Of course, negative out-flow (in-flow) FDI subsidy \(s(s_k)\) means out-flow (in-flow) FDI tax.

Taking into consideration that (3)-(10) hold at the equilibrium in the third stage and that (11)-(14) hold at the equilibrium in the second stage, the first-order conditions for the home welfare maximization are given by:

\[
-\frac{\partial k}{\partial s} s - \frac{\partial k^*}{\partial s} s_k - \frac{\partial k}{\partial s} A + \frac{\partial k^*}{\partial s} (B + H) = 0,
\]

(16)

and

\[
-\frac{\partial k}{\partial s_k} s - s_k \frac{\partial k^*}{\partial s_k} - k^* - \frac{\partial k}{\partial s_k} A + \frac{\partial k^*}{\partial s_k} (B + H) = 0,
\]

(17)

where \(A = p'(Z)Z \frac{\partial Z}{\partial k} > 0\), \(B = \left\{ p''(Z^*) (T + Y) \frac{\partial Y^*}{\partial k} - p'(Z)Z \frac{\partial Z}{\partial k^*} \right\} > 0\) and \(H = p'(Z)X \left( \frac{\partial Y^*}{\partial k^*} + \frac{\partial X^*}{\partial k^*} \right) < 0\). To avoid tedious yet inessential arguments, it is also assumed here that the second-order conditions hold in the neighborhood of the equilibrium. Then, denoting
the optimal home out-flow and in-flow subsidies by \( s^* \) and \( s_k^* \) that simultaneously satisfy
the first-order conditions of (16) and (17), and considering (9) and (10), we get:
\[
  s^* = k^* \frac{\partial k^*}{\partial s} \Omega - A, \quad \text{and} \quad s_k^* = \left( B + H - k^* \frac{\partial k^*}{\partial s} \Omega \right).
\] (18)
Since \( k^* \frac{\partial k^*}{\partial s} \Omega \) and \( A \) (\( B \) and \( H - k^* \frac{\partial k^*}{\partial s} \Omega \)) have opposite signs each other and these
signs depend on all slopes of the firms' reaction curves in the second and third stages, it is
impossible to judge definitely the signs of the optimal FDI subsidies, \( s^* \) and \( s_k^* \), of the home
government in general. Therefore, we judge them by dividing the situation into cases that
depend on the slopes of the FDI reaction curves in the second stage and cases that depend on
the slopes of the output and export reaction curves in the third stage. However, since it is
very tedious and unnecessary to discuss all the cases, we examine four typical cases that
depend on the slopes of the FDI reaction curves in the second stage: (1) \( \frac{\partial k}{\partial s} \) and \( \frac{\partial k^*}{\partial s} \) are
both large, (2) \( \frac{\partial k}{\partial s} \) and \( \frac{\partial k^*}{\partial s} \) are both small, (3) \( \frac{\partial k}{\partial s} \) is small and \( \frac{\partial k^*}{\partial s} \) is large, and (4) \( \frac{\partial k}{\partial s} \) is
large and \( \frac{\partial k^*}{\partial s} \) is small.

Case 1 corresponds to the case in which the marginal profit of the foreign firm with
respect to the foreign FDI is very sensitive to both the home and foreign FDIs, that is,
\( |\Pi_{kk}^*| \rightarrow \infty \) and \( |\Pi_{k*k}^*| \rightarrow \infty \). Hence, considering (13) and (18), the effects of a change in
the home out-flow subsidy, \( s \), on the home and foreign FDIs, \( k \) and \( k^* \), are both large
enough to give \( k^* \frac{\partial k}{\partial s} > (B + H)\Omega \) and \( k^* \frac{\partial k^*}{\partial s} > A\Omega \), which in turn implies \( s^* > 0 \) and \( s_k^* < 0 \). Case 2 is the opposite case of case 1, and \( \frac{\partial k}{\partial s} \approx 0 \) and \( \frac{\partial k^*}{\partial s} \approx 0 \) hold. Thus,
substituting these results into (18), we obtain \( s^* < 0 \) but the sign of \( s_k^* \) is ambiguous. In case
3, while marginal profit of the foreign firm with respect to the foreign FDI is very sensitive to
the home FDI, it is not sensitive to the foreign FDI, that is, $|\Pi_{k^*}^\prime| \approx \infty$ and $|\Pi_{k^*}^\prime| \approx 0$, which entails $\frac{\partial k}{\partial s} \approx \infty$ and $\frac{\partial k^*}{\partial s} \approx 0$. Therefore, considering this result and (18), one gets $s^* > 0$ but the sign of $s^*_k$ is ambiguous. Case 4 is the opposite case of case 3. Then, $\frac{\partial k}{\partial s} \approx 0$ and $\frac{\partial k^*}{\partial s} \approx \infty$ hold. Consequently, taking into consideration these results and (18) one gets definitely $s^* < 0$ and $0 < s^*_k$.

Since the same reasoning as is employed in obtaining (18) is also applied to the optimal foreign FDI subsidies, the above arguments yield the next proposition:

**Proposition 3:** (i) While the optimal home (foreign) out-flow FDI subsidy is positive in the case in which a marginal profit of the foreign (home) firm with respect to foreign (home) FDI is very sensitive to home (foreign) FDI, it is negative in the opposite case. (ii) The optimal home (foreign) in-flow FDI subsidy is negative in the case where a marginal profit of the foreign (home) firm with respect to foreign (home) FDI is very sensitive to foreign (home) FDI but unresponsive to home (foreign) FDI. However, it is ambiguous in the case where a marginal profit of the foreign (home) firm with respect to foreign (home) FDI is not sensitive to foreign (home) FDI.

It is widely believed that while the optimal out-flow FDI subsidy is negative, the in-flow FDI subsidy is positive, since negative out-flow FDI subsidy prevents out-flow FDI and positive in-flow FDI subsidy promotes in-flow FDI (see Proposition 2). However, Proposition 3 demonstrates that such a belief is not always true. In the actual world, some countries that are unable to pay a positive subsidy by cash offer other avenues for in-flow FDI such as no
profit taxes and/or use of land with no lent for preparation years, but such political
instruments are not optimal in some cases. The governments must pay careful attention to
the circumstances they face when determining the optimal FDI subsidies.

Establishing a model that is similar to the present model except for the assumptions of
constant parent plants (the home and foreign ADI levels are both zero) and of no capital
constraints, Ishii (2003) found that, while the signs of the optimal home and foreign in-flow
FDI subsidies are ambiguous, the optimal home and foreign out-flow FDI subsidies are
always negative, since we get \( \frac{\partial k^*}{\partial s} = \frac{\partial x^*}{\partial k^*} = 0 \) in such the case. Hence, it seems that Ishii's
proposition is a special case of Proposition 3. However, it is impossible to say that the optimal
FDI subsidies presented in this paper are always more appropriate those proposed in Ishii's
paper. It depends on which model is a more appropriate match to the actual economy. Hence,
the governments must select those policies derived from the model that have greater real-
world validity when adopting FDI subsidies.

Proposition 3, when taken together with Propositions 1 and 2, indicates that FDI
subsidies may be used as strategic policies if they are used carefully. Propositions 1 and 2
combine to show that the effects of changes in the FDI on firms' output-exports and the
effects of changes in the FDI subsidies on firms' FDI are all definitely determined, though
Proposition 3 states that the signs of the optimal FDI subsidies are not judged definitely.
Therefore, the governments can use FDI subsidies as a means to implement strategic policies,
provided that they pay careful attention to the way in which they are used.

6. Concluding Remarks

Establishing a generalized model of an international Cournot industry that consists of the
home and foreign firms that choose endogenously both the locations and sizes of FDI (and ADI) under capital constraints, we first investigated the effects of FDI subsidies on firms' output, export and FDI choices and then discussed the optimal FDI subsidies. As a result, we have obtained some interesting results. However, since they are summarized as propositions in previous sections, we do not repeat them here, and refer only to some of their general characteristic features.

We found that the governments can use FDI subsidies to implement strategic policies, provided that careful attention is paid to prevailing real-world conditions. Even if the governments cannot guess the optimal levels of FDI subsidies from a theoretical point of view (Proposition 3), they can gather all information necessary for determining the optimal FDI subsidies in the real world, according to the theoretical suggestions presented in this paper. Since it is certain what the effects of changes in the FDIIs on firms' output-export choices and the effects of changes in the FDI subsidies on firms' FDI decisions will be (Proposition 2), the governments can effectively manage the in-flow and out-flow FDI levels by combining the in-flow and out-flow FDI subsidies.

Furthermore, it could be shown, though it is not discussed explicitly in the present paper, that the FDI subsidies do not have such drastic effects on the firms' plant choices as are indicated by Markusen, et al. (1993 and 1995). In this paper, since the firms can endogenously determine the plant sizes in the second stage, the costs for constructing their plants are also control variables though these are fixed when the firms decide their output-export levels in the third stage. Therefore, the firms can vary a certain fraction of their plant construction costs by changing their plant sizes when the governments change their FDI subsidies. In such a case, the firms would adjust their plant sizes smoothly before they drastically open or close the plants of constant sizes.
Finally, the following point from Proposition 3 should be emphasized. It seems quite natural that the governments should be responsible for controlling FDI flows in order to improve their own economic welfare. Then, the governments must intervene in all the firms' FDI choices from the standpoint of effective resource allocation, and thus the out-flow FDI tax and the in-flow FDI subsidy are regarded as inevitable policies. As a result, there is a possibility that the international economy would drift away from a free economy, contrary to the findings of Janeba (1998) and Ishii (2001) that the world economy approaches to a free trade economy when firms have their subsidiary plants in their rival's country.

Of course, the present model cannot explain all aspects of the multi-nationals' trade-FDI choices and the government FDI subsidies, and it is clearly irrelevant to some industries or some policies. In order to investigate other aspects appropriately, it is necessary to extend the model so as to include such aspects correctly. The present model assumes homogenous goods, fixed sizes of parent plants, and a three-stage game of a Cournot duopoly. However, when examining the Beltrand duopoly for example, an assumption of heterogeneous goods would be much more plausible. Moreover, this paper excludes other types of political instruments that may have some effects on firms' FDI flows. It would be interesting and useful to compare the effectiveness of the FDI subsidies with those of some other policies.
Appendices

Appendix 1.

Here we examine the theoretical plausibility of the assumption that in a homogeneous good model the subsidiaries supply their products only to the countries where they are located.

Suppose that the home subsidiary also supplies its products to the foreign and home countries. Then the profit of the home multinational firm defined by (1) is replaced by

$$\Pi = p(X + T^* + Y^* + y)(X + y) + p^*(X^* + T + Y + y^*)(T + Y) - C(X + T) - G(T)$$

$$- c(k)(y + y) - v(k) - h(y) + (s + s^*; k) \tag{1}$$

where $y$ is the home subsidiary's exports, $h(y)$ is its export cost function with $h'(y) > 0$, and other notations and functions are all the same as in section 2. In this case, the control variables of the home multinational firm are $X$, $T$, $Y$ and $y$ in (7). Thus, assuming an inner equilibrium, the first-order condition of the home multinational firm is given by

$$p'(Z)(X + y) + p(Z) - C = 0, \tag{i}$$

$$p'^*(Z)(T + Y) + p^*(Z^*) - G'(T) - C = 0, \tag{ii}$$

$$p'^*(Z)(T + Y) + p^*(Z^*) - c(k) = 0, \tag{iii}$$

$$p'(Z)(X + y) + p(Z) - h'(y) - c(k) = 0. \tag{iv}$$

where the second-order condition is assumed to be satisfied, for simplification. Thus, the first-order condition given by (i)-(iv) presents:

$$g'(T) + h'(y) = 0. \tag{A.1}$$

However, this is inconsistent with the positive marginal export costs. This implies, from a theoretical point of view, that there is no possibility that the home subsidiary supplies its products to both of the foreign and home countries in a homogeneous good model. Therefore,
considering the theoretical reasoning from (A.1) and the empirical observations of some multinational firms in the real world (see footnote 4), it is reasonable to assume, for present purposes, that the home subsidiary sells its products only in the foreign country but does not export back them to the home country. The same reasoning applies to the foreign multinational firm.

Appendix 2.

From (4) and (5), one gets, as a condition under which exports of the home multinational firm is positive,

\[ G'(T) + C\left(\bar{K} - k\right) = c(k). \]  \hspace{1cm} (A.2)

This shows that the home multinational firm chooses its exports \( T \) so as to equate the (effective) unit export cost, \( G'(T) + C\left(\bar{K} - k\right) \), to the unit production cost of its subsidiary, \( c(k) \). One might indicate that since \( C\left(\bar{K} - k\right) \) and \( c(k) \) are both given to the home multinational firm in the third stage, \( T \) becomes zero (a corner solution) when \( C\left(\bar{K} - k\right) \) exceeds \( c(k) \). However, while \( C\left(\bar{K} - k\right) \) and \( c(k) \) are parameters in the third stage, they are not parameters but control variables in the second stage. Therefore, the home multinational firm that plans to supply its products to the foreign country via the two routes of exporting and overseas production decides \( C\left(\bar{K} - k\right) \) and \( c(k) \) in the second stage so as to choose positive exports in the third stage. The same reasoning applies to the foreign multinational firm.

Appendix 3.

Here, we will show only the derivation of (10), since that of (9) is obtained by exchanging variables with and without asterisks. The effects of changes in \( k \) and \( k^* \) on \( X, T^* \) and
\( Y^* \) are respectively obtained by taking the total differential of (7), (8) and (3):

\[
\begin{pmatrix}
\Delta_{11} & \Delta_{12} & \Delta_{13} \\
\Delta_{21} & \Delta_{22} & \Delta_{23} \\
\Delta_{31} & \Delta_{32} & \Delta_{33}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial T^*}{\partial k} \\
\frac{\partial T^*}{\partial k^*} \\
\frac{\partial X}{\partial k}
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
-\frac{\partial k^*}{\partial k} - \frac{\partial k^*}{\partial k^*}
\end{pmatrix},
\]  
(A.3)

and

\[
\begin{pmatrix}
\Delta_{11} & \Delta_{12} & \Delta_{13} \\
\Delta_{21} & \Delta_{22} & \Delta_{23} \\
\Delta_{31} & \Delta_{32} & \Delta_{33}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial T^*}{\partial k^*} \\
\frac{\partial Y^*}{\partial k^*} \\
\frac{\partial X}{\partial k^*}
\end{pmatrix}
= \begin{pmatrix}
-\frac{\partial k^*}{\partial k^*} \\
\frac{\partial k^*}{\partial (k^*)} \\
0
\end{pmatrix},
\]  
(A.4)

where \( \Delta_{11} = p^*(Z)\left(T^* + Y^*\right) + 2p'Z - G^*(T^*) \), \( \Delta_{12} = \Delta_{21} = \Delta_{22} = p^*(Z)\left(T^* + Y^*\right) + 2p'Z \), \( \Delta_{13} = \Delta_{23} = p^*(Z)\left(T^* + Y^*\right) + p'Z \), \( \Delta_{31} = \Delta_{32} = p^*(Z)X + p'Z \), \( \Delta_{33} = p^*(Z)X + 2p'Z \) and \( \Delta = -G^*(T^*)p'Z\left(p^*(Z) + 3p'(Z)\right) < 0 \). Thus, while we can obtain \( \frac{\partial T^*}{\partial k}, \frac{\partial Y^*}{\partial k^*} \) and \( \frac{\partial X}{\partial k} \) from (A.3), we can get \( \frac{\partial T^*}{\partial k^*}, \frac{\partial Y^*}{\partial k^*} \) and \( \frac{\partial X}{\partial k^*} \) from (A.4). Similarly, from (4), (5) and (9), we can derive \( \frac{\partial T}{\partial k}, \frac{\partial T}{\partial k^*}, \frac{\partial Y}{\partial k}, \frac{\partial Y}{\partial k^*}, \frac{\partial X}{\partial k}, \frac{\partial X}{\partial k^*} \) and \( \frac{\partial X^*}{\partial k} \).
Footnotes

1. As is well known, not only FDI subsidies, but also trade policies such as tariffs, export subsidies and voluntary export restraints, and even corporate taxes and consumption taxes, affect the optimal FDI levels chosen by multinational firms. However, since political instruments, apart from the FDI subsidies, have their own purposes it is rare that they are used to control the FDI levels directly. Furthermore, the effects of these policies on the FDIs have been already analyzed in many papers including Brander and Spencer (1987), Flamm and Reiss (1993), Hillman and Ursprung (1993), Konishi, Saggi and Weber (1999) and Williamson (1986). Therefore, this paper concentrates on the effects of FDI subsidies on the firms' optimal FDI choices.

2. Markusen, Edward and Oleswiler (1993) established a model that considers the firms' endogenous plant locations when analyzing the optimal environmental pollution policies. However, they implicitly assumed that firms are 'footloose', as indicated by Motta and Thisse (1994), and that firms' plant sizes are exogenously given. However, in analyzing multinationals that already have parent plants in their own countries and intend to construct subsidiary plants in other countries, the above assumptions are not plausible.

3. The present model also depends on Motta and Thisse (1994), Brander and Spencer (1987) and Hoel (1997). This paper assumes, following Motta and Thisse, that both of the home and foreign firms already have parent plants in their own countries when the game begins.

4. For example, while the Toyota automobile company in Japan supplies its products to Japan and the U.S.A., Toyota U.S.A. sells almost all of its products in the U.S.A. Further, some researches have found that many parent firms export their products to the countries in which their subsidiaries are constructed (see Belderbos and Sleuwaegen (1996), Blonigen
(2001) and Baldwin and Ottaviano (2001), for example. With respect to the theoretical explanation, see Appendix 1.

5. The strictly increasing and convex plant construction cost functions and the constant unit production costs both hold when the production functions of production facilities are strictly increasing and concave with respect to capital stocks (= plant scales) and homogenous to degree one with respect to other factors except for capital stocks, respectively.

6. Though detailed discussions are omitted here, it is easily shown that the increasing marginal export costs, \( G'(T) > 0 \) and \( G''(T^*) > 0 \), are necessary in the neighborhood of the equilibrium in order to ensure both of the second-order conditions for maximizing home and foreign firms' profits and the stability conditions of the industry equilibrium in the second stage (see the Routh theorem).

7. When the subsidiary plant sizes are both exogenously fixed, the present model reduces to that exploited by Ishii (2001).

8. With respect to the plausibility assuming the inner solution, see Appendix 2.

9. The second-order condition for maximizing profit of the home (foreign) multinational firm is that the Hessian \( H \) (\( H^* \)) is negative definite in the neighborhood of the equilibrium, where, adopting notations of \( D_{11} = p'(Z)X + 2p'(Z), \ D_{22} = p''(Z^*)(T+Y) + 2p''(Z^*) \)

\[-G'(T) and \quad D_{23} = D_{32} = D_{33} = p''(Z^*)(T+Y) + 2p''(Z^*) \]

\[
H = \begin{pmatrix}
D_{11} & 0 & 0 \\
0 & D_{22} & D_{32} \\
0 & D_{32} & D_{33}
\end{pmatrix}
\]

And, \( H^* \) is given by exchanging notations with * and those without * in H.
10. In (11), if the reasonable assumptions of \( \lim_{k \to 0} v'(k) = 0 \) and \( \lim_{K \to 0} v'(K) = 0 \) are additionally adopted, then a corner solution of \( k = 0 \) or a corner solution of \( K = 0 \) is excluded. Since the same reasoning is applied to (12), a corner solution of \( k^* = 0 \) or a corner solution of \( K^* = 0 \) is also excluded. This paper analyzes the inner solution case because the corner solution case is trivial.
References


