The Optimal Ownership Share of A Multinational Firm in An International Cournot Duopoly

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Abstract

A simple three-stage game model of an internationally asymmetric Cournot duopoly consisting of a multinational firm (the home firm) and a national firm (the foreign firm) is exploited to examine the optimal ownership share of the multinational firm and its economic implications. Thus, this paper finds, as the main proposition, that the optimal ownership share of the multinational firm is always equal to unity (less than unity) when its parent firm and its subsidiary act cooperatively (non-cooperatively). It also shows that the optimal ownership share chosen by the home multinational firm equals the socially optimal ownership share, and thus the government intervention is unnecessary.

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1. Introduction

Nowadays multi-nationalization of firms is very common in many industries. There exist many reasons why firms become multinational. Some firms establish subsidiaries to secure more stable material supplies and/or to employ cheaper inputs including labors, and others conduct overseas production to exploit better markets and/or to mitigate trade conflicts. In any cases it is quite natural that multinational firms would face several issues that are much tougher than do national firms, since the formers consist of parent firms and subsidiaries located in different countries with different economic conditions, respectively.

One of such tougher issues to be solved by the multinational firm is the decision as to the ownership share at which the parent firm owns its subsidiary (say the ownership share decision, henceforth). This is because the ownership share has simultaneously multiple relations with the firm's other decisions. The ownership share is not only the repatriation ratio of the subsidiary's profit but also the allotment share of its cost. The home parent firm must weigh the benefit of a larger repatriation of subsidiary's profit resulting from a larger ownership share, against the disadvantage of having to pay higher costs for its subsidiary.

Furthermore, the ownership share has a bearing on subsidiary's independence in its decision making. It is widely known that while a parent firm with a unity of ownership share can perfectly manage its subsidiary, a parent firm with a less-than-unity ownership share can have only partial control over its subsidiary's management. Therefore, it is quite natural that the ownership share is one of the most significant endogenous control variables to the multinational firm.

Indeed, various levels of ownership share will be observed among multinational firms in these days. Some parent firms totally own their subsidiaries, while others partially own them. Why are their ownership shares different from firm to firm? In fact, are there any close relations between the parent firm's ownership share and the subsidiary's independence? These are essential problems to be addressed for a proper understanding of characteristic features of multinational. With this in mind, it is necessary and useful to establish a model of the multinational firm that determines endogenously the ownership share as one of its own control variables. But, as far as I know, there is not such a model so far[1]. This paper will present a model that considers ownership share as one of the control variables of the multinational firm and will examine the optimal ownership share of the multinational firm.

As for the reasons why the home firm builds its subsidiary in the foreign country and conducts overseas production, this paper will explicitly consider the lower production costs in the foreign country compared to those in the home country, the higher export costs compared to the production costs in the foreign country, and the possibility of trade conflicts between the home and foreign countries. In this paper, the first of the above is captured by a difference between the production costs of the home parent firm and subsidiary, and the latter two are reflected in a difference between the export costs of the home parent firm and the production costs of the home subsidiary.

When the home firm decides how many goods it produces and exports (say the output-export decision, henceforth) it is very significant for the home firm to have determined whether the home parent firm behaves cooperatively or non-cooperatively with its subsidiary (say the cooperation decision, henceforth). This is why when the home parent firm acts cooperatively (non-cooperatively) with its subsidiary, the parent firm and its subsidiary cooperatively (independently) choose their outputs and sales so as to maximize their joint (own) profits, and thus economic results are quite different from each other.

It is also clear that the cooperation decision is related to the independence of the subsidiary. When the parent firm and its subsidiary act cooperatively they are regarded as a multinational firm having a single headquarters. Then, the subsidiary would be subordinate to the instructions of its parent firm. On the other hand, when the parent firm and its subsidiary behave non-cooperatively they are taken as independent firms with separate headquarters, respectively. In that case, the subsidiary can determine its own output, without instructions from its parent firm. Therefore, the home parent firm will consider the cooperation decision as significant as the ownership share decision. This paper will also investigate relations between these decisions.

In order to investigate the issues mentioned above, this paper will extend the reciprocal trade two-stage game model of an international Cournot duopoly exploited by Brander and Krugman [1983] and Brander and Spencer [1985] (say B-K-S model, henceforth) and establish a simple three-stage game model of an internationally asymmetric Cournot duopoly consisting of a multinational firm (the home firm) and a national firm (the foreign firm). The reasons why this paper extends the B-K-S model are that such a model is better than any other types of models for analyzing imperfectly competitive multinational firms in the present days and that it is easily extended to discuss multinational firms in a perfectly competitive market.

Though the preset model depends on the B-K-S model, there exist some essential differences between two' models. First, in the B-K-S model, both the home and foreign firms consist of merely the parent firms that produce homogenous goods only in their own countries and supply these goods to the both countries, respectively. By contrary, in the present model, while the foreign firm has only its parent firm in the foreign country, the home firm constructs its subsidiary in the foreign to produce and sale homogenous goods there, in addition to the parent firm in the home country. Second, while the B-K-S model is a two-stage game model, the present model depends on a three-stage game model. This is why the home firm in the present model faces the decisions to be solved in more multiple stages. Furthermore, trade policies such as import tariffs, export subsidies, and so on, are not considered in the present model since they are not essential to the ownership share discussion.

As is immediately clear from the above statements, both the home and foreign firms in this paper have already the parent firms in their own countries when the game begins, and then only the home firm becomes a multinational firm by building its subsidiary in the foreign firm. Therefore, the industry in question is an internationally asymmetric duopoly that consists of the home firm with a subsidiary and the foreign firm with no subsidiary. This delimitation of the scope of the analysis, while it may seem to be unduly strict at first sight, is not so in reality, when it is recognized that such asymmetric industries are just as popular as symmetric ones in the real world (for example, the US Toyota sales almost all its products in the USA).

The rest of this paper is organized as follows. Section 2 will establish a two-stage game model of an internationally asymmetric Cournot duopoly consisting of a home multinational firm and a foreign national firm. The home firm faces not only the output-export decision but also the ownership share decision and the cooperation decision, while the foreign firm makes merely the output-export decision. Then, using this model, sections

3 and 4 will first examine the optimal output-export decision, the optimal ownership share decision and the optimal cooperation decision, and then investigate the relationships between these optimal decisions. As a result, it will show that when the home parent firm acts cooperatively (non-cooperatively) with the home subsidiary the optimal ownership share of the home firm is always equal to (less than) unity and that if the industry is perfectly competitive, the optimal ownership share of the home multinational firm is always unity.

Moreover, section 5 will examine the socially optimal ownership share which maximizes the home welfare and demonstrate that while the socially optimal ownership share is generally equal to the optimal ownership share chosen by the home firm, the two differ from each other if the firm's production causes external effects such as environmental damages and the home firm is a non-cooperative multinational firm. Finally, concluding remarks will be presented in section 6.

2. Assumptions and Basic Model

This section will establish a very simple model of an internationally asymmetric Cournot duopoly that consists of a home multinational firm with a subsidiary and a foreign national firm with no subsidiary. In order to focus on the optimal ownership share choice of a multinational firm, some other issues, such as transfer pricing, technological transfer or licensing and tax heavens, will be omitted. Furthermore, several assumptions will be adopted to simplify analysis, while taking care to ensure that the essence of the paper is retained.

Suppose that the home and foreign firms initially produce a homogeneous good in their own countries and supply these products to the home and foreign countries, and then only the home firm becomes a multinational firm, with a subsidiary that produces the same good in the foreign country. Consequently, while the foreign firm is still a national firm having only a parent firm in the foreign country, the home firm is a multinational firm having a parent firm in the home country and a subsidiary in the foreign country. It is assumed that though both these parent firms supply their products to both the home and foreign countries, while the home subsidiary supplies its products only to the foreign country.

As a matter of fact, the home firm faces several kinds of decisions until it becomes a multinational firm and produces goods. The first is whether the home firm should be a multinational firm through constructing its subsidiary in the foreign country (the subsidiary construction decision). The second is what share of ownership the home parent should have in its subsidiary (the ownership share decision). The third is whether the home parent should act cooperatively or non-cooperatively with its subsidiary (the cooperation decision). And, finally the home firm must decide how the home parent and subsidiary should choose output and export levels (the output-export decision). This paper, however, will assume that the home firm has already made the first decision and discuss the latter three decisions. Then, the home firm faces the cooperation decision, the ownership share decision, and the output-export decision.

This paper will illustrate such an internationally asymmetric duopoly as a model of the three-stage game incorporating the sub-game perfect equilibrium. That is, only the home firm makes the cooperation decision in the first stage and the ownership share decision in the second stage. And, in the third stage, the home and foreign firms simultaneously choose their optimal output and export levels, given the cooperation and ownership share decisions of the home firm in the first stage and the rival's output-export decisions, respectively. In order to solve these decisions, this paper will adopt a procedure of backward induction.

In the third stage, the internationally duopolistic industry consists of three types of firms, i.e., the home parent firm, the home subsidiary, and the foreign firm, which compete as Cournot quantity-setters in home and foreign markets that are assumed to be segregated from each other. While the home parent firm and the foreign firm produce respectively a homogenous good in their own countries and supply their products to both the home and foreign markets, the home subsidiary produces the same goods in the foreign country and supplies all of its products only to the foreign market. Thus, the inverse demand functions of the home and foreign markets are respectively given by $p(X) = p(X_1+X_2)$ with $p'(X)<0$ and $p^*(X^*) = p^*(X_1^*+X_2^*+X_f^*)$ with $p^{*\prime}(X^*)<0$, where p (p^*) is the home (foreign) price, X (X^*) is total sales in the home (foreign) country, X_1 (X_2^*) is domestic sales of the home (foreign) parent firm, X_1^* (X_2) is exports of the home (foreign) parent firm, and X_f^* is output (= sales) of the home subsidiary in the foreign country.

Since differences in production and export costs of the home and foreign firms are regarded as central reasons why only the home firm has its subsidiary in the foreign country, this paper will explicitly introduce not only production costs but also export costs into the model. However, it will not here examine the conditions under which only the home firm becomes a multinational firm, but will assume that such conditions are satisfied, in order to avoid rather tedious but unnecessary calculations. Therefore, it is assumed that, under such conditions, the production costs of the home parent firm, the foreign firm and the home subsidiary are respectively specified by $c_1(X_1+X_1^*)$, $c_2\left(X_2+X_2^*\right)$ and $c_f X_f^*$, where c_1, c_2 and c_f are respectively unit costs for producing homogeneous goods that are all assumed to be positive constants[2].

Furthermore, though neither the home parent nor the foreign firm incurs any plant construction costs, since their production plants have already been constructed, the home subsidiary must additionally incur a plant construction cost C_F since it is newly constructed when the home firm becomes a multinational firm[3]. This is one of the most significant factors affecting the subsidiary construction decision of the home firm when it plans to be a multinational firm. In practice, the lower plant construction cost would increase the firm's incentive to commit to greater overseas production and/or to build a bigger subsidiary. However, since this paper focuses on the optimal ownership share choice of the home firm, the subsidiary's plant size is regarded as given. Then, the plant construction cost C_F is also regarded as positive and constant.

On the other hand, the export cost function of the home parent and foreign firms are respectively given by $g_1(X_1^*)$ and $g_2(X_2)$ whose marginal costs are strictly positive and increasing, i.e., $g_1'\left(X_1^*\right)>0$, $g_1''\left(X_1^*\right)>0$, $g_2'\left(X_2\right)>0$ and $g_2''\left(X_2\right)>0$. Though detail explanations are omitted in order to save space, the increasing marginal export costs, i.e., $g_1''\left(X_1^*\right)>0$ and $g_2''\left(X_2\right)>0$, will ensure all of the second-order conditions for maximizing home and foreign firms' profits and the stability conditions of the industry equilibrium in the second stage[4].

Now, under the presumptions mentioned above, profit Π_1 of the home parent firm is defined as the sum of the profit earned by the home plant and the profit repatriated from its subsidiary. Therefore, the profit of the home parent firm is

$$\Pi_1 = \{p(X_1+X_2)X_1 + p^*(X_1^*+X_2^*+X_f^*)X_1^* - c_1(X_1+X_1^*) - g_1(X_1^*)\}$$

$$+ \alpha\{p^*(X_1^*+X_2^*+X_f^*)X_f^* - c_f X_f^* - C_F\}. \quad (1)$$

In the definition of home parent profit of (1), $\{p(X_1+X_2)X_1 + p^*(X_1^*+X_2^*+X_f^*)X_1^* - c_1(X_1+X_1^*) - g_1(X_1^*)\}$ is the profit earned by the home plant, $\{p^*(X_1^*+X_2^*+X_f^*)X_f^* - c_f X_f^* - C_F\}$ is the profit of the subsidiary's plant, and α denotes the ownership share at which the home parent owns its subsidiary.

Hence, the second term expressed as $\alpha\{p^*(X_1^*+X_2^*+X_f^*)X_f^* - c_f X_f^* - C_F\}$ is a fraction of the home subsidiary's profit, which the home subsidiary repatriates to the home parent firm. However, considering that $\alpha\{c_f X_f^* + C_F\}$ represents a fraction of the subsidiary's total costs, it is obvious that while the home parent firm can obtain a part of its subsidiary's profit, it must incur a part of its subsidiary's cost at a ratio equal to its ownership share. This is the reason why the home parent firm ambivalently worries about what ratio of ownership share to choose.

Furthermore, the profit of the home parent firm is rewritten as

$$\Pi_1 = p(X_1+X_2)X_1 + p^*(X_1^*+X_2^*+X_f^*)(X_1^* + \alpha X_f^*)$$

$$- c_1(X_1 + X_1^*) - g_1(X_1^*) - \alpha c_f X_f^* - \alpha C_F. \quad (1)'$$

By considering this expression of (1)', it is easy to understand that the home subsidiary transfers a part of its output to its parent firm at a lower marginal cost (which is regarded as the transfer price) than the market price (for more exact arguments, see (4c) for example). This being so, the present model may also be regarded as including the transfer pricing problem of the multinational firm.

The sufficient condition under which the home firm becomes a multinational firm is that its parent's profit Π_1 is larger than its profit obtained in the case where the home firm remains as a national firm. Without a subsidiary, the home firm's profit Π_1^N is defined as

$$\Pi_1^N = p(X_1+X_2)X_1 + p^*(X_1^*+X_2^*)X_1^* - c_1(X_1+X_1^*) - g_1(X_1^*).$$

Hence, the decision whether the home firm builds its subsidiary in the foreign country or not depends on whether Π_1 is larger than Π_1^N at equilibrium. In the case when the former is bigger than the latter, the home firm builds its subsidiary in the foreign country to be a multinational firm, but in the opposite case the home firm chooses not to be a multinational firm. Since this paper assumes that the home firm has already determined to be a multinational firm, it is regarded that $\Pi_1^N < \Pi_1$ holds at equilibrium. Consequently, under this condition, the home firm will choose the optimal ownership share α^e from the range of

$$0 < \alpha \leq 1. \tag{2}$$

The post-repatriation profit of the home subsidiary is

$$\Pi_f = (1 - \alpha)\{p^*(X_1^*+X_2^*+X_f^*)X_f^* - c_f X_f^* - C_F\}, \tag{3}$$

where $(1 - \alpha)$ is the share of the profit that the home subsidiary can retain by running its own plant. When $0 < \alpha \leq 1$ holds, the necessary condition under which the home firm chooses to be a multinational firm is that the post-repatriation profit of the subsidiary (denoted by Π_f^e, henceforth) is positive at equilibrium: $\Pi_f^e > 0$, or the pre-repatriation profit of the subsidiary is positive at equilibrium: $\Pi_f^e/(1 - \alpha) > 0$.

On the other hand, the profit of the foreign firm is defined as

$$\Pi_2 = p(X_1+X_2)X_2 + p^*(X_1^*+X_2^*+X_f^*)X_2^* - c_2(X_2+X_2^*) - g_2(X_2). \tag{4}$$

Clearly, the foreign firm's profit consists only of the profit of its parent plant. Therefore, in the second stage the foreign firm chooses its output $(X_2^*+X_2)$ and its export X_2 so as to maximize its own profit Π_2 defined as (4), given all of output-export levels chosen by the home firm and other parameters.

3. Equilibrium Output and Export Levels in the Third Stage

In the third stage, when the home and foreign firms respectively choose their optimal output-export levels, they have quite different plant-production structures. While the foreign firm has only a parent firm, the home firm has a parent firm and a subsidiary. Therefore, though the foreign firm can just maximize profit defined as (4), the home firm cannot merely maximize profit given by (1) and/or (3). This is why the maximization problem of the home firm depends on whether the home firm is a cooperative multinational firm whose home parent firm and subsidiary act cooperatively, or is a non-cooperative multinational firm whose home parent firm and subsidiary behave non-cooperatively.

Since the home firm's cooperation decision and ownership share decision are respectively determined in the first stage and in the second stage, both the home and foreign firms act, given these decisions of the home firm, in the second stage when they choose the optimal output-export levels. It is here assumed that when the home firm is a cooperative (non-cooperative) multinational firm, its parent firm and subsidiary choose output-export levels cooperatively (non-cooperatively) so as to maximize the sum of their profits (their own respective profits) in the second stage. Therefore, this paper will discuss separately these cases in this section.

3.1. The Non-cooperative Case

Following the definition presented above, when the home firm is a non-cooperative multinational firm, the home parent firm chooses X_1 and X_1^* so as to maximize Π_1 in (1), given other variables, and the home subsidiary determines X_f^* so as to maximize Π_f in (4), given other variables, since they are both Cournot quantity-setters in the home and foreign markets. Hence, the first-order conditions for maximizing the home parent firm's profit are given by

$$p'(X)X_1 + p(X) - c_1 = 0, \tag{5a}$$

$$p^{*\prime}(X^*)(X_1^* + \alpha X_f^*) + p^*(X^*) - g_1'(X_1^*) - c_1 = 0. \tag{5b}$$

And, the first-order condition for maximizing the home subsidiary's profit is

$$p^{*\prime}(X^*)X_f^* + p^*(X^*) - c_f = 0. \tag{5c}$$

Furthermore, since the foreign firm decides X_2 and X_2^* so as to maximize its profit Π_2 in (4), given other variables, the first-order conditions for maximizing the foreign firm's profit are

$$p^{*\prime}(X^*)X_2^* + p^*(X^*) - c_2 = 0, \tag{6a}$$

$$p'(X)X_2 + p(X) - g_2'(X_2) - c_2 = 0. \tag{6b}$$

Since the second-order conditions for maximizing the firms' profits are all satisfied under the conditions mentioned in the previous section, (5a), (5b), (5c), (6a) and (6b) are respectively taken to be the reaction curves of the home parent firm, the home subsidiary and the foreign firm in the output-export decisions of the second stage.

The Cournot-Nash industry equilibrium of the international duopoly in question is given by a pair of X_1, X_2, X_f^*, X_1^* and X_2^* satisfying simultaneously all equations in (5) and (6). However, it is obvious that the equilibrium levels of X_1 and X_2 are obtained by solving (5a) and (6b), while the industry equilibrium levels of X_f^*, X_1^* and X_2^* are given by solving (5b), (5c) and (6b) since the separation feature holds between the former two and the latter three variables at the industry equilibrium.

Since (5a) and (6b) do not include the ownership share α, the equilibrium levels of X_1 and X_2 are both independent of α. Therefore, one gets easily

$$\frac{\partial X_1}{\partial \alpha} = \frac{\partial X_2}{\partial \alpha} = 0. \tag{7a}$$

However, since the ownership share α is included in the equation system defined by (5b), (5c) and (6b), the equilibrium levels of X_f^*, X_1^* and X_2^* depend on the ownership share

α. Then, deriving $\frac{\partial X_1^*}{\partial \alpha}$, $\frac{\partial X_f^*}{\partial \alpha}$ and $\frac{\partial X_2^*}{\partial \alpha}$ from this equation system and considering the features of the inverse demand function and the cost functions, one has (see Appendix):

$$\frac{\partial X_1^*}{\partial \alpha} = \frac{-p^{*\prime}(X^*)X_f^*}{\{(1+\alpha)p^{*\prime}(X^*)-g_1^{\prime}(X_1^*)\}} < 0, \tag{7b}$$

$$\frac{\partial X_f^*}{\partial \alpha} = \frac{p^{*\prime}(X^*)X_f^*}{\{(1+\alpha)p^{*\prime}(X^*)-g_1^{\prime}(X_1^*)\}} > 0, \tag{7c}$$

and

$$\frac{\partial X_2^*}{\partial \alpha} = 0. \tag{7d}$$

Then, these results are summarized as followings:

Proposition 1: In the third-stage equilibrium of the internationally asymmetric duopoly where the home firm is a non-cooperative multinational firm and the foreign firm is a national firm, a rise in the ownership share α reduces the home firm's export X_1^* and increases the home subsidiary's output X_f^*, and *vice versa*. However, it would not affect the home firms' domestic sales X_1, the foreign firms' domestic sales X_2^* and the foreign firm's export X_2.

Furthermore, from (7b), (7c) and (7d) one can easily obtain

$$\frac{\partial X_1^*}{\partial \alpha} + \frac{\partial X_f^*}{\partial \alpha} = 0, \text{ and } \frac{\partial X_1^*}{\partial \alpha} + \frac{\partial X_f^*}{\partial \alpha} + \frac{\partial X_2^*}{\partial \alpha} = 0. \tag{7e}$$

This indicates that though the effects of a change in α on X_1^* and X_f^* have the opposite signs, their absolute values are the same each other. Therefore, a change in α would not affect the home firm's total sales (X_1^* + X_f^*) in the foreign market. Furthermore, it is demonstrated from (7a)-(7e) that the home and foreign firms' domestic sales X_1 and X_2^*, their total outputs $(X_1 + X_1^* + X_f^*)$ and $(X_2^* + X_2)$, and the home and foreign consumption $(X_1 + X_2)$ and $(X_1^* + X_f^* + X_2^*)$ are all independent of a change in α.

3.2. The Cooperative Case

When the home firm is a cooperative multinational firm, the home parent firm and its subsidiary cooperatively choose the parent firm's output X_1, its export X_1^* and the subsidiary's output X_f^* so as to maximize their joint profit $\Pi_1^c = \Pi_1 + \Pi_f$, given other variables included in this joint profit. Substituting Π_1 and Π_f given by (1) and (3) into the right side of $\Pi_1^c = \Pi_1 + \Pi_f$ respectively, the joint profit Π_1^c of the home firm is

$$\Pi_1^c = p(X_1+X_2)X_1 + p^*(X_1^*+X_2^*+X_f^*)X_1^* - c_1(X_1+X_1^*) - g_1(X_1^*)$$
$$+\{p^*(X_1^*+X_2^*+X_f^*)X_f^* - c_f X_f^* - C_F\}. \quad (8)$$

Therefore, since the second-order conditions always hold under the assumptions adopted in this paper, the optimal levels of X_1, X_1^* and X_f^* chosen by the home firm are obtained by solving the following first-order conditions:

$$p'(X)X_1 + p(X) - c_1 = 0, \quad (9a)$$

$$p^{*'}(X^*)(X_1^* + X_f^*) + p^*(X^*) - g_1'(X_1^*) - c_1 = 0, \quad (9b)$$

$$p^{*'}(X^*)X_f^* + p^*(X^*) - c_f = 0. \quad (9c)$$

In this equation system, (9a), (9b) and (9c) are respectively the domestic output reaction function, the export reaction function and the foreign output reaction function of the home firm in the output-export decision of the second stage. Clearly, the ownership share α of the home firm does not appear in these reaction functions. Moreover, since the foreign firm's profit is still given by (3), the first-order conditions of the foreign firm in the second stage are the same as (6a) and (6b). Therefore, the Cournot-Nash equilibrium in the internationally duopolistic industry, consisting of the cooperative (home) multinational firm and the foreign firm, is given by a pair of X_1, X_2, X_f^*, X_1^* and X_2^* satisfying simultaneously all equations in (6) and (9).

As has been already indicated in the non-cooperative case, the separation feature also holds between the former two variables, X_1 and X_2, and the latter three variables, X_f^*, X_1^* and X_2^*, at the industry equilibrium. Hence, while the industry equilibrium levels of X_1 and X_2 are given by solving simultaneously (6b) and (9a), the industry equilibrium levels of X_1^*, X_2^* and X_f^* are obtained by solving (6a), (9b) and (9c) simultaneously. However, it is easily shown that since the equation system of (9) does not include the ownership share α, the equilibrium levels of X_1, X_2, X_1^*, X_2^* and X_f^* are all independent of α:

$$\frac{\partial X_1}{\partial \alpha} = \frac{\partial X_2}{\partial \alpha} = \frac{\partial X_1^*}{\partial \alpha} = \frac{\partial X_2^*}{\partial \alpha} = \frac{\partial X_f^*}{\partial \alpha} = 0. \tag{10}$$

Then, (10) presents:

Proposition 2. When the home firm is a cooperative multinational firm the optimal ownership share α chosen by the home firm in the second-stage would not have any effects on the home and foreign firms' domestic sales X_1 and X_2^*, their exports X_1^* and X_2, and the home subsidiary's output X_f^* in the third stage industry equilibrium.

It is also demonstrated from (10) that a change in α would have no effects on the home firm's total sales (X_1^* + X_f^*) in the foreign market, the home parent total output

$(X_1 + X_1^*)$, the home and foreign firms' total outputs $(X_1 + X_2^* + X_f^*)$ and $(X_1^* + X_2)$, and the home and foreign consumption $(X_1 + X_2)$ and $(X_1^* + X_2^* + X_f^*)$, when the home firm is a cooperative multinational firm.

4. The Optimal Choices of the Home Firm in the Second Stage

In the second stage, since the foreign firm does not have its subsidiary, it does not make the ownership share decision. Therefore, the home firm can choose its optimal ownership share without considering any reactions from the foreign firm. However, since the home firm consists of its parent firm and subsidiary, it is necessary for the home firm to establish a rule to determine the optimal level of α.

When the home firm consists of its parent firm and subsidiary three types of decision rule are considered to be typical: (1) the parent firm chooses the ownership share, (2) the subsidiary decides the ownership share, and (3) the parent firm and the subsidiary determine the ownership share by negotiation. Among these decision rules, the second rule seems to be unrealistic, since the parent firm and the subsidiary exchange positions in this case.

Furthermore, since the parent firm is generally the first proposer and the final authorizer in its subsidiary's construction decision, it has a much stronger position compared to the subsidiary. Indeed, while the parent firm has a responsibility to organize its subsidiary, it has the powerful right to affect its subsidiary's existence. The parent firm and its subsidiary are not equal partners when the home firm initially determines the parent firm's share of ownership in its subsidiary. This demonstrates that the first rule seems to be more plausible than the last as a means of making the ownership share decision. Therefore, this paper will adopt the first rule, adopting the last on another occasion[6].

Of course, the first rule does not imply that the parent firm completely neglects the subsidiary's opinions and/or wishes. There may well be negotiation and an exchange of informations concerning important issues, between the parent firm and its subsidiary. The first rule just indicates that the parent firm makes the final decision in the ownership share decision. Therefore, it is assumed that the home parent firm chooses α so as to maximize its profit Π_1 in the second stage, given the cooperative decision in the first stage.

4.1. The Non-cooperative Case

Since (7a)-(7e) hold when the home firm is a non-cooperative multinational firm, the first-order condition for the ownership share decision in the first stage is given by[5]

$$\frac{\partial \Pi_1}{\partial \alpha} = p^{*\prime}(X^*)X_1^* \frac{\partial X_f^*}{\partial \alpha} + \frac{\Pi_f^*}{1-\alpha} = 0. \tag{11}$$

Now, taking into consideration $\Pi_f^* > 0$ (from the condition under that the home firm has its subsidiary), $p^{*\prime}(X^*)X_1^* \frac{\partial X_f^*}{\partial \alpha} < 0$ (from (7c)) and $p^{*\prime}(X^*) < 0$ (from a feature of the inverse demand function), one gets from (11):

$$\alpha^* < 1. \tag{11)'$$

Therefore, (11)' presents, together with (2),

$$0 < \alpha^* < 1. \tag{12}$$

This shows that when the home firm is a non-cooperative multinational firm its optimal ownership share α^* is positive but less than unity.

4.2. The Cooperative Case

On the other hand, (10) holds when the home firm is a cooperative multinational firm Therefore, differentiating Π_1 with respect to α and taking account of (9), (10) and features of the inverse demand functions in the home and foreign markets, the first-order condition for the ownership share choice in the first stage is expressed as[7]

$$\frac{\partial \Pi_1}{\partial \alpha} = \Pi_f^* > 0. \tag{13}$$

This states that the home parent firm likes its ownership share to be as large as possible, because the large the ownership share becomes, the larger the home parent profit. However, the ownership share cannot exceed unity as is shown by (2), due to the physical restriction. Then, (2) and (13) present

$$\alpha^{*} = 1, \tag{14}$$

which shows that when the home firm is a cooperative multinational firm the optimal ownership share α^{*} chosen by its parent firm is always unity.

Now, one can summarize the arguments in 4.1 and 4.2 as the following main proposition:

Proposition 3: While the optimal ownership share of the cooperative multinational firm always equals unity, the optimal ownership share of the non-cooperative multinational firm is always less than unity, while remaining positive.

Though this paper does not explicitly consider a perfectly competitive industry, $p^{*\prime}(X^{*})$ = 0 holds when the industry in question is perfectly competitive. Thus, substituting this condition into (11),

$$\frac{\partial \Pi_1}{\partial \alpha} = \frac{\Pi_f^{*}}{1-\alpha} > 0.$$

holds, even if the home firm is a non-cooperative multinational firm. Then, this inequation demonstrates, together with (2), that when the international industry in question is perfectly competitive, the optimal ownership share of the home parent firm is always unity. But, when the optimal ownership share is unity, the home firm maximizes the joint profits consisting of its own profit and its subsidiary's profit, as is shown in the previous section. It follows that when the industry is perfectly competitive, the home firm is always a cooperative multinational firm with the unity ownership share.

On the other hand, when the industry in question is a Cournot duopoly, (12) holds if the home firm is a non-cooperative multinational firm, and (14) holds if the home firm is a non-cooperative multinational firm. Therefore, while the non-cooperative multinational home firm partially owns its subsidiary but perfectly leaves the subsidiary's output choice completely up to its discretion, the cooperative multinational home firm perfectly owns its subsidiary and would not allow a discretionary element on the part of the subsidiary in its output-export decisions of the second stage.

5. The Optimal Cooperation Decision in the First Stage

Following the same reasoning as just applied in a case of the ownership share decision, it is assumed here that the home parent firm is also the final commander in the cooperation decision. Thus, the home parent firm makes the cooperation decision so as to maximize its own profit.

Compared to the ownership share decision and the output-export decision of the home firm, its cooperation decision is easier to make. Since the cooperation decision is merely an alternative choice whether the parent firm acts cooperatively or non-cooperatively with its subsidiary, it is unnecessary to use any analytical calculations. The home firm only compares its equilibrium profits (denote by Π_1^n and Π_1^c) in the non-cooperative and cooperative cases and chooses the larger profit.

Sections 3 and 4 have already discussed the equilibrium levels of X_1, X_2, X_f^*, X_1^*, X_2^* and α in the non-cooperative and cooperative cases, respectively. Then, the equilibrium profits, Π_1^n and Π_1^c, of the home parent firm in the non-cooperative and cooperative cases are obtained through substituting these equilibrium variables into Π_1 in (1), respectively. Consequently, the home parent firm chooses to acts non-cooperatively (cooperatively) with its subsidiary when $\Pi_1^n > \Pi_1^c$ ($\Pi_1^n < \Pi_1^c$) holds. This is all that the home parent firm concerns.

However, there exists one point that should be emphasized here. That is regarded with the relations between the cooperation decision, the ownership share decision and the output-export decision of the home parent firm. When the home parent firm decides to act cooperatively (non-cooperatively) with its subsidiary in the first stage, it always chooses the unity (less-than-unity) ownership share in the second stage, as is obvious from proposition 3. And, in the third stage, the home parent firm with unity (less-than-unity) ownership share maximizes joint (its own) profit, cooperatively (non-cooperatively) with its subsidiary. Therefore, when home firm is a cooperative (non-cooperative) multinational firm, the home parent firm always behaves cooperatively (non-cooperatively) with its subsidiary, and its optimal ownership share is always less-than-unity (equal to unity). There is no possibility that the cooperative (non-cooperative) parent home firm chooses the less-than-unity (unity) ownership share and maximize its own profit (the joint profit including its subsidiary's profit).

6. The Socially Optimal Ownership Share and Policies

Though the previous section has analyzed the privately optimal ownership share α^* chosen by the home multinational firm, it is not clear whether α^* is equal to the socially optimal ownership share α_S^* that maximizes the home country's welfarer. It has been widely believed that the equilibrium achieved by the imperfectly competitive market would not generally maximize the social welfare. This section will examine this problem.

The welfare of the home country (denote by HW) is defined as the sum of consumer's surplus and producer's surplus:

$$\mathrm{HW} = \int_0^X p(z)dz - p(X)X + \Pi_1,$$

where $\int_0^X p(z)dz - p(X)X$ is the home consumer' surplus, Π_1 is the home producer's surplus (= profit of the home parent firm), and X (= $X_1 + X_2$) is the total consumption in the home country. Then, taking into consideration that X_1 and X_2 are both always independent of α from (7a) and (10), the first-order condition for maximizing the home country's welfare HW is

$$\frac{\partial HW}{\partial \alpha} = \frac{\partial \Pi_1}{\partial \alpha} = 0, \tag{15a}$$

when the home firm is a non-cooperative multinational firm, but it is given by

$$\frac{\partial HW}{\partial \alpha} = \frac{\partial \Pi_1}{\partial \alpha} = \Pi_f^* > 0, \tag{15b}$$

when the home firm is a cooperative multinational[8]. It is easily demonstrated that (15a) and (15b) are respectively equal to (11) and (13). This indicates:

Proposition 4. The privately optimal ownership share is always equal to the socially optimal ownership share.

When the socially optimal ownership share α_S^* is always equal to the privately optimal ownership share α^{*}, the optimal ownership share chosen by the home firm would not distort the socially optimal resource allocation. Therefore, there is no reason why the home government should intervene in the ownership share choice by the home parent firm even if the market is imperfectly competitive.

7. Concluding Remarks

With respect to the actual international economy, it is very interesting to note that many multinational parent firms partially own their subsidiaries and entrust some decisions to them. In the subsidiary construction decision, the parent firms could choose the ownership share of unity, but, in fact, many parent firms do not choose such a share and allow the subsidiaries independence in their own management. This fact illustrates that partial ownership of the subsidiary by its parent firm and the subsidiary's independence in its management are closely related to each other. This paper has investigated this relationship theoretically.

This paper first established a simple three-stage game model of an internationally asymmetric Cournot duopoly, which consists of a home firm with a subsidiary in the foreign country and a foreign firm with no subsidiary. Then, using such a model, it has analyzed the optimal ownership share choice of the home firm and presented some interesting results, which are summarized as propositions 1 - 4. Importantly, it emerged that the ownership share is one of the endogenous variables chosen by the home firm and is closely related to its cooperation decision and the output-export decision.

For example, proposition 3 proposes that the optimal ownership share of the home firm is always less than (equal to) unity when its parent firm and subsidiary behave non-cooperatively (cooperatively). This implies, in turn, that the non-cooperative (cooperative) multinational firm would not choose the unity (less than unity) ownership share. Furthermore, propositions 1 and 2 show that the output-export decisions of the home and foreign firms also depend on the home firm's ownership share choice and its cooperation decision. Finally, proposition 4 means that the government intervention is unnecessary in the ownership share decisions of the multinational firm.

Of course, in the actual world one might observe some phenomena that run counter to the propositions presented in this paper. This is because some international industries in the actual world are much more complicated than the industry modeled in this paper. This paper has omitted some issues such as transfer pricing, technological transfer or spillover, tax heaven that are pertinent to the multinational firm, in order to focus on the ownership share choice of the multinational home firm.

Furthermore, this paper has implicitly presumed that when the home parent firm partially owns its subsidiary (that is, the optimal ownership share of the home parent firm is less than unity) the other part of its subsidiary is not owned by the foreign firm, but by the subsidiary itself. However, in the actual world there is a possibility that the foreign firm partially owns the home subsidiary. And, under some conditions, the foreign firm also becomes a multinational firm by building its subsidiary in the home country. In such a case, the industry in question becomes an internationally symmetric duopoly. Thus, generalized models modifying some of these points would reinforce the propositions derived in this paper.

Appendix

Derivations of (7b), (7c) and (7d)

The equilibrium of the industry where the home firm is a non-cooperative multinational firm are given by pairs of X_1^*, X_2^* and X_f^* satisfying simultaneously the following equations:

$$p^{*'}(X^*)(X_1^* + \alpha X_f^*) + p^*(X^*) - g_1'(X_1^*) - c_1 = 0, \tag{5a}$$

$$p^{*'}(X^*)X_f^* + p^*(X^*) - c_f = 0, \tag{5b}$$

$$p^{*'}(X^*)X_2^* + p^*(X^*) - c_2 = 0. \tag{6a}$$

Thus, the effects of a change in α on X_1^*, X_2^* and X_f^* in the industry equilibrium are obtained by differentiating totally all these equations with respect to α:

$$\begin{pmatrix} \Delta_{11}^* & \Delta_{12}^* & \Delta_{13}^* \\ \Delta_{21}^* & \Delta_{22}^* & \Delta_{23}^* \\ \Delta_{31}^* & \Delta_{32}^* & \Delta_{33}^* \end{pmatrix} \begin{pmatrix} \partial X_1^* / \partial \alpha \\ \partial X_f^* / \partial \alpha \\ \partial X_2^* / \partial \alpha \end{pmatrix} = \begin{pmatrix} -p^{*'}(X^*)X_f^* \\ 0 \\ 0 \end{pmatrix}, \tag{A1}$$

where $\Delta_{11}^* = p^{*''}(X^*)(X_1^* + \alpha X_f^*) + 2p^{*'}(X^*) - g_1''(X_1^*)$, $\Delta_{12}^* = p^{*''}(X^*)(X_1^* + \alpha X_f^*) + (1 + \alpha)p^{*'}(X^*)$, $\Delta_{13}^* = p^{*''}(X^*)(X_1^* + \alpha X_f^*) + p^{*'}(X^*)$, $\Delta_{21}^* = \Delta_{22}^* = p^{*''}(X^*)(X_1^* + X_{f*}) + 2p^{*'}(X^*)$, $\Delta_{23}^* = p^{*''}(X^*)(X_1^* + X_{f*}) + p^{*'}(X^*)$, $\Delta_{31}^* = \Delta_{32}^* = p^{*''}(X^*)X_2^* + p^{*'}(X^*)$, $\Delta_{33}^* = p^{*''}(X^*)X_2^* + 2p^{*'}(X^*)$, and $\Delta = p^{*'}(X^*)\{(1+\alpha)p^{*'}(X^*) - g_1''(X_1^*)\}\{p^{*''}(X^*) + 3p^{*'}(X^*)\}$. Then, calculating $\frac{\partial X_1^*}{\partial \alpha}$, $\frac{\partial X_f^*}{\partial \alpha}$ and $\frac{\partial X_2^*}{\partial \alpha}$ from (A1) and substituting all features of the inverse demand functions and the cost functions into the results, one gets:

$$\frac{\partial X_1^*}{\partial \alpha} = \frac{-p^{*'}(X^*)X_f^*}{\{(1+\alpha)p^{*'}(X^*) - g_1^{'}(X_1^*)\}} < 0,$$

$$\frac{\partial X_f^*}{\partial \alpha} = \frac{p^{*'}(X^*)X_f^*}{\{(1+\alpha)p^{*'}(X^*) - g_1^{'}(X_1^*)\}} > 0, \text{ and}$$

$$\frac{\partial X_2^*}{\partial \alpha} = 0.$$

Footnotes

1. It is impossible to list here, all of many papers that have discussed multinational firms; but the following are closely related to the present paper: Ethier (1986), Ethier and Markusen (1991), Glass and Saggi (2002), Horn and Persson (2001), Horstmann and Markusen (1987a), and Yanagawa (1994). They examine the difference between foreign direct investment and licensing and/or the difference of the ownership structure between foreign direct investment and licensing by establishing static or dynamic models. They, however, do not consider the ownership share choices of multinational firms.
2. This means that all of the firms' production functions are homogeneous of degree one.
3. For example, Markusen, Morey and Olewiler (1993), Motta and Thisse (1994), Markusen, Morey and Olewiler (1995), and Hoel (1997) have presented models of multinational firms where the plant construction is endogenous. They, however, implicitly assumed that the ownership shares of multinational firms are always positive and constant.
4. It has often been indicated that the average export cost might be decreasing in some cases. However, the increasing marginal export cost is compatible with this indication when the export cost includes a fixed part. For example, when the total export cost g consists of a fixed cost F and a variable cost $h(X)$, the average export cost AT is given by $AT = g/X = \{F + h(X)\}/X$. Then, differentiating this with respect to X, we have $AT' = \{h'(X)X - F - h(X)\}/X^2$. Hence, AT' is always negative in the case when a fixed cost F is large enough.
5. The second-order condition is satisfied if Π_f is large enough and/or the absolute value of $p^{*}(X^{*})$ is small enough, though it is not certain whether this condition always holds. Hence, I assume that this condition holds in the neighborhood of the industry equilibrium.
6. For similar arguments, see Grossman and Hart (1986), for example.
7. Since (10) holds in the case where the home firm is a cooperative multinational firm, the positive subsidiary's profit $\Pi_f > 0$ ensures that the second-order condition always holds in the industry equilibrium.
8. When the second-order condition for maximizing the home firm's profit is satisfied, the

second-order condition for maximizing the home country's welfare always holds.

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