Foreign Direct Investment
and FDI Subsidies
in An International Cournot Duopoly

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Abstract
A simple three-stage game model of an international Cournot duopoly, consisting of home and foreign multinational firms, is exploited to examine the optimal subsidies for foreign direct investments (FDIs). Firms endogenously determine their FDI sizes, and the governments provide FDI subsidies to firms, according to their FDI sizes. Thus, this paper demonstrates that out-flow and in-flow FDI subsidies have different effects on firms' decisions of FDIs and output-exports and that the FDI subsidies are used as strategic policies. It also finds that the optimal FDI subsidies are not always zero even when FDIs are endogenously chosen and depend on whether labor assessments are included in social welfare functions.

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1. Introduction

Many papers, initiated by Brander and Krugman (1983), and Brander and Spencer (1984 and 1985), have established various types of imperfectly competitive trade models (henceforth, ICT models) analyzing trade subsidy policies and have proposed that it is an effective strategic policy to subsidize exports and that the optimal level of subsidy is not zero. Then, introducing foreign direct investments (henceforth, FDIs) into the ICT models, several recent papers have exploited some imperfectly competitive trade and foreign direct investment models (henceforth, ICTF models) and demonstrated that the export subsidy policy loses its effectiveness as a strategic trade policy and its optimal level becomes zero in circumstances where the firms endogenously choose both exports and FDIs.

For examples of ICTF models, see Janeba (1998) and Ishii (2001). Janeba (1998) has modified a third country trade model presented by Brander and Spencer (1985) so as to consider firms’ choices of plant locations (through FDIs) and found that if firms can freely shift their production plants between the home and foreign countries, the optimal export subsidy becomes zero. Moreover, Ishii (2001) has extended a reciprocal trade model exploited by Brander and Krugman (1983), and Brander and Spencer (1984) so as to include firms’ overseas production (by FDIs) and found that the optimal export subsidy is zero when firms produce goods not only in their own countries but also in their rivals’ countries.¹

Though the ICTF models are regarded as more general and realistic than the ICT
models because they have considered firms’ FDIs (or plant locations) as well as exports, they have implicitly assumed that FDI sizes (or plant sizes) are given exogenously, and thus have never considered the FDI subsidies. However, in the present world, firms determine FDI sizes endogenously and many governments provide FDI subsidies. Therefore, it is necessary and useful to establish a more generalized ICTF model, in which firms and governments endogenously determine FDI sizes and FDI subsidies, respectively, and to investigate the effectiveness of providing FDI subsidies as a strategic policy and the optimal levels of such subsidies.

In order to investigate the issues mentioned above, this paper will establish a very simple generalized ICTF model, by extending the reciprocal trade model exploited by Brander and Krugman (1983), and Brander and Spencer (1994) (henceforth, the B-K-S ICT model) so as to include firms’ FDIs and governments’ FDI subsidies. Hence, the framework of the present ICTF model is almost the same as that of B-K-S ICT model, but there exist three essential differences between these models, among others. The first is whether the firms produce goods only in their own countries or in both the home and foreign countries. The second is whether governments give firms export subsidies or FDI subsidies. And, the last is whether the game has two stages or three.

In the B-K-S ICT model, the home and foreign firms have their plants producing homogenous goods only in their own countries, and supply these goods to both countries. By contrast, in the present ICTF model, the home and foreign firms have their production plants not only in their own countries but also in their rivals'
countries (henceforth, the parent plants and the subsidiary plants, respectively). Clearly, the firms in this paper are multinational firms that use FDIs when constructing their subsidiary plants in the rivals' countries. Therefore, the firms in this model will act quite differently from the national firms in the B-K-S ICT model.

Furthermore, while the B-K-S ICT model discusses export subsidies, the present model considers FDI subsidies. While export subsidies are used to control international commodity flows, FDI subsidies are adopted to manage international FDI flows. However, since changes in the international FDI flows are finally accompanied by changes in the international commodity flows, they cause changes in international output-export structures. Therefore, FDI subsidies affect not only international FDI flows but also, in the end, international output-export structures. In this paper, I will differentiate explicitly between the subsidies for out-flow FDIs and the subsidies for in-flow FDIs (henceforth, the out-flow FDI subsidies and the in-flow FDI subsidies, respectively), because they have quite different effects on the FDIs, as is soon shown.

Finally, while the B-K-S ICT model assumes a two-stage game, the present model adopts a three-stage game. This is why the present model is extended to include the endogenous choices of the firms' FDIs or subsidiaries' plant sizes. Taking into consideration that plant construction takes considerable time, a three-stage game is more plausible for analyzing the optimal FDI subsidies for multinational firms' FDIs.

In what follows, an international Cournot duopoly such as that mentioned above is
used to illustrate a generalized ICTF model of a three-stage game incorporating sub-

game perfect equilibrium. That is, in the first stage the home and foreign
governments determine simultaneously the optimal FDI subsidies. Then, in the
second stage, the home and foreign firms respectively choose their own FDIs, given
the FDI subsidies decided by the governments in the first stage and the rivals' FDIs.
Finally, in the third stage, the home and foreign firms non-cooperatively determine
optimal output-export levels, given the FDI subsidies decided in the first stage, the
FDIs chosen in the second stage and the rival's output-export decisions, respectively.
This paper will adopt a backward induction for solving the issues mentioned above.

Thus, this paper will present the following propositions as its main findings. The
first is that while a change in the out-flow (in-flow) FDI subsidy has a positive effect
on the out-flow (in-flow) FDI, a change in the out-flow (in-flow) FDI subsidy has no
effect on the in-flow (out-flow) FDI. The second is that if the government does not
assess labor employment the optimal out-flow FDI subsidy is zero, but the optimal
in-flow FDI subsidy is positive. The last is that while the optimal out-flow FDI
subsidy is negative, the optimal in-flow FDI subsidy is positive when the government
assesses labor employment. It also shown that when the firms can vary smoothly
their subsidiary plant sizes in the rivals' countries, changes in the FDI subsidies don't
have such drastic effects, as is indicated by Markusen, Morey and Olewiler (1993 and

The rest of this paper is organized as follows. Section 2 will establish a
generalized ICTF model of an international Cournot duopoly, as explained above. Then, using such a model, sections 3 will analyze the firms' output-export choices, and section 4 will investigate the firms' FDI determinations. And, section 5 will discuss the optimal FDI subsidies. Finally, concluding remarks will be presented in section 6.

2. Assumptions and Basic Model

This section will establish a very simple generalized ICTF model of an international Cournot duopoly consisting of home and foreign firms that have, respectively, not only parent plants in their own countries but also subsidiary plants, as a result of FDIs, in their rivals' countries. Although the present ICTF model depends on the B-K-S ICT model, as was mentioned above, there are several differences between these models. So, it is worthwhile explaining the essential features of the present model in some detail.

Suppose that the home and foreign firms have already fixed the sizes of parent plants producing homogenous goods and then plan to construct new subsidiary plants producing the same goods by FDI in their rivals' countries, respectively. Therefore, while the firms' parent plant sizes are exogenously fixed, their subsidiary plant sizes are endogenously determined through FDIs.³

When there already exist parent plants, the home and foreign firms have no construction costs for their parent plants. However, they must incur construction
costs, \( v \) and \( v^* \), for their subsidiary plants since these plants are newly constructed. It is quite natural to consider that construction costs for the home and foreign subsidiary plants are increasing and strictly convex functions of the home and foreign firms' FDIs, \( k \) and \( k^* \), respectively. Then, these subsidiary plant construction cost functions are respectively denoted as \( v(k) \) (with \( v'(k) > 0 \) and \( v'(k) > 0 \)) and \( v^*(k^*) \) (with \( v^*(k^*) > 0, \; v^*(k^*) > 0 \)) (henceforth, the notations with asterisks, *, respectively denote the foreign variables corresponding to the home variables). Though the home and foreign subsidiary plant construction costs are fixed costs in the third stage when the firms choose output-export levels, they are variable costs in the second stage when the firms determine their FDIs. Consequently, it will soon be shown that changes in the FDI subsidies would not have such drastic effects on the firms' plant choices, as demonstrated by Markusen et. al (1993 and 1995).

Furthermore, while unit production cost, \( C \) (\( C^* \)), of the home (foreign) parent plant is independent of its FDI since its parent plant is already constructed when the game begins, unit production costs, \( c \) (\( c^* \)) of the home (foreign) subsidiary plant depends on its FDI, \( k \) (\( k^* \)), in the second stage. Generally, since the firm's subsidiary plant size increases as its FDI rises and the firm's unit production cost reduces as its plant size increases (due to scale merits, for example), then unit production costs of firms' subsidiary plants are decreasing functions of their own FDIs, respectively. Therefore, unit production costs, \( c \) and \( c^* \), of the home and foreign subsidiary plants
are given respectively by functions such as $c(k)$ with $c'(k) < 0$ and $c^*(k^*)$ with $c^*(k^*) < 0$. Furthermore, this paper assumes that these functions are strictly convex, $c'(k) > 0$ and $c^*(k^*) > 0$, and that all unit production costs, $C, C^*, c$ and $c^*$, of firms' parent and subsidiary plants are independent of their outputs in the third stage, for simplicity of analysis.

By contract, the export cost function of the home and foreign parents are given respectively by $G(T)$ and $G^*(T^*)$ whose marginal costs are strictly positive and increasing, i.e., $G'(T) > 0$, $G^*(T^*) > 0$, $G'(T) > 0$ and $G^*(T^*) > 0$. Though detail explanations are omitted in order to save space, the increasing marginal export costs, i.e., $G'(T) > 0$ and $G^*(T^*) > 0$ will ensure that all the second-order conditions for maximizing home and foreign firms' profits and the stability conditions of the industry equilibrium in the second stage are met.  

It is assumed that the home and foreign markets are segregated from each other and that while the home and foreign firms supply goods produced by the parent plants to both countries, they sell goods produced by the subsidiary plants only in the rivals' countries where the subsidiary plants are constructed, respectively. Thus, both the home and foreign firms have two routes to supply their goods to the rivals' country, i.e., exports and overseas production. These assumptions, while they may seem to be unduly strict at first sight, are not so in reality, when it is recognized that such
industries are quite popular in the real world. Thus, the home and foreign inverse
demand functions are given respectively by \( p(Z) = p(X + T^* + Y^*) \) with \( p'(Z) < 0 \)
and \( p^*(Z^*) = p^*(X^* + T + Y) \) with \( p^*(Z^*) < 0 \), where \( p \) and \( p^* \) are respectively
the home and foreign prices, \( X \) and \( T \) (\( X^* \) and \( T^* \)) are respectively domestic sale
and export of the home (foreign) parent plant, and \( Y(Y^*) \) is output (= sale) of the
home (foreign) subsidiary plant.

It is finally supposed that the home and foreign governments provide, respectively,
the per-unit FDI subsidies, \( s \) and \( s^* \), for out-flow FDIs and the per-unit FDI
subsidies, \( s_k \) and \( s_k^* \), for in-flow FDIs, which are determined so as to maximize their
own welfares in the first stage and are unchanged thereafter. Consequently, \( s \), \( s^* \),
\( s_k \) and \( s_k^* \) are all regarded as constant to the firms in the second and third stages.
However, as is clear from the definitions, the home firm's FDI is regarded as the
out-flow FDI by the home country but as the in-flow FDI by the foreign country. And
the foreign firm's FDI is considered as the out-flow FDI by the foreign country but as
the in-flow FDI by the home country. Therefore, henceforth, let \( s \), \( s_k \), \( s^* \), \( s_k^* \) and \( s^*_k \)
be, respectively, the home out-flow FDI subsidy, the home in-flow FDI subsidy, the
foreign out-flow FDI subsidy, and the foreign in-flow FDI subsidy.

Now, under the assumptions and features explained above, profits, \( \Pi \) and \( \Pi^* \), of
the home and foreign firms are defined respectively as
\[ \Pi = \{ p(Z)X + p^*(Z^*)Y - C(X + T) - G(T) \} \]
\[ + \{ p^*(Z^*)Y - c(k)Y - v(k) \} + (s + s_k^*)k, \]  \hspace{1cm} (1)

and

\[ \Pi^* = \{ p^*(Z^*)X^* + p(Z)Y^* - C^*(X^* + T^*) - G^*(T^*) \} \]
\[ + \{ p(Z)Y^* - c^*(k^*)Y^* - v^*(k^*) \} + (s^* + s_k^*)k^*. \]  \hspace{1cm} (2)

where \( Z = X + T + Y \) and \( Z^* = X^* + T + Y \) are total sales (= consumptions) in the home and foreign countries, respectively. In (1) (and (2)), the first and second terms braced \{ \} are, respectively, profits of the home (foreign) parent and subsidiary plants, and the last term is revenues from the governments’ subsidies.\(^5\)

The home and foreign firms in a Cournot industry act independently to maximize their own profits defined by (1) and (2), respectively, and the home and foreign governments determine respectively their FDI subsidies so as to maximize their own welfares, as defined in section 5. This paper will solve these problems from the third stage equilibrium to the first stage equilibrium by using a method of backward induction.

3. Optimal Output-Export Choices in the Third Stage

This section will discuss the firms' output-export decisions at the industry
equilibrium. In the third stage, while the control variables of the home and foreign firms are their parent plant outputs, $X$ and $X^*$, their exports, $T$ and $T^*$, and their subsidiary plant output, $Y$ and $Y^*$, respectively. Since the Cournot firms act non-cooperatively, the home firm chooses $X$, $T$ and $Y$ so as to maximize its profit defined as (1), given the foreign firm's output-export levels and all other exogenous variables in the third stage. Therefore, the home firm's first-order conditions for maximizing its profit are given by

$$p(Z) + p'(Z)X - C = 0,$$  \hspace{1cm} (3)

$$p^*(Z^*) + p^*(Z^*)(T + Y) - G'(T) - C = 0,$$  \hspace{1cm} (4)

$$p^*(Z^*) + p^*(Z^*)(T + Y) - c(k) = 0,$$  \hspace{1cm} (5)

and the second-order conditions always hold under the conditions mentioned above. Therefore, the optimal levels of $X$, $T$ and $Y$ chosen by the home firm are obtained by solving (3)-(5).

Similarly, the foreign firm chooses $X^*$, $T^*$ and $Y^*$ so as to maximize its profit defined as (2), given the home firm's output-export levels and all exogenous variables in the third stage. Then, the foreign firm's first-order conditions for maximizing its profit are given by

$$p^*(Z^*) + p''(Z^*)X^* - C^* = 0,$$  \hspace{1cm} (6)

$$p(Z) + p'(Z)(T^* + Y^*) - G''(T^*) - C^* = 0,$$  \hspace{1cm} (7)
\[ p(Z) + p'(Z)(T^* + Y^*) - c'(k^*) = 0. \]  

(3)

And, the second-order conditions are always satisfied under the conditions presented in the previous section. Thus, it is possible to obtain the optimal levels of \( X^* \), \( T^* \) and \( Y^* \) of the foreign firm by solving (6)-(8).

In these first-order conditions presented above, (3) and (6) are the output-reaction functions of the home and foreign parent plants, (4) and (6) are the export-reaction functions of the home and foreign parent plants, and (5) and (8) are the output-reaction functions of the home and foreign subsidiary plants, respectively. Since the goods produced by the home and foreign firms are regarded as strategically substitutive, the following conditions are satisfied:

\[ p'(Z) + p'(Z)X < 0, \quad p''(Z^*) + p''(Z^*)(T + Y) < 0, \]  

(9)

and

\[ p''(Z^*) + p''(Z^*)X^* < 0, \quad p'(Z) + p'(Z)(T^* + Y^*) < 0. \]  

(10)

It follows that the reaction curves of the home and foreign firms mentioned above are all downward sloping.

The Cournot-Nash industry equilibrium of the international duopoly in the third stage is given by \( X, T, Y, X^*, T^* \) and \( Y^* \) that simultaneously satisfy the equation system consisting of (3)-(8). However, it is immediately obvious that while the equilibrium levels of \( X, T^* \) and \( Y^* \) are determined by solving (3), (7) and (8), those of \( X^*, T \) and \( Y \) are obtained by solving (4), (5) and (6):
\[ X = X(C, C^*, k^*), \quad T^* = T^*(C, C^*, k^*), \quad Y^* = Y^*(C, C^*, k^*), \]

and

\[ X^* = X^*(C, C^*, k), \quad T = T(C, C^*, k), \quad Y = Y(C, C^*, k). \]

Thus, this kind of separation theorem, the conditions of demand functions and the features of cost functions mentioned above combine to show that the industry equilibrium in the third stage is locally stable, but the possibility of multiple equilibrium cannot be excluded. So, it is here assumed that there exists a stable Cournot-Nash equilibrium in the third stage in order to make the following comparative statics analyses meaningful.

Differentiating totally all of (3)-(8) with respect to \( k \) and taking account of the features and conditions of demand functions and cost conditions mentioned above, one gets the effects of a change in the home firms' FDI, \( k \), on the industry equilibrium, \( X, T, Y, X^*, T^* \) and \( Y^* \):

\[
\frac{\partial T}{\partial k} = \frac{c'(k)}{G'(T)} < 0, \quad \text{(11.i)}
\]

\[
\frac{\partial Y}{\partial k} = -\frac{c'(k)G'(T)\left[p^*(Z^*)X^* + 2p^*(Z^*)\right]}{\Delta k} - \frac{c'(k)}{G'(T)} > 0, \quad \text{(11.ii)}
\]

\[
\frac{\partial X^*}{\partial k} = \frac{c'(k)G'(T)\left[p^*(Z^*)X^* + p^*(Z^*)\right]}{\Delta k} < 0, \quad \text{(11.iii)}
\]

\[
\frac{\partial T^*}{\partial k} = \frac{\partial Y^*}{\partial k} = \frac{\partial X^*}{\partial k} = 0, \quad \text{(11.iv)}
\]

where \( \Delta^* = -G'(T)p^*(Z^*)\left[p^*(Z^*)Z^* + 3p^*(Z^*)\right] < 0 \). Similarly, differentiating totally all of (3)-(8) with respect to \( k^* \) and taking into consideration the same features
and conditions as presented above, one obtains the effects of a change in the foreign firms' FDI, \( k^* \), on the industry equilibrium, \( X, T, Y, X^*, T^* \) and \( Y^* \):

\[
\frac{\partial T^*}{\partial k^*} = \frac{c^*(k^*)}{G^*(T^*)} < 0, \quad (12.i)
\]

\[
\frac{\partial Y^*}{\partial k^*} = - \frac{c^*(k^*)G^*(T^*)}{\Delta} \left[ p^*(Z)X + 2p^*(Z) \right] - \frac{c^*(k^*)}{G^*(T^*)} > 0, \quad (12.ii)
\]

\[
\frac{\partial X}{\partial k^*} = \frac{c^*(k^*)G^*(T^*)}{\Delta} \left[ p^*(Z)X + p^*(Z) \right] < 0, \quad (12.iii)
\]

\[
\frac{\partial T}{\partial k^*} = \frac{\partial Y}{\partial k^*} = \frac{\partial X^*}{\partial k^*} = 0, \quad (12.iv)
\]

where \( \Delta = - G^*(T^*)p^*(Z)(p^*(Z)+3p^*(Z)) < 0 \).

Therefore, these results of (11) and (12) are paraphrased as:

Proposition 1. A rise in the home (foreign) firm's FDI increases output of the home (foreign) subsidiary plant, but reduces both exports of the home (foreign) parent plant and output of the foreign (home) parent plant, and vice versa. However, it has no effects on home (foreign) parent plant's output, foreign (home) parent plant's exports and foreign (home) subsidiary plant's output.

Furthermore, (11) presents the following relations:

\[
\frac{\partial T}{\partial k} + \frac{\partial Y}{\partial k} > 0, \quad \frac{\partial T^*}{\partial k} + \frac{\partial X}{\partial k} < 0, \quad \frac{\partial T}{\partial k} + \frac{\partial X^*}{\partial k} > 0, \quad \frac{\partial T^*}{\partial k} + \frac{\partial Y^*}{\partial k} > 0, \quad (13)
\]

Then, one can present:
Corollary.

A rise in the home firm's FDI increases home firm's total overseas sales, home firm's total output and foreign consumption, but reduces home firm's domestic output, foreign firm's domestic output and foreign firm's total output, and *vice versa*. However, it has no effect on home consumption and foreign firm's total overseas sales.

Since the effects of a change in the foreign firms' FDIs are parallel to those of a change in the home firm's FDI, as is easily understood from (11) and (12), this corollary is also true even when the words "home" and "foreign" are interchanged.

While \( \frac{\partial Z^*}{\partial k} > 0 \) and \( \frac{\partial Z}{\partial k^*} > 0 \) imply that the FDI improves consumption level of the host country, \( \frac{\partial Z}{\partial k} = \frac{\partial Z}{\partial k} = 0 \) demonstrate that the FDI does not change the consumption level of the guest country. On the other hand, while \( \frac{\partial Y}{\partial k} + \frac{\partial T^*}{\partial k} + \frac{\partial X^*}{\partial k} \)

> 0 and \( \frac{\partial Y^*}{\partial k} + \frac{\partial T}{\partial k^*} + \frac{\partial X}{\partial k^*} > 0 \) mean that the FDI increases the output level of the host country, \( \frac{\partial Y}{\partial k} + \frac{\partial T}{\partial k} + \frac{\partial X}{\partial k} < 0 \) and \( \frac{\partial Y^*}{\partial k^*} + \frac{\partial T^*}{\partial k^*} + \frac{\partial X^*}{\partial k^*} < 0 \) indicate that the FDI results in a reduction of the output level of the guest country. Therefore, the FDI is welcomed by both consumers and firm in the host country, but not welcomed by the firm in the guest country.

When taken together, proposition 1 and corollaries 1 & 2 show that if the governments can manage the firms' FDIs by changing their FDI subsidies, they can use their FDI subsidies as strategic policies for controlling firms' market shares and
revenues. So, in the next section, we will investigate whether the governments can control the firms' FDIs by means of their FDI subsidies.

4. Equilibrium FDI Levels in the Second Stage

In the second stage, the Cournot home and foreign firms non-cooperatively choose their FDIs so as to maximize their own profits, given the governments' FDI subsidies determined in the first stage, the rival's FDI level in the second stage, the firms' optimal output-export levels expected to be chosen in the third stage, and the parameters included in (1) and (2), respectively. Therefore, taking into consideration that the conditions of (3)-(8) will always hold in industry equilibrium of the third stage, the first-order conditions for maximizing the home and foreign firms' profits in the second stage are given respectively by

\[ p^*(Z^*) \frac{\partial X^*}{\partial k} (T + Y) - c'(k)Y - v'(k) + (s + s^*_k) = 0, \quad (14) \]

and

\[ p^*(Z) \frac{\partial X}{\partial k} (T^* + Y^*) - c^*(k^*)Y^* - v^*(k^*) + (s^* + s_k) = 0. \quad (15) \]

Though it is not sure whether the second-order conditions, \( \frac{\partial^2 \Pi}{\partial k^2} < 0 \) and \( \frac{\partial^2 \Pi^*}{\partial k^{*2}} < 0 \), always hold for all ranges of \( k \) and \( k^* \), it is here assumed, for simplicity, that both of these second-order conditions hold in neighborhood of the industry equilibrium.

The Cournot-Nash industry equilibrium in the second stage is given by \( k \) and \( k^* \) that simultaneously satisfy (14) and (15). As is indicated in the previous section,
though there is a possibility of multiple industry equilibrium, it is also assumed here that there exists a Cournot-Nash industry equilibrium in the second stage. However, taking account of (11), (12) and (13), it is easily shown that

\[
\frac{\partial^2 \Pi}{\partial k^2} = 0, \quad \text{and} \quad \frac{\partial^2 \Pi^*}{\partial k \partial k^*} = 0 \tag{16}
\]

hold in the industry equilibrium of the second stage. Therefore, these equations given by (16) and the second-order conditions, \(\frac{\partial^2 \Pi}{\partial k^2} < 0\) and \(\frac{\partial^2 \Pi^*}{\partial k^2} + \frac{\partial^2 \Pi^*}{\partial k^*} \), of the home and foreign firms combine to ensure the stability of the industry equilibrium in the second stage.

It is immediately obvious from (14) and (15) that the industry equilibrium FDI levels, \(k\) and \(k^*\), are both functions of out-flow and in-flow FDI subsidies, \(s, s^*\), \(s_k\) and \(s_k^*\), which are determined by the home and foreign governments in the first stage. However, it is easily recognized from (14) and (15) that while the home out-flow FDI subsidy, \(s\), and the foreign in-flow FDI subsidy, \(s_k^*\), have the same effects on the industry equilibrium, the foreign out-flow FDI subsidy, \(s^*\), and the home in-flow FDI subsidy, \(s_k\), have the same effects on the industry equilibrium. Therefore, the relations among \(k, k^*, s, s^*, s_k\) and \(s_k^*\) are given the following functions,

\[
k = k (s + s_k^*, s^* + s_k) \quad \text{and} \quad k^* = k^* (s + s_k^*, s^* + s_k),
\]

which, in turn, shows that it is impossible to discern the effects of changes in \(s\) and \(s_k^*\) on the equilibrium, and the effects of changes in \(s^*\) and \(s_k\) on the equilibrium. Let us investigate these effects more strictly in the rest of this section.

Differentiating (14) and (15) totally with respect to \(s\) (\(s_k^*\)), one can obtain the
effects of a change in the home out-flow FDI subsidy (the foreign in-flow FDI subsidy) on the industry equilibrium. Therefore, considering the second-order conditions, these effects are derived as:

\[
\frac{\partial k^*}{\partial s} = \frac{\partial k^*}{\partial s^*_k} = 0, \text{ and } \frac{\partial k}{\partial s} = \frac{\partial k^*}{\partial s^*_k} = -\frac{1}{\left(\frac{\partial^3 \Pi}{\partial k^2}\right)} > 0. \tag{17}
\]

It follows that a rise (reduction) in the home out-flow FDI subsidy or the foreign in-flow FDI subsidy raises (reduces) the home firm's FDI, but does not affect the foreign firm's FDI.

On the other hand, differentiating totally (14) and (15) with respect to \( s^* (s_k) \), deriving \( \frac{\partial k^*}{\partial s}, \frac{\partial k^*}{\partial s^*_k}, \frac{\partial k}{\partial s}, \text{ and } \frac{\partial k^*}{\partial s^*_k} \) respectively, and considering the firms' second-order conditions, the effects of the foreign out-flow FDI subsidy (the home in-flow FDI subsidy) on the industry equilibrium are given as:

\[
\frac{\partial k}{\partial s^*_k} = \frac{\partial k}{\partial s^*_k} = 0, \text{ and } \frac{\partial k^*}{\partial s^*_k} = \frac{\partial k^*}{\partial s^*_k} = -\frac{1}{\left(\frac{\partial^3 \Pi}{\partial k^2}\right)} > 0. \tag{18}
\]

It follows that an increase (a reduction) in the foreign out-flow FDI subsidy or the home in-flow FDI subsidy increases (reduces) the foreign firm's FDI, but does not change the home firm's FDI.

Now, the results of (17) and (18) are summarized together with as the following proposition:
Proposition 2. In the industry equilibrium in the second stage, the home (foreign) firm's FDI is increasing in the home (foreign) out-flow FDI subsidy and the foreign (home) in-flow FDI subsidy, but it is independent of the foreign (home) out-flow FDI subsidy and the home (foreign) in-flow FDI subsidy.

This proposition demonstrates that while neither of the home or foreign governments can manage the rival firm's FDI by changing only their own out-flow FDI subsidy, they can control both of the home and foreign firms' FDIs by appropriate mixing of their out-flow and in-flow FDI subsidies. Therefore, propositions 1 and 2 combine to show that there is a possibility that the governments can adopt the FDI subsidies as strategic policies. However, the effectiveness of the FDI subsidies as strategic policies depends on the concepts of welfare adopted by the governments in the first stage. The next section will investigate such issues.

5. Determinations of Optimal FDI Subsidies in the First Stage

In the first stage, the home and foreign governments determine the optimal levels of their FDI subsidies so as to maximize their respective welfare. Therefore, it is essentially important what kinds of welfare concepts are adopted when the governments determine the optimal FDI subsidies. This section will adopt two kinds of welfare concept, that is, one based on ordinal social surplus and the other based on extended social surplus including assessments for labor employment. 7

Since FDIs are expected to affect labor employments, it is quite natural that the
governments consider such effects on employment when determining the optimal FDI subsidies. Particularly, in countries suffering large unemployment, FDIs are regarded as ways to modify such unemployment. This section will first analyze the optimal FDI subsidy determinations based on the two concepts of welfare mentioned above, and then it will examine the differences between optimal FDI subsidies based on these two welfare concepts.

5.1 Optimal FDI Subsidies without Labor Assessment

The net welfare, $W$, of the home country that is based on the ordinal social surplus is defined as:

$$W = \left( \int_0^Z p(\theta) d\theta - p(Z)Z \right) + \Pi - sk - s_k^*, \quad (19)$$

where the first term, $\left( \int_0^Z p(\theta) d\theta - p(Z)Z \right)$, is the home consumer's surplus, the second term, $\Pi$, is the home firm's profit, the third term, $sk$, is the home out-flow FDI subsidy payment, and the last term, $s_k^*$, is the home in-flow FDI subsidy payment. Suppose that the home government knows all features of these terms and chooses the home out-flow and in-flow subsidies, $s$ and $s_k$, so as to maximize the home net welfare defined by (19) in the first stage.

Taking into consideration that (17) and (18) hold at the industry equilibrium in the second stage, the first-order conditions for maximizing the home net welfare in the first stage are given by
\[
\frac{\partial W}{\partial s} = - \frac{\partial k}{\partial s} = 0. \tag{20}
\]

and
\[
\frac{\partial W}{\partial s_k} = - \frac{\partial k^*}{\partial s_k} \left( p'(Z)(r^* + y^*) \frac{\partial Z}{\partial k^*} + \frac{s^*}{\varepsilon} + s_k \right) = 0, \tag{21}
\]

where \( \varepsilon \) is the elasticity of the home in-flow FDI subsidy for the foreign firm's FDI. Then, assuming, for simplicity, that the second-order conditions for maximizing the home net welfare are satisfied in the industry equilibrium in the first stage, the optimal home out-flow and in-flow subsidies, \( s^* \) and \( s_k^* \), are given respectively by solving the first-order conditions given by (20) and (21). Hence, considering that (17), (18), (20) and (21) hold at the equilibrium, the optimal levels of the home out-flow and in-flow FDI subsidies are given respectively by:

\[
s^* = 0, \quad \text{and} \quad s_k^* = - \frac{p'(Z)(r^* + y^*) \frac{\partial Z}{\partial k^*}}{1 + \frac{1}{\varepsilon}} > 0. \tag{22}
\]

Simultaneously, similar reasoning is applied to the optimal foreign FDI subsidy determinations since the determinations of optimal home and foreign FDI subsidies are symmetrical. Hence, denoting respectively the optimal foreign out-flow and in-flow subsidies as \( s'^{**} \) and \( s_k'^{**} \), one can present:

\[
s'^{**} = 0, \quad \text{and} \quad s_k'^{**} = - \frac{p'^{*}(Z^*)(r + y)^* \frac{\partial Z^*}{\partial k^*}}{1 + \frac{1}{\varepsilon^*}} > 0, \tag{23}
\]
where $\varepsilon^* = \left( \frac{s^*_k}{k} \left( \frac{\partial k^*}{\partial s^*_k} \right) \right)$ is the elasticity of the foreign in-flow FDI subsidy for the home firm's FDI.

Thus, (22) and (23) are paraphrased as the next proposition:

Proposition 3: When governments maximize the net welfare based on the ordinal social surplus, while the optimal home and foreign out-flow FDI subsidies are both zero, the optimal home and foreign in-flow FDI subsidies are both positive.

The asymmetry between the optimal out-flow and in-flow FDI subsidies stems from the asymmetry between the effects of changes in the out-flow and in-flow FDI subsidies on the firms' FDIs (see proposition 2). While a change in $s$ or $s^*_k$ affects $k$ positively, but does not affect $k^*$, a change in $s^*$ or $s^*_k$ affects $k^*$ positively, but does not affect $k$. Moreover, it is shown from proposition 3 that the optimal FDI subsidy for both governments is to give nothing to the out-flow FDIs (laissez-faire) and to give the positive subsidy to the in-flow FDIs when they do not assess labor employment.

5.2 Optimal FDI Subsidies Considering Labor Assessment

In the previous subsection, the governments decide the optimal FDI subsidies without considering labor employment. However, if there is unemployment, the governments give particular assessments to labor employments as well as the social surplus of the industry when they decide the optimal FDI subsidies. This subsection will examine the optimal FDI subsidy choices by the governments in such a case.

The home total labor employment of the industry in question is sum of labor, $L$, 

22
employed by the home parent plant and labor, \( l \). employed by the foreign subsidiary plant. Then, assuming that labor employments by these plants depend on their outputs, \( X + T \), and \( Y^* \), respectively, \( L \) and \( l \) are given respectively by the following employment functions:

\[
L = L(X + T) \text{ with } L'(X + T) > 0, \text{ and } l = l(Y^*) \text{ with } l'(Y^*)
\]  

(24)

Then, the net welfare, \( W_L \), of the home country when the home government considers employment assessments as well as the social surplus in the industry is given by:

\[
W_L = \left( \int_0^Z p(\theta) d\theta - p(Z)Z \right) + \Pi - sk - s_k k^* + w \left( L(X + T) + l(Y^*) \right),
\]  

(25)

where \( w \) denotes the home wage rate that is assumed constant because unemployment is high or the labor market is perfectly competitive. Though the difference between the net home welfares defined by (19) and (25) depends on whether or not they include the labor assessment, the net home welfare given by (25) seems to be more realistic in the present world where unemployment is one of significant economic issue to be solved.

Similar to the previous subsection, assuming that the second-order conditions are all satisfied, the optimal out-flow and in-flow FDI subsidies determined by the home government are obtained by solving the following first-order conditions:

\[
\frac{\partial W}{\partial s} = - s \frac{\partial k}{\partial s} + w L'(X + T) \frac{\partial T}{\partial k} \frac{\partial k}{\partial s} = 0,
\]  

(26)
\[
\frac{\partial W}{\partial s_k} = - \frac{\partial k^*}{\partial s_k} \left( p \left( Z \left( T^* + Y^* \right) \right) \frac{\partial Z}{\partial k^*} + \frac{s_k}{e} + s_k \right)
- w \left( L \left( X + T \right) \frac{\partial X}{\partial k^*} + l \left( Y^* \right) \frac{\partial Y^*}{\partial k^*} \right) = 0.
\]

Therefore, under the conditions of (17), (18) and (24), the optimal home out-flow FDI subsidy, \( s_{L^*} \), is derived as:

\[
s_{L^*} = w L \left( X + T \right) \frac{\partial T}{\partial k} < 0,
\]

and, the optimal home in-flow DI subsidy, \( s_{L_h}^* \), is given by:

\[
s_{L_h}^* = - \frac{p \left( Z \left( T^* + Y^* \right) \right) \frac{\partial Z}{\partial k^*} - w \left( L \left( X + T \right) \frac{\partial X}{\partial k^*} + l \left( Y^* \right) \frac{\partial Y^*}{\partial k^*} \right)}{1 + \frac{1}{e}}.
\]

Clearly, the sign of \( s_{L_h}^* \) is ambiguous under the conditions of (17), (18) and (24), because the effect of a change in the foreign FDI subsidy on the home parent plant employment, \( L \left( X + T \right) \frac{\partial X}{\partial k^*} \), has the opposite sign to the effect of a change in the foreign FDI on the foreign subsidiary plant employment, \( l \left( Y^* \right) \frac{\partial Y^*}{\partial k^*} \). However, since the foreign FDI is generally regarded as a factor that increases labor employment in the home country, it is very plausible to assume that the absolute value of the former is smaller than that of the latter. Therefore, \( s_{L_h}^* \) is positive and larger than \( s_{L^*} \) in the previous subsection.

Since the arguments mentioned above are also applied to the determination of the optimal foreign FDI subsidies, the optimal foreign out-flow and in-flow FDI...
subsidies, $s_{L^*}^*$ and $s_{I^*}^*$, are given by attaching asterisks to variables with no asterisks and removing asterisks from variables with asterisks in (28) and (29). Thus, we have:

$$s_{L^*}^* = w^* L^* (X^* + T^*) \frac{\partial T^*}{\partial k^*} < 0,$$

$$s_{I^*}^* = -\frac{p^* (Z^* T + Y) \frac{\partial Z^*}{\partial k} - w^* \left[ L^* (X^* + T^*) \frac{\partial X^*}{\partial k} + i^* (Y) \frac{\partial Y}{\partial k} \right]}{1 + \frac{1}{e^*}} > 0,$$

where $w^*$ is the wage rate in the foreign country.

Consequently, (28), (29), (30) and (31) combine to present:

Proposition 4. When governments consider employment as well as the total surplus in their own countries, both of the optimal home and foreign out-flow FDI subsidies are always negative, but both of the optimal home and foreign in-flow FDI subsidies are positive and larger than those without considering employment.

The asymmetry between optimal out-flow and in-flow FDI subsidies also holds in this case, as in the previous subsection. However, while proposition 3 demonstrates that the optimal policies for the governments are not to intervene in the out-flow FDIs with the FDI subsidies and to give the positive subsidies to the in-flow FDIs when the government does not consider employment, proposition 4 indicates that the optimal policies for the governments are to impose tax on the out-flow FDIs and to give the larger positive FDI subsidies to the in-flow FDIs when the government considers
employment as well as social surplus.

It is also shown, from these propositions 3 and 4, that governments generally tend to reinforce intervention in the FDI flows when they consider employment than when they do not. While the optimal out-flow FDI subsidies are zero in the case without employment assessments, they are negative in the case with employment assessments. And, the optimal in-flow FDI subsidies are larger in the latter case than in the former case. As a result, the differences between out-flow and in-flow FDI subsidies become larger in the latter case than in the former. Therefore, in the case where the governments emphasize labor employment in their countries, they will provide FDI subsidies that promote the in-flow FDI and restrain the out-flow FDI.

One theoretical suggestion derived from propositions 3 and 4 is that, if the government adopts the FDI subsidy policies supported by proposition 3, it considers the net welfare defined as (19), but if it provides the FDI subsidies supported by proposition 4, it adopts the net welfare given by (25). Though it is impossible to make a definite decision as to whether the governments should adopt the FDI subsidy policies supported by proposition 3 or 4, it seems that, these days, many governments adopt the FDI subsidy policies supported by proposition 4. If this is so, one can say that the net welfare defined by (25) is more plausible as the objective function of the government.
6. Concluding Remarks

Establishing a generalized ICTF model of an international Cournot industry where a home firm and a foreign firm are both multinationals, this paper has investigated the effectiveness of FDI subsidies as strategic policies and the optimal levels of FDI subsidies. It has assumed that while their parent plant sizes both are exogenously fixed, their subsidiary plant sizes both are endogenously determined by their FDIs. It has also assumed that goods produced by their parent plants are supplied to both the home and foreign countries, while goods produced by their subsidiary plants are sold only in their rivals' countries.

As a result, this paper has presented several interesting results. The first is that the government can use FDI subsidies as strategic policies by appropriate mixing of the out-flow and in-flow FDI subsidies. The second is that while the optimal out-flow FDI subsidy is zero when the government maximizes an ordinal welfare excluding employment assessment, its optimal level becomes negative when the government adopts an extended welfare that includes employment assessment. The last is that the optimal in-flow FDI subsidy is always positive, not only when the government adopts an ordinal welfare without labor assessment but also when it adopts an extended welfare with labor assessment, but the optimal in-flow subsidy is larger in the latter case than in the former.

However, when the firms can endogenously determine their subsidiary plant sizes, while the construction costs of these plants are fixed when the firms choose their
output-export levels in the third stage, these costs are variable when the firms determine their FDIs in the second stage. That is, firms determine plant construction costs endogenously by changing their FDI levels in the second stage. Therefore, it is also shown that when the firms can vary smoothly their FDI levels for changes in the FDI subsidies, any changes in the FDI subsidies don’t have such drastic effects, as is indicated by Markusen, Morey and Olewiler (1993 and 1995), Motta and Thisse (1994), and Hoel (1997).  

Finally, the following point should be emphasized. In the actual world, governments in some countries suffering from labor unemployment regard the effects of the FDI subsidies on the labor employment as being just as significant as those of the FDI subsidies on the social surplus. In such countries, as the government’s objective the extended welfare considering employment assessments becomes more plausible than the traditional welfare concept without labor assessments. Then, as is shown by the proposition 4, government interventions of the negative out-flow FDI subsidy (= the out-flow FDI tax) and of the positive in-flow FDI subsidy are always welcomed by the residents of countries with labor unemployment. As a result, there is a possibility that the international economy will be far from free, contrary to the findings of Janeba (1998) and Ishii (2001) that the world economy approaches to a free economy when firms can freely construct their plants, even in their rivals’ countries.

There are several ways to extend and/or generalize the ICTF model in this paper.
The present model assumes a homogenous goods, fixed parent plant size, three-stage game of a Cournot industry. Moreover, it does not consider environmental effects and other issues such as transfer prices, technological spillovers and so on, which are specific to multinationals. A model considering these aspects might lead to the modification of propositions presented in this paper.
Appendix

Derivation of (11) and (12)

Here, we will show only the derivation of (12), since that of (11) is obtained by exchanging variables with and without asterisks. The effects of a change in $k^*$ on $X^*$, $T^*$ and $Y^*$ are obtained by differentiating totally equations (5), (6) and (1) with respect to $k^*$:

\[
\begin{pmatrix}
\Delta_{11} & \Delta_{12} & \Delta_{13} \\
\Delta_{21} & \Delta_{22} & \Delta_{23} \\
\Delta_{31} & \Delta_{32} & \Delta_{33}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial T^*}{\partial k^*} \\
\frac{\partial Y^*}{\partial k^*} \\
\frac{\partial X}{\partial k^*}
\end{pmatrix} = \begin{pmatrix} 0 \\ c^*(k^*) \\ 0 \end{pmatrix},
\]

(A1)

where $\Delta_{11} = p^*(Z)(T^* + Y^*) + 2p^*(Z) - G^*(T^*)$, $\Delta_{12} = \Delta_{21} = \Delta_{22} = p^*(Z)(T^* + Y^*) + 2p^*(Z)$, $\Delta_{13} = \Delta_{23} = p^*(Z)(T^* + Y^*) + p^*(Z)$, $\Delta_{31} = \Delta_{32} = p^*(Z)X + p^*(Z)$, $\Delta_{33} = p^*(Z)X + 2p^*(Z)$ and $\Delta = -G^*(T^*)p^*(Z)(p^*(Z) + 3p^*(Z)) < 0$.

Then, obtaining $\frac{\partial T^*}{\partial k^*}$, $\frac{\partial Y^*}{\partial k^*}$ and $\frac{\partial X}{\partial k^*}$ from (A1) and substituting all features of demand functions and cost functions into the results, one gets (12):

\[
\frac{\partial T^*}{\partial k^*} = -\frac{c^*(k^*)p^*(Z)(p^*(Z)Z + 3p^*(Z))}{\Delta} < 0,
\]

\[
\frac{\partial Y^*}{\partial k^*} = -\frac{c^*(k^*)G^*(T^*)(p^*(Z)Z + 2p^*(Z))}{\Delta} - \frac{\partial T^*}{\partial k^*} > 0,
\]

\[
\frac{\partial X}{\partial k^*} = \frac{c^*(k^*)G^*(T^*)(p^*(Z)Z + p^*(Z))}{\Delta} < 0.
\]
Footnotes

1. Markusen, Edward and O'Leuiler (1993 and 1995), Motta and Thisse (1994), and Hoel (1997) have also considered the firms' endogenous plant locations in their papers analyzing the optimal environmental pollution policies. However, they have assumed implicitly that the production plant sizes of the firms are exogenously given and constant. On the other hand, Bond and Samuelson (1989), Bucovetsky and Wilson (1991) and Gordon (1992) have examined the effects of capital income in perfectly competitive markets.

2. The present model also depends on Motta and Thisse (1994), Brander and Spencer (1987) and Hoel (1997). This paper assumes, as has been assumed by Motta and Thisse, that both of the home and foreign firms have already established parent plants in their own countries when the game begins. Further, it adopts not only a traditional concept of welfare, which excludes employment assessments, but also an extended concept of welfare that includes employment assessments that are considered by Brander and Spencer (1984), and Hoel (1997).

3. Motta and Thisse (1994) have considered a model where firms are already established in their mother country when the game begins, though their purpose is to discuss the optimal environmental taxes.

4. It has often been indicated that the average export cost might decrease in some cases. However, the increasing marginal export cost is compatible with this indication when the export cost includes a fixed part. For example, when the
total export cost $G$ consists of a fixed cost $F$ and a variable cost $h(T)$, the average export cost $AT$ is given by $AT = G/T = (F + h(T))/T$. Then, differentiating this with respect to $T$, we have $AT' = (h'(T)T - F - h(T))/T^2$.

Hence, $AT'$ is always negative in the case when a fixed cost $F$ is large enough. Furthermore, it is shown in this paper, following the Routh theorem, that $G'(T) > 0$ and $G''(T^*) > 0$ are necessary to ensure the second-order conditions for maximizing firms' profits and the local stability conditions of the industry equilibrium in the third stage.

5 For example, the U.A. Toyota sells almost all its products in the U.S.A.

6. When the firms' subsidiary plant sizes are both exogenously fixed, the present ICTF model reduces to the ICTF model exploited by Ishii (2001).

7. Brander and Spencer (1987) and Hoel (1997) have also adopted a concept of welfare that includes labor assessment.

8. Though Markusen, Morey and Olewiler (1993 and 1995), Motta and Thisse (1994), and Hoel (1997) have established models in which the firms construct plants in different countries or regions, they considered only taxes imposed on firms' outputs, but not taxes imposed on the plant constructions as anti-pollution policies. However, the analysis in this paper demonstrates that FDI subsidies or taxes are also effective as anti-pollution policies.
References


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