早稲田大学現代政治経済研究所

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Naohiro Takahashi
Waseda University, Tokyo

No.E1404

Working Paper Series

Institute for Research in
Contemporary Political and Economic Affairs

Waseda University
169-8050 Tokyo,Japan

A Simple Model for the Damage Caused by Ungrounded Rumors

Naohiro Takahashi[*]
Waseda University, Tokyo
E-mail: naohiro_t@fuji.waseda.jp

May 28, 2014

Abstract

"*Fuhyo-Higai*" is modeled as a result of social learning where Bayes-rational consumers can wait and see how others behave in order to learn from it before making their own decision. The social learning model, allowing heterogeneous beliefs in the quality of a good and endogenous timing of each purchase, suggests that *Fuhyo-Higai* can be mitigated by less risk-aversion of consumers or less heterogeneity of beliefs, which makes rational consumers better off. Furthermore, it gives us another suggestion: as it is expected that more consumers behave irrationally, *Fuhyo-Higai* will be relieved.

1. Introduction

This paper primarily explores what sparks off *Fuhyo-Higai*, where people seem irrationally to forgo their purchase. It may be that: a wild rumor gains currency, and a frantic rush occurs among some individuals to avoid a hazard. Then the rest may develop a collective mind which makes them follow the hysterical herd without rationally thinking about what they really want to do. But is that always true? Our answer is "Not necessarily". Our model, assuming rational risk-averse consumers, suggests collective *rational* behavior gives rise to *Fuhyo-Higai*. In the wake of a risky event, rational consumers would rather "wait and see what others do" than commit themselves because a choice of actions taken by others is worth evaluating to see how risky the situation might be. Once we look upon *Fuhyo-Higai* as such a mechanism of rational decision-making, we can delineate several intriguing, constructive ideas. Most interesting is that irrational people, who always forgo their purchase regardless of risk, not necessarily aggravate *Fuhyo-Higai*; on the

[*] The author would like to thank Hideki Konishi for helpful suggestions and comments.

contrary, the expectation that more consumers are irrational could encourage more rational people to buy a good immediately, for rational people come to rely less on learning from a choice of actions taken by others. Another implication, which is less intriguing but might be more important in political terms, is that less risk-aversion or less heterogeneous beliefs can moderate *Fuhyo-Higai*.

What is *Fuhyo-Higai*? When a society faces a risk event, its government primarily has to design public policies to reduce the risk level to its adequate level, at which almost no one is adversely affected by a hazard. Even though the risk is successfully lowered by the policies to its permissible level, however, people at times keep showing strong concern and shy away from the risk. Such a case will impose enormous costs to the society, in the forms of less trade or consumption of even safety-secured goods, or stricter (often too strict) regulation to respond to social repercussions.

The aforementioned case is usually called in Japan *Fuhyo-Higai*.[1] Sekiya (2011) gives a definition for the word:

- the economic damage sparked by the phenomenon of people giving up consumption, sightseeing, or trade because they view as dangerous an essentially safety-secured thing – a food, a product, an area, or a firm

Because *Fuhyo-Higai* involves uncertainty, people might want to observe other people's behavior before taking their own actions. This kind of interaction, or social learning, has been studied in psychology and economics. In the field of economics, Bikhchandani, Hirshleifer, and Welch (1992) develop a model for an *informational cascade*, where all individuals herd independent of their private information signals. In the model, discrete signals and the exogenous order of individual's decision making assumed, the public belief, built by the history of observing actions of others, dominates an individual's private belief with a positive probability. Then it is optimal that he just follows the behavior of the preceding individual, disregarding his own private information. A generalization of the model is made by Smith and Sørensen (2000), with signals distributed continuously. Basing their model on a Markov process for social learning and a martingale for the likelihood ratio of public belief conditional on the state of nature, they verify that the belief converges almost surely to a fixed point, meaning "*a herd always occurs eventually* (Chamley 2004)."

[1] The oil spill from a Russian tanker Nakhodka in January 1997, the radiation leak at a nuclear power plant in Tokaimura, Ibaraki in September 1999, the outbreaks of BSE (Bovine Spongeform Encephalopathy), Avian Influenza, and SARS (Severe Acute Respiratory Syndrome) in the 2000s and the disaster of a Fukushima nuclear plant in March 2011 are, to name but a few, said to cause *Fuhyo-Higai* in Japan.

Drawing upon these discussions about social learning, we set up a simple theoretical model for *Fuhyo-Higai*, in order to speculate about its process and causes. The remainder of the paper is structured as follows. Section 2 formularizes a decision-making problem of consumers to give a common framework for our analysis. Section 3 presents a one-period model for consumers' decision making without social learning. Section 4 extends the model to a two-period model which allows consumers to choose to delay their purchasing decision to period 2 and observe an action taken by others. We are intrigued by the questions: "Are consumers better off or worse off by their waiting?" "What causes more consumers to wait?" "Can we find a way to mitigate *Fuhyo-Higai*?" Section 5 discusses these questions. Section 6 concludes the paper.

2. The Framework for our Model

Suppose that consumers are making a decision about whether or not to buy a good whose quality, θ, is uncertain. θ is the realization of a normal random variable with the mean zero and the variance $1/\rho_\theta$. Once it is realized, the value of θ is assumed to be invariable. Prior to the decision, consumer $i \in \{1, \cdots, N\}$ receives one signal s_i once and for all throughout our model. The signal is conditional on θ with a noise, which is independent across consumers and identically normally distributed, and independent of θ:

$$s_i = \theta + \varepsilon_i \qquad with\ \varepsilon_i \sim N\left(0,\ \ \frac{1}{\rho_\varepsilon}\right). \tag{2.1}$$

Note that ρ_ε represents the precision of a signal.

On receipt of a private signal, consumer i updates her belief about the distribution of θ via Bayes's rule. Let us denote the probability density function of ε_i by $f(\varepsilon_i)$, the prior probability density function of θ by $g(\theta)$, and the conditional probability density function of θ given s_i by $g(\theta|s_i)$, where

$$g(\theta|s_i) = \frac{f(s_i - \theta)g(\theta)}{\int f(s_i - \theta)g(\theta)d\theta}. \tag{2.2}$$

As is well known, the distribution with $g(\theta|s_i)$ is normal with the mean $\rho_\varepsilon s_i/(\rho_\theta + \rho_\varepsilon)$ and the variance $1/(\rho_\theta + \rho_\varepsilon)$. Seeing from the fact that the mean is increasing in s_i with the variance unchanged, we have the following claim:

Claim 1. *The probability distribution with* $g(\theta|s_i)$ *first-order stochastically dominates the distribution with* $g(\theta|s_i')$ *iff* $s_i > s_i'$.

On the basis of the posterior belief, then, she chooses one of the two actions: buying or staying. If she chooses to buy a good, she will gain the payoff of $\pi(\theta)$; otherwise, she will gain nothing. As for the payoff function, we make the following assumption:

Assumption 1. *The payoff function,* $\pi(\theta)$*, satisfies the following properties:*

- $\pi(\theta)$ *is increasing and concave in* θ.
- $\theta\pi(\theta) > 0$ *for all* $\theta \neq 0$, *and* $\pi(\theta) = 0$ *for* $\theta = 0$.
- *Its unconditional expected value is negative, i.e.,* $\int \pi(\theta)g(\theta) < 0$.

The first property implies that consumers are risk-averse. The second indicates that events with $\theta < 0$ are risky. The third suggests that no consumer buys a good without receipt of a signal.

3. A Simple One-period Model

Assume first the case without social learning: consumer i chooses either action, "buy" or "stay", only on the basis of her private signal. Her expected payoff from "buy" of her choice is:

$$E(\pi|s_i) = \int \pi(\theta)g(\theta|s_i)d\theta, \tag{3.1}$$

and her expected payoff from "stay" of her choice is zero. An immediate consequence of Claim 1 is that $E(\pi|s_i)$ is increasing in s_i. Thus there exists a unique critical point s^* such that:

$$E(\pi|s_i) \geq 0 \ \ iff \ \ s_i \geq s^*, \tag{3.2}$$

and it leads to the following proposition:

Proposition 1. *Let* s^* *be such that* $E(\pi|s^*) = 0$. *Then consumer* i*'s optimal strategy is characterized by a threshold and is monotone in her signal such that:*

she buys a good if and only if $s_i \geq s^*$.

Regarding the sign of s^*, it could be either positive or negative. When $s_i = 0$, the posterior distribution with $g(\theta|0)$ has the same mean and a higher precision compared with the prior distribution with $g(\theta)$. Since consumer i is risk-averse, we know $\int \pi(\theta)g(\theta|0)d\theta > \int \pi(\theta)g(\theta)d\theta\,(< 0)$ but the sign of $\int \pi(\theta)g(\theta|0)d\theta$ is still ambiguous. To make our analysis simple, here we assume:

Assumption 2. *A consumer who receives a private signal of zero expects a negative payoff, i.e.,*

$$\int \pi(\theta)g(\theta|0)d\theta < 0. \tag{3.3}$$

Assumption 2 implies that $s^* > 0$. It is validated if ρ_ε is sufficiently small relative to the degree of concavity of $\pi(\theta)$.

4. A Simple Two-period Model[2]

Since most consumers are not quite certain of their own private belief with scant information, they have an incentive to get more information about the risk; they may want to wait and see how things go. Extending the informational cascade model of Bikhchandani, Hirshleifer, and Welch (1992) with the preordered actions, Chamley (2004) presents a model of the waiting game with heterogeneous beliefs and endogenous timing of investments. Each agent chooses when to invest and observes the number of investments by others in each period. In the model, a symmetric equilibrium depends on the intertemporal arbitrage between the opportunity cost of delay and the value of information gained from observation.

The setting of our model and its equilibrium analysis basically follow the model in Chamley (2004). Making use of his model, we would rather focus on: what factor affects the value of equilibrium-threshold and determines *Fuhyo-Higai*.

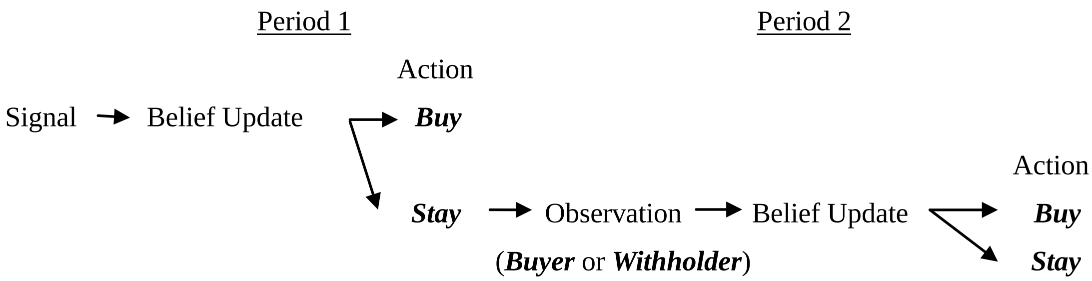
4.1 The Model

Consumer i opts for a strategy to maximize her expected payoff, where she is allowed to make at most one irrevocable purchase in either period 1 or 2 of her choice. As with the one-period model,

[2] See Appendix G for a three-period model. Basically we have the same kind of argument in a three-period model, where more consumers prefer to wait in period 1 than in a two-period model.

she receives a signal s_i at the beginning of period 1. If she chooses "buy" in period 1, her game is over at that point. If she chooses "stay" in period 1, she is given an opportunity to observe a choice of action taken by another:[3] "buy" or "stay". On the basis of the original signal and the observation, she can update her belief and make a final decision whether or not to buy a good in period 2. This social learning process is presented in Figure 1.

Figure 1. Social learning process



This game of a two-period model is characterized by the interaction of the probability of observing another consumer's past choice of action and her own choice of action in period 1.

4.2 Decision in period 2

To solve an equilibrium of the game, let us define a symmetric monotone strategy:

> ***Definition.*** **(Symmetric monotone strategy [SMS])** *Let s be the common threshold for consumer $j \in \{1, \cdots, N\}$. Consumer j buys a good immediately in period 1 if and only if $s_j \geq s$.*

We will solve the game backward. Assuming a SMS, consumer $i \neq j$ comes upon two possible events in period 2: observing an action of "buy" taken by consumer j with $s_j \geq s$ or observing an action of "stay" taken by j with $s_j < s$. In the former event, consumer i updates the density of her belief into

$$g(\theta | s_j \geq s, s_i) = \frac{[1 - F(s - \theta)]g(\theta | s_i)}{\int [1 - F(s - \theta)]g(\theta | s_i) d\theta} \tag{4.1}$$

and her expect payoff from "buy" is

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[3] We relax this assumption into more encounters in 5.5.

$$E(\pi|s_j \geq s, s_i) = \int \pi(\theta) g(\theta|s_j \geq s, s_i) d\theta. \quad (4.2)$$

Likewise in the latter, the density of her belief and her expected from "buy" are formulated respectively as

$$g(\theta|s_j < s, s_i) = \frac{F(s-\theta) g(\theta|s_i)}{\int F(s-\theta) g(\theta|s_i) d\theta} \quad (4.3)$$

and

$$E(\pi|s_j < s, s_i) = \int \pi(\theta) g(\theta|s_j < s, s_i) d\theta. \quad (4.4)$$

Notice that in either event, she opts to buy a good if and only if her expected payoff is non-negative.

With regard to these densities, we have the following claim:[4]

Claim 2. *The probability distribution with* $g(\theta|s_j \geq s, s_i)$ *first-order stochastically dominates the distribution with* $g(\theta|s_i)$, *and the probability distribution with* $g(\theta|s_i)$ *first-order stochastically dominates the distribution with* $g(\theta|s_j < s, s_i)$.

An immediate consequence of Claim 2 is the following:

$$E(\pi|s_j < s, s_i) < E(\pi|s_i) < E(\pi|s_j \geq s, s_i) \quad for\ any\ s_i. \quad (4.5)$$

With some calculation we also have the following claims:[5]

Claim 3. *The probability distribution with* $g(\theta|s_j \geq s, s_i)$ *first-order stochastically dominates the distribution with* $g(\theta|s_j \geq s, s_i')$ *iff* $s_i > s_i'$. *Likewise, the probability distribution with* $g(\theta|s_j < s, s_i)$ *first-order stochastically dominates the distribution with* $g(\theta|s_j < s, s_i')$ *iff* $s_i > s_i'$.

Claim 4. *The probability distribution with* $g(\theta|s_j \geq s, s_i)$ *first-order stochastically dominates the distribution with* $g(\theta|s_j \geq s', s_i)$ *iff* $s > s'$.

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[4] See Appendix A for its proof.

[5] See Appendix B for the proof of Claim 3, and Appendix C for the proof of Claim 4.

Now let us proceed to the necessary condition for a positive expected payoff. Consider a marginal consumer whose expected payoff is zero when she delays her decision to period 2 and encounters a past buyer. From Claim 3 and Claim 4, we find the marginal consumer's private signal has the following properties:

Lemma 1. *Given a common threshold* s, *let* $\hat{b}(s)$ *denote a function such that* $E\big(\pi\big|s_j \geq s, \hat{b}(s)\big) = 0$. *Then,* $\hat{b}(s)$ *has the following properties:*

- $\hat{b}(s)$ is unique for each s and $E\big(\pi|s_j \geq s, s_i\big) > 0$ if and only if $s_i > \hat{b}(s)$.
- $\hat{b}(s)$ is decreasing in s.[6]

Suppose that consumer i delays her decision to period 2 and encounters a consumer who made a purchase in period 1. Lemma 1 implies that she buys a good if and only if $s_i \geq \hat{b}(s)$.

4.3 Decision in period 1

Now let us go back to a decision-making in period 1. Let $Pr\big(s_j \geq s|s_i\big)$ denote the probability that consumer i encounters a consumer who made a purchase in period 1, where

$$Pr\big(s_j \geq s|s_i\big) = \int [1 - F(s-\theta)]g(\theta|s_i)d\theta. \tag{4.6}$$

By the same token, the probability that consumer i encounters a consumer who withholds a purchase in period 1 is

$$Pr\big(s_j < s|s_i\big) = \int F(s-\theta)g(\theta|s_i)d\theta. \tag{4.7}$$

In this inter-temporal decision-making, we assume a discount rate as $\delta \in (0,1)$. Then consumer i opts to buy a good in period 1 if and only if

$$\begin{aligned}\frac{1}{\delta}E(\pi|s_i) \geq\ & Pr\big(s_j \geq s|s_i\big)max\big\{E\big(\pi|s_j \geq s, s_i\big),\quad 0\big\} \\ & +Pr\big(s_j < s|s_i\big)max\big\{E\big(\pi|s_j < s, s_i\big),\quad 0\big\},\end{aligned} \tag{4.8}$$

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[6] Claim 4 implies that for any $s > s'$, we have $E\left(\pi|s_j \geq s, \hat{b}(s')\right) > E\left(\pi|s_j \geq s', \hat{b}(s')\right) = E\left(\pi|s_j \geq s, \hat{b}(s)\right) = 0$. From Claim 3, we find $\hat{b}(s') > \hat{b}(s)$, and therefore $\hat{b}(s)$ is decreasing in s.

the right hand side of which represents the expected payoff from delaying her decision to period 2.

Now let us define a threshold signal by $b(s)$ such that (4.8) holds with equality at $s_i = b(s)$. Then we can state that $b(s)$ is characterized by the following condition:[7]

$$E\left(\pi|b(s)\right) = \delta Pr\left(s_j \geq s|b(s)\right) E\left(\pi|s_j \geq s, b(s)\right) > 0, \tag{4.9}$$

which can be rephrased as:[8]

$$\int \pi(\theta) g\left(\theta|b(s)\right) d\theta = \delta \int \pi(\theta)[1 - F(s - \theta)] g\left(\theta|b(s)\right) d\theta > 0. \tag{4.10}$$

We find the following lemma for $b(s)$:

Lemma 2. *Given a common threshold s, let $b(s)$ denote a function of s satisfying (4.10). Then, $b(s)$ has the following properties:*

- $b(s)$ is unique for each s and satisfies $b(s) > s^*$.
- $b(s)$ is continuous and $lim_{s \to +\infty} b(s) = s^*$.
- $b(s)$ is decreasing in s if $b(s) \leq \frac{\rho_\theta + \rho_\varepsilon}{\rho_\varepsilon} s$.

The fact that $b(s) > s^*$ in the first statement follows from the fact that $E\left(\pi|b(s)\right) > 0 = E(\pi|s^*)$. The continuity of $b(s)$ in the second statement follows straightforwardly from the definition. As for the limiting value, the reason is as follows. As s goes to infinity, the term $\int \pi(\theta)[1 - F(s - \theta)] g\left(\theta|b(s)\right) d\theta$ approaches zero for any s_i, which implies that $b(s)$ responds such that the left hand side of (4.10) goes to zero as well. Since $\int \pi(\theta) g(\theta|s^*) d\theta = 0$, the statement follows. Regarding the uniqueness of $b(s)$ and the third statement, we need a little more involved proof, given in Appendix E. Note that the sufficient condition in third statement can be intuitively interpreted as follows: when consumer i delays her decision, she will encounter another consumer whose signal is not so bad as her own signal times $\rho_\varepsilon / (\rho_\theta + \rho_\varepsilon)$.

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[7] Notice that when $E\left(\pi|s_j < s, s_i\right) > 0$, $E\left(\pi|s_j \geq s, s_i\right) > 0$ from (4.5). Since $Pr\left(s_j \geq s|s_i\right)E\left(\pi|s_j \geq s, s_i\right) + Pr\left(s_j < s|s_i\right)E\left(\pi|s_j < s, s_i\right) = E(\pi|s_i)$, we can easily see the equality of (4.8) does not hold owing to δ. Thus, $E\left(\pi|s_j < s, b(s)\right) \leq 0$. The right hand side of (4.9) is strictly positive owing to (4.5).

[8] See Appendix D for its proof.

We also have the following proposition:[9]

Proposition 2-1. *Given a common threshold s, consumer i's optimal strategy in period 1 is a monotone strategy and is characterized by $b(s)$ such that:*

she buys a good in period 1 if and only if $s_i \geq b(s)$; otherwise, she delays her decision, and buys it in period 2 if and only if she encounters a consumer who made a purchase in period 1.

4.4 SMS Equilibrium

We will confine our analysis to a SMS equilibrium with a critical point s^{**}, in which consumer $i \in \{1, \cdots, N\}$ buys a good in period 1 if and only if $s_i \geq s^{**}$; otherwise she delays her decision. When she delays her decision, she buys it in period 2 if and only if her signal satisfies $s_i \geq \hat{b}(s^{**})$ and she encounters a consumer who made a purchase in period 1. Such a critical point is characterized by $b(s^{**}) = s^{**}$. Thus we have the following proposition:

Proposition 2-2. *Let s^{**} be an equilibrium-threshold. Then s^{**} is the solution of the arbitrage equation between opportunity cost of delay and option value of delay:*

$$\int \pi(\theta) g(\theta|s^{**}) d\theta = \delta \int \pi(\theta)[1 - F(s^{**} - \theta)] g(\theta|s^{**}) d\theta > 0, \tag{4.11}$$

where s^{**} always exists and uniquely defined as far as $s^{**} > 0$.

The equilibrium threshold s^{**} always exists and uniquely defined[10] from the fact in Lemma 2 that $b(s^{**})$ is decreasing in s^{**}.[11] We also have the following propositions for s^{**}:

Proposition 3. *s^{**} is greater than s^*, the critical point in a one-period model.*

Proposition 4. *s^{**} increases with the discount rate δ.*[12]

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[9] See Appendix E for its proof.

[10] Multiple equilibria can be attained if we assume that a consumer becomes more risk averse in period 2 than in period 1, and she expects a certain fixed amount of reward only when she buys a good in period 1, where in (4.11) L.H.S. becomes relatively flatter than R.H.S. and only L.H.S. shifts to the left. For example, suppose her payoff function in period 1 $\pi_1(\theta) = 1 - exp(-1/2\,\theta)$, and the one in period 2 $\pi_2(\theta) = 1 - exp(-2\theta)$. When she also expects a reward of 0.05 in buying a good in period 1, three distinct equilibria occur.

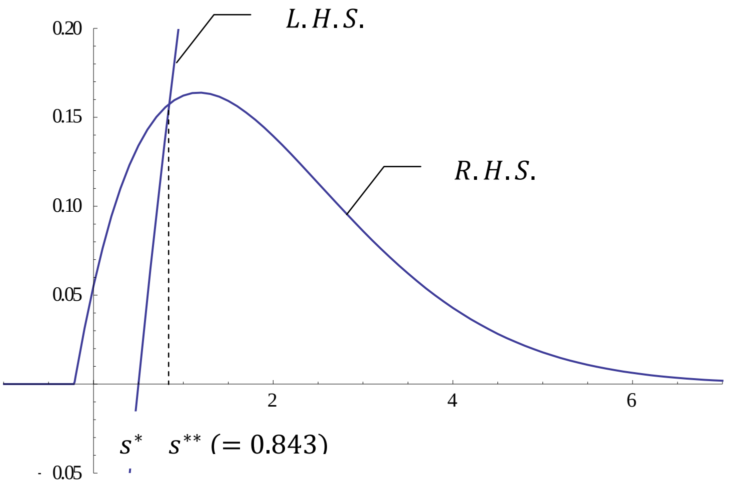
[11] Note that for $s^{**} > 0$, the inequality $b(s^{**}) < (\rho_\theta + \rho_\varepsilon)s^{**}/\rho_\varepsilon$ holds.

[12] Let $\Omega(s, s_i) = \int \pi(\theta) g(\theta|s_i) d\theta - \delta \int \pi(\theta)\,[1 - F(s - \theta)] g(\theta|s_i) d\theta$. Note that s^{**} satisfies $\Omega(s^{**}, s^{**}) = 0$. From the results of Appendix E, we have $d\Omega/ds^{**} > 0$ whenever $s^{**} > 0$. Since $\partial\Omega/\partial\delta < 0$, $ds^{**}/d\delta > 0$ holds.

Proposition 3 follows from the fact that $b(s^{**}) > s^* = lim_{s\to+\infty} b(s)$. Recall that if ρ_ε is sufficiently small relative to the degree of concavity of $\pi(\theta)$ so that $\int \pi(\theta) g(\theta|0) d\theta < 0$, as we assumed in (3.3), it is guaranteed that $s^{**} > s^* > 0$. This lemma demonstrates that the population of consumers who buy in period 1 gets smaller with the opportunity of waiting and seeing another consumer's past behavior. Proposition 4 on a comparative statics result is intuitively true because a larger discount rate leads to more option value of delay, which encourages more consumers to delay their decisions.

The decision problem is illustrated in Figure 2, here being assumed the concave function of $\pi(\theta) = 1 - exp(-\theta)$. The discount rate is assumed to be close to one, being here 0.9. It is because consumers do not care much if they miss the chance of buying now the produce (e.g. the rice produced in Fukushima), where its substitute is readily accessible (e.g. the rice produced in other areas). As is clear from Claim 1, $L.H.S.$ of (4.11), which represents the expected payoff from buying a good in period 1, is monotonically increasing. $R.H.S.$, which represents the expected payoff from delaying a decision, forms a hump shape; it turns to decreasing because while $E(\pi|s_i)$ is monotonically increasing in s_i, $b(s)$ is decreasing in s. Since for any s_i, $\int \pi(\theta)[1 - F(s - \theta)] g\big(\theta|b(s)\big) d\theta$ approaches zero as s goes up, $R.H.S.$ finally goes to zero.

Figure 2. Equilibrium-threshold



$\theta \sim N(0,1)$, $\varepsilon_i \sim N(0,1)$, $\delta = 0.9$, $\pi(\theta) = 1 - exp(-\theta)$

5. Discussions

5.1 Social Welfare Analysis

We will consider what effect a change in s^{**} has on the expected payoff to each consumer. In our two-period model, consumers fall into three groups: those with $s_i \geq s^{**}$, called type 1, who buy a good in period 1, those with $s_i \in \left[\hat{b}(s^{**}), s^{**}\right]$, called type 2, who withhold their purchase and buy a good in period 2 when having a chance encounter with a past buyer, and those with $s_i < \hat{b}(s^{**})$, called type 3, who will never buy a good. Needless to say, the expected payoffs to type 1 and type 3 are not affected by a change in s^{**}, and thus we will focus on type 2 consumers.

The expected payoff to a type 2 consumer i, denoted by $E_2(s^{**}, s_i)$, is

$$E_2(s^{**}, s_i) = Pr\left(s_j \geq s^{**} | s_i\right) E\left(\pi | s_j \geq s^{**}, s_i\right) = \int \pi(\theta)[1 - F(s^{**} - \theta)] g(\theta | s_i) d\theta, \quad (5.1)$$

where $s_i \in \left[\hat{b}(s^{**}), s^{**}\right]$. As s^{**} increases, the probability of encountering a past buyer decreases, whereby $E_2(s^{**}, s_i)$ decreases. On the other hand, given that she encounters a past buyer, the rise in s^{**} increases her conditional expected payoff, whereby $E_2(s^{**}, s_i)$ increases. In this way, a higher s^{**} has the two contradicting effects on her expected payoff, but, as we show below, the former negative effect outruns the latter positive effect.

Proposition 5. *Consumers with* $s_i \in \left[\hat{b}(s^{**}), s^{**}\right]$, *who delay their decision and buy a good when meeting a past buyer, are worse off in response to an increase in* s^{**}.

(proof)

Differentiating $E_2(s^{**}, s_i)$ with respect to s^{**} yields

$$\frac{\partial E_2(s^{**}, s_i)}{\partial s^{**}} = -\int \pi(\theta) f(s^{**} - \theta) g(\theta | s_i) d\theta < 0, \quad (5.2)$$

where, as shown in Appendix E in detail, the inequality holds for type 2 consumers taking $s = s^{**}$. ■

In the context of *Fuhyo-Higai*, the increase in s^{**} makes worse off not only type 2 consumers, but also producers even more devastatingly. The discount rate of producers is generally far smaller than consumers, for if it leaves unsold, a good, especially perishables, may have to be dumped away, which allows us to assume their discount rate is close to zero. It suggests that producers, whose payoff is approximately proportional to the number of goods sold in period 1, are likely to be much worse off by consumers delaying their decisions.

5.2 Risk-aversion

Risk perceptions vary from hazard to hazard. If a consumer confronts a risk caused by fatal or unknown hazards (e.g. carcinogen, radiation, genetically modified organisms) or involved with vivid symptoms (e.g. "Mad-cow" disease, Ebola "hemorrhagic" fever), she may emotionally fear the risk. Conversely, she might perceive a risk as lower if the risk event is of such common occurrence that she is accustomed to facing the risk (e.g. car accidents, food poisoning from eggs or mushrooms). In either way, risk perceptions seem to exert a profound influence on decision-making. Lowenstein et al. (2001) states in their "risk-as-feelings" hypothesis that decision-making under uncertainty is likely to have an *all or none* characteristic that may be sensitive to the *possibility* rather than the *probability* of negative consequences.

Based on the hypothesis, we regard the risk perception as independent of the probability involved in the quality of a good; in the simplest way, a consumer's risk perception can be explained by her risk-bearing attitude. If a hazard appears fatal, unknown, or vivid, she will become more risk-averse. However, if the hazard is articulated based on facts, which allows her to compare it with other hazards and turn her eyes to the probability, she may become less risk-averse; that is, her payoff function becomes less concave. For simplicity, let us assume here we deal with a small group of consumers, among whom their risk-aversion remains homogeneous but could be changeable simultaneously by the same degree. Risk-aversion can be defined in terms of their payoff functions as follows: Suppose consumers modify their payoff function, $\pi_1(\theta)$, into $\pi_2(\theta)$. The statement that consumers become more risk-averse is equivalent to the statement that we can find an increasing, concave function $\psi(\cdot)$ such that $\pi_2(\theta) = \psi\big(\pi_1(\theta)\big)$. Then we have the following proposition:

Proposition 6. *As consumers become less risk-averse, s^{**} decreases.*

(proof)

Let us define

$$\Omega(s, s_i) = \int \pi(\theta) g(\theta|s_i) d\theta - \delta \int \pi(\theta)\,[1 - F(s-\theta)] g(\theta|s_i) d\theta$$

$$= \int \pi(\theta)\{1 - \delta[1 - F(s-\theta)]\} g(\theta|s_i) d\theta. \qquad (5.3)$$

Suppose that consumers become more risk-averse, modifying their payoff function $\pi_1(\theta)$ into $\pi_2(\theta)$. Then, we find the equilibrium-thresholds s_1^{**} and s_2^{**} for $\pi_1(\theta)$ and $\pi_2(\theta)$ respectively such that:

$$\Omega_1(s_1^{**}, s_1^{**}) = \int \pi_1(\theta)\{1-\delta[1-F(s_1^{**}-\theta)]\}g(\theta|s_1^{**})d\theta = 0, \tag{5.4}$$

$$\Omega_2(s_2^{**}, s_2^{**}) = \int \pi_2(\theta)\{1-\delta[1-F(s_2^{**}-\theta)]\}g(\theta|s_2^{**})d\theta = 0. \tag{5.5}$$

We want to show that $\Omega_2(s_1^{**}, s_1^{**}) < 0$, which must be followed by $s_1^{**} < s_2^{**}$ owing to $\Omega_2(s^{**}, s^{**})$ is increasing in s^{**}. With ψ, $\Omega_2(s_1^{**}, s_1^{**})$ can be described as

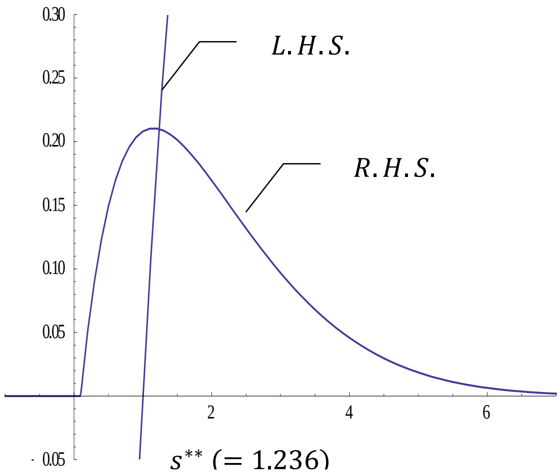
$$\Omega_2(s_1^{**}, s_1^{**}) = \int \psi(\pi_1(\theta))\{1-\delta[1-F(s_1^{**}-\theta)]\}g(\theta|s_1^{**})d\theta. \tag{5.6}$$

Since $\psi(\cdot)$ is an increasing, concave function and $\psi(0)=0$ owing to our assumption that $\pi_1(0)=\pi_2(0)=0$, we have $\Omega_2(s_1^{**}, s_1^{**}) < \Omega_1(s_1^{**}, s_1^{**}) = 0$. ■

Intuitively speaking, delaying a purchasing decision to period 2 is safer than making a purchase in period 1 because the former enables a consumer to eliminate the worst case, where she buys a good if she encounters a consumer who did not buy it in period 1. Thus, a more risk-averse consumer prefers delaying her decision to period 2.

Figure 3 (1) and (2) depict the fact. Notice that in the representation of $\pi(\theta) = 1 - exp(-a\theta)$ for $a > 0$, it is well-known that the Arrow-Pratt coefficient of absolute risk aversion at θ, denoted by $r_A(\theta,\pi)$, is equal to a for all θ. As the coefficient goes down, $L.H.S.$, which is the riskier, shifts to the left whereas $R.H.S.$, which is the safer, becomes smaller. The resultant effect makes the equilibrium-threshold go down.

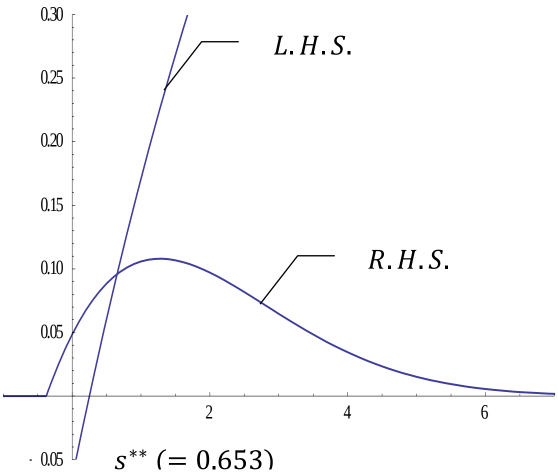
Figure 3 (1). $r_A(\theta,\pi) = 2.0$



$\theta \sim N(0,1), \varepsilon_i \sim N(0,1), \delta = 0.9$

$\pi(\theta) = 1 - exp(-2\theta)$

Figure 3 (2). $r_A(\theta,\pi) = 0.5$



$\theta \sim N(0,1), \varepsilon_i \sim N(0,1), \delta = 0.9$

$\pi(\theta) = 1 - exp(-0.5\theta)$

5.3 Precision of a signal

Precision of a signal received by a consumer partly depends on her way to gain the relevant information or her attitude toward the source of information. After a risk event occurred, she may vigorously read the reference books or watch the relevant news from various sources so that she can get a handle on the risk. This may lead to a smaller noise.[13] Or she may be rather lazy or indifferent and swallow whole a story she has heard by chance, which could result in a larger noise. The degree of trust in the source of information may also constitute a noise. If she does not trust the source, she tends to add a larger noise to the signal received by her than another who receives the same signal but trusts the source more. In this way, her level of knowledge about the risk and her degree of trust in the sender of information will affect her "subjective probability" of the risk event.

What if a group of consumers somehow change their attitude toward the way of acquiring information or the source of the information so that they gain a more accurate signal, or they construct less heterogeneous beliefs? Assuming again a small group of consumers, who could change their common attitude simultaneously by the same degree, we have the following proposition:

Proposition 7. *As precision of a signal becomes higher,* s^{**} *decreases.*

(proof)

Let $F_L(\cdot)$ and $F_H(\cdot)$ denote the distribution functions of the noises with precision ρ_L and ρ_H, and $g_L(\theta|s_i)$ and $g_H(\theta|s_i)$ denote the conditional probability density functions given s_i with ρ_L and ρ_H, respectively, where $\rho_L < \rho_H$. Then we can describe equilibrium thresholds as:

$$\Omega_{\mathrm{L}}(s_L^{**}, s_L^{**}) = \int \pi(\theta) g_L(\theta|s_L^{**}) d\theta - \delta \int \pi(\theta)\,[1 - F_L(s_L^{**} - \theta)] g_L(\theta|s_L^{**}) d\theta = 0, \quad (5.7)$$

$$\Omega_{\mathrm{H}}(s_H^{**}, s_H^{**}) = \int \pi(\theta) g_H(\theta|s_H^{**}) d\theta - \delta \int \pi(\theta)\,[1 - F_H(s_H^{**} - \theta)] g_H(\theta|s_H^{**}) d\theta = 0. \quad (5.8)$$

Consider $\Omega_{\mathrm{H}}(s_L^{**}, s_L^{**})$ such that:

$$\Omega_{\mathrm{H}}(s_L^{**}, s_L^{**}) = \int \pi(\theta) g_H(\theta|s_L^{**}) d\theta - \delta \int \pi(\theta)\,[1 - F_H(s_L^{**} - \theta)] g_H(\theta|s_L^{**}) d\theta. \quad (5.9)$$

We want to show that $\Omega_{\mathrm{H}}(s_L^{**}, s_L^{**}) > 0$, which must be followed by $s_L^{**} > s_H^{**}$ owing to

[13] Receiving more relevant news might not necessarily guarantee a smaller noise. Signals from the mass media are not only factual but also inferential, value-attached, symbolic, and sensational. If most of the media are identically biased, they could rather amplify a noise. Here we assume that the bias is normally distributed. We put aside this significant issue, what role the media play in *Fuhyo-Higai*, for future discussion.

$\Omega_{\rm H}(s^{**}, s^{**})$ is increasing in s^{**}. Then the following inequality holds:

$$\frac{\int \pi(\theta)\,[1 - F_H(s_L^{**} - \theta)]g_H(\theta|s_L^{**})d\theta}{\int \pi(\theta)\,[1 - F_L(s_L^{**} - \theta)]g_L(\theta|s_L^{**})d\theta} < \frac{\int \pi(\theta)g_H(\theta|s_L^{**})d\theta}{\int \pi(\theta)g_L(\theta|s_L^{**})d\theta}. \quad (5.10)$$

Since the distribution with $g_H(\theta|s_L^{**})$ has a higher mean when $s_L^{**} > 0$ and higher precision than the distribution with $g_L(\theta|s_L^{**})$, the distribution with $g_H(\theta|s_L^{**})$ first-order stochastically dominates the distribution with $g_L(\theta|s_L^{**})$. In addition, because $F_H(\cdot)$ is normal with the same mean as $F_L(\cdot)$ and higher precision than $F_L(\cdot)$, for an increasing concave function $\pi(\theta)$, $F_H(\cdot)$ second-order stochastically dominates $F_L(\cdot)$, which implies

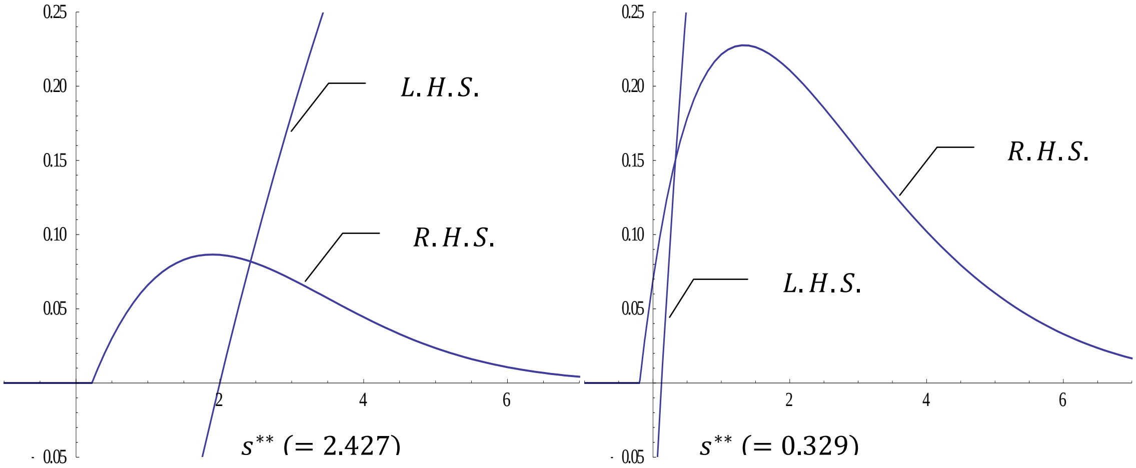
for $\theta > s_L^{**}$, $1 - F_L(s_L^{**} - \theta) > 1 - F_H(s_L^{**} - \theta)$, and

for $\theta < s_L^{**}$, $1 - F_H(s_L^{**} - \theta) > 1 - F_L(s_L^{**} - \theta)$.

They indicate that in the left hand side of (5.10), the numerator puts a heavier weight than the denominator when $\theta < s_L^{**}$, and a lighter weight when $\theta > s_L^{**}$, where $s_L^{**} > 0$. Considering the assumption that $\pi(\theta)$ is an increasing concave function such that $\theta\pi(\theta) > 0$ for all $\theta \neq 0$, (5.10) holds and $\Omega_{\rm H}(s_L^{**}, s_L^{**}) > 0$ follows. ■

As in Figure 4 (1) and (2), higher precision of a signal increases both the expected payoff from buying a good in period 1 and the expected payoff from delaying a decision; $L.H.S.$ shifts to the left and $R.H.S.$ becomes larger. On the assumption that a consumer is risk-averse and $\theta\pi(\theta) > 0$, however, the former effect outweighs the latter. By virtue of the fact, the equilibrium-threshold goes down.

Figure 4 (1). $\rho_\varepsilon = 0.5$ **Figure 4 (2).** $\rho_\varepsilon = 2.0$



$\theta \sim N(0,1), \varepsilon_i \sim N(0,2), \delta = 0.9$

$\pi(\theta) = 1 - exp(-\theta)$

$\theta \sim N(0,1), \varepsilon_i \sim N(0,0.5), \delta = 0.9$

$\pi(\theta) = 1 - exp(-\theta)$

5.4 Irrational Consumers

It is a common assumption that there exist irrational consumers, who always forgo a purchase regardless of their magnitude of private signals. In the real world, this kind of behavior is true of "zero-risk" pursuers, who try to shy away from any slight risk once they perceive it. A higher percentage of irrational consumers definitely decrease the probability that rational consumers encounter past buyers. It seems that the situation is compounded by more irrational consumers, but a higher *expected* percentage of irrational consumers moderate *Fuhyo-Higai*.

Suppose that in the wake of a risky event, a relatively large proportion of consumers behave irrationally. People do not know the proportion and have two equally likely estimates of the proportion: the right estimate, α_H, and the wrong estimate, α_L, where people underestimate the proportion; that is, they think most of them might behave rationally. Let $\bar{\alpha}$ denote the expected value, i.e., $\bar{\alpha} = E(\alpha)$. Without any relevant information on the proportion, the expected value will be what they would expect by chance: $\bar{\alpha} = (\alpha_H + \alpha_L)/2$. Suppose the government estimates the proportion correctly, by conducting a survey on consumers' attitude toward a risk, for example. If the government announces that the right estimate is actually α_H, the expected value will be such that $\bar{\alpha} = \alpha_H$. We will see how the rise in the expected value impinges on an equilibrium-threshold. Note that here is also assumed that α_H and α_L are independent of θ or ε_i.

If a consumer i comes across an irrational consumer after her choice of delay, she knows she will learn nothing about the quality of a good, and no value will be generated. Thus, given $\bar{\alpha}$, an equilibrium-threshold, denoted by s^{IR}, is defined by:

$$\int \pi(\theta) g(\theta | s^{IR}) d\theta = \delta(1 - \bar{\alpha}) \int \pi(\theta)[1 - F(s^{IR} - \theta)] g(\theta | s^{IR}) d\theta > 0. \tag{5.11}$$

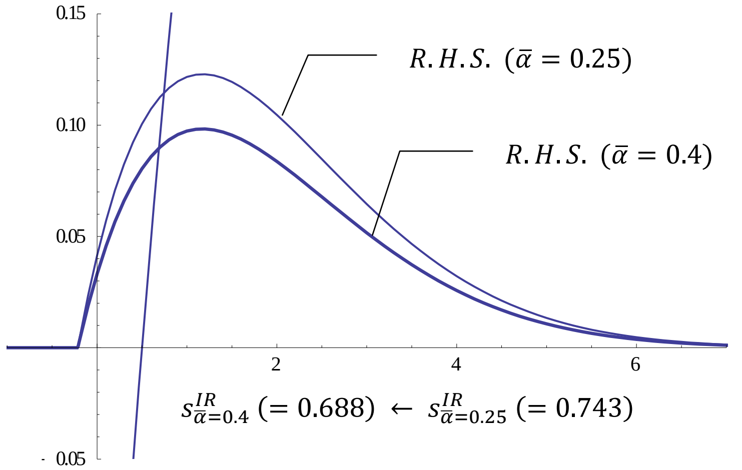
As is clear from (5.11), $R.H.S.$ of the equation becomes smaller with an increase in $\bar{\alpha}$. An immediate consequence of it is as follows:

> ***Proposition 8.*** *Suppose that there exist some irrational consumers in the population who always forgo their purchase. Then, an equilibrium-threshold decreases as the expected proportion of irrational consumers increases.*

Figure 5 illustrates the proposition. Let $\alpha_H = 0.4$ and $\alpha_L = 0.1$. Her initial expected value $\bar{\alpha}$ is 0.25. In response to the government announcement, $\bar{\alpha}$ increases to 0.4. Since her chance to encounter rational consumers, whose past action is informative on the quality of the good, decreases,

$R.H.S.$ shrinks. Consequently, the equilibrium-threshold goes down. The rise in the expected proportion of irrational consumers leads rational consumers to rely less on their observation, changing the situation for the better.

Figure 5. The change in the expected proportion of irrational consumers



$\theta \sim N(0,1),\ \varepsilon_i \sim N(0,1), \delta = 0.9,\ \pi(\theta) = 1 - exp(-\theta)$

5.5 More Encounters

So far we have assumed only one encounter in each period. This assumption might be relevant for a case in which consumers make a first-hand contact of face-to-face communication when observing an action. In reality, however, they often read a newspaper reporting, say, "According to our public opinion poll, 43 % of the consumers stopped eating the produce," where they observe a large number of actions at one time. To consider this kind of case, let us assume more encounters in each period.

Suppose a consumer encounters all other $N-1$ consumers in a two-period model. The arbitrage equation between the expected payoff from the choice of buying a good in period 1 and the expected payoff from the choice of delaying the decision will be:

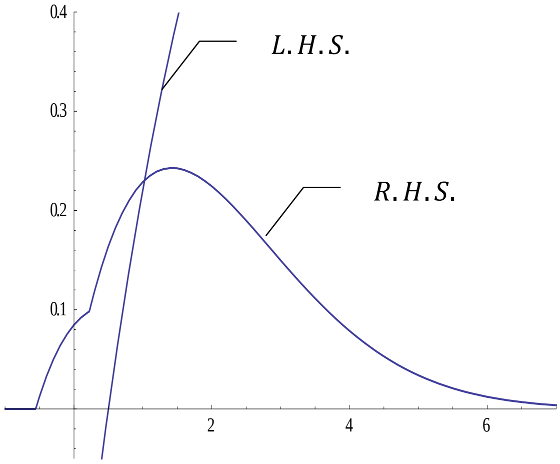
$$\int \pi(\theta) g(\theta | s^{**}(N)) d\theta$$
$$= \delta \times \sum_{n=0}^{N-1} Pr(X = n\ | s^{**}(N)) \times max\left(\int \pi(\theta) g_n(\theta | s^{**}(N)) d\theta\,,\quad 0\right), \quad (5.12)$$

where $s^{**}(N)$ is an equilibrium-threshold, X is a random variable of the population of past buyers a consumer encounters, $g_n(\theta | s^{**}(N))$ is a density of her belief given that she has encountered n past buyers, and

$$Pr(X = n|s_i) = \frac{(N-1)!}{n!\,(N-1-n)!}\int [1 - F(s^{**}(N) - \theta)]^n [F(s^{**}(N) - \theta)]^{N-1-n} g(\theta|s_i) d\theta.$$

Though it seems difficult to identify the shape of the option value of delay, $R.H.S.$ of (5.12), we can get an idea of its property. As N gets larger, waiting and seeing becomes more informative on θ, and $R.H.S.$ becomes larger relative to $L.H.S.$ As a result, the equilibrium-threshold gets higher. Figure 6 (1) and (2) exemplify the idea with two cases in which a consumer has two encounters and three encounters respectively. In the case of two encounters, a consumer with a lower signal expects a positive payoff only if she encounters two past buyers. When her signal is higher enough, she expects a positive payoff even if she encounters only one past buyer, to say nothing of the case where she encounters two past buyers. The graph of $R.H.S.$ in Figure 6 (1) has a kink at which her expected payoff from a chance of encountering only one past buyer becomes positive. Likewise, the graph of $R.H.S.$ in Figure 6 (2) has two kinks. As you can see from both figures, two encounters, compared with only one encounter shown in Figure 2, make $R.H.S.$ larger, and three encounters make $R.H.S.$ even larger. The resultant effect leads to greater equilibrium thresholds. We would say that more encounters make *Fuhyo-Higai* worse.

Figure 6 (1). Two encounters

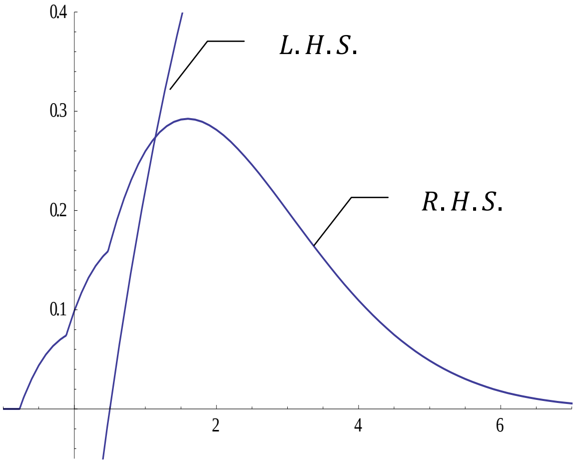


$s^{**} = 1.024$

$\theta \sim N(0,1)$, $\varepsilon_i \sim N(0,1)$, $\delta = 0.9$,

$\pi(\theta) = 1 - exp(-\theta)$

Figure 6 (2) Three encounters



$s^{**} = 1.141$

$\theta \sim N(0,1), \varepsilon_i \sim N(0,1), \delta = 0.9,$

$\pi(\theta) = 1 - exp(-\theta)$

Let us consider the extreme case where any consumer can obtain perfect information on θ and realize the true quality of a good if she delays her decision to period 2. This case can be asymptotically attained if the population $N \to \infty$, and moreover she can observe not only all $N-1$ actions of others but also their received signals are somehow revealed truthfully to her. Suppose a consumer chooses to delay her decision to period 2. Then she can find the realized value of θ at the end of period 1. Recall $\pi(\theta) = 0$ for $\theta = 0$. She will buy the good in period 2 if and only if the realized $\tilde{\theta}$ is such that $\tilde{\theta} \geq 0$. The arbitrage equation is described as follows:

$$\int_{-\infty}^{+\infty} \pi(\theta) g(\theta|s^P) d\theta = \delta \int_{0}^{+\infty} \pi(\theta) g(\theta|s^P) d\theta. \tag{5.13}$$

Let $L.H.S.^P$ denote the left hand side and $R.H.S.^P$ the right hand side of (5.13). If $\delta = 1$, $R.H.S.^P$ is larger than $L.H.S.^P$ for any s_i and

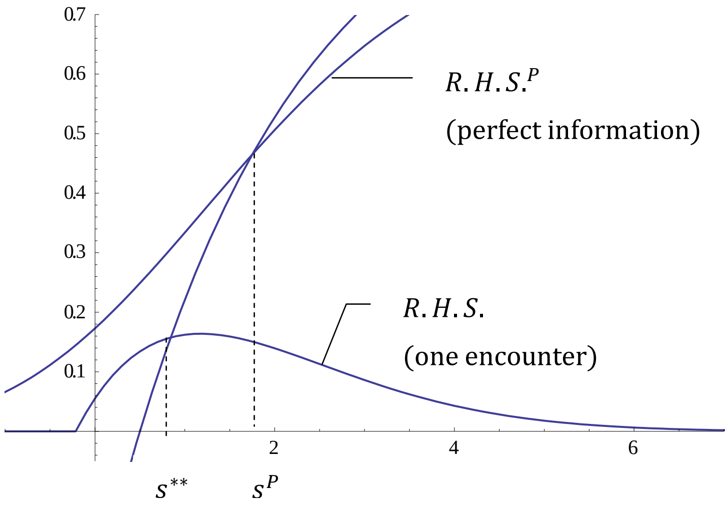
$$\lim_{s_i \to +\infty} (R.H.S.^P - L.H.S.^P) = 0. \tag{5.14}$$

Because observation gives perfect information on θ, a private signal is less effective to the $R.H.S.^P$ than $L.H.S.^P$: [14]

$$\frac{\partial L.H.S.^P}{\partial s_i} > \frac{\partial R.H.S.^P}{\partial s_i}. \tag{5.15}$$

Thus we have a unique equilibrium-threshold $s^P > s^{**}$, as shown in Figure 7.

Figure 7. Perfect information in period 2



(= 0.843) (= 1.749)

$\theta \sim N(0,1)$, $\varepsilon_i \sim N(0,1)$, $\delta = 0.9$, $\pi(\theta) = 1 - exp(-\theta)$, $where\ \tilde{\theta} = 0$

[14] See Appendix F for its proof.

6. Concluding Remarks

Using our models, we find the following implications:

- *Fuhyo-Higai* occurs through social learning by rational risk-averse consumers, where consumers prefer to wait and see a choice of actions taken by others before making their own decisions. It makes some of the consumers worse off, and producers much worse off.

- Less risk-aversion or higher precision of a signal is likely to mitigate *Fuhyo-Higai*. Furthermore, the rise in the expected proportion of irrational consumers also moderates *Fuhyo-Higai*. More chances of observation, on the other hand, worsen *Fuhyo-Higai*.

Probably consumers' choices of behavior are, in the real world, mostly not irrational but rational even in the situation where *Fuhyo-Higai* is taking place. They are not hypnotized nonthinkers, nor do they do anything "wrong" in the sense of rationality. They wait and see how others behave just because they want more precise information or prefer a less risky option, possibly bearing in mind an old saying from a Shakespeare: "Discretion is the better part of valor." At the same time, however, this "rational" decision makes some of the consumers and probably all producers worse off.

So what can the government do against *Fuhyo-Higai*? Since a lot of consumers make a rational choice, it would be extremely difficult for the government to eradicate *Fuhyo-Higai*. However, our model suggests that the government could mitigate the damage by providing consumers with the relevant scientific information on risks as precisely and sufficiently as possible. Not only does it help rational consumers independently form a sound private belief with the information and rely less on observation of others while waiting; it keeps rational consumers from holding too strong risk-aversion. Moreover, it is useful to conduct, not in a state of emergency but regularly, a survey on consumers' attitude toward an existing or hypothetical risk. It gives consumers an opportunity to rehearse their independent decision-making for an actual risk event. It also helps consumers grasp how large the population of irrational consumers is, forestalling an underestimate of the population.

To make our model more persuasive and suggestive, we still leave tasks to get on with. The source of signals, among others, is an issue we need to discuss more deeply. In this paper, without identifying the source of signals, we assume consumers receive a single signal once and for all. In fact, however, the source of signal mainly involves the media and the government, and they would send multiple signals. Moreover, the media could give inferential, value-attached, symbolic, and sensational signals to consumers, rather than factual ones. Then what effect the government can put on consumers' behavior if it adds a new factual signal to them? We will leave this important discussion for the future.

Appendix A (***Proof of Claim 2***)

Recall that we denote the p.d.f. of θ by $g(\theta)$ and the p.d.f. of ε_i by $f(\varepsilon_i)$. Let $F(\varepsilon) = \int f(\varepsilon)d\varepsilon$. Then we can write the joint p.d.f. of s_i and θ as:

$$h(s_i,\theta) = f(s_i - \theta)g(\theta). \qquad (A-1)$$

The conditional p.d.f. of θ given s_i is:

$$g(\theta|s_i) = \frac{h(s_i,\theta)}{\int h(s_i,\theta)d\theta} = \frac{f(s_i-\theta)g(\theta)}{\int f(s_i-\theta)g(\theta)d\theta}. \qquad (A-2)$$

Under a SMS with the common threshold s,

$$Pr\big(s_j \geq s|\theta\big) = 1 - F(s-\theta), \qquad (A-3)$$

$$Pr\big(s_j < s|\theta\big) = F(s-\theta). \qquad (A-4)$$

By use of (A-2) to (A-4), we have:

$$g\big(\theta|s_i,s_j \geq s,s\big) = \frac{h\big(s_i,s_j \geq s\ ,\theta\big)}{\int h\big(s_i,s_j \geq s,\theta\big)d\theta} = \frac{f(s_i-\theta)[1-F(s-\theta)]g(\theta)}{\int f(s_i-\theta)[1-F(s-\theta)]g(\theta)d\theta}$$

$$= \alpha_1(s_i,s)[1-F(s-\theta)]g(\theta|s_i),$$

where

$$\alpha_1(s_i,s) = \frac{\int f(s_i-\theta)g(\theta)d\theta}{\int f(s_i-\theta)[1-F(s-\theta)]g(\theta)d\theta}, \qquad (A-5)$$

$$g\big(\theta|s_i,s_j < s,s\big) = \frac{h\big(s_is_j < s,,\theta\big)}{\int h\big(s_i,s_j < s,\theta\big)d\theta} = \frac{f(s_i-\theta)F(s-\theta)g(\theta)}{\int f(s_i-\theta)F(s-\theta)g(\theta)d\theta}$$

$$= \alpha_0(s_i,s)F(s-\theta)g(\theta|s_i),$$

where

$$\alpha_0(s_i,s) = \frac{\int f(s_i-\theta)g(\theta)d\theta}{\int f(s_i-\theta)F(s-\theta)g(\theta)d\theta}. \qquad (A-6)$$

Notice that:

$$\int g(\theta|s_i)d\theta = \int g\big(\theta|s_i,s_j \geq s,s\big)d\theta = \int g\big(\theta|s_i,s_j < s,s\big)d\theta = 1. \qquad (A-7)$$

Since $1-F(s-\theta)$ is strictly increasing and $F(s-\theta)$ is strictly decreasing in θ, we have:

$$\int_{-\infty}^{\hat{\theta}} g\big(\theta|s_i,s_j \geq s,s\big)d\theta < \int_{-\infty}^{\hat{\theta}} g(\theta|s_i)d\theta \quad for\ any\ \hat{\theta} \in (-\infty,\infty), \qquad (A-8)$$

$$\int_{-\infty}^{\hat{\theta}} g(\theta|s_i)d\theta < \int_{-\infty}^{\hat{\theta}} g\big(\theta|s_i,s_j < s,s\big)d\theta \quad for\ any\ \hat{\theta} \in (-\infty,\infty), \qquad (A-9)$$

which concludes the proof.

■

Appendix B (***Proof of Claim 3***)

Let us denote an arbitrary function of θ by $v(\theta)$, instead of $g(\theta)$. Then we can write the joint p.d.f. of θ and s_i as:

$$h(\theta, s_i) = f(s_i - \theta)v(\theta). \qquad (B-1)$$

Then the conditional p.d.f. of θ given s_i is:

$$h(\theta|s_i) = \frac{f(s_i - \theta)v(\theta)}{\int f(s_i - \theta)v(\theta)d\theta}. \qquad (B-2)$$

We want to show that for every $\hat{\theta} \in (-\infty, +\infty)$:

$$\int_{-\infty}^{\hat{\theta}} h(\theta|s_i)d\theta < \int_{-\infty}^{\hat{\theta}} h(\theta|s_i')d\theta \quad iff \quad s_i > s_i'. \qquad (B-3)$$

Let $H(s_i, \hat{\theta})$ denote $\int_{-\infty}^{\hat{\theta}} h(\theta|s_i)d\theta$ and $p(s_i)$ denote $\int f(s_i - \theta)v(\theta)d\theta$.

Differentiating $H(s_i, \hat{\theta})$ w.r.t. s_i, we have:

$$\frac{\partial H}{\partial s_i} = \int_{-\infty}^{\hat{\theta}} \frac{\partial h(\theta|s_i)}{\partial s_i} d\theta, \qquad (B-4)$$

$$\frac{\partial h(\theta|s_i)}{\partial s_i} = \frac{f'(s_i - \theta)v(\theta)}{p(s_i)} - \frac{f(s_i - \theta)v(\theta)p'(s_i)}{p(s_i)^2} = \frac{f(s_i - \theta)v(\theta)}{p(s_i)}\left\{\frac{f'(s_i - \theta)}{f(s_i - \theta)} - \frac{p'(s_i)}{p(s_i)}\right\}$$

$$= \left\{\frac{f'(s_i - \theta)}{f(s_i - \theta)} - \frac{p'(s_i)}{p(s_i)}\right\} h(\theta|s_i). \qquad (B-5)$$

Note that $\int h(\theta|s_i)d\theta = 1$. Thus we have:

$$\frac{\partial \int h(\theta|s_i)d\theta}{\partial s_i} = \int \frac{\partial h(\theta|s_i)}{\partial s_i} d\theta = 0. \qquad (B-6)$$

Substituting $(B-5)$ into $(B-6)$, we have

$$\int \frac{\partial h(\theta|s_i)}{\partial s_i} d\theta = \int \left\{\frac{f'(s_i - \theta)}{f(s_i - \theta)} - \frac{p'(s_i)}{p(s_i)}\right\} h(\theta|s_i)d\theta = \int \frac{f'(s_i - \theta)}{f(s_i - \theta)} h(\theta|s_i)d\theta - \frac{p'(s_i)}{p(s_i)}$$

$$= 0. \qquad (B-7)$$

Recall $f(*)$ is the p.d.f. of the noise, that is,

$$f(\varepsilon) = \frac{\sqrt{\rho_\varepsilon}}{\sqrt{2\pi}} e^{-\rho_\varepsilon \frac{\varepsilon^2}{2}}. \qquad (B-8)$$

Then

$$\frac{f'(s_i - \theta)}{f(s_i - \theta)} = -\rho_\varepsilon \varepsilon = \rho_\varepsilon(\theta - s_i). \qquad (B-9)$$

From $(B-7)$ and $(B-9)$,

$$\frac{p'(s_i)}{p(s_i)} = \int \frac{f'(s_i-\theta)}{f(s_i-\theta)} h(\theta|s_i)d\theta = \int \rho_\varepsilon(\theta - s_i)h(\theta|s_i)d\theta$$

$$= -\rho_\varepsilon s_i + \rho_\varepsilon \int \theta h(\theta|s_i)d\theta. \qquad (B-10)$$

Substituting $(B-9)$ and $(B-10)$ into $(B-5)$, we have:

$$\frac{\partial h(\theta|s_i)}{\partial s_i} = \left\{\rho_\varepsilon(\theta - s_i) + \rho_\varepsilon s_i - \rho_\varepsilon \int \theta h(\theta|s_i)d\theta\right\} h(\theta|s_i) = \rho_\varepsilon[\theta - E(\theta|s_i)]\, h(\theta|s_i).$$

$$(B-11)$$

Going back to $(B-4)$, we have:

$$\frac{\partial H}{\partial s_i} = \int_{-\infty}^{\hat\theta} \frac{\partial h(\theta|s_i)}{\partial s_i} d\theta = \rho_\varepsilon \int_{-\infty}^{\hat\theta} [\theta - E(\theta|s_i)]\, h(\theta|s_i)d\theta. \qquad (B-12)$$

Notice that $\theta - E(\theta|s_i)$ is increasing in θ, and as $\hat\theta \to +\infty$, the R.H.S. converges to zero. Hence, for every $\hat\theta \in (-\infty, +\infty)$, we have:

$$\frac{\partial H}{\partial s_i} = \int_{-\infty}^{\hat\theta} \frac{\partial h(\theta|s_i)}{\partial s_i} d\theta < 0. \qquad (B-13)$$

Therefore, for arbitrary $v(\theta)$, $h(\theta|s_i)$ first-stochastically dominates $h(\theta|s_i')$ iff $s_i > s_i'$.
By taking $v(\theta)$ such that:

$$v(\theta) = g(\theta|s_i, s_j \geq s, s) = \frac{f(s_i-\theta)[1-F(s-\theta)]g(\theta)}{\int f(s_i-\theta)[1-F(s-\theta)]g(\theta)d\theta} \quad or$$

$$v(\theta) = g(\theta|s_i, s_j < s, s) = \frac{f(s_i-\theta)F(s-\theta)g(\theta)}{\int f(s_i-\theta)F(s-\theta)g(\theta)d\theta},$$

Claim 3 holds. ■

Appendix C (*Proof of Claim 4*)

Consider a likelihood ratio of $g(\theta|s_j \geq s, s_i)$ to $g(\theta|s_j \geq s', s_i)$ as

$$\frac{g(\theta|s_j \geq s, s_i)}{g(\theta|s_j \geq s', s_i)} = \frac{\int[1-F(s'-\theta)]g(\theta|s_i)d\theta}{\int[1-F(s-\theta)]g(\theta|s_i)d\theta} \times \frac{1-F(s-\theta)}{1-F(s'-\theta)}. \qquad (C-1)$$

Recall the well-established fact that if the likelihood ration is increasing in θ, then the FOSD property holds. Since the first term on the right hand side is independent of θ, we can focus on the second term. Taking its log and differentiating it, we have:

$$\frac{\partial}{\partial\theta} ln\left[\frac{1-F(s-\theta)}{1-F(s'-\theta)}\right] = \frac{f(s-\theta)}{1-F(s-\theta)} - \frac{f(s'-\theta)}{1-F(s'-\theta)} > 0. \qquad (C-2)$$

The inequality holds because $f(\varepsilon)/[1-F(\varepsilon)]$ is strictly increasing in ε from the property of the hazard ratio of a normal distribution. This concludes the proof. ■

Appendix D

By use of $(A-5)$, we transform $Pr(s_j \geq s|s_i)E(\pi|s_j \geq s, s_i)$ into:

$$Pr(s_j \geq s|s_i)E(\pi|s_j \geq s, s_i) = \int [1-F(s-\theta)]g(\theta|s_i)d\theta \times \int \pi(\theta)g(\theta|s_j \geq s, s_i)d\theta$$

$$= \int [1-F(s-\theta)]g(\theta|s_i)d\theta$$

$$\times \frac{1}{\int [1-F(s-\theta)]g(\theta|s_i)d\theta} \int \pi(\theta)\,[1-F(s-\theta)]g(\theta|s_i)d\theta$$

$$= \int \pi(\theta)\,[1-F(s-\theta)]g(\theta|s_i)d\theta. \qquad (D-1)$$

Appendix E (***Proof that $b(s)$ is unique and decreasing in s if $(s) \leq (\rho_\theta+\rho_\varepsilon)s/\rho_\varepsilon$, and consumer i chooses a monotone strategy***)

Consumer i will buy a good in period 1 if and only if:

$$\int \pi(\theta)g(\theta|s_i)d\theta \geq \delta \int \pi(\theta)[1-F(s-\theta)]g(\theta|s_i)d\theta. \qquad (E-1)$$

Let the left hand side of $(E-1)$ be L.H.S., and the right hand side R.H.S. For the uniqueness of $b(s)$ and the claim that she chooses a monotone strategy, we want to show that:

$$\left.\frac{\partial L.H.S.}{\partial s_i}\right|_{s_i=b(s)} > \left.\frac{\partial R.H.S.}{\partial s_i}\right|_{s_i=b(s)}. \qquad (E-2)$$

Let $\Omega(s, s_i) = L.H.S. - R.H.S.$ Then we have:

$$\Omega(s, s_i) = \int \pi(\theta)g(\theta|s_i)d\theta - \delta \int \pi(\theta)\,[1-F(s-\theta)]g(\theta|s_i)d\theta$$

$$= \int \pi(\theta)\{1-\delta[1-F(s-\theta)]\}g(\theta|s_i)d\theta. \qquad (E-3)$$

Manipulating $(B-11)$, we have:

$$\frac{\partial g(\theta|s_i)}{\partial s_i} = \rho_\varepsilon[\theta - E(\theta|s_i)]\, g(\theta|s_i). \qquad (E-4)$$

Differentiating Ω w.r.t. s_i and evaluating it at $s_i = b(s)$, we have:

$$\left.\frac{\partial\Omega}{\partial s_i}\right|_{s_i=b(s)} = \rho_\varepsilon \int \left[\theta - E\big(\theta|b(s)\big)\right]\pi(\theta)\{1-\delta[1-F(s-\theta)]\}g\big(\theta|b(s)\big)d\theta. \qquad (E-5)$$

Substituting $s_i = b(s)$ into $(E-3)$, we have by the definition of $b(s)$:

$$\Omega\big(s, b(s)\big) = \int \pi(\theta)\{1-\delta[1-F(s-\theta)]\}g\big(\theta|b(s)\big)d\theta = 0. \qquad (E-6)$$

By use of $(E-5)$, $(E-6)$ can be written as:

$$\left.\frac{\partial\Omega}{\partial s_i}\right|_{s_i=b(s)} = \rho_\varepsilon \int \theta\pi(\theta)\{1-\delta[1-F(s-\theta)]\}g\big(\theta|b(s)\big)d\theta > 0, \qquad (E-7)$$

the sign of which follows from the assumption that $\theta\pi(\theta) > 0$ for $\theta \neq 0$, and the fact that $1-\delta[1-F(s-\theta)] > 0$ for all θ and $\rho_\varepsilon > 0$. It follows that $b(s)$ is unique and that consumer i chooses a monotone strategy.

To check to see if $b(s)$ is decreasing in s if $b(s) \leq (\rho_\theta + \rho_\varepsilon)s/\rho_\varepsilon$, we want to show that for $b(s) \leq (\rho_\theta + \rho_\varepsilon)s/\rho_\varepsilon$, we have:

$$b'(s) = -\left.\frac{\partial\Omega/\partial s}{\partial\Omega/\partial s_i}\right|_{s_i=b(s)} < 0. \qquad (E-8)$$

Consider the sign of

$$\frac{\partial\Omega}{\partial s} = \delta\int \pi(\theta)f(s-\theta)g(\theta|s_i)d\theta = \delta\int f(s-\theta)g(\theta|s_i)d\theta \int \pi(\theta)g(\theta|s,s_i)d\theta \qquad (E-9)$$

at $s_i = b(s)$, where

$$g(\theta|s,s_i) = \frac{f(s-\theta)g(\theta|s_i)}{\int f(s-\theta)g(\theta|s_i)d\theta} \qquad (E-10)$$

is a p.d.f. conditional on $s_j = s$ and s_i. From this expression we notice that the sign of $\partial\Omega/\partial s$ depends on that of $\int \pi(\theta)g(\theta|s,s_i)d\theta$, the expected payoff conditional on $s_j = s$ and s_i. As is clear from the signal extraction theory, the distribution with $g(\theta|s,s_i)$ is normal with the mean $[\rho_\varepsilon/(\rho_\theta + 2\rho_\varepsilon)](s+s_i)$ and the precision $\rho_\theta + 2\rho_\varepsilon$. Compared with the mean $[\rho_\varepsilon/(\rho_\theta+\rho_\varepsilon)]s_i$ and the precision $\rho_\theta + \rho_\varepsilon$ of the distribution with $g(\theta|s_i)$, the precision is always higher. Accordingly, unless the mean is smaller, i.e.,

$$\frac{\rho_\varepsilon}{\rho_\theta + 2\rho_\varepsilon}(s+s_i) \geq \frac{\rho_\varepsilon}{\rho_\theta+\rho_\varepsilon}s_i, \qquad (E-11)$$

or equivalently,

$$s \geq \frac{\rho_\varepsilon}{\rho_\theta+\rho_\varepsilon}s_i, \qquad (E-12)$$

then we have

$$\int \pi(\theta)g(\theta|s,s_i)d\theta > \int \pi(\theta)g(\theta|s_i)d\theta \qquad (E-13)$$

because $\pi(\theta)$ is assumed to be concave, owing to the second-order stochastic dominance. Since $\int \pi(\theta)g\big(\theta|b(s)\big)d\theta > 0$, we have

$$\left.\frac{\partial\Omega}{\partial s}\right|_{s_i=b(s)} > 0 \qquad (E-14)$$

if $(E-12)$ holds for $s_i = b(s)$, the condition of which is reduced to

$$b(s) \le \frac{\rho_\theta + \rho_\varepsilon}{\rho_\varepsilon} s. \qquad (E-15)$$

Therefore, if $(E-15)$ holds, $(E-8)$ is verified by $(E-7)$ and. $(E-14)$. ■

Appendix F (***Proof of (5.15)***)

Recall $\pi(\theta)$ is increasing in θ. From Claim 1, we have:

$$\frac{\partial\, L.H.S.^P}{\partial s_i} > 0 \; and \; \frac{\partial\, R.H.S.^P}{\partial s_i} > 0. \qquad (D-1)$$

Now

$$\frac{\partial L.H.S.^P}{\partial s_i} = \int_{-\infty}^{0} \pi(\theta)\frac{\partial g(\theta|s_i)}{\partial s_i}d\theta + \int_{0}^{+\infty} \pi(\theta)\frac{\partial g(\theta|s_i)}{\partial s_i}d\theta$$

$$= \int_{-\infty}^{0} \pi(\theta)\frac{\partial g(\theta|s_i)}{\partial s_i}d\theta + \frac{1}{\delta}\frac{\partial R.H.S.^P}{\partial s_i}. \qquad (D-2)$$

From Claim 1 again, the first term is positive. Thus,

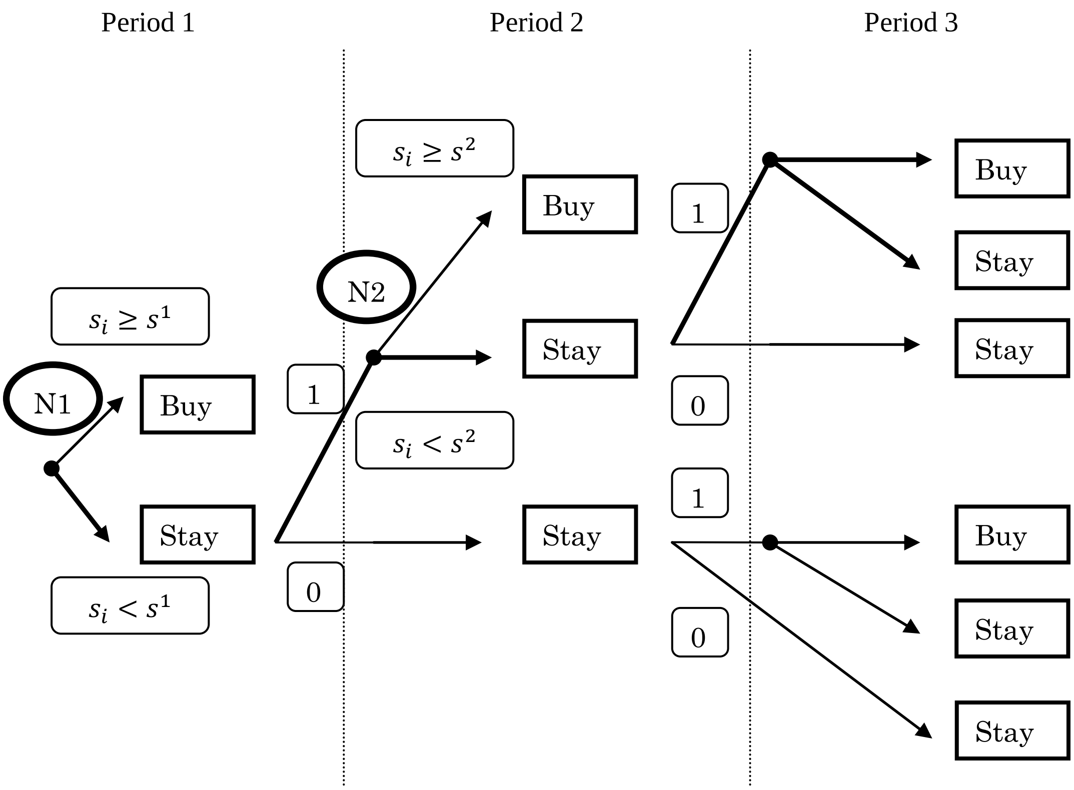
$$\frac{\partial R.H.S.^P}{\partial s_i} = \delta\left[\frac{\partial L.H.S.^P}{\partial s_i} - \int_{-\infty}^{0} \pi(\theta)\frac{\partial g(\theta|s_i)}{\partial s_i}d\theta\right] < \frac{\partial L.H.S.^P}{\partial s_i}. \qquad (D-3)$$

■

Appendix G (***A three-period model***)

A three-period model gives a consumer one more opportunity to make an observation. It suggests that her expected payoff from delaying her decision to period 2 increases relatively more than that in a two-period model. As a result, "*Fuhyo-Higai*" will become worse in a three-period model.

The decision-making tree is described below, where the numerical "1" means that a consumer encounters a past buyer, and "0" means she encounters a past withholder. Notice that she can arbitrage at two points in this model: Node 1 and Node 2, denoted by N1 and N2 respectively in the figure. Let s^1 denote the critical point at N1; that is, she will buy a good if and only if $s_i \geq s^1$ at N1. Likewise, let s^2 denote the critical point at N2; that is, she will buy it if and only if $s_i \geq s^2$ at N2. To illustrate, suppose at N1 stands consumer i with $s_i < s^1$. She will choose to delay her decision to period 2. If she encounters a past buyer, called consumer j, she will proceed to N2. At this point, she will buy a good if and only if $s^1 > s_i \geq s^2$. Suppose not, and she will delay her decision to period 3. At the end of period 3, then, she will be given one more chance to encounter a consumer, called consumer k. When consumer k is a past buyer, she will buy the good in period 3 if the expected payoff from buying it is positive; otherwise, she will not make a purchase in the end (The bold line in the figure follow the above illustration).



Now consider her decision-making backward. At Node 2, where she chose to delay her decision to period 2 and has already had an encounter with a past buyer j, the equilibrium threshold s^2 satisfies the following arbitrage equation:

$$\delta\int\pi(\theta)g\big(\theta|s_j\geq s^1,s^2\big)d\theta$$

$$=\delta^2\Big[Pr(s_k\geq s^1|s^2)\int\pi(\theta)g\big(\theta|s_j\geq s^1,s_k\geq s^1,s^2\big)d\theta$$

$$+Pr(s^1>s_k\geq s^2|s^2)\int\pi(\theta)g\big(\theta|s_j\geq s^1,s^1>s_k\geq s^2,s^2\big)d\theta\Big]$$

$$(E-1)$$

The first term in the right hand side represents the expected payoff from delaying her decision encountering a past buyer k who made a purchase in period 1, and the second term represents the one from delaying her decision and encountering a past buyer k who made a purchase in period 2. When she chose to stay in period 1 and has had an encounter with a past withholder, on the other hand, her expected payoff is:

$$max\Big\{\delta^2\Big[Pr(s_k\geq s^1|s_i)\int\pi(\theta)g\big(\theta|s_j<s^1,s_k\geq s^1,s_i\big)d\theta$$

$$+Pr(s^1>s_k\geq s^2|s_i)\int\pi(\theta)g\big(\theta|s_j<s^1,s^1>s_k\geq s^2,s_i\big)d\theta\Big],\qquad 0\Big\}.$$

$$(E-2)$$

Let us define $\hat{B}(s)$ such that:

$$\int\pi(\theta)g\left(\theta|s_j\geq s^1,s_k\geq s^2,\hat{B}(s)\right)d\theta=0. \qquad (E-3)$$

A consumer with $s_i<\hat{B}(s)$ will not choose to buy the good even if she encounters two past buyers. Now let us go back to Node 1. By use of $(E-1)$ and $(E-2)$, we can define two equilibrium-thresholds, depending on her signal, at Node 1 as follows:

- For $s_i\geq s^2$, there exists an equilibrium-threshold s^{1a} such that:

$$\int\pi(\theta)g(\theta|s^{1a})d\theta$$

$$=\delta Pr\big(s_j\geq s^{1a}|s^{1a}\big)\int\pi(\theta)g\big(\theta|s_j\geq s^{1a},s^{1a}\big)d\theta$$

$$+\delta^2 Pr\big(s_j<s^{1a}|s^{1a}\big)$$

$$\times\ max\Big\{\delta^2\Big[Pr(s_k\geq s^{1a}|s^{1a})\int\pi(\theta)g\big(\theta|s_j<s^{1a},s_k\geq s^{1a},s^{1a}\big)d\theta$$

$$+Pr(s^{1a}>s_k\geq s^2|s_i)\int\pi(\theta)g\big(\theta|s_j<s^{1a},s^{1a}>s_k\geq s^2,s^{1a}\big)d\theta\Big],0\Big\}.$$

$$(E-4a)$$

- For $\hat{B}(s)<s_i<s^2$, there exists an equilibrium-threshold s^{1b} such that:

$$\int \pi(\theta) g(\theta | s^{1b}) d\theta$$

$$= \delta^2 Pr(s_j \geq s^{1b} | s^{1b}) \Big[Pr(s_k \geq s^{1b} | s^{1b}) \int \pi(\theta) g(\theta | s_j \geq s^{1b}, s_k \geq s^{1b}, s^{1b}) d\theta$$

$$+ Pr(s^{1b} > s_k \geq s^2 | s^{1b}) \int \pi(\theta) g(\theta | s_j \geq s^{1b}, s^{1b} > s_k \geq s^2, s^{1b}) d\theta \Big]$$

$$+ \delta^2 Pr(s_j < s^{1b} | s^{1b})$$

$$\times \max \Big\{ \delta^2 \Big[Pr(s_k \geq s^{1b} | s^{1b}) \int \pi(\theta) g(\theta | s_j < s^{1b}, s_k \geq s^{1b}, s^{1b}) d\theta$$

$$+ Pr(s^{1b} > s_k \geq s^2 | s^{1b}) \int \pi(\theta) g(\theta | s_j < s^{1b}, s^{1b} > s_k \geq s^2, s^{1b}) d\theta \Big], 0 \Big\}. \qquad (E-4b)$$

As compared with R.H.S. in the two-period model, as in (4.11), R.H.S. in $(E-4a)$ is larger by the second term, with L.H.S. unchanged. Thus we have:

$$s^{**} < s^{1a}. \qquad (E-5)$$

As for s^{1b}, a consumer with $s^{1b} \leq s_i < s^2$ has the strategy of "buy a good at Node 1, delay her decision at Node 2". Because this is out of the equilibrium path, we can disregard it, and every consumer with $s_i < s^2$ chooses to delay her decision to period 2. In summary, more consumers choose to delay the decision in a three-period model.

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