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The Political Economy of Social Security Funding: Why Social VAT Reform?

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Abstract

Recently, taxation reforms entailing a "social" valued-added tax (VAT), i.e., a social security reform shifting funding from traditional wage-based taxation to consumption taxation, have been obtaining political support in some developed countries; e.g. Japan, France, Denmark, and Germany. This paper analyzes the political economy of social security funding in an overlappinggenerations economy. In particular, we consider how population aging influences the choice of wage or consumption tax financing by focusing on their differential impact on inter- and intragenerational redistribution. Our results show that population aging may drastically alter the political equilibrium, in that if the population growth rate is higher than the interest rate, wage taxation is the only equilibrium choice, but if it is lower, multiple equilibria are likely to emerge, in which the introduction of consumption taxation emerges as an alternative equilibrium choice.

JEL classification numbers: D78, H55

Key words: political economy of social security; consumption tax; structure-induced equilibrium

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1 Introduction

This paper examines the political economy of a change in social security funding. Recently, countries such as Japan, France, Germany, and Denmark, among others, have considered the idea of a "social" value-added tax (VAT) as a way to shift financial sources from traditional wage-based social security contributions to consumption taxes. Denmark has already implemented a social VAT reform, while Germany has raised its VAT by 3% as a way to lower the labor costs of domestic firms. In France, a social VAT reform, though not yet implemented, has remained controversial at least since the last presidential election. Lastly, Japan increased its consumption tax rate from 5% to 8% as of April 1, 2014, with a pledge to place all additional revenues in the public pension budget.

The increasing popularity of consumption tax financing is the result of several social and economic motives. One is the international competitiveness of domestic industries. For example, one argument has been that the international competitiveness of domestic industries has weakened or will become weaker through increasing labor costs associated with the rise in social security contributions. Concerns for intergenerational fairness appears to have more, or at least equal, importance.¹ This motive emphasizes that retirees, especially richer retirees, should share the burden of population aging with the working generation. In fact, this appears to have played a role already in shaping tax policies on social security transfer in developed countries. As Adema, From and Ladaique (2011) argue, most OECD countries employ the indirect taxation of consumption out of benefit income. This enables governments to receive back a considerable amount of public social expenditure as well as the direct taxation of benefit income.²

Table 1 illustrates the extent of diversification in the composition of funding for social pro-

¹In Japan, the concern for intragenerational fairness is also emphasized in the call for consumption tax funding. The reason is that a substantial share of the working population, especially those in their 20s or 30s, are not paying national pension contributions, despite their being mandated.

²According to their estimates of *net* total social expenditure including taxation and private social spending, international differences in the ratio of social expenditure to GDP are less than what we usually observe for *gross* social expenditure. For example, in terms of gross social expenditure, France, Sweden, and Denmark are the three largest social spenders in the OECD, but in terms of net total social expenditure, Sweden ranks fourth and Denmark ninth.

tection expenditure across several European countries and Japan.³

Looking at the share of social contributions in total receipts, we can see that these differ significantly, ranging from only 23.5% in Denmark to 66.5% in the Netherlands. It is certain that most social contributions are from wage-based taxes such as payroll taxes. As the reverse, the share of government contributions is also very variable, ranging from just 24.3% in the Netherlands to 74.2% in Denmark. Naturally, government contributions consist of various revenue sources, including consumption, individual income, corporate income, and capital income taxes, bond issues, and so on. It is important to note that these components include taxes that are theoretically expected to have excise tax effects; that is, part of their burden is shifted to commodity prices and effectively paid by consumers.⁴ To the extent this excise tax effect prevails, the share of government contributions has to be included in any rough estimate of consumption tax financing.

Table 1 also shows how differently social contributions are allocated between employers and protected persons across these countries. In Sweden, for example, employers pay more than three times the amount that protected persons do, whereas employers and protected persons share payments almost equally in countries such as the Netherlands, Germany, and Denmark. Japan also belongs to this latter category of countries. If we accept the conventional, but theoretically somewhat questionable, argument that social contributions paid by employers but not those paid by employees are likely to be shifted to commodity prices, then we may also include the share of employer contributions when estimating the size of consumption tax financing.⁵

The question is why does the composition of social protection receipts differ so much across countries? In addition, why has consumption tax financing become so popular of late in some countries? Lastly, why are the social contributions of employees and employers shared so differently across countries? This paper attempts to respond to these questions from a political economy perspective. We place an analytical focus on inter- and intragenerational conflict over the choice of social security funding. For this purpose, we construct an overlapping-generations

³The social protection programs underlying the figures in Table 1 include not only old-age pension programs but also health care, unemployment, housing and social assistance programs.

⁴See Atkinson and Stiglitz (1980, Ch. 6) for the excise tax effect of corporate income taxes.

⁵Theoretically, when markets are perfectly competitive, the share-out ratios should make no difference in equilibrium prices and resource allocation. This argument, however, seems to be hardly acceptable in practice.

Nation	Social Contributions			Government	Other receipts*
	Total	Employer contributions	Protected persons' contributions	contributions	
Netherlands	66.5	32.4	34.1	24.3	9.1
Austria	64.3	37.6	26.7	34.0	1.7
France	63.3	43.0	20.3	34.7	2.0
Germany	63.1	33.5	29.6	35.2	1.7
Hungary	56.0	35.9	20.1	40.8	3.2
Japan ^{**}	52.0	26.9	25.1	37.6	10.5
Italy	53.1	38.2	14.8	45.3	1.6
Finland	47.4	35.4	12.0	46.1	6.6
Sweden	45.2	35.6	9.6	52.6	2.2
United Kingdom	44.0	31.2	12.8	47.9	8.1
Denmark	23.5	11.8	11.7	74.2	2.4

Table 1: The composition of social protection receipts in 2011, %

Data source: Eurostat for European countries and Financial Statistics of Social Security (published by the National Institute of Population and Social Security Research) for Japan.

Note: (*) Other receipts include transfers from reserves. (**) The data for Japan are based on the International Labor Organization (ILO) criterion, which differs somewhat from that used by Eurostat.

model of workers and retirees with different level of income and wealth. Using this model, we analyze the outcome of majority voting taking place over the choice of wage and consumption taxation to finance social security benefits.

The choice of wage and consumption tax financing causes different redistributive effects within the same generation as well as across generations. To highlight these effects, suppose that all tax revenues go into a social security program providing a flat benefit for each retiree. Consumption tax financing then redistributes income not only among workers but also among retirees. The beneficiaries are poor workers and poor retirees, who receive more than they pay, whereas rich retirees and rich workers pay more than they receive. In contrast, wage tax financing redistributes income only across workers. This is because all retirees receive benefits without paying wage taxes. The beneficiaries from wage tax financing thus consist of poor workers and all retirees. Therefore, as far as the intragenerational redistribution effects are concerned, a majority coalition including rich retirees will support a proposal for replacing consumption tax financing with wage tax financing. As a result, wage tax financing appears to be a unique outcome in the political choice of social security funding.

However, the differential effects of the two taxes on the cost of intergenerational transfers also play a critical role in this political choice. Wage tax financing entails intergenerational transfers in transferring tax revenues from workers to retirees. The amount of intergenerational transfers also increases with a higher wage tax. Consumption tax financing, on the other hand, transfers a smaller amount across generations than wage tax financing, even when they yield the same revenues. This is because both workers and retirees pay consumption taxes, but only those taxes paid by workers are transferred to retirees.⁶ Moreover, a higher consumption tax decreases the wage tax revenues transferred to retirees to the extent that it discourages labor supply. Importantly, in a dynamically efficient economy, where the population growth rate is lower than the interest rate, intergenerational transfers are more expensive than private savings for workers to finance their postretirement consumption. As demographic aging advances, a larger population of workers may prefer replacing wage tax financing with consumption tax financing because the latter reduces the amount of costly intergenerational transfers.

⁶This further implies that under consumption tax financing, workers have to save more for their postretirement consumption to pay for consumption taxes.

In this paper, we consider a voting model that determines a wage tax, a consumption tax, and the size of the social security benefit per retiree financed by these taxes. Generally, no Condorcet winner arises in majority voting over multiple issues. Following Conde-Ruiz and Galasso (2005), we employ the concept of a structure-induced equilibrium invented by Shepsle (1979) to aggregate policy preferences over wage and consumption taxes and to describe the political equilibrium. Specifically, we suppose that a wage tax rate and a consumption tax rate are put separately to the vote and each Condorcet winner is selected with the other tax rate taken as given.

We find the decisive worker-voter is poorer in the voting over a wage tax than in the voting over a consumption tax. This is because all retirees always support a proposal for increasing the size of benefits under wage tax financing. As poorer workers gain more from redistribution in the social security program, the majority coalition allows the social security benefit per retiree to grow under wage tax financing. Given that a wage tax finances such a large benefit, a majority coalition is less likely to emerge to support a proposal of introducing a consumption tax to increase the benefit further. We show that in a dynamically efficient economy with a small margin prevailing between population growth and the interest rate, as well as in a dynamically inefficient economy, wage tax financing is a unique structure-induced equilibrium outcome. The result is that as conventionally observed in many countries, only wage taxes finance social security benefits.

As population aging proceeds, however, the policy preferences for social security funding change.

Regarding wage tax financing, even poor workers prefer a lower wage tax because wage taxation yields smaller marginal revenues due to a smaller labor force. On the other hand, because of the reduced population of worker–voters with an aging population, the political influence of retiree–voters strengthens to the extent that a proposal for a higher wage tax tends to gain more support. Although these two changes tend to counteract each other, we show that if the density of the median worker–voter is sufficiently large, which seems quite natural in a standard wage income distribution, the effect working through the political influence of retirees is outweighed; that is, population aging shifts aggregate policy preferences toward a lower wage tax.

In terms of financing through a consumption tax, we find that poor and middle-class workers are inclined to prefer a higher consumption tax as population aging proceeds. This is because for them, funding social security with intergenerational transfers becomes more expensive. Poor and middle-class retirees also prefer a higher consumption tax to compensate them for wage tax revenues lost because of the smaller labor force. Thus, aging shifts the aggregate policy preferences toward a higher consumption tax.

Combining these effects, we argue that population aging may drastically change the political equilibrium. We show that in a dynamically efficient economy with a large margin between the population growth rate and the interest rate, multiple structure-induced equilibria potentially arise to finance social security: only the wage taxation, only the consumption taxation, or their combination may fund social security. Consumption tax financing thus emerges as an equilibrium outcome in response to population aging. This result explains the recent increase in the political popularity of "social" VAT reform in some countries. The coexistence of three types of equilibrium also explains the observed international diversity in the composition of social security funding, as shown earlier in Table 1. We also show that with a sufficiently large margin between the population growth rate and the interest rate, there is no majority coalition that supports wage tax financing. As a result, in this situation, only the consumption taxation funds social security.

To the best of our knowledge, no existing study addresses the public choice of social security funding with a combination of different taxes. In the literature on the political economy of income redistribution and social security, including Browning (1975), Meltzer and Richard (1981), Hu (1982), Boadway and Wildasin (1989), Cooley and Soares (1999), Tabellini (2000), Razin, Sadka, and Swagel (2002) among others, almost all employ models with only wage taxation to finance government spending for redistribution. As an exception, the model in Conde-Ruiz and Galasso (2005), on which we base our model, employs two different wage taxes—namely, a social security tax for transfers to retirees and an income redistribution tax for transfers to poor workers–for explaining the coexistence of the two social transfer programs.

The remainder of the paper is organized as follows. Section 2 sets out the model. Section 3 examines majority voting over wage tax rates, taking the consumption tax rate as given. Here we show that under a plausible condition, population aging lowers the Condorcet-winner wage tax rate despite retirees becoming more politically influential. Section 4 considers majority voting over consumption tax rates, taking the wage tax rate as given. Here we show that population aging increases the Condorcet-winner consumption tax rate. Using the results of the two previous sections, Section 5 analyzes the structure-induced equilibria in which wage and consumption tax

rates are Condorcet winners given the other rate, and examines the impact of population aging on the equilibrium outcomes. Section 6 concludes the paper.

2 The model

2.1 Brief description of the economy

We consider an economy with two overlapping generations, workers and retirees, in which the population grows at the rate of n > 0. To highlight the redistribution effects of a social security program, we assume that every individual works only when young and consumes only after retirement. Worker *i* has E_i units of leisure as her initial endowment, out of which she supplies N_i units of labor. She consumes C_i units of numeraire goods after retirement, financing it out of her savings, interest income, and social security benefits. Every retiree receives a flat social security benefits is in a pay-as-you-go fashion by either wage tax revenues, consumption tax revenues, or both. The wage tax rate is denoted as τ_w , and the net consumption tax rate is $\hat{\tau}$. We convert the latter into a gross consumption tax rate $\tau_c := \hat{\tau}/(1+\hat{\tau})$. There are no intrafamily transfers, such as bequests. The interest rate and the wage rate are assumed to be constant, the former denoted as r and the latter normalized to unity.

2.2 Workers and retirees

Worker i faces a lifetime budget constraint,

$$(1+\hat{\tau})C_i = (1+r)(1-\tau_w)N_i + (1+\hat{\tau})B,$$

where $0 \le N_i \le E_i$ and the term $(1 + \hat{\tau})B$ represents nominal social security benefits. Using a net consumption tax rate $\tau_c := \hat{\tau}/(1 + \hat{\tau})$, we can rewrite the budget constraint as

$$C_i = (1+r)(1-\tau_c)(1-\tau_w)N_i + B.$$

The result is that consumption taxation affects a worker's budget constraint in the same way as wage taxation despite their differences in the timing of payment.

The worker's leisure endowment, E_i , which reflects her innate ability, depends on her luck, which follows a cumulative distribution function, $F(E_i)$, with support $[\underline{E}, \overline{E})$. The mean E and the median E_m are respectively defined by $E = \int_{\underline{E}}^{\overline{E}} x f(x) dx$ and $F(E_m) = 1/2$, where f(x) denotes the probability density function. The ability distribution is left skewed, as in Figure 1, yielding $E_m < E$.



Figure 1: Distribution of ability

Worker i's utility function is specified as

$$u_i = \frac{C_i}{1+\delta} + \ln(E_i - N_i),$$

where δ is the rate of time preference. We set $\delta = r$ for simplicity. We also assume that <u>E</u> is sufficiently large to ensure $N_i > 0$ for every individual in equilibria. We then respectively obtain her labor supply and indirect utility as follows:

$$N_i = E_i - \frac{1}{(1 - \tau_w)(1 - \tau_c)}$$

and

$$U_i = (1 - \tau_c)(1 - \tau_w)E_i - \ln(1 - \tau_w)(1 - \tau_c) + \frac{B}{1 + r}.$$
(1)

Her savings are written as $A_i = (1 - \tau_w)N_i$. The distribution of before-tax wage income, which is equal to the labor supply, is as skewed as the ability distribution.

Suppose that worker i has retired after saving A_i . Her utility in the retirement period comes only from her consumption. We write this as

$$V_i = (1 - \tau_c)(1 + r)A_i + B.$$
(2)

2.3 Social security system

Wage taxes and consumption taxes finance social security expenditures. No revenues remain for the future. From the budget constraint, we express the social security benefit per retiree in real terms as

$$B = \tau_c (1+r)A + \tau_w (1+n)N, \tag{3}$$

where

$$N = N(\tau_w, \tau_c) := E - \frac{1}{(1 - \tau_w)(1 - \tau_c)}$$
(4)

and

$$A = (1 - \tau_w)N(\tau_w, \tau_c) \tag{5}$$

are labor supply per worker and savings per retiree, respectively. The first term in (3) is consumption tax revenue (except for taxes paid by consumption out of social security benefits), and the second term is wage tax revenue. If the tax rates are constant over time, we can write (3) as

$$B = B(\tau_w, \tau_c) := [(1+r)\tau_c(1-\tau_w) + (1+n)\tau_w]N(\tau_w, \tau_c).$$
(6)

3 Wage tax financing

Society has to determine a wage tax rate, a consumption tax rate, and the size of a social security benefit per retiree funded by these taxes. Considering the government's budget, the public choice reduces to determining the two tax rates. We assume that society puts each tax rate separately to majority voting, where every voter votes taking the other tax rate as given.

3.1 Policy preferences of retirees

Let us first consider retirees' preferences over wage tax rates, taking the consumption tax rate as given. Plugging (3) into (2), we obtain retiree *i*'s utility function as

$$V_i(\tau_w, \tau_c) := (1+r)[(1-\tau_c)A_i + \tau_c A] + (1+n)\tau_w N(\tau_w, \tau_c).$$
(7)

Given that A_i and A are predetermined, every retiree's most preferred wage tax rate is τ_w^o , at which the per-worker wage tax revenue, $\tau_w N(\tau_w, \tau_c)$, is maximized. Retirees share the same preferences because wage tax financing causes no redistribution among them. More specifically, from the first-order condition,

$$\frac{\partial V_i}{\partial \tau_w} = (1+n) \left(N(\tau_w, \tau_c) + \tau_w \frac{\partial N}{\partial \tau_w} \right) = 0, \tag{8}$$

their most preferred wage tax rate is obtained as

$$\tau_w^o = \tau_w^o(\tau_c) := 1 - \frac{1}{\sqrt{(1 - \tau_c)E}}.$$
(9)

This tax rate decreases as the consumption tax rate increases. We also find that retirees' preferences are single peaked because $\partial^2 V_i / \partial \tau_w^2 = -2/[(1 - \tau_c)(1 - \tau_w)^3] < 0.$

3.2 Policy preferences of workers

Next, consider workers' policy preferences. In contrast to retirees, wage tax financing redistributes income from rich to poor workers.

Let us examine how a higher wage tax affects worker *i*'s utility. Following the literature, we assume policy commitment in that workers expect that the tax rate determined in voting today will not change in their retirement period.⁷ From (1), worker *i*'s utility is then written as

$$U_i(\tau_w, \tau_c) := (1 - \tau_c)(1 - \tau_w)E_i - \ln(1 - \tau_w)(1 - \tau_c) + \frac{B(\tau_w, \tau_c)}{1 + r}.$$
(10)

Differentiating (6) and (10), we have

$$\frac{\partial U_i}{\partial \tau_w} = -(1 - \tau_c)N_i + \frac{1}{1 + r}\frac{\partial B}{\partial \tau_w} \tag{11}$$

and

$$\frac{\partial B}{\partial \tau_w} = (1+n) \left(N + \tau_w \frac{\partial N}{\partial \tau_w} \right) + (1+r) \tau_c \left(-N + (1-\tau_w) \frac{\partial N}{\partial \tau_w} \right).$$
(12)

From (12), we observe that a higher wage tax produces two effects on the amount of social security benefit that workers expect to receive after retirement. First, as shown by the first term in (12), a higher wage tax rate increases wage tax revenues in the next period when workers today get retired. Second, as shown by the second term, this also reduces consumption tax revenues in the next period by reducing savings today.

⁷In Appendix C we relax this assumption following Conde-Ruiz and Galasso (2005) and show that the same equilibrium outcomes are realized in the subgame-perfect equilibria of an infinitely repeated voting game.

Combining (11) and (12), we obtain

$$\frac{\partial U_i}{\partial \tau_w} = (1 - \tau_c)(E - E_i) + [1 - (1 - \tau_w)(1 - \tau_c)]\frac{\partial N}{\partial \tau_w} - \frac{r - n}{1 + r} \left(N + \tau_w \frac{\partial N}{\partial \tau_w}\right).$$
(13)

The first term represents the intragenerational redistribution effect. This favors workers with less ability than the mean (and hence, workers poorer than the average). The second term, which is negative for all workers, represents the loss due to aggravated distortions in labor supply.⁸ The third term shows the loss arising from the intergenerational redistribution. In fact, this is negative in as far as a higher wage tax rate increases wage tax revenues. As is well known, this is because intergenerational income transfers are dynamically inefficient in an economy with r > n.

Let τ_w^y be worker *i*'s most-preferred wage tax rate. Developing and rearranging (13) yields the first-order condition,

$$\frac{\partial U_i}{\partial \tau_w} = -(1-\tau_c)E_i + \left(\frac{1+n}{1+r} - \tau_c\right)E + \frac{1}{1-\tau_w}\left(1 - \frac{1+n}{(1+r)(1-\tau_c)(1-\tau_w)}\right) \le 0.$$
(14)

With the strict inequality, $\tau_w^y = 0$. This condition allows us to write this as a function of E_i , τ_c , and n:

$$\tau_w^y = \tau_w^y(E_i, \tau_c, n). \tag{15}$$

Whenever $\tau_w^y > 0$, the second-order condition, $\partial^2 U_i / \partial \tau_w^2 < 0$, must be met. The condition reduces to

$$1 - \frac{2(1+n)}{(1+r)(1-\tau_w)(1-\tau_c)} < 0, \tag{16}$$

which is guaranteed if (1 + n)/(1 + r) > 1/2. We assume this throughout the analysis to ensure that workers' preferences, as well as those of retirees, are single peaked.

Workers' most-preferred wage tax rates have the following properties. First, as shown by (13), $\tau_w^y = 0$ for $E_i \ge E$ whenever $r \ge n$. Rich workers are then better off without a social security program if it makes for a dynamically inefficient transfer. Second, τ_w^y is decreasing in E_i . That is, richer workers have lower most-preferred wage tax rates. This follows from $\partial^2 U_i / \partial E_i \partial \tau_w = -(1 - \tau_c) < 0.$

⁸Because $1 - (1 - \tau_w)(1 - \tau_c)$ is the difference between the before- and after-tax wage rates, multiplying it by the change in the labor supply yields the change in the deadweight loss in the labor supply.

3.3 Voting equilibrium

Suppose that majority voting takes place to determine a wage tax rate with the consumption tax rate taken as given. We can invoke the median voter theorem to find a Condorcet-winner wage tax rate given that all voters' preferences are single peaked.

Comparing the policy preferences between retirees and workers, we have $\tau_w^y(E_i, \tau_c, n) < \tau_w^o(\tau_c)$ because τ_w^o is the tax rate that maximizes wage tax revenues. Combined with the assumption of n > 0, it results that the decisive voter is among the workers. If we denote her ability by E_w , this is determined by $1 + (1 + n)F(E_w) = (2 + n)/2$, or

$$F(E_w) = \frac{n}{2(1+n)}.$$
(17)

From this condition, it is clear that $E_w < E_m$. Taking account of the fact that E_w increases with n, we write the Condorcet-winner wage tax rate, τ_w^* , as a function of τ_c and n:

$$\tau_w^* = \tau_w^y(E_w, \tau_c, n) := \tau_w^*(\tau_c, n).$$
(18)



Figure 2: Condorcet-winner wage tax rate

Figure 2 depicts the distribution of the most-preferred wage tax rates to explain diagrammatically the nature of the Condorcet winner. The rectangular areas represent the population share of workers and retirees who most prefer $\tau_w = 0$ and $\tau_w = \tau_w^o$, respectively. The height of the graph at each tax rate between zero and τ_w^o corresponds to the density of workers who prefer it most. The hump-shaped schedule is drawn by reversing the right and left of the ability distribution and extending it upward by a factor of 1 + n. The Condorcet-winner wage tax rate is determined at the level that equalizes the areas to the left and to the right under the graph.

3.4 Comparative statics

Let us consider the effects of population aging and a higher consumption tax on the Condorcetwinner wage tax rate, assuming that $\tau_w^* > 0$.

Consider first the effect of population aging, which is identified as a decrease in n in our model. Population aging affects the Condorcet-winner wage tax rate in opposite directions:

$$\frac{\partial \tau_w^*}{\partial n} = \frac{\partial \tau_w^y}{\partial n} + \frac{\partial \tau_w^y}{\partial E_i} \frac{\partial E_w}{\partial n}$$

First, it reduces τ_w^* by lowering the most-preferred tax rate of the decisive worker-voter. This effect is captured by the first term, which is positive if $\tau_w^y > 0.^9$ The intuitive reason is that aging increases the costs of intergenerational transfer. Second, aging increases τ_w^* by strengthening the political influence of retirees. This effect is captured by the second term, which is negative if $\tau_w^y > 0.^{10}$ Because the population of retirees increases relative to workers, a poorer worker-voter, who prefer a higher wage tax rate, becomes decisive in the voting.

Because these two effects counteract each other, the way that population aging affects the Condorcet-winner wage tax rate is generally ambiguous. Nonetheless, if the density of the decisive worker-voter, $f(E_w)$, is sufficiently large, which is quite natural in the most commonly observed wage income distribution, such as in Figure 1, the first effect is likely to outweigh the second. As a result, population aging decreases the Condorcet-winner wage tax rate.¹¹

Consider next the effect of a higher consumption tax rate. The effect has three channels, and in total, it turns out to decrease unambiguously the Condorcet-winner wage tax rate. First, a higher consumption tax induces poor workers to prefer a lower wage tax because the extent of income redistribution across workers through wage tax financing effectively becomes smaller. In other words, a higher consumption tax leads wage tax financing to be less beneficial to poor

 ${}^{\scriptscriptstyle 10}{\rm This \ follows \ given \ }\partial\tau^y_w/\partial E_i<0 \ {\rm and \ }\partial E_w/\partial n=1/[2(1+n)^2f(E_w)]>0.$

¹¹This observation is similar to that by Razin, Sadka, and Swagel (2002). They used an overlapping generations model with human capital formation and showed that population aging may lead to a downsizing in the size of the welfare state. They also tested this hypothesis with data for the US and 12 European countries over the period 1965–92 to obtain a positive empirical result. See also Disney (2007), Simonovits (2007), and Galasso and Profeta (2007), among others, for the controversies that their paper initiated.

⁹Differentiating (11) with respect to *n* and making use of (14) yields $\partial^2 U_i / \partial n \partial \tau_w = [(1 - \tau_c)N_i + \tau_c E]/(1+n) > 0$ when $\tau_w = \tau_w^y$.

workers because they have to pay a larger part of their benefits back to the government through consumption taxation. Second, a higher consumption tax induces every worker to prefer a lower wage tax because it aggravates labor market distortions. Third, and conversely, a higher consumption tax induces every worker to prefer a higher wage tax because it decreases the size of costly intergenerational transfers owing to the smaller wage tax revenue collected. As a matter of fact, the second effect is shown to outweigh the third, and a higher consumption tax reduces the Condorcet-winner wage tax rate;

$$\frac{\partial \tau_w^*}{\partial \tau_c} = \frac{\partial \tau_w^y}{\partial \tau_c} < 0$$

whenever $\tau_w^* > 0.^{12}$ The following proposition summarizes the observations obtained in this section.

Proposition 1 Suppose that $\tau_w^* > 0$. (i) The way that population aging affects the Condorcetwinner wage tax rate is then generally ambiguous. Provided that the density of the decisive worker-voter is sufficiently large, population aging then lowers the Condorcet-winner wage tax rate. (ii) A higher consumption tax rate unambiguously decreases the Condorcet-winner wage tax rate.

4 Financing by consumption tax

4.1 Policy preferences of retirees

Consider first retirees' preferences over consumption tax rates. Given a wage tax rate, (3) and (7), we show that retiree j's most-preferred consumption tax rate, τ_c^o , satisfies

$$\frac{\partial V_j}{\partial \tau_c} = -(1+r)A_j + \frac{\partial B}{\partial \tau_c} \le 0, \tag{19}$$

¹²If we look at (13), the three channels correspond to the changes in the three terms on the right-hand side. Differentiating (11) with respect to τ_c , we have

$$\frac{\partial^2 U_i}{\partial \tau_c \partial \tau_w} = -(E - E_i) - \frac{1 + n}{(1 + r)(1 - \tau_w)^2 (1 - \tau_c)^2}$$

The first term reflects the effect through the first channel, and the second term reflects the net effect through the second and third channels. Given that the decisive worker-voter has $E_i < E$, both terms are negative.

and $\tau_c^o = 0$ with the strict inequality. Retirees, whose asset holdings are predetermined, expect the effect of a higher consumption tax on the size of a benefit per retiree to be

$$\frac{\partial B}{\partial \tau_c} = (1+r)A + (1+n)\tau_w \frac{\partial N}{\partial \tau_c}.$$
(20)

The first term shows the increase in consumption tax revenues per retiree, and the second term shows the reduction in wage tax revenues per retiree. Substituting (20) into (19) yields

$$\frac{1}{1+r}\frac{\partial V_j}{\partial \tau_c} = A - A_j - \frac{(1+r)\tau_w}{(1+r)(1-\tau_c)^2(1-\tau_w)} \le 0.$$
(21)

Thus, $\tau_c^o = 0$ for every retiree wealthier than the average. Furthermore, as $\partial^2 V_j / \partial \tau_c^2 < 0$ is guaranteed, each retiree's policy preferences are single peaked.

Suppose that the same wage tax rate was applied to retiree j's earnings when young. If retiree j was a worker with ability E_j , then $A - A_j = (1 - \tau_w)(E - E_j)$, and hence from (21), her most-preferred consumption tax rate, τ_c^o , is determined as a function of E_j , τ_w , and n:

$$\tau_c^o = \tau_c^o(E_j, \tau_w, n). \tag{22}$$

It is intuitively straightforward that τ_c^o is decreasing in E_i ; the richer retiree has the lower mostpreferred consumption tax rate.¹³

4.2 Policy preferences of workers

Consider next workers' preferences over consumption tax rates, assuming policy commitment as in the case of wage tax financing. Differentiation of (10) shows that a higher consumption tax rate affects worker i's utility and her social security benefit as follows:

$$\frac{\partial U_i}{\partial \tau_c} = -(1 - \tau_w)N_i + \frac{1}{1 + r}\frac{\partial B}{\partial \tau_c}$$
(23)

and

$$\frac{\partial B}{\partial \tau_c} = (1+r)A + (1+r)(1-\tau_w)\tau_c \frac{\partial N}{\partial \tau_c} + (1+n)\tau_w \frac{\partial N}{\partial \tau_c}.$$
(24)

Comparing (20) with (24), the only difference is the second term in (24). This reflects workers' expectations about how consumption tax revenues in their retirement period will respond to the change in their current labor supply.

¹³This observation follows from $\partial^2 V_j / \partial E_j \partial \tau_c = -(1+r)(1-\tau_w) < 0$ and $\partial^2 V_j / \partial \tau_c^2 < 0$.

Plugging (24) into (23) yields

$$\frac{\partial U_i}{\partial \tau_c} = (1 - \tau_w)(E - E_i) + [1 - (1 - \tau_w)(1 - \tau_c)]\frac{\partial N}{\partial \tau_c} - \frac{(r - n)\tau_w}{1 + r}\frac{\partial N}{\partial \tau_c}.$$
(25)

Similar to (14), the first term represents the intragenerational redistribution, the second represents the distortionary effect on the labor supply, and the third represents the effect associated with the intergenerational transfer. The second term is always negative. The third term is positive when r > n, because a higher consumption tax decreases intergenerational transfers by reducing wage tax revenues. However, the second term outweighs the third, and thus their net effect is negative, as shown below.

Arranging the terms in (25), we find that worker *i*'s most-preferred consumption tax rate, τ_c^y , satisfies

$$\frac{\partial U_i}{\partial \tau_c} = (1 - \tau_w)(E - E_i) - \frac{(1 + n)\tau_w}{(1 + r)(1 - \tau_c)^2(1 - \tau_w)} - \frac{\tau_c}{(1 - \tau_c)^2} \le 0,$$
(26)

and $\tau_c^y = 0$ with the strict inequality. Note that $\partial^2 U_i / \partial \tau_c^2 < 0$ is guaranteed, and thus workers' preferences are single peaked. (26) allows us to write τ_c^y as a function of E_i , τ_w , and n:

$$\tau_c^y = \tau_c^y(E_i, \tau_w, n). \tag{27}$$

From (26), $\tau_c^y = 0$ for workers richer than the average. Moreover, it is intuitively straightforward that τ_c^y is decreasing in E_i ; the richer worker has the lower most-preferred consumption tax rate.¹⁴

4.3 Voting equilibrium

Suppose that majority voting takes place to determine a consumption tax rate, given a wage tax rate. To find a Condorcet winner, we first consider the pairing of a worker and a retiree whose most-preferred consumption tax rates coincide.

Comparing (21) and (26), we observe that the most-preferred consumption tax rate is the same for worker i and retiree j if and only if their abilities satisfy

$$E_j = E^o(E_i, \tau_w, n) := E_i + \frac{\tau_c^y}{(1 - \tau_w)(1 - \tau_c^y)^2}.$$
(28)

Combining a worker of ability E_i and a retiree of ability $E^o(E_i, \tau_w, n)$, we can aggregate the policy preferences. Importantly, the abilities matched satisfy

$$E^{o}(E_{i},\tau_{w},n) \ge E_{i},\tag{29}$$

¹⁴This observation follows from $\partial^2 U_i / \partial E_i \partial \tau_c = -(1 - \tau_w) < 0.$

with the equality holding if and only if $\tau_c^y(E_i, \tau_w, n) = 0$. Recall here that the most-preferred consumption tax rate is decreasing in ability for workers as well as for retirees. The above inequality then implies that retirees prefer a higher consumption tax rate than workers if they have the same ability. In other words, for any given consumption tax rate, the proportion of retirees who prefer raising it is greater than that of workers. This is because retirees do not care about how a higher consumption tax today affects consumption tax revenues in the next period.

Figure 3 diagrammatically explains how to identify a Condorcet winner. The upper graph in Figure 3 represents the distribution of retirees' most-preferred consumption tax rates, and the lower graph represents that of workers. The respective rectangles show the proportions of retirees and workers who most prefer a zero tax rate. Though both graphs have a similar shape, the retirees' graph is located more to the right than the workers' graph. This reflects the fact that retirees prefer a higher consumption tax than workers with the same ability. In addition, the height of the retirees' graph is contracted by 1/(1+n), compared with that of the workers owing to population growth. The Condorcet-winner consumption tax rate, τ_c^* , is determined at the level separating the two graphs such that the sum of the areas on each side is equalized.



Figure 3: Voting equilibrium with consumption taxes

To formalize the above diagrammatic exposition, aggregate the population of workers and retirees with the same preferences, making use of (28). The ability of the median worker-voter, E_c^* , is then determined by

$$F(E^{o}(E_{c}^{*},\tau_{w},n)) + (1+n)F(E_{c}^{*}) = \frac{2+n}{2}.$$
(30)

The first term on the left-hand side is the proportion of retirees voting for increasing consumption taxes and the second term is that of workers. Generally, E_c^* depend on n and τ_w . Hence, we denote it by $E_c^* = E_c^*(\tau_w, n)$. The Condorcet-winner consumption tax rate is written as

$$\tau_c^* = \tau_c^y(E_c^*, \tau_w, n) := \tau_c^*(\tau_w, n).$$
(31)

Note that because of (29), $E_c^* < E_m$ whenever $\tau_c^* > 0$.

4.4 Comparative statics

Consider first the effects of population aging on the preferences of retirees and workers, and then on the Condorcet-winner consumption tax rate.

Population aging induces retirees to favor a higher consumption tax rate irrespective of how wealthy they are. This is owing to a smaller labor force. Every retiree expects to receive a larger amount of benefits from a consumption tax increase than before because it causes a smaller reduction in wage tax revenues. Population aging thus leads workers to prefer a higher consumption tax rate. Recall that aging makes intergenerational transfers more costly for workers. A higher consumption tax is then beneficial to every worker because it decreases the amount of intergenerational transfers by reducing the labor supply and thus the wage tax revenues transferred to retirees. Moreover, aging changes the position of the decisive voter and makes a poorer workervoter decisive. Recall from the inequality in (29) that retirees have more intense preferences for a consumption tax increase than workers do. As a result, population aging, or strengthening retirees' political influence, decreases E_c^* . With these three effects combined, the Condorcet-winner consumption tax rate becomes unambiguously higher as population ages.

Consider next the effect of a higher wage tax rate. First, retirees prefer a lower consumption tax rate when there is a higher wage tax rate. The reason is that a consumption tax increase reduces wage tax revenues more with a higher wage tax. A wage tax increase also induces workers to prefer a lower consumption tax, symmetrically as a consumption tax increase induces them to prefer a lower wage tax.¹⁵ Overall, as all voters change their preferences in the same direction,

 $^{^{15}}$ See footnote 12.

the Condorcet-winner consumption tax rate decreases in response to a higher wage tax rate.

The following proposition summarizes our observations in this section.

Proposition 2 Suppose that $\tau_c^* > 0$. (i) Population aging increases the Condorcet-winner consumption tax rate. (ii) A wage tax increase lowers the Condorcet-winner tax rate.

Proof: See appendix A.

5 Political economy of social security funding

5.1 Structure-induced equilibria

We now analyze the public choice of the pair of the two tax rates. No Condorcet winner generally exists in voting over multiple issues without restricting either voters' policy preferences or the structure of political decision making.¹⁶ Taking the latter course, we use the notion of structure-induced equilibrium, first introduced by Shepsle (1979). The focus of the analysis is on how the equilibrium outcome responds to a change in the population growth rate.

Suppose that a wage tax rate and a consumption tax rate are within the jurisdictions of separate committees in a legislature.¹⁷ We assume that the members of each committee reflect all voters' policy preferences in society with no biases—the case that Shepsle (1979) refers to as "the committee of the whole." Each committee determines a tax rate within its jurisdiction by majority voting, taking the other tax rate as given. A pair of the two tax rates, (τ_c^e, τ_w^e) , is a structure-induced equilibrium if and only if τ_w^e is a Condorcet winner given τ_c^e and vice versa; that is, $\tau_c^e = \tau_c^*(\tau_w^e, n)$ and $\tau_w^e = \tau_w^*(\tau_c^e, n)$.

Let us start with a diagrammatic analysis. Figures 4 and 5 depict the wage-tax reaction curve $W_1W_2\tau_c$, which plots the relationship of the two taxes satisfying (18) with a given population growth rate. The Condorcet-winner wage tax rate decreases in response to a higher consumption tax rate and is equal to zero when the consumption tax rate exceeds the threshold rate at W_2 . The reaction curve coincides with the schedule of the most-preferred wage tax rates of a worker with ability E_w .

¹⁶See, e.g., Persson and Tabellini (2000).

¹⁷In Japan, the Committee on Health, Welfare, and Labor in the lower house (as well as that in the upper house) has jurisdiction over wage-based social security contributions, while the Committee on Financial Affairs has jurisdiction over consumption taxes.

Similarly, the consumption tax reaction curve $C_1C_2\tau_w$ depicts the relationship satisfying (31) with the same population growth rate. The Condorcet-winner consumption tax rate decreases in response to a higher wage tax rate and is equal to zero when the wage tax rate exceeds the threshold rate at C_2 . In contrast to the wage tax reaction curve, the consumption tax reaction curve does not coincide with the schedule of a particular worker's most-preferred consumption tax rates, because the decisive worker-voter changes, depending on the wage tax rates, as shown in (28).



Figure 4: A unique structure-induced equilibrium

If r > n at a small margin or if $n \ge r$, the wage tax reaction curve is located above the consumption tax reaction curve, as depicted in Figure 4. The structure-induced equilibrium is then uniquely determined at W_1 , where the social security benefits are financed only by wage taxation. The equilibrium size of social security benefits is driven so large that a coalition of poor workers and poor retirees who may support the introduction of consumption tax financing cannot constitute a majority. The economic advantage of consumption tax financing is limited because the costs of intergenerational transfer are not very large for workers. Conversely, if only consumption tax financing is employed to fund social security benefits, the size is so unsatisfactory for poor workers and all retirees that they can form a majority coalition to support a proposal introducing wage tax financing.

As the population growth rate decreases, the positions of the two schedules change. If the effect of retirees' strengthened political influence does not change the median worker-voter's ability very much, the wage tax reaction curve shifts to the left. The consumption tax reaction curve unambiguously moves upward. If the population growth rate becomes sufficiently small, and thus r > n holds with a sufficiently large margin, the reaction curves are then likely to cross each other, as illustrated in Figure 5. There then occur three different types of structure-induced equilibrium. The social security program in equilibrium employs wage taxation only at W_1 , consumption taxation only at C_1 , and both at C_3 .



Figure 5: Multiple structure-induced equilibria

Comparing Figures 4 and 5, we demonstrate that consumption tax financing may emerge as a political equilibrium outcome in an aging society. This is because population aging induces the society to shift the revenue source of its social security program from wage taxation to consumption taxation, totally or partially, in equilibria like C_1 and C_3 in Figure 5. The economic condition behind such a shift is that with slower population growth, a wage tax increase contributes less to the social security budget, and the costs of intergenerational transfer become higher at the same time. Politically, this leads an increasing population of poor workers and poor retirees that support a proposal of introducing consumption tax financing. It also induces an increasing population of middle-class and rich workers to oppose a wage tax increase. In spite of all retirees' preferring a wage tax increase, the population of poor workers whose preferences align with their own is too small to form a majority coalition. Backed by such a politico-economic interaction, the society introduces consumption taxes to fund its social security program.

Nonetheless, as shown in W_1 in Figure 5, it is also possible that a social security program continues to be financed only with wage taxation, even if population aging proceeds. Wage tax financing is one of the political equilibrium outcomes unless the population growth rate becomes sufficiently low to make the two reaction curves intersect only on the vertical axis. If a wage tax rate increases to the median worker-voter's ideal level, no majority coalition can support the introduction of consumption tax financing, although it does help society to save the costs of intergenerational transfer.

5.2 Formal presentation and simulation

To present these findings formally, we require some new notation. First, taking r and n as given, let τ_w^c be the wage tax rate at C_2 , and let τ_c^w be the consumption tax rate at W_2 in Figures 4 and 5. Precisely, these are defined by $\tau_w^c := \min\{\tau_w | \tau_c^*(\tau_w, n) = 0\}$ and $\tau_c^w := \min\{\tau_c | \tau_w^*(\tau_c, n) = 0\}$. Second, let $E_c := E_c^*(0, n)$ be the ability held by the median worker-voter in voting over consumption taxes in the absence of wage taxation. It is already clear that $E_w < E_c < E_m$. Finally, define two threshold ability levels,

$$E_H := E_m - \frac{r-n}{1+r} \left(E - \frac{1}{1-\tau_w^c} \right)$$

and

$$E_L := E_c - \frac{r - n}{(1 + r)(1 - \tau_c^c)} \left(E - \frac{1}{1 - \tau_c^c} \right)$$

Then the results of the diagrammatic analysis above are formally stated in the following proposition.

Proposition 3 (i) A structure-induced equilibrium with $\tau_w^e > 0 = \tau_c^e$ exists if and only if $E_w \leq E_H$. (ii) A structure-induced equilibrium with $\tau_w^e = 0 < \tau_c^e$ exists if and only if $E_w \geq E_L$. (iii) A structure-induced equilibrium with $\tau_w^e > 0$ and $\tau_c^e > 0$ exists simultaneously with the equilibria in (i) and (ii) if and only if $E_L < E_w < E_H$.

Proof: See Appendix B.

As illustrated in Figure 4, a social security program financed entirely by wage taxation is a unique political equilibrium outcome in a society with $n \ge r$, where $E_H \ge E_m > E_w$ and $E_L \ge E_c > E_w$ hold. Conversely, population aging must proceed to establish n < r for consumption tax financing to emerge as a political equilibrium outcome. The situation illustrated in Figure 5 occurs if and only if $E_L < E_w < E_H$.¹⁸ A social security program whose funding is only by consumption taxation is a unique political outcome if r is sufficiently high relative to n to establish $E_L < E_H < E_w$.

Finally, we simulate the three patterns of equilibria to verify the above findings. Suppose that one period corresponds to 25 years and write worker *i*'s ability as $E_i = \underline{E} + z_i$. We let $\underline{E} = 1.2$ and z_i follow a log-normal distribution with mean 0.5 and standard deviation 0.35.¹⁹ This distribution produces E = 2.95 and $E_m = 2.85$. The annual interest rate is held fixed at 3%, and the annual population growth rate is chosen out of three options, 1.5%, 2%, and 2.5%.

Table 2 provides the simulation results.

 Table 2: Simulation results

Case 1:	$r = 1.03^{25} - 1, n = 1.02^{25} - 1$
	$(E_L, E_w, E_H) = (2.34, 2.42, 2.45)$
	$(\tau_w^e, \tau_c^e) = (0.137, 0), (0.077, 0.038), (0, 0.126)$
Case 2:	$r = 1.03^{25} - 1, n = 1.025^{25} - 1$
	$(E_L, E_w, E_H) = (2.56, 2.47, 2.64)$
	$(\tau_w^e, \tau_c^e) = (0.204, 0)$
Case 3:	$r = 1.03^{25} - 1, n = 1.015^{25} - 1$
	$(E_L, E_w, E_H) = (2.14, 2.36, 2.29)$
	$(\tau_w^e, \tau_c^e) = (0, 0.130)$

In Case 1, where the annual population growth rate is 2%, we obtain $E_L < E_w < E_H$, and the three types of equilibrium arise. In the respective equilibria, funding is through a 13.7% wage tax,

 $^{\mbox{\tiny 18}} {\rm As}$ shown in Appendix B, the equilibrium with $\tau^e_w > 0$ and $\tau^e_c > 0$ is unique if it exists.

¹⁹This ability distribution yields a before-tax wage distribution with quartile dispersion coefficient (3rd quartile – 1st quartile)/(2×median) = 0.23 and decile dispersion coefficient (9th decile – 1st decile)/(2× median) = 0.45. These are close to the corresponding values in the Japanese wage distribution for males aged 40 to 44 years. According to the Basic Survey on Wage Structure 2013, they are 0.23 and 0.48, respectively.

a 12.6% consumption tax, and a combination of a 7.7% wage tax and a 3.8% consumption tax. In Case 2, where the annual population growth rate is 2.5%, we obtain $E_w < E_L < E_H$, and the funding in the unique equilibrium is via a 20.4% wage tax. In Case 3, where the annual population growth rate is 1.5%, we obtain $E_L < E_H < E_w$, and the funding in the unique equilibrium is via a 13.0% consumption tax.

6 Concluding remarks

This paper analyzed the political economy of social security funding by wage and consumption taxes using a model of majority voting in an overlapping-generations framework. We placed the analytical focus on the difference in the distributional impacts between the two taxes as well as the costs of intergenerational transfer. We showed that as population aging proceeds, consumption tax financing tends to be included as part of an equilibrium outcome, while wage tax financing also continues to be used. Such equilibrium multiplicity explains why the revenue sources for social security vary across countries as well as why "social" VAT reform has recently become so popular in some countries.

Our theoretical insights provide some interesting testable hypotheses. First, a social security program is more likely to be funded by taxes whose burdens are expected to be shifted to consumers as population aging proceeds in a society. Second, admitting the conventional argument that employer-pays social security contributions shift to prices more than employee-pays contributions, a society with slower population growth tends to increase the share-out ratio of the former. We leave the empirical analysis of these hypotheses to future research.

Our analysis also has an implication for the recent debates on the effect of aging on the size of the welfare state. Recently, Razin, Sadka, and Swagel (2002) theoretically argued that population aging may serve to downsize the size of the welfare state, by taking account of the trade-off between a political and an economic effect; namely, on the one hand, aging makes stronger the political power of the old, and on the other hand, it increases the cost of redistribution. They also tested this hypothesis with data for the US and 12 European countries over the period 1965–92 and provided some empirical evidence. Their analysis initiated theoretical and empirical debates by several studies, including Disney (2007), Simonovits (2007), and Galasso and Profeta (2007), among others. These studies, however, do not take account of the shift in financing methods. Our analysis suggests that the shift from wage tax financing to consumption tax financing will make the impact of population aging on the size of the welfare state more ambiguous than these analyses concur. This is because in the steady state of a dynamically efficient economy, consumption taxation can collect larger revenues than wage taxation when imposed at the same rate because the former has a broader tax base. As argued in this analysis, however, we have to consider a political factor in that wage tax financing makes a poorer worker decisive; other things being equal, wage tax financing leads to a larger social security benefit per retiree. With these counteracting effects, the shift to consumption tax financing adds an additional ambiguity to the overall effect of population aging on the size of the welfare state.

We also leave several other extensions of the model to future research. First, our model assumes policy commitment in that the tax policies determined today will not change in the future. One way to relax this assumption is to make the voting game dynamic and solve its subgame-perfect structure-induced equilibrium, a solution concept introduced by Conde-Ruiz and Galasso (2005). In Appendix C, we show that every equilibrium outcome described in the main text is realized in the equilibrium of an infinitely repeated voting game. Second, we have assumed a small open economy with fixed rates of wages and interest. By extending the framework to a model with endogenous growth, we would be able to obtain richer insights about the relationship between social security funding and economic growth.

Appendix A: Proof of proposition 2

Suppose $\tau_c^y(E_i, \tau_w, n) > 0$ and $\tau_w > 0$. Then, differentiating (26) yields

$$\frac{\partial \tau_c^y}{\partial E_i} = \frac{1 - \tau_w}{U_{cc}} < 0,$$

$$\frac{\partial \tau_c^y}{\partial \tau_w} = \frac{1}{U_{cc}(1 - \tau_c^y)^2 (1 - \tau_w)^2} \left[\frac{1 + n}{1 + r} (1 + \tau_w) + \tau_c^y (1 - \tau_w) \right] < 0,$$

and

$$\frac{\partial \tau_c^y}{\partial n} = \frac{\tau_w}{(1+r)U_{cc}(1-\tau_c^y)^2(1-\tau_w)} < 0,$$

where

$$U_{cc} := \frac{\partial^2 U_i}{\partial \tau_c^2} = -\frac{1}{(1 - \tau_c^y)^3} \left[\frac{2(1+n)}{1+r} \frac{\tau_w}{1 - \tau_w} + 1 + \tau_c \right] < 0.$$

To prove (i), differentiate (30) with respect to n, and we have

$$\left[f(E^{o})\frac{\partial E^{o}}{\partial E_{i}} + (1+n)f(E_{c}^{*})\right]\frac{\partial E_{c}^{*}}{\partial n} = \frac{1}{2} - F(E_{c}^{*}) - f(E^{o})\frac{\partial E^{o}}{\partial n} > 0.$$
(A.1)

The sign follows because $F(E_c^*) < F(E_m) = 1/2$ and

$$\frac{\partial E^o}{\partial n} = \frac{\partial E_j}{\partial \tau_c^y} \frac{\partial \tau_c^y}{\partial n} < 0,$$

the latter of which comes from (28) given $\partial E_j / \partial \tau_c^y > 0$ and $\partial \tau_c^y / \partial n \leq 0$. From (28), on the other hand, we have

$$\frac{\partial E^o}{\partial E_i} = 1 + \frac{1 + \tau_c^y}{(1 - \tau_w)(1 - \tau_c^y)^3} \frac{\partial \tau_c^y}{\partial E_i} = -\frac{2(1 + n)\tau_w}{(1 + r)U_{cc}(1 - \tau_c^y)^3(1 - \tau_w)} > 0.$$

Thus, the bracketed term on the left-hand side of (A.1) must be positive, and hence $\partial E_c^*/\partial n > 0$.

From (31), then,

$$\frac{\partial \tau_c^*}{\partial n} = \frac{\partial \tau_c^y}{\partial E_i} \frac{\partial E_c^*}{\partial n} + \frac{\partial \tau_c^y}{\partial n} < 0$$

whenever $\tau_c^* > 0$.

The proof of (ii) is quite similar. Differentiating (30) with respect to τ_w yields

$$\left[f(E^{o})\frac{\partial E^{o}}{\partial E_{i}} + (1+n)f(E_{c}^{*})\right]\frac{\partial E_{c}^{*}}{\partial \tau_{w}} = -f(E^{o})\frac{\partial E^{o}}{\partial \tau_{w}} > 0$$
(A.2)

whenever $\tau_c^* > 0$, because from (28)

$$\begin{split} \frac{\partial E^o}{\partial \tau_w} &= \frac{\tau_c^y}{(1 - \tau_c^y)^2 (1 - \tau_w)^2} + \frac{1}{1 - \tau_w} \left[\frac{1}{(1 - \tau_c^y)^2} + \frac{2\tau_c^y}{(1 - \tau_c^y)^3} \right] \frac{\partial \tau_c^y}{\partial \tau_w} \\ &= \frac{(1 + n)[1 + \tau_w + \tau_c^y (1 - \tau_w)]}{(1 + r)(1 - \tau_c^y)^5 (1 - \tau_w)^3 U_{cc}} < 0 \end{split}$$

whenever $\tau_c^y > 0$. Given that the bracketed term on the left-hand side of (A.2) is positive, we obtain $\partial E_c^* / \partial \tau_w > 0$. From (31), then,

$$\frac{\partial \tau_c^*}{\partial \tau_w} = \frac{\partial \tau_c}{\partial E_i} \frac{\partial E_c^*}{\partial \tau_w} + \frac{\partial \tau_c}{\partial \tau_w} < 0$$

whenever $\tau_c^* > 0$. ||

Appendix B: Proof of proposition 3

Let τ_w^w and τ_w^c be the wage tax rates at W_1 and C_2 in Figure 4, respectively. Formally, they are defined by $\tau_w^w := \tau_w^*(0, n)$ and $\tau_w^c := \min\{\tau_w | \tau_c^*(\tau_w, n) = 0\}$. If we let $T_w^w := 1/(1 - \tau_w^w)$ and $T_w^c := 1/(1 - \tau_w^c)$ to simplify the notations, then the equilibrium conditions, (14), (26), (28), and (30) yield

$$kE - E_w^* + T_w^w (1 - kT_w^w) = 0 \tag{A.3}$$

and

$$E - E_m + kT_w^c (1 - T_w^c) = 0, (A.4)$$

where k := (1 + n)/(1 + r). In (A.4), we make use of the fact that $E_c^* = E_m$ when $\tau_c^y = 0$, following from (28). An equilibrium with $\tau_w^e > 0 = \tau_c^e$ exists if and only if $T_w^w \ge T_w^c$. Because (A.4) is quadratic, subtracting (A.4) from (A.3) after substituting T_w^c for T_w^w in (A.3), we can rewrite the necessary and sufficient condition for $T_w^w \ge T_w^c$ into

$$E_w \le E_H := E_m - (1 - k)(E - T_w^c).$$
 (A.5)

Similarly, let τ_w^c and τ_c^c be the consumption tax rates at W_2 and C_1 in Figure 4. Their formal definitions are $\tau_c^w := \min\{\tau_c | \tau_w^*(\tau_c, n) = 0\}$ and $\tau_c^c := \tau_c^*(0, n)$. Let us denote $T_c^w := 1/(1 - \tau_c^w)$ and $T_c^c := 1/(1 - \tau_c^c)$ for simplicity. Then the equilibrium conditions, (14), (26), (28), and (30) yield

$$E - E_w + (k-1)ET_c^w + T_c^w(1 - kT_c^w) = 0$$
(A.6)

and

$$E - E_c + T_c^c (1 - T_c^c) = 0. (A.7)$$

The existence of an equilibrium with $\tau_c^e > 0 = \tau_w^e$ is guaranteed if and only if $T_c^c \ge T_c^w$. Subtracting (A.7) from (A.6) after substituting T_c^c for T_c^w in (A.6) reduces the condition to

$$E_w \ge E_L := E_c - (1 - k)T_c^c (E - T_c^c).$$
(A.8)

Rewriting the equilibrium conditions, (14), (26), and (30), we find that if an equilibrium with $\tau_w^e > 0$ and $\tau_c^e > 0$ exists, then the tax rates, τ_w and τ_c , and the ability of the median worker-voter

in voting on consumption tax, E_c^* , are determined through the following system of equations:

$$E - E_w + E(k-1)T_c + T_cT_w(1 - kT_cT_w) = 0$$
(A.9)

$$E - E_c^* + kT_c^2 T_w (1 - T_w) + T_c T_w (1 - T_c) = 0$$
(A.10)

and

$$F(E^{o}) + (1+n)F(E_{c}^{*}) = \frac{2+n}{2},$$
(A.11)

where $T_w := 1/(1-\tau_w)$ and $T_c := 1/(1-\tau_c)$. The definition of E^o is given by $E^o = E + kT_c^2T_w(1-T_w)$, which we obtain from (26) and (28) in the case of $\tau_c^y > 0$. Then, subtracting (A.10) from (A.9) yields

$$(1-k)T_c(E-T_cT_w) = E_c^* - E_w.$$

Given that $E - T_c T_w > 0$ owing to a positive labor supply and $E_c^* > E_w$, it follows that k < 1 is necessary for the existence of an equilibrium with $\tau_w^e > 0$ and $\tau_c^e > 0$.

We next show that as illustrated in Figure 5, the consumption tax reaction curve is steeper than the wage tax reaction curve at their intersection.

Differentiating (A.9), we have the slope of the wage tax reaction curve,

$$\frac{\partial T_c}{\partial T_w} = -\frac{T_c \lambda}{T_w \lambda + E(k-1)} < 0,$$

where $\lambda := 1 - 2kT_cT_w < 0$ owing to the second-order condition spelled out in (16). Similarly, differentiating (A.10) and (A.11), and rearranging the terms, we obtain the slope of the consumption tax reaction curve,

$$\frac{\partial T_c}{\partial T_w} = -\frac{T_c\lambda+\alpha}{T_w\lambda+\beta} < 0,$$

where

$$\alpha := T_c^2(k-1) + \frac{f(E^o)}{(1+n)f(E_c^*)}kT_c^2(1-2T_w) < 0$$

and

$$\beta := 2T_w T_c(k-1) + \frac{2f(E^o)}{(1+n)f(E_c^*)} kT_w T_c(1-T_w) < 0.$$

Simple calculation then demonstrates that the consumption tax reaction curve is steeper, because

$$\frac{T_c\lambda+\alpha}{T_w\lambda+\beta} - \frac{T_c\lambda}{T_w\lambda+E(k-1)} = \frac{\Delta}{(T_w\lambda+\beta)(T_w\lambda+E(k-1))} > 0,$$

where

$$\Delta := T_c(k-1)\{\lambda(E - T_wT_c) + ET_c(k-1)\} + \frac{kf(E^o)T_c^2}{(1+n)f(E_c^*)}\{-2\lambda T_w + E(k-1)(1-2T_w))\} > 0.$$

Finally, given that the consumption tax reaction curve is steeper, it follows that the equilibrium with $\tau_w^e > 0$ and $\tau_c^e > 0$ is unique and that it exists if and only if $T_w^w > T_w^c$ and $T_c^c > T_c^w$. This condition is reduced to $E_L < E_w < E_H$. ||

Appendix C: Subgame perfection of the equilibrium outcomes

Following Conde-Ruiz and Galasso (2005), we show that every structure-induced equilibrium obtained in the main text under the assumption of policy commitment is established as a subgame-perfect equilibrium outcome in an infinitely repeated voting game without policy commitment.

Suppose that a voting game takes place in each period, where each worker and retiree announces a pair of wage and consumption tax rates. Let τ_{wt} and τ_{ct} be the voting outcomes in period $t \geq 1$, which are defined respectively as the medians of wage and consumption tax rates announced by voters. Let h_1 be the history at the start of the game, and let h_t be one at the start of period t. The latter is a combination of h_1 and the outcomes having been realized until period t. The set H collects all possible histories, and H^c contains only h_1 and h_t such that $\tau_{ws} = \tau_w^e$ and $\tau_{cs} = \tau_c^e$ for all $s \leq t - 1$. We will denote by H_t the set of possible histories until period t.

Each voter's strategy in period t is a mapping from H_t to the set of the pairs of the two tax rates. Let $\sigma_i^o(h_t)$ be the strategy of a retiree with ability E_i when voting in period t. Following (7), her payoff function is defined as

$$V_{it} = (1+r)[A_i - \tau_{ct}(1-\tau_{wt-1})(E_i - E)] + (1+n)\tau_{wt}N(\tau_{wt}, \tau_{ct+1}),$$

where A and A_i are constant, satisfying $A_i < A$ if and only if $E_i < E$. Similarly, let $\sigma_i^y(h_t)$ be the strategy of a worker with ability E_i when voting in period t and, following (10), define her payoff function as

$$U_{it} = (1 - \tau_{ct+1})(1 - \tau_{wt})E_i - \ln(1 - \tau_{wt})(1 - \tau_{ct+1}) + \frac{B_{t+1}}{1 + r},$$

where

$$B_{t+1} = (1+r)\tau_{ct+1}(1-\tau_{wt})N(\tau_{wt},\tau_{ct+1}) + (1+n)\tau_{wt+1}N(\tau_{wt+1},\tau_{ct+2}).$$

Now we will show that every structure-induced equilibrium (τ_w^e, τ_c^e) presented in proposition 3 is established as a stationary subgame-perfect equilibrium outcome of the infinitely repeated voting game by the combination of strategies, $\sigma_i^o(h_t) = (\tau_{wi}^o(h_t), \tau_{ci}^o(h_t))$ and $\sigma_i^y(h_t) = (\tau_{wi}^y(h_t), \tau_{ci}^y(h_t))$ for $t \ge 1$, such that

$$\tau_{wi}^o(h_t) = \tau_w^o(\tau_c^e), \quad \tau_{ci}^o(h_t) = \tau_c^e \tag{A.12}$$

for $h_t \in H_t$ and $E_i \in [\underline{E}, \overline{E}]$;

$$\tau_{wi}^{y}(h_{t}) = \begin{cases} \tau_{w}^{e} \text{ if } h_{t} \in H^{c} \\ 0 \text{ otherwise,} \end{cases} \quad \tau_{ci}^{y}(h_{t}) = \begin{cases} \tau_{c}^{e} \text{ if } h_{t} \in H^{c} \\ 0 \text{ otherwise} \end{cases}$$
(A.13)

for $E_i \in [\underline{E}, E_c^e]$; and

$$\tau_{wi}^{y}(h_{t}) = \begin{cases} \tau_{w}^{y}(E_{i}, \tau_{c}^{e}, n) & \text{if } h_{t} \in H^{c} \\ 0 & \text{otherwise,} \end{cases} \quad \tau_{ci}^{y}(h_{t}) = \begin{cases} \tau_{c}^{y}(E_{i}, \tau_{w}^{e}, n) & \text{if } h_{t} \in H^{c} \\ 0 & \text{otherwise.} \end{cases}$$
(A.14)

for $E_i \in (E_c^e, \overline{E}]$, where E_c^e is the ability level that the equilibrium median worker-voter has in voting on consumption tax rates, implicitly defined by $\tau_c^y(E_c^e, \tau_w^e, n) = \tau_c^e$.

These strategies have the following properties. First, as (A.12) shows, concerning voting on wage tax rates, the equilibrium strategy of a retiree stipulates the same behavior as she chooses in the structure-induced equilibrium with policy commitment. As regards consumption tax rates, every retiree votes for τ_c^e , whatever happens in the past. Second, as (A.13) shows, the votes of workers with $E_i \leq E_c^e$ cluster at the pair of tax rates realized in the structure-induced equilibrium with policy commitment, as long as it has been repeatedly realized in the past, and otherwise they all vote for abolishing the social security system. Third, as (A.14) shows, workers with $E_i > E_c^e$ vote as described in the text whenever the outcome (τ_w^e, τ_c^e) has been repeatedly realized, but otherwise they will vote for abolishing the social security system. Under these strategies, (τ_{wt}, τ_{ct}) = (τ_w^e, τ_c^e) if $h_t \in H^c$, and otherwise (τ_{wt}, τ_{ct}) = (0,0).

Let us check whether these strategies constitute a subgame-perfect equilibrium, assuming that even a single vote can affect the voting outcome.

To begin with, consider the strategy of retirees. If the consumption tax rate is τ_c^e , they all want to increase the wage tax rate above τ_w^e because $\tau_w^o(\tau_c^e) > \tau_w^e$. To do this, they have to increase the votes for the tax rates higher than τ_w^e . However, their votes are already higher than the level, and thus there is no room for them to change the voting outcome. With respect to the consumption tax rate, retirees with $E_i < E^o(E_c^e, \tau_w^e)$ want to increase it above τ_c^e . However, they cannot manipulate the voting outcome in their desired direction because they already vote for τ_c^e . A similar reasoning applies to the voting behavior of retirees with $E_i > E^o(E_c^e, \tau_w^e)$.

Now turn to the strategy for workers. First, in the case of $h_t \in H^c$, a similar reasoning applies. There is no room for each worker to manipulate the voting outcome in period t in her desired direction because she already votes in that way. Next, suppose that workers with $E_i \leq E_w^e$ strategically voted for a wage tax rate below τ_w^e and successfully reduced τ_{wt} in period t. Then, the voting behavior stipulated in (A.13) will yield $\tau_{wt+1} = \tau_{ct+1} = 0$ in period t + 1. This means that these workers receive no social security benefits. If so, their best outcome in period t is $\tau_{wt} = 0$. However, even when this happens, $U_i(\tau_w^e, \tau_c^e) \ge U_i(0, 0)$ holds for the following reason. First, $U_i(\tau_w^e, \tau_c^e) \ge U_i(0, \tau_c^e)$ for $E_i \le E_w^e$ because of the single-crossing property of the utility function. Second, $U_i(0, \tau_c^e) \ge U_i(0, 0)$ for $E_i < E_c^e$ because $\tau_c^e < \tau_c^y(E_i, 0, n)$ and $\partial^2 U_i/\partial \tau_c^2 < 0$. Accordingly, they have no incentive to deviate from (A.13). Regarding workers with $E_i > E_w^e$, because the majority of votes cluster at τ_w^e , they cannot manipulate the voting outcome even if they change their votes on the wage tax rates. A similar reasoning applies to the voting on the consumption tax rates, and the above strategies result to form a subgame-perfect Nash equilibrium. ||

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