Waseda Economics
Working Paper Series

Technological Unemployment, the Pigou Effect, and Monetary Growth

Tomohiro Inoue
Graduate School of Economics
Waseda University
and
Eiji Tsuzuki
Graduate School of Economics
Waseda University

Graduate School of Economics
Waseda University

1-6-1, Nishiwaseda, Shinjuku-ku, Tokyo
169-8050 JAPAN

Waseda Economics
Working Paper Series

Technological Unemployment, the Pigou Effect, and Monetary Growth

Tomohiro Inoue
Graduate School of Economics
Waseda University
and
Eiji Tsuzuki
Graduate School of Economics
Waseda University

Number 08-001
September 2008

Graduate School of Economics
Waseda University

1-6-1, Nishiwaseda, Shinjuku-ku, Tokyo

169-8050 JAPAN

Technological Unemployment, the Pigou Effect, and Monetary Growth*

Tomohiro Inoue[†] Eiji Tsuzuki[‡]

September 4, 2008

Abstract

In this paper, we model an economy in which technological change continues to increase potential supply constantly while the Pigou effect can adjust the market, and analyze the influence that technological change and monetary growth have on employment. We obtain the following result. When the monetary growth rate equals the technological change rate, full employment persists. When the monetary growth rate is less than the technological change rate, underemployment and deflation persist. In contrast, when the monetary growth rate is more than the technological change rate, excess employment and inflation persist.

Keywords: Effective demand shortage, Technological change, Monetary growth, Money-in-utility model, Monopolistic competition

JEL Classification Number: E20; E40; O10

---

*We are grateful to Professor Kazuyuki Sasakura.

[†]Graduate school of Economics, Waseda University, 1-6-1 Nishiwaseda, Shinjuku-ku, Tokyo 169-8050, Japan. E-mail: inouetomo@gmail.com

[‡]Graduate school of Economics, Waseda University, 1-6-1 Nishiwaseda, Shinjuku-ku, Tokyo 169-8050, Japan. E-mail: etsu@ruri.waseda.jp.

1 Introduction

Conventional macroeconomic theory is divided into long-run and short-run types. The long-run theory analyzes mainly supply factors: technological change and capital stock. The short-run theory usually analyzes demand factors.

Unless effective demand shortage vanishes eventually, the long-run theory which excludes demand factors is not valid. Many economists think that it vanishes since a market has an automatic adjustment mechanism such as the Pigou effect [1] .

According to Kalecki (1944) which was the earliest criticism for the Pigou effect, the debt-deflation effect [2] sets off the Pigou effect. However Tobin (1980) has stated that the Pigou effect is dominant in the long-run, while the debt-deflation effect is dominant in the short-run [3] .Thus some of old Keynesian economists have thought that full employment occurs in the long-run. New Keynesian economists have also thought so. They have believed in the long-run money neutrality and Neoclassical growth theory, which assumes full employment [4].

In contrast, Post Keynesian has argued about the persistent underemployment. Kalecki emphasized the work of the debt-deflation effect, as stated above. Robinson (1953) criticized the Neoclassical production function, which implies the capital-labor substitutability. Robinson (1973) urged the importance of the uncertainty that is distinct from risk, and Davidson (1991) insisted that the uncertainty causes the persistent underemployment. In short, they have not made any standard assumptions. Hence they have been frequently disregarded by mainstream economists.

We dare to imagine an economy in which agents perform dynamic optimization with perfect-foresight, capital and labor are substitutable, and the Pigou effect works dominantly. Is there no need to concern us about the persistent underemployment in such an economy? Are there any oversights [5] ?

---

[1] The Pigou effect is the action that a rise in real money balance increases consumption during deflation. The Pigou effect has been argued by Pigou (1943,1947), Patinkin (1965), Hahn (1965), etc.

[2] Before that, Fisher (1933) had already presented the debt-deflation effect. For several years, studies such as Asada (2004,2006) have focused on the effect.

[3] Tobin also indicated the possibility that the Pigou effect does not work sufficiently even in the long-run.

[4] Mankiw stated, "economists today are more interested in the long-run equilibrium"(Mankiw (1992) : pp.561). "Although there is continuing debate about whether the Solow model provides an adequate description of economic growth, the model is rarely criticized as being too classical"(pp.561). "Old classical economists, such as David Hume, asserted that money was neutral in the long run but not in the short run. This is exactly the position held by new Keynesians"(pp.563).

[5] Ono (2001) has argued that underemployment state can persist under the condition that the marginal utility

Now we date back to the effective demand theory of pre-Keynes temporarily. Malthus argued about the influence of technological change on effective demand shortage. Labor-saving technological change does not increase demand while it increases supply [6]. As a result, effective demand runs short, and technological unemployment occurs [7]. Though in 20th century Keynes (1936) excavated the unemployment problem due to effective demand shortage suggested by Malthus, Keynes excluded the influence of technological change.

From that time onward, despite much attention to effective demand shortage, few studies have been concerned with the influence of technological change on effective demand shortage [8], since the influence has been considered to be temporary and insignificant. If technological change were one-off, the Pigou effect would surely extinguish the influence eventually.

However, technology has been improving constantly in modern times. If technological change continues to increase potential supply while the Pigou effect increases consumption demand, the market adjustment may not end. Though the consumption demand chases the potential supply, the potential supply would go on running away like a road mirage. Confirming whether the Pigou effect really fails to extinguish effective demand shortage requires the model in which both technological change and market adjustment caused by the Pigou effect continue to occur simultaneously.

The short-run theory generally assumes that the technological change rate is zero, and the price is rigid or sluggish; thus the adjustment speed is zero or positive. The long-run theory assumes that the technological change rate is positive, and the price is flexible; thus the adjustment speed is infinite.

In the real economy, however, both the technological change rate and the adjustment speed are usually positive and finite. We would like to model such an economy faithfully, and confirm

of real money balance is not zero, even in the standard model. Ono got to the point that many economists had missed out. We attempt to indicate another oversight.

[6]Malthus wrote, "The three great causes most favourable to production are, accumulation of capital, fertility of soil, and inventions to save labour. They all act in the same direction; and as they all tend to facilitate supply, without reference to demand, it is not probable that they should either separately or conjointly afford an adequate stimulus to the continued increase of wealth"(Malthus (1836): Vol.6, pp.288).

[7]In 19th century, technological unemployment was also argued by Sismondi (1819), Ricardo (1821), etc.

[8]Over the past few decades, several studies such as Aghion and Howitt (1994), Postel-Vinay (2002) have been made on technological unemployment. They focused on the mismatch unemployment. It is different from the unemployment due to effective demand shortage presented by Malthus (1836) and this paper. Though Pasinetti (1981,1993) have been concerned with the influence of technological change on effective demand shortage, the studies have not been brought to public attention.

whether underemployment state persists in the model.

We make a Money-in-utility model [9] under perfect-foresight dynamics, introduce the wage adjustment function into it, suppose a monopolistically competitive market to represent explicitly optimum pricing by firms under the given wage [10] , and exclude the debt-deflation effect. The production function is Cobb=Douglas function. A firm can adjust its labor input and capital input to the appropriate level instantly.

If unemployment occurs in this model, the nominal wage declines sluggishly according to the wage adjustment function. In response to the wage decline, the firms bring down a price to the optimum level instantly. The rise in real money balance during the deflation increases consumption demand. On the other hand, technological change continues to increase potential supply constantly. Therefore, underemployment state may persist.

In this regard, however, whether persistent underemployment occurs depends not only on the technological change rate but also on the monetary growth rate, since monetary growth as well as deflation increases real money balance, and helps market adjustment. Hence we also examine the optimal rate of monetary growth that can maintain full employment.

In the next section, we present a model. Section 3 describes the steady state of the model, and Section 4 analyzes the dynamic stability of the steady state. The last section concludes.

2 Model

General view of the model

As illustrated in Figure 1, suppose that there is an economy consisting of three types of agent: household, firm, and retailer. There are numerous firms in the economy, and each firm produces differentiated goods. All of the goods are assumed to be measured in a common physical unit. Firm i produces goods i. Both of firms and goods are continuously distributed, and their total numbers are each normalized at unity, i.e. $i \in [0, 1]$. Similarly, the households and retailers are also continuously distributed, and their total numbers are each normalized at unity. In Figure 1, only one representative retailer and one representative houshoeld are described.

[9]Money-in-utility models have been developed by Sidrauski (1967), Brock (1974,1975), Obstfeld and Rogoff (1983), Siegel (1983), Wan and Yip (1992), etc.

[10]In this paper, the steady-state employment rate does not depend on the degree of monopolization. Our main purpose to suppose a monopolistically competitive market is not to analyze the influence of the monopolization on unemployment but to make the pricing mechanism clear.

Firm i sells retailers goods i. Let y_i be the quantity of goods i. A retailer puts together differentiated goods into final goods. The final goods are available for both consumption and investment. The retailer sells households the final goods as consumption goods by c, and sells firms the final goods as investment goods by I. Let y be the total quantity of final goods; thus $y = c + I$ holds. Since the retailers are assumed to be in a perfectly competitive market, they never make a profit [11].

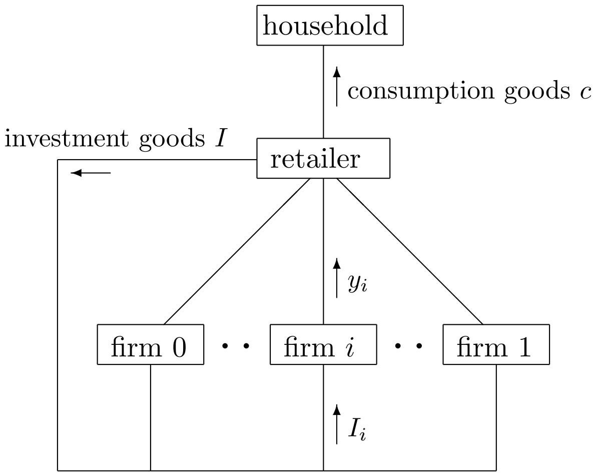

Figure 1: Abstract figure of the model

Retailers

A Retailer puts together differentiated goods into final goods, based on Dixit=Stiglitz function [12]. The quantity of final goods y is given by

$$y = \left[\int_0^1 y_i^{\frac{\phi-1}{\phi}} di \right]^{\frac{\phi}{\phi-1}} \tag{2.1}$$

where y_i is the quantity of goods i. ϕ represents the elasticity of substitution among goods. We assume that ϕ is constant over time, and $\phi > 1$.

[11] We assume that the retailer does not require any capital and labor for aggregating and selling.

[12] See Dixit and Stiglitz (1977) and Blanchard and Kiyotaki (1987).

Let p_i denote the price of goods i set by firm i. When the retailer minimizes its cost,

$$y_i = \left(\frac{p_i}{p}\right)^{-\phi} y \tag{2.2}$$

holds (see Appendix A.1), where p is the price of final goods. p is given by

$$p = \left[\int_0^1 p_i^{1-\phi} di\right]^{\frac{1}{1-\phi}}. \tag{2.3}$$

All retailers sell the final goods at this price.

Firms

Assuming the production function be Cobb=Douglas type, the quantity of goods i: y_i is

$$y_i = k_i^{\alpha} h_i^{1-\alpha} \tag{2.4}$$

where $0 < \alpha < 1$. k_i is the capital stock of firm i. h_i is the quantity of effective labor, which is defined as $h_i \equiv z l_i$. l_i is the quantity of labor employed by firm i, and z is the level of technology. z in period t: $z(t)$ is given by

$$z(t) = z(0)e^{gt} \quad (g > 0) \tag{2.5}$$

where g is the technological change rate.

The instantaneous real profit of firm i: Π_i is

$$\Pi_i = \frac{p_i y_i - W h_i - p I_i}{p} \tag{2.6}$$

where W denotes the nominal wage rate, which means nominal wage per unit of effective labor in this paper. I_i denotes the quantity of investment goods that the retailers sell firm i. For simplicity, we assume that investing never requires adjustment cost.

The discounted present value of firm i: V_i is given by

$$V_i = \int_0^{\infty} \Pi_i(t) e^{-\int_0^t r(s)ds} dt \tag{2.7}$$

$$= \int_0^{\infty} \left[\frac{p_i(t)y_i(t) - W(t)h_i(t) - p(t)I_i(t)}{p(t)}\right] e^{-\int_0^t r(s)ds} dt \tag{2.8}$$

where r is the real interest rate. For simplifying the model, suppose that the capital depreciation rate is zero; thus we have

$$\dot{k}_i = I_i. \tag{2.9}$$

When firm i maximizes V_i,

$$p_i = \frac{\phi}{\phi - 1} \frac{1}{1-\alpha} \left(\frac{h_i}{k_i} \right)^{\alpha} W \tag{2.10}$$

holds (see Appendix A.2). This equation implies that the price of goods i: p_i is equal to the marginal cost $[1/(1-\alpha)](h_i/k_i)^{\alpha} W$ multiplied by the markup rate $\phi/(\phi - 1)$.

We assume that all firms have the same initial capital stock $k(0)$. Now all firms have the same quantities of effective labor and capital stock, because all firms behave with the same equations. Let h be such a quantity of effective labor, and k be such a quantity of capital stock; thus $h_i = h$ and $k_i = k$ hold. Therefore, we have

$$p_i = \frac{\phi}{\phi - 1} \frac{1}{1-\alpha} \hat{h}^{\alpha} W \tag{2.11}$$

where $\hat{h}$ is defined as $\hat{h} \equiv h/k$. From Eq. (2.11), all differentiated goods have the same price. Hence, from Eq. (2.3), $p_i = p$ holds.

We now can rewrite Eq. (2.11) as

$$p = \frac{\phi}{\phi - 1} \frac{1}{1-\alpha} \hat{h}^{\alpha} W \tag{2.12}$$

or

$$\frac{\phi}{\phi - 1} w = (1-\alpha) \hat{h}^{-\alpha}. \tag{2.13}$$

This equation implies that the real wage rate $w \equiv W/p$ multiplied by the markup rate $\phi/(\phi - 1)$ is equal to the marginal product of labor $(1-\alpha)\hat{h}^{-\alpha}$ [13].

Since all prices of differentiated goods are same, from Eq. (2.2), all demands of differentiated goods are also same. From Eq. (2.1), $y_i = y$ holds. Furthermore, all quantities of effective labor

[13] As the elasticity of substitution among goods ϕ approaches infinity, the markup rate $\phi/(\phi - 1)$ approaches 1, and the market becomes perfectly competitive. In this case, Eq. (2.13) becomes $w = (1-\alpha)\hat{h}^{-\alpha}$ implying that the real wage is equal to the marginal product of labor.

employed by firms are same, so that all labor quantities are also same. Let l be such a labor quantity; thus $l_i = l$ holds. Similarly, $\Pi_i = \Pi$, $V_i = V$, and $I_i = I$ hold.

The stocks issued by firms are divided equally and owned by households. All of profit Π is paid to the households as dividend. Let d be real dividend; thus $d = \Pi$ holds. Using the nominal price of stock Q, we define the real price of stock q as $q \equiv Q/p$. q is equivalent to the discounted present value of firm V, so that $q = V$ holds. The real interest rate r is defined as

$$r \equiv \frac{\dot{q} + d}{q} = \frac{\dot{V} + \Pi}{V}. \tag{2.14}$$

When the firms maximize their profit, the following equation also holds (see Appendix A.3).

$$\frac{\phi}{\phi - 1} r = \alpha \hat{h}^{1-\alpha} \tag{2.15}$$

This equation implies that the real interest rate r multiplied by the markup rate $\phi/(\phi - 1)$ is equal to the marginal product of capital $\alpha \hat{h}^{1-\alpha}$ [14] . Defining $\beta \equiv \alpha(\phi - 1)/\phi$, Eq. (2.15) can be rewritten as

$$r = \beta \hat{h}^{1-\alpha}. \tag{2.16}$$

Households

In this economy, there exist numerous households that live infinitely. A household obtains utility from both consumption and money balance in each period. Let $u(c)$ be the utility of consumption c, and $v(m)$ be the utility of real money balance m. We suppose $u' > 0, u'' < 0$ and $v' > 0, v'' < 0$ for these functions.

The household maximizes lifetime utility:

$$\int_0^\infty [u(c) + v(m)] e^{-\rho t} dt \quad (\rho > 0) \tag{2.17}$$

where ρ is the discount rate of household.

The real asset balance of household a consists of the real money balance m and the real price of stock q, thus

$$a \equiv m + q \tag{2.18}$$

[14] If the elasticity of substitution among goods ϕ approaches infinity, Eq. (2.15) becomes $r = \alpha \hat{h}^{1-\alpha}$. This equation implies that the real interest rate is equal to the marginal product of capital.

holds. The nominal asset balance of household A is increased by stock return or wage income, and decreased by consumption. Thus, we have

$$\dot{A} = \dot{Q} + p\Pi + Wh - pc. \tag{2.19}$$

Using Eq. (2.18), Eq. (2.19) is rewritten as

$$\dot{a} = \dot{q} + \Pi + wh - c - \pi m \tag{2.20}$$

where π is the inflation rate defined as $\pi \equiv \dot{p}/p$. Substituting Eq. (2.14) and Eq. (2.18) into Eq. (2.20) yields

$$\dot{a} = ra + wh - c - Rm \tag{2.21}$$

where R is the nominal interest rate defined as $R \equiv r + \pi$.

Now we set the Hamilton function:

$$\mathcal{H} = u(c) + v(m) + \mu(ra + wh - c - Rm) \tag{2.22}$$

where μ is the costate variable of a. The first order conditions are given by

$$\frac{\partial \mathcal{H}}{\partial c} = u'(c) - \mu = 0 \tag{2.23}$$

$$\frac{\partial \mathcal{H}}{\partial m} = v'(m) - \mu R = 0 \tag{2.24}$$

$$\dot{\mu} = -\frac{\partial \mathcal{H}}{\partial a} + \rho\mu = (\rho - r)\mu. \tag{2.25}$$

The transversality condition is given by

$$\lim_{t\to\infty} \mu(t)a(t)e^{-\rho t} = \lim_{t\to\infty} \mu(t)[q(t) + m(t)]e^{-\rho t} = 0. \tag{2.26}$$

Using Eq. (2.23), Eq. (2.24), and Eq. (2.25), we have

$$\eta_c \frac{\dot{c}}{c} + \rho + \pi = R = \frac{v'(m)}{u'(c)} \tag{2.27}$$

where $\eta_c \equiv -u''(c)c/u'(c)$ is the elasticity of the marginal utility of consumption [15]. For simplifying calculations, we use the logarithmic utility functions; more precisely we assume $u(c) = \ln c$ and $v(m) = \ln m$. Therefore, Eq. (2.27) is rewritten as

$$\frac{\dot{c}}{c} + \rho + \pi = R = \frac{c}{m}. \tag{2.28}$$

[15] In Ono (2001), Eq. (2.27) is called Keynes's rule.

Employment rate and inflation rate

Let l^n be the quantity of employed labor at full employment [16]. The elasticity of labor supply to wage is assumed to be zero, so that l^n is constant over time.

Define employment rate ε as $\varepsilon \equiv l/l^n$. l is the quantity of employed labor, as stated above. $\varepsilon < 1$ implies underemployment, $\varepsilon = 1$ is full employment, and $\varepsilon > 1$ is excess employment. Define the quantity of effective labor at full employment h^n as $h^n \equiv zl^n$. We have

$$\varepsilon \equiv \frac{l}{l^n} = \frac{h}{h^n}. \tag{2.29}$$

The change rate of W: π_w is written as

$$\pi_w \equiv \frac{\dot{W}}{W} = \gamma\left(\frac{h}{h^n} - 1\right) = \gamma(\varepsilon - 1) \quad (where\ \gamma > 0) \tag{2.30}$$

where γ denotes the wage adjustment speed. Eq. (2.30) is the wage adjustment function, and represents the wage Phillips curve. This equation implies that the change rate of wage rate depends on the relation between the supply and the demand for effective labor.

From Eq. (2.12), we obtain

$$\pi \equiv \frac{\dot{p}}{p} = \frac{\dot{W}}{W} + \alpha\left(\frac{\dot{\hat{h}}}{\hat{h}}\right) = \gamma(\varepsilon - 1) + \alpha\left(\frac{\dot{\hat{h}}}{\hat{h}}\right). \tag{2.31}$$

Suppose that financial authorities increase money supply M by constant rate θ; thus $\dot{M}/M = \theta$ holds where $\theta \geq 0$. Since m is given by $m \equiv M/p$, we have

$$\frac{\dot{m}}{m} = \frac{\dot{M}}{M} - \frac{\dot{p}}{p} = \theta - \pi. \tag{2.32}$$

3 Steady state

The dynamic system consists of Eq. (2.4), Eq. (2.16), Eq. (2.28), Eq. (2.31), and Eq. (2.32) is put together into the following system of differential equations S (see Appendix A.4 for the

[16] l^n is somewhat less than the total quantity of labor. For example, mismatch unemployment causes the difference. The labor quantity l can exceed the labor quantity at full employment l^n, but can not exceed the total quantity. In this paper, we omit the economic state in which the aggregate demand require a labor quantity more than the total quantity.

derivation of the system)

$$(S) \qquad \begin{aligned}
\frac{\dot{\chi}}{\chi} &= \chi - (\theta + \rho) &(3.1)\\
\frac{\dot{\hat{c}}}{\hat{c}} &= r - \rho - \frac{\dot{k}}{k} = \beta \hat{h}^{1-\alpha} - \rho - (\hat{h}^{1-\alpha} - \hat{c}) = (\beta - 1)\hat{h}^{1-\alpha} - \rho + \hat{c} &(3.2)\\
\frac{\dot{\hat{h}}}{\hat{h}} &= \frac{1}{\alpha}(\pi - \gamma\varepsilon + \gamma) = \frac{1}{\alpha}(\chi - \beta\hat{h}^{1-\alpha} - \gamma\upsilon\hat{h} + \gamma) &(3.3)\\
\frac{\dot{\upsilon}}{\upsilon} &= \frac{\dot{\varepsilon}}{\varepsilon} - \frac{\dot{\hat{h}}}{\hat{h}} = \left(\frac{\dot{\hat{h}}}{\hat{h}} + \frac{\dot{k}}{k} - \frac{\dot{h^n}}{h^n}\right) - \frac{\dot{\hat{h}}}{\hat{h}} = \frac{\dot{k}}{k} - g = \hat{h}^{1-\alpha} - \hat{c} - g &(3.4)
\end{aligned}$$

where $\chi \equiv c/m$, $\hat{c} \equiv c/k$, $\upsilon \equiv \varepsilon/\hat{h}$. The variables evaluated in the non-trivial steady state are

$$\begin{aligned}
\chi^* &= \theta + \rho &(3.5)\\
\hat{c}^* &= \frac{\rho + g}{\beta} - g = \frac{1}{\alpha}\frac{\phi}{\phi - 1}(\rho + g) - g &(3.6)\\
\hat{h}^* &= \left(\frac{\rho + g}{\beta}\right)^{\frac{1}{1-\alpha}} = \left[\frac{1}{\alpha}\frac{\phi}{\phi - 1}(\rho + g)\right]^{\frac{1}{1-\alpha}} &(3.7)\\
\upsilon^* &= \varepsilon^*/\hat{h}^* = \frac{\gamma + \theta - g}{\gamma}/\hat{h}^* = \frac{\gamma + \theta - g}{\gamma}\left[\frac{1}{\alpha}\frac{\phi}{\phi - 1}(\rho + g)\right]^{\frac{1}{\alpha - 1}} &(3.8)
\end{aligned}$$

(see Appendix A.5) where the variables with $*$ denote the steady-state value.

In this steady state, from $\dot{\upsilon}/\upsilon = 0$ and Eq. (3.4), $\dot{k}/k = g$ holds. $\dot{c}/c = g$, $\dot{m}/m = g$, and $\dot{h}/h = g$ also hold.

The steady-state employment rate ε^* is given by

$$\varepsilon^* = \hat{h}^*\upsilon^* = \hat{h}^*\frac{\gamma + \theta - g}{\gamma}/\hat{h}^* = \frac{\gamma + \theta - g}{\gamma}. \tag{3.9}$$

Hence, $\varepsilon^* = 1$ holds, under the condition that the wage adjustment speed γ is infinite. We obtain the following proposition.

Proposition 1

If the wage adjustment speed were infinite, the money superneutrality for employment would hold, and full employment would persist.

Neoclassical economics usually supposes that the adjustment speed is infinite. In what follows, however, we assume that γ is finite, considering that the nominal wage rate is sluggish in the real economy. Now we have the following proposition using the technological change rate g, the monetary growth rate θ, and the steady-state employment rate ε^*.

Proposition 2

(i) When $g < \theta$, $\varepsilon^* > 1$ holds, underemployment occurs in the steady state.

(ii) When $g = \theta$, $\varepsilon^* = 1$ holds, full employment occurs in the steady state.

(iii) When $g > \theta$, $\varepsilon^* < 1$ holds, excess employment occurs in the steady state.

In the case of $\gamma + \theta < g$, there is no steady state in which the employment rate is positive. This implies that the economy is unstable if technological change rate g is remarkably high. In this paper, we exclude such a case; thus assume $\gamma + \theta > g$.

From Eq. (3.9), we also find that the steady-state employment rate ε^* changes depending on the monetary growth rate θ. Therefore, we can state:

Proposition 3

If the wage adjustment speed is finite, the money superneutrality for employment does not hold in the steady state.

From Eq. (2.31), inflation rate π^* becomes

$$\pi^* = \gamma(\varepsilon^* - 1) + \alpha \left(\frac{\dot{\hat{h}}}{\hat{h}} \right) = \gamma \frac{\gamma + \theta - g - \gamma}{\gamma} + 0 = \theta - g. \tag{3.10}$$

Thus, inflation rate π^* is given by the difference between the monetary growth rate θ and the technological change rate g. Thus we obtain:

Proposition 4

(i) When $g < \theta$, $\pi^* > 0$ holds, inflation occurs in the steady state.

(ii) When $g = \theta$, $\pi^* = 0$ holds, the price is constant in the steady state.

(iii) When $g > \theta$, $\pi^* < 0$ holds, deflation occurs in the steady state.

Combining Proposition 2 and Proposition 4, we have as follows

Proposition 5

(i) When $g < \theta$, boom and inflation persist.

(ii) When $g = \theta$, full employment and the constant price persist.

(iii) When $g > \theta$, stagnation and deflation persist.

Incidentally, let us derive the nominal interest rate R^* and the real interest rate r^* [17] . They are

$$R^* = \frac{\dot{c}}{c} + \rho + \pi^* = g + \rho + \theta - g = \rho + \theta \tag{3.11}$$

$$r^* = \frac{\dot{c}}{c} + \rho = g + \rho. \tag{3.12}$$

R^* depends on the monetary growth rate θ, and r^* depends on the technological change rate g [18] . Therefore we have:

Proposition 6

The money superneutrality for real interest rate holds in the steady state.

[17] The steady-state real price of stock q^* is derived in Appendix A.6. In this steady state, the transversality condition is satisfied, and the proof of it is presented in Appendix A.7.

[18] The model in this paper and the model in Siegel (1983) are similar in the respect that they are Money-in-utility models with technological change. Therefore, the version of models presented in Siegel (1983) that introduces logarithmic utility function results in Eq. (3.10), Eq. (3.11), and Eq. (3.12). Only, the models do not produce Eq. (3.9) because Siegel excludes effective demand shortage.

4 Stability of the steady state

In this section, we analyze the dynamic stability of the steady state. The Jacobi matrix J_1 of system S is given by

$$J_1 = \begin{bmatrix} 2\chi - (\theta+\rho) & 0 & 0 & 0 \\ 0 & [(\beta-1)\hat{h}^{1-\alpha}-\rho]+2\hat{c} & (\beta-1)(1-\alpha)\hat{h}^{-\alpha}\hat{c} & 0 \\ \frac{1}{\alpha}\hat{h} & 0 & \frac{1}{\alpha}\left[(\chi+\gamma)-(2-\alpha)\beta\hat{h}^{1-\alpha}-2\gamma\upsilon\hat{h}\right] & -\frac{1}{\alpha}\gamma\hat{h}^2 \\ 0 & -\upsilon & (1-\alpha)\hat{h}^{-\alpha}\upsilon & \hat{h}^{1-\alpha}-\hat{c}-g \end{bmatrix}$$

As $\chi^* = \theta+\rho$, $c^* = \psi - g$, $h^* = \psi^{\frac{1}{1-\alpha}}$, and $\upsilon^* = \frac{\gamma+\theta-g}{\gamma}\psi^{\frac{-1}{1-\alpha}}$, Jacobi matrix J_1^* evaluated in the steady state is given by

$$J_1^* = \begin{bmatrix} \theta+\rho & 0 & 0 & 0 \\ 0 & \psi-g & (\beta-1)(1-\alpha)\psi^{\frac{-\alpha}{1-\alpha}}(\psi-g) & 0 \\ \frac{1}{\alpha}\psi^{\frac{1}{1-\alpha}} & 0 & (g+\rho)-\frac{1}{\alpha}(\theta+\gamma+\rho) & -\frac{1}{\alpha}\gamma\psi^{\frac{2}{1-\alpha}} \\ 0 & -\frac{\gamma+\theta-g}{\gamma}\psi^{\frac{-1}{1-\alpha}} & (1-\alpha)\frac{\gamma+\theta-g}{\gamma}\psi^{\frac{-1-\alpha}{1-\alpha}} & 0 \end{bmatrix}$$

This Jacobi matrix is decomposable, and the characteristic equation $\Delta_1(\lambda)$ is written as

$$\Delta_1(\lambda) = (\theta+\rho-\lambda)|J_2^* - \lambda I| = (\theta+\rho-\lambda)(\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3) \tag{4.1}$$

where

$$J_2^* = \begin{bmatrix} \psi-g & (\beta-1)(1-\alpha)\psi^{\frac{-\alpha}{1-\alpha}}(\psi-g) & 0 \\ 0 & (g+\rho)-\frac{1}{\alpha}(\theta+\gamma+\rho) & -\frac{1}{\alpha}\gamma\psi^{\frac{2}{1-\alpha}} \\ -\frac{\gamma+\theta-g}{\gamma}\psi^{\frac{-1}{1-\alpha}} & (1-\alpha)\frac{\gamma+\theta-g}{\gamma}\psi^{\frac{-1-\alpha}{1-\alpha}} & 0 \end{bmatrix} \tag{4.2}$$

$$\begin{aligned} a_1 &= -trace J_2^* = -\frac{\partial\dot{\hat{c}}}{\partial\hat{c}^*} - \frac{\partial\dot{\hat{h}}}{\partial\hat{h}^*} \\ &= -\left[(\psi+\rho) - \frac{1}{\alpha}(\theta+\gamma+\rho)\right] \end{aligned} \tag{4.3}$$

$$
\begin{aligned}
a_2 &= \begin{vmatrix} \frac{\partial \dot{\hat{c}}}{\partial \hat{c}^*} & \frac{\partial \dot{\hat{c}}}{\partial \hat{h}^*} \\ 0 & \frac{\partial \dot{\hat{h}}}{\partial \hat{h}^*} \end{vmatrix} + \begin{vmatrix} \frac{\partial \dot{\hat{c}}}{\partial \hat{c}^*} & 0 \\ \frac{\partial \dot{v}}{\partial \hat{c}^*} & 0 \end{vmatrix} + \begin{vmatrix} \frac{\partial \dot{\hat{h}}}{\partial \hat{h}^*} & \frac{\partial \dot{\hat{h}}}{\partial v^*} \\ \frac{\partial \dot{v}}{\partial \hat{h}^*} & 0 \end{vmatrix} \\
&= (\psi - g)\left[(\rho + g) - \frac{1}{\alpha}(\gamma + \theta + \rho)\right] + \frac{1-\alpha}{\alpha}\psi(\gamma + \theta - g) \qquad (4.4)
\end{aligned}
$$

$$
\begin{aligned}
a_3 &= -detJ_2^* = -\left[\frac{\partial \dot{\hat{c}}}{\partial \hat{c}^*}\frac{\partial \dot{v}}{\partial h^*} - \frac{\partial \dot{\hat{c}}}{\partial \hat{h}^*}\frac{\partial \dot{v}}{\partial c^*}\right]\frac{\partial \dot{\hat{h}}}{\partial v^*} \\
&= -(\rho + g)\left[\frac{\rho + g}{\beta} - g\right](\gamma + \theta - g)\frac{1-\alpha}{\alpha}. \qquad (4.5)
\end{aligned}
$$

It is straightforward to find one eigenvalue of $\Delta_1(\lambda)$. Let λ_0 denotes it, $\lambda_0 = \theta + \rho > 0$. For the remaining eigenvalues, we check only their sign. In general, there are the following relationships between the coefficients of characteristic equation and the eigenvalues λ_i $(i = 1, 2, 3)$ [19] .

$$
\begin{aligned}
&a_1 = -(\lambda_1 + \lambda_2 + \lambda_3) \\
&a_2 = \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 \\
&a_3 = -\lambda_1\lambda_2\lambda_3 \\
&a_1a_2 - a_3 = -(\lambda_1 + \lambda_2)(\lambda_2 + \lambda_3)(\lambda_3 + \lambda_1)
\end{aligned} \qquad (4.6)
$$

Here $a_2 < 0$ and $a_3 < 0$ hold (see Appendix A.8 for the proof). Since $a_3 < 0$, we find that there is one positive real eigenvalue (let λ_1 denotes this) and two eigenvalues (λ_2 and λ_3) whose real parts have the same sign. Moreover, with $a_2 < 0$, we find that both λ_2 and λ_3 have a negative real part. Consequently, we obtain the following proposition.

Proposition 7

The characteristic equation $\Delta_1(\lambda)$ has two positive real eigenvalues and two eigenvalues with a negative real part.

χ, c, $\hat{h}$, and v are all control variables. Hence, there are innumerable paths to the steady state, and the economy is indeterminable. However, it is stable in the sense that it converges to the steady state.

[19]See e.g. Gandolfo (1997) Chaper 16.

5 Conclusion

In this paper, we model an economy in which technological change continues to increase potential supply constantly while the Pigou effect can adjust the market, and analyzed the influence that technological change and monetary growth have on employment. We obtained the following result.

If the adjustment speed of nominal wage were infinite, the money superneutrality for employment would hold, and full employment would persist. If the adjustment speed of nominal wage is finite, the long-run money superneutrality for employment does not hold. Even in this case, when the monetary growth rate equals the technological change rate, full employment persists. When the monetary growth rate is less than the technological change rate, underemployment and deflation persist. In contrast, when the monetary growth rate is more than the technological change rate, excess employment and inflation persist.

Since the long-run money superneutrality for employment does not hold, financial authorities can control effective demand in the long-run. Without controlling it adequately, full employment does not persist. Nevertheless Neoclassical growth theory eliminates financial policy as well as fiscal policy. The theory seems to represent the laisser-faire economy that maintains full employment automatically.

We say nothing to rabid Neoclassical economists, who consider the adjustment speed is infinite in the real economy. Instead we raise a question about New Keynesian's thesis. They believe in the long-run money superneutrality [20] and Neoclassical growth theory which assumes full employment, even though they consider that the adjustment speed is finite in the real economy. This paper made it clear that the New Keynesian's belief is rather paradoxical [21] .

Contrary to New Keynesian, we state that financial policy is essential even in the long-run. Financial authorities should make the monetary growth rate equal the technological change rate. This assertion resembles Friedman's k% rule. However, Friedman insisted on the rule to prevent not underemployment but deflation. Instead we make the assertion to prevent not only deflation but also underemployment. In addition, we do not disallow a discretionary financial policy unlike Friedman. We consider only that the monetary growth rate is approximately equal to the technological change rate in the long-term trend, with continuing to control money supply

[20]The long-run money supernuetrality for real interest rate holds in this paper, too. However we should focus not only on the superneutrality for real interest rate but also on the superneutrality for employment.

[21]The conclusion of this paper depends on the version of Phillips curve. Hence New Keynesian's thesis is not absolutely paradoxical, but not truism.

adequately [22].

To put it the other way around, without increasing money supply sufficiently, underemployment persists in the long-run. The modern economy in which technology continues to improve constantly always has the possibility of a crisis of technological unemployment. The crisis seldom turns into a reality, since monetary growth usually renders the adverse effect of technological change harmless. Therefore people ignore the harm that technological change has secretly. However, with slumping of money supply, technological unemployment will surface and create serious economic problems [23] .

A Appendix

A.1

To derive Eq. (2.2) , solve the following isoperimetric problem.

$$\min \quad E \equiv \int_0^1 p_i y_i di \tag{A.1}$$

$$\text{subject to} \quad \left[\int_0^1 y_i^{\frac{\phi-1}{\phi}} di\right]^{\frac{\phi}{\phi-1}} = y \tag{A.2}$$

E is the total expenditure of a retailer.

A.2

We derive Eq. (2.10). Firm i maximizes Eq. (2.8) subject to Eq. (2.4) and Eq. (2.9). The Hamilton function of this problem is

$$\mathcal{H} = (p_i y_i - W h_i - p I_i)/p + \lambda(y_i - k_i^{\alpha} h_i^{1-\alpha}) + \mu I_i \tag{A.3}$$

[22] It is just a theoretical thesis that the optimal rate of monetary growth equals the technological change rate. For practical purposes, we should include the downward rigidity of wage and the business cycle, which are excluded in this paper.

[23] The "Heisei recession" in Japan is an actual example of such a problem. Today, many economists consider that demand factors caused the Heisei recession. Some of empirical researches regard the slumping of money supply as the critical factor of the recession. This regard consists with the conclusion of this paper, which a low rate of monetary growth causes long-run deflationary recession.

where λ is Lagrange multiplier with constraint equation (2.4), μ is the costate variable of k. From Eq. (2.2), Eq. (A.3) can be rewritten as

$$\mathcal{H} = \left[p_i\left(\frac{p_i}{p}\right)^{-\phi} y - Wh_i - pI_i\right]/p + \lambda\left[\left(\frac{p_i}{p}\right)^{-\phi} y - k_i^{\alpha} h_i^{1-\alpha}\right] + \mu I_i. \tag{A.4}$$

The first-order conditions are

$$\frac{\partial \mathcal{H}}{\partial p_i} = \left[(1-\phi)\frac{p_i^{-\phi}}{p^{-\phi}} y\right]/p + \lambda(-\phi)\frac{p_i^{-\phi-1}}{p^{-\phi}} y = \frac{1-\phi}{p} y_i - \lambda\phi\frac{1}{p_i} y_i = 0 \tag{A.5}$$

$$\frac{\partial \mathcal{H}}{\partial h_i} = -\frac{W}{p} - (1-\alpha)\left(\frac{k_i}{h_i}\right)^{\alpha} \lambda = 0 \tag{A.6}$$

$$\frac{\partial \mathcal{H}}{\partial I_i} = -1 + \mu = 0 \tag{A.7}$$

$$\dot{\mu} = -\frac{\partial \mathcal{H}}{\partial k_i} + r\mu = \lambda\alpha\left(\frac{k_i}{h_i}\right)^{\alpha-1} + r\mu. \tag{A.8}$$

From Eq. (A.5) and Eq. (A.6), we obtain

$$p_i = \frac{\phi}{\phi-1}\frac{1}{1-\alpha}\left(\frac{h_i}{k_i}\right)^{\alpha} W. \tag{A.9}$$

A.3

Eq. (2.15) is derived from Eq. (A.6), Eq. (A.7), Eq. (A.8), and Eq. (2.13).

A.4

First, we derive Eq. (3.1). Eq. (2.28) can be rewritten as

$$\frac{\dot{c}}{c} = R - \pi - \rho = r - \rho. \tag{A.10}$$

From $\pi = c/m - r$,

$$\frac{\dot{m}}{m} = \theta - \pi = \theta - \frac{c}{m} + r. \tag{A.11}$$

Using Eq. (A.10) and (A.11) yields

$$\frac{\dot{c}}{c} - \frac{\dot{m}}{m} = r - \rho - \left(\theta - \frac{c}{m} + r\right) = \frac{c}{m} - \theta - \rho. \tag{A.12}$$

This equation can be rewritten as

$$\frac{\dot{\chi}}{\chi} = \chi - \theta - \rho. \tag{A.13}$$

Second, we derive Eq. (3.2). Dividing Eq. (2.4) by k yields

$$\frac{y}{k} = \left(\frac{h}{k}\right)^{1-\alpha} \tag{A.14}$$

or

$$\hat{y} = \hat{h}^{1-\alpha}. \tag{A.15}$$

From $y = c + I$ and $I = \dot{k}$, we obtain

$$\frac{y}{k} = \frac{c}{k} + \frac{I}{k} = \frac{c}{k} + \frac{\dot{k}}{k} \tag{A.16}$$

or

$$\hat{y} = \hat{c} + \frac{\dot{k}}{k}. \tag{A.17}$$

From this equation and Eq. (A.15),

$$\frac{\dot{k}}{k} = \hat{h}^{1-\alpha} - \hat{c}. \tag{A.18}$$

Using this equation, Eq. (A.10), and Eq. (2.16) yields

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{\dot{c}}{c} - \frac{\dot{k}}{k} = r - \rho - \frac{\dot{k}}{k} = \beta\hat{h}^{1-\alpha} - \rho - (\hat{h}^{1-\alpha} - \hat{c}) = (\beta - 1)\hat{h}^{1-\alpha} - \rho + \hat{c}. \tag{A.19}$$

Eq. (3.3) is derived from Eq. (2.31) and Eq. (2.16).

Using Eq. (A.18) and

$$\varepsilon = \frac{h}{h^n} = \frac{\hat{h}k}{h^n} \tag{A.20}$$

yields Eq. (3.4).

A.5

Since $\dot{\chi}/\chi = 0$, from Eq. (3.1)

$$\chi = \theta + \rho. \tag{A.21}$$

Since $\dot{\upsilon}/\upsilon = 0$, from Eq. (3.4)

$$\hat{h}^{1-\alpha} - \hat{c} = g. \tag{A.22}$$

Since $\dot{\hat{c}}/\hat{c} = 0$, from Eq. (3.2)

$$\beta\hat{h}^{1-\alpha} - \rho - \hat{h}^{1-\alpha} + \hat{c} = 0. \tag{A.23}$$

Substituting Eq. (A.22) into Eq. (A.23) gives

$$\beta\hat{h}^{1-\alpha} = g + \rho \tag{A.24}$$

or

$$\hat{h} = \left(\frac{g+\rho}{\beta}\right)^{\frac{1}{1-\alpha}}. \tag{A.25}$$

Since $\dot{\hat{h}}/\hat{h} = 0$, Eq. (3.3) becomes

$$\chi - \beta\hat{h}^{1-\alpha} - \gamma\upsilon\hat{h} + \gamma = 0. \tag{A.26}$$

Substituting Eq. (A.21) and Eq. (A.24) into Eq. (A.26) gives

$$(\theta + \rho) - (g + \rho) - \gamma\upsilon\hat{h} + \gamma = 0 \tag{A.27}$$

or

$$\theta - g - \gamma\upsilon\hat{h} + \gamma = 0. \tag{A.28}$$

Therefore υ is

$$\upsilon = \frac{\gamma + \theta - g}{\gamma}/\hat{h}. \tag{A.29}$$

A.6

From Eq. (2.6) and $y = c + I$, the real profit of firm Π becomes

$$\Pi = \frac{pc + pI - Wh - pI}{p} = c - wh. \tag{A.30}$$

Substituting Eq. (2.13) into Eq. (A.30) gives

$$\Pi = c - \frac{\phi - 1}{\phi}(1-\alpha)\hat{h}^{-\alpha}h. \tag{A.31}$$

From Eq. (A.31), the steady-state profit Π^* becomes

$$\begin{aligned} \Pi^* &= c^* - \frac{\phi - 1}{\phi}(1-\alpha)(\hat{h}^*)^{-\alpha}h^* \\ &= \left[\hat{c}^* - \frac{\phi - 1}{\phi}(1-\alpha)(\hat{h}^*)^{-\alpha}\hat{h}^*\right]k^*. \end{aligned}$$

As k^* increase at a rate of g, Π^* also increases at a rate of g. The steady-state real price of stock q^* is

$$q^*(t) = \int_t^\infty \Pi^*(s)e^{-r^*(s-t)}ds. \tag{A.32}$$

From Eq. (3.12), Eq. (A.32) can be rewritten as

$$\begin{aligned} q^*(t) &= \int_t^\infty \Pi^*(t)e^{g(s-t)}e^{-(g+\rho)(s-t)}ds \\ &= \frac{\Pi^*(t)}{\rho}. \end{aligned} \tag{A.33}$$

This equation implies that the steady-state real price of stock q^* also increases at a rate of g.

A.7

We prove that Eq. (2.26) representing the transversality condition is satisfied in the steady state. X is defined as $X \equiv \mu m e^{-\rho t}$, and Z is defined as $Z \equiv \mu q e^{-\rho t}$. Moreover, g_x is defined as the change rate of X, and g_z is defined as the change rate of Z. If both of g_x and g_z are negative in the steady state, the transversality condition is satisfied.

g_x is

$$g_x \equiv \frac{\dot{X}}{X} = \frac{\dot{\mu}}{\mu} + \frac{\dot{m}}{m} - \rho = -\frac{\dot{c}}{c} + \theta - \pi - \rho$$
$$= -g + \theta - \gamma\varepsilon + \gamma - \rho. \tag{A.34}$$

Substituting Eq. (3.9) into Eq. (A.34) gives

$$g_x^* = -g + \theta - \gamma\frac{\gamma + \theta - g}{\gamma} + \gamma - \rho = -\rho \tag{A.35}$$

where g_x^* is g_x in the steady state. Therefore $g_x^* < 0$ holds.

Since Eq. (2.28), $r = (\dot{c}/c) + \rho$ holds. From this equation and Eq. (2.14), g_z becomes

$$g_z \equiv \frac{\dot{Z}}{Z} = \frac{\dot{\mu}}{\mu} + \frac{\dot{q}}{q} - \rho = -\frac{\dot{c}}{c} + (r - \frac{\Pi}{q}) - \rho$$
$$= -\frac{\dot{c}}{c} + \frac{\dot{c}}{c} + \rho - \frac{\Pi}{q} - \rho = -\frac{\Pi}{q}. \tag{A.36}$$

Substituting Eq. (A.33) into Eq. (A.36) gives

$$g_z^* = -\frac{\Pi^*}{\Pi^*/\rho} = -\rho. \tag{A.37}$$

where g_z^* is g_z in the steady state. Therefore $g_z^* < 0$ holds.

Since $g_x^* < 0$ and $g_z^* < 0$, the transversality condition is satisfied.

A.8

We prove $a_2 < 0$ and $a_3 < 0$. Eq. (4.4) can be rewritten as

$$a_2 = (\psi - g)\left[(\rho + g) - \frac{1}{\alpha}(\gamma + \theta + \rho)\right] + \frac{1-\alpha}{\alpha}\psi(\gamma + \theta - g)$$
$$= (\psi - g)\left[\frac{\alpha - 1}{\alpha}\rho + \frac{\alpha - 1}{\alpha}g - \frac{1}{\alpha}(\gamma + \theta - g)\right] + \frac{1-\alpha}{\alpha}\psi(\gamma + \theta - g)$$
$$= (\psi - g)\frac{\alpha - 1}{\alpha}(\rho + g) + \left[\frac{1}{\alpha}g - \psi\right](\gamma + \theta - g). \tag{A.38}$$

From $\psi = \frac{1}{\alpha}\frac{\phi}{\phi - 1}(\rho + g)$,

$$a_2 = \left[\frac{1}{\alpha}\frac{\phi}{\phi - 1}(\rho + g) - g\right]\frac{\alpha - 1}{\alpha}(\rho + g) + \left[\frac{1}{\alpha}g - \frac{1}{\alpha}\frac{\phi}{\phi - 1}(\rho + g)\right](\gamma + \theta - g).$$

Since $1/\alpha > 1$, $\phi/(\phi - 1) > 1$, $(\alpha - 1)/\alpha < 0$, and $\gamma + \theta - g > 0$, $a_2 < 0$ holds.

a_3 can be rewritten as

$$\begin{aligned} a_3 &= -(\rho + g)\left[\frac{\rho + g}{\beta} - g\right](\gamma + \theta - g)\frac{1 - \alpha}{\alpha} \\ &= -(\rho + g)\left[\frac{\rho}{\beta} + \frac{1 - \beta}{\beta}g\right](\gamma + \theta - g)\frac{1 - \alpha}{\alpha}. \end{aligned} \tag{A.39}$$

Since $\beta \equiv \alpha(\phi - 1)/\phi < 1$, $a_3 < 0$ holds.

References

[1] Aghion, Philippe and Howitt, Peter [1994] “Growth and Unemployment,” Review of Economic Studies, 61, pp.477-494.

[2] Asada, Toichiro [2004] “Price Flexibility and Instability in a Macrodynamic Model with a Debt Effect,” Journal of International Economic Studies, 18, pp. 41-60

[3] Asada, Toichiro [2006] “Inflation Targeting Policy in a Dynamic Keynesian Model with a Debt Effect: A Japanese Perspective,” in Chiarella, R. and Franke, P. Flaschel and W. Semmler, editors, Quantitative and Empirical Analysis of Nonlinear Dynamic Macromodels, Elsevier, pp.517-544.

[4] Blanchard, Olivier J. and Kiyotaki, Nobuhiro [1987] “Monopolistic Competition and the Effects of Aggregate Demand,” The American Economic Review, 77, pp.647-666.

[5] Brock, William A. [1974] “Money and Growth: The Case of Long run Perfect Foresight,” International Economic Review, 15, pp.750-777.

[6] Brock, William A. [1975] “A Simple Perfect Foresight Monetary Model,” Journal of Monetary Economics, 1, pp.133-150.

[7] Davidson, Paul [1991] Controversies in Post Keynesian Economics, Edward Elgar.

[8] Dixit, Avinash K. and Stiglitz, Joseph E. [1977] “Monopolistic Competition and Optimum Product Diversity,” American Economic Review, 67, 3, pp 297-308.

[9] Fisher, Irving [1933] “The Debt-Deflation Theory of Great Depressions,” Econometrica, 1, 4, pp.337-357.

[10] Gandolfo, Giancarlo [1997] Economic Dynamics, Study edition, Springer-Verlag.

[11] Hahn, Frank H. [1965] “On Some Problems of Proving the Existence of an Equilibrium in a Monetary Economy,” in Hahn and Brechling, editors, Theory of Interest Rates, pp.126-135.

[12] Kalecki, Michal [1944] “Professor Pigou on ‘The Classical Stationary State’ A Comment,” The Economic Journal, 54, 1, pp.131-132.

[13] Keynes, John M. [1936] The General Theory of Employment, Interest and Money, The Macmillan Press.

[14] Malthus, Robert T. [1836] Principles of Political Economy, 2nd edn. (The Works of Thomas Robert Malthus, Vol.5-6, William Pickering, 1986).

[15] Mankiw, N. Gregory [1992] “The Reincarnation of Keynesian Economics,” European Economic Review, 36, pp.559-565.

[16] Obstfeld, Maurice and Rogoff, Kenneth [1983] “Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out?,” Journal of Political Economy, 91, pp.675-687.

[17] Ono, Yoshiyasu [2001] “A Reinterpretation of Chapter 17 of Keynes’s General Theory: Effective Demand Shortage Under Dynamic Optimization,” International Economic Review, 42, pp.207-236.

[18] Pasinetti, Luigi L. [1981] Structural Change and Economic Growth: a theoretical essay on the dynamics of the wealth of nations, Cambridge University Press.

[19] Pasinetti, Luigi L. [1993] Structural Economic Dynamics: A Theory of the Economic Consequences of Human Learning, Cambridge University Press.

[20] Patinkin, Don [1965] Money, Interest, and Prices, New York.

[21] Pigou, Arthur C. [1943] “The Classical Stationary State,” The Economic Journal, 53, 212, pp.343-351

[22] Pigou, Arthur C. [1947] “Economic Progress in a Stable Environment,” Economica, 14, 55, pp.180-188.

[23] Postel-Vinay, Fabien [2002] "The Dynamics of Technological Unemployment," International Economic Review, 43, 3, pp.737-760.

[24] Ricardo, David [1821] Principles of Political Economy and Taxation, 3rd ed., New York, Everyman's Library.

[25] Robinson, John [1953] "The Production Function and the Theory of Capital," The Review of Economic Studies, 21, 2, pp.81-106.

[26] Robinson, John [1973] "What has become of the Keynesian Revolution?," After Keynes, Basil Blackwell, pp.1-11.

[27] Sidrauski, Miguel [1967] "Rational Choice and Patterns of Growth in a Monetary Economy," American Economic Review, 57, pp.534-544.

[28] Siegel, Jemmy J. [1983] "Technological Change and the Superneutrality of Money," Journal of Money, Credit and Banking, 15, pp.363-367.

[29] Sismondi, Simonde de J.C.L. [1819] Nouveaux principes d'economie politique, ou de la Richesse dans ses rapports avec la population.

[30] Tobin, James [1980] Asset Accumulation and Economic Activity: Reflections on contemporary macroeconomic theory, University of Chicago Press.

[31] Wang, Ping and Yip, Chong K. [1992] "Alternative Approaches to Money and Growth," Journal of Money, Credit and Banking, 24, pp.553-562.