Estimating a Cooperative Game of Bargaining: The Case of Government Formation*

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Abstract

This paper estimates a cooperative bargaining model by using data from government formation. We compare our results with those of Adachi and Watanane (2008), who use a non-cooperative bargaining model. Although the estimates of the ministerial ranking are similar in Adachi and Watanabe's (2008) and our studies, the relative weight for the Prime Minister is estimated *lower* based on our cooperative bargaining models. In addition, our Vuong test suggests that cooperative formulation has a better fit to the observed data than Adachi and Watanabe's (2008) non-cooperative formulation does.

Keywords: Government Formation; Ministerial Weights; The Shapley-Shubik Power Index; Structural Estimation.

JEL classification: C71; C72; C78.

1 Introduction

In this paper, we estimate a cooperative bargaining game by using data from government formation. This issue is related to an important question in political economics: how do ministerial posts differ in their importance? Our main focus is to obtain parameter estimates of relative ministerial weights in parliamentary democracies in the case of Japan. We compare our results with those of Adachi and Watanane (2008), who use a non-cooperative bargaining model, and argue that our cooperative bargaining formulation has a better fit.

It would not be unnatural that one expects that cooperative games give more robust results because they do not depend on the details of the timings and rules of government formation. As Osborne and Rubinstein (1994, pp.255-6) state, "a coalitional model is distinguished from a noncooperative model primarily by its focus on what groups of players can

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achieve rather than what individual players can do and by the fact that it does not consider the details of how groups of players function internally." Osborne and Rubinstein (1994, p.256) also emphasize that either of the two approaches should not be viewed as superior or more basic, and that "each of them reflects different kinds of strategic considerations and contributes to our standing of strategic reasoning." Thus, we think that our comparison *based on empirical results* is important in its own right. In the present paper, we compare the results from a cooperative setting with the ones from Adachi and Watanabe's (2008) noncooperative setting.

Among a number of solution concepts for cooperative games, the Shapley value gives us one specific set of payments for coalition members, which are deemed fair. We use the solution concept by Shapley and Shubik (1954), a modified version of the Shapley value, as well as a more familiar concept by Nash (1950). We show that all of these cooperative bargaining model have a better fit than to the observed data than Adachi and Watanabe's (2008) non-cooperative formulation does. It is also verified that the relative weight for the Prime Minister is estimated *lower* based on our cooperative bargaining models, though the estimates of the ministerial ranking are similar in Adachi and Watanabe's (2008) and our studies. In addition, our Vuong (1989) test suggests that cooperative formulation has a better fit to the observed data than Adachi and Watanabe's (2008) non-cooperative formulation does.

Our results are encouraging. Our method would be applied to analyze issues that are well suitable to the concept of cooperative games. One such important area is the economics of the family. So far, researchers in this area rely mainly on Nash bargaining solutions (see, e.g., Del Boca and Flinn (2006)) when they consider allocation in a household (a husband and a wife). The concept of cooperative games seems fitted to issues in families.

The rest of the paper is organized as follows. Section 2 explains our cooperative formulation of government formulation. We consider two solution concepts: the Nash solution and the Shapley-Shubik power index. Then, after econometric specification is presented in Section 3, we show empirical results in Section 4. We present Vuong's (1989) likelihood ratio test to compare our formulation with Adachi and Watanabe's (2008) noncooperative formulation. We also consider the modified Shapley-Shubik Index. Section 5 concludes the paper.

2 Cooperative Game of Government Formation

We model government formation as a weighted majority game $\Gamma(\theta)$, where θ denotes a vector of model primitives. Throughout the paper, we consider a complete information environment. Thus, each element in θ is observable to all of the players in game. Let $N = \{1, .., n\}$ be the set of players. Let $v = (v_1, ..., v_n)$ be the vector of players' payoffs.

2.1 The Nash Solution

The Nash solution $v = (v_1, ..., v_n)$ is the solution that maximizes the product of the difference in each player's payoff when the negotiation is agreed on and that when it breaks down. More formally, it is obtained by solving

$$\max \prod_{i=1}^{N} (v_i - c_i)^{p_i}, \tag{1}$$

where v_i is player i's payoff when the negotiation is agreed on, and c_i is that when the negotiation breaks down, where $v_i > c_i$ for any $i = 1, ..., N$. Player i's bargaining power is captured by p_i. Because the Nash solution concept is axiomatically constructed, and thus is free of how a negotiation proceeds.[1]

Proposition 1 *The solution is given by*

$$\frac{v_1 - c_1}{p_1} = \cdots = \frac{v_n - c_n}{p_n}.$$

Proof. Notice first that

$$\max \prod_{i=1}^{n} (v_i - c_i)^{p_i}$$
$$\Leftrightarrow \max \sum_{i=1}^{n} p_i log(v_i - c_i).$$

Using the normalization

$$\sum_{i=1}^{n} v_i = 1,$$

we can rewrite expression (1) as

$$\max \sum_{i=1}^{n-1} p_i log(v_i - c_i) + p_n log(1 - \sum_{i=1}^{n-1} v_i - c_n)$$

By solving this maximization problem, we have

$$\frac{\partial}{\partial v_i} \left(\sum_{i=1}^{n-1} p_i log(v_i - c_i) + p_n log(1 - \sum_{i=1}^{n-1} - c_n) \right) = 0,$$

which leads to

$$\frac{v_i - c_i}{p_i} - \frac{v_n - c_n}{p_n} = 0$$

Thus, we obtains

$$\frac{v_1 - c_1}{p_1} = \cdots = \frac{v_n - c_n}{p_n}.$$

■

An important issue here is how to set $c = (c_1, ..., c_n)$. When symmetric two-person Nash solution is analyzed, the maxmin values or the Nash equilibrium is customarily set as a breakdown point. There is no such a custom in the analysis of asymmetric n-person Nash solution. In our study, we assume that the breakdown point is where each faction obtains payoff zero, because, e.g., the party breaks down. Thus, $c = (0, ..., 0)$.

[1] The research question of what kind of negotiation process yields the Nash solution has been extensively studied in the research program "Nash Program." While Rubinstein (1982) proposed such a process in two-person bargaining games, it is still unclear what kind of negotiation process yields the Nash solution in a general n-person bargaining games. See, e.g., Okada (2007) for an application of asymmetric n-person Nash solution.

We also assume that the bargaining power is given by

$$p_i = \frac{w_i}{\sum_{l=1}^{n} w_l},$$

where w_i captures the "relative dominance" of player i (to be specified later in Section 3). This assumption is based on Gamson (1961), which is also used by Adachi and Watanabe (2008).

Proposition 2 *The solution is given by*

$$\frac{v_1 - c_1}{p_1} = \cdots = \frac{v_n - c_n}{p_n}.$$

Proof. By inserting

$$c_i = 0$$

and

$$p_i = \frac{w_i}{\sum_{l=1}^{n} w_l}$$

into

$$\frac{v_1 - c_1}{p_1} = \cdots = \frac{v_n - c_n}{p_n},$$

we have

$$v_i = \frac{w_i}{\sum_{l=1}^{n} w_l}.$$

■

2.2 The Shapley-Shubik Power Index

We call a set $S \subseteq N$ that is formed for some joint action a *coalition*. We define coalition S's payoff from their joint action by $v(S)$. The function $v(\cdot)$ is called a *characteristic function*. A game expressed by the set of players, N, and the characteristic function v is called a characteristic function form game, (N, v).

A characteristic function form game, (N, v), where the characteristic function of any coalition S takes value 0 or 1 is called *voting game*. Coalition S is called a *winning coalition* if the alternative that all players in coalition S vote for is passed. The collection of all such S's is denoted by W. If the alternative voted by a coalition is not passed, the coalition is called a *losing coalition*. These relationships are succinctly summarized as

$$v(S) = \begin{cases} 1 \text{ if } S \subseteq W \\ 0 \text{ otherwise.} \end{cases}$$

The Shapley-Shubik index is application by Shapley and Shubik (1954) of the Shapley value (Shapley (1953)) to the voting game. Let S be a losing coalition. After voter i is added, the coalition S becomes $S \cup \{i\}$. If the coalition $S \cup \{i\}$ is a winning coalition, then voter i is the one who changes the losing coalition to the winning coalition. Such a voter i is called a *pivotal voter*. The number of the permutation $N = \{1, 2, ..., n\}$ is $n!$. The expectation of voter i being pivotal if each permutation is assumed to realize with the same probability is called the *Shapley-Shubik index*. It is given by

$$\phi_i = \frac{1}{n!} \sum s!(n - s - 1)!,$$

where $s = |S|$ and $n = |N|$. In practice, it is computationally burdensome to calculate the Shapley-Shubik index. We use Tomoki Matsui's website[2] that bases the work of Matsui and Matsui (1998).

The Shapley-Shubik is often interpreted as the influence that a voter exercises in an election. In our studies, however, the Shapley-Shubik index should be interpreted the Shapley value with the characteristic function takes value either zero or one. The Shapley index is ex-ante evaluation of how much payoff a player gains in each game. The Shapley-Shubik index is ex-ante evaluation of how much each player gains in each voting game. More specifically, we assume that each faction ex-ante predicts that it will receive the payoff that corresponds to the Shapley-Shubik index from a voting game (e.x. a presidential election in the LDP). We assume that the source of payoff comes from allocation of ministerial weights. We also assume that the difference between the ex-ante payoff evaluation (i.e., the Shapley-Shubik index) and the payoff that each faction gains from allocation of ministerial weights is treated as the residual term, ϵ_i.

2.3 Adachi and Watanabe's (2008) Noncooperative Formulation

In contrast to the present study, Adachi and Watanabe (2008) propose a non-cooperative bargaining game of government formulation là Baron and Ferejohn (1989). First, player i is (randomly or nonrandomly) selected as proposer. Player i, then, proposes a ministerial allocation as well as monetary transfer to all other players. Each non-proposer independently agrees or disagrees with the proposal. If all non-proposers unanimously agree it, then the bargaining games ends. However, at least one non-proposer disagrees, then the bargaining moves on to the next stage, where a new proposer is *randomly* selected. The game continues in the same manner until the agreement is made. Adachi and Watanabe (2008) assume round-by-round time discounting, and the time discount factor is denoted by $\delta \in [0, 1)$. The recognition probability of player j is selected as a proposer is specified by

$$\frac{w_j \exp(\alpha w_j)}{\sum_{l=1}^{n} w_l \exp(\alpha w_l)},$$

where w_j expresses the "relative dominance" of player j (as seen in subsection 2.1; to be specified later in Section 3), and $\alpha \geq 0$ captures the scale effect. Then, as a direct application of Eraslan's (2002) result, proposer i's equilibrium payoff is unique and it is given by

$$v_i = 1 - \sum_{j \neq i} \delta \frac{w_j \exp(\alpha w_j)}{\sum_{l=1}^{n} w_l \exp(\alpha w_l)},$$

while non-proposer j's equilibrium payoff is also unique and it is given by

$$v_j = \delta \frac{w_j \exp(\alpha w_j)}{\sum_{l=1}^{n} w_l \exp(\alpha w_l)}.$$

3 Econometric Specification

Now, we decompose player i's ex-post payoff (unobservable to researchers), y_i, into the part related to an observable (to researchers) part and an unobservable part. Specifically, we employ the following specification:

$$v_i = x_i \beta + \epsilon_i.$$

[2] http://www.misojiro.t.u-tokyo.ac.jp/tomomi/voting/voting.html

when the solution concept is the Nash solution, and

$$\phi_i = x_i \beta + \epsilon_i.$$

when the Shapley-Shubik Power Index is used, where $x_i = [x_{i1}, ..., x_{ik}]$ denotes player i's ministerial allocation, $\beta = [\beta_1, \ldots, \beta_k]$ is the vector of ministerial weights, and ϵ_i is monetary transfer (possibly negative) that player i obtains. We assume that x_{ij} is a dummy variable that takes 1 (if player i obtains the post of minister j) or 0 (otherwise). We normalize the ministerial weights by assuming that

$$\sum_{j=1}^{k} \beta_j = 1$$

and $0 \leq \beta_j \leq 1$. We also assume that the vector of monetary transfers satisfies

$$\sum_{i=1}^{n} \varepsilon_i = 0,$$

which means that the budget must balance within the players. From these assumptions, we have

$$\sum_{i=1}^{n} v_i = 1.$$

A natural imposition on player i's payoff is that it must be nonnegative: $v_i \geq 0$. Thus, it is that $-1 \leq \varepsilon_i \leq 1$ for any player i.

3.1 Data

The data used for this study is the same one in Adachi and Watanabe (2008) with the following two additions: (1) numbers of diet members in each faction for each government formation and (2) majority quotas ("q" in the model) at the time of each government formation. The sample covers the ministerial allocation in the period from 1958 to 1993, when the Liberal Democratic Party (LDP) maintained a majority in the House of Representatives. It is fairly natural to assume that a player in the game of in this period was each faction in the LDP (see Adachi and Watanabe (2008) for a discussion of this assumption).

3.2 Maximum Likelihood Estimation

We assume that ε_i is distributed according to the Beta distribution. With the Beta distribution, one can set the upper and lower bounds for the random variable, in contrast to the Normal distribution that allows infinite values. Remember that the primitive and the normalization impose these restrictions on the range of ε_i. Its probability density function is given by

$$f_x = \frac{1}{B(q,r)} \frac{(x-a)^{q-1}(b-x)^{r-1}}{(b-a)^{q+r-1}} \qquad a \leq x \leq b$$

where q and r are parameters that determine the form of the distribution, and $B(q,r)$ is the Beta function:

$$B(q,r) = \int_0^1 x^{q-1}(1-x)^{r-1}.$$

In our study, the upper bound is one and the lower bound is negative one, and thus the probability density function is given by

$$f_x = \frac{1}{B(q,r)} \frac{(x+1)^{q-1}(1-x)^{r-1}}{2^{q+r-1}} \qquad -1 \le x \le 1$$

When the Nash solution concept is used, the likelihood function L is given by

$$L(\beta, \sigma | w_{i,t}, x_{i,t}) = \prod_{t=1}^{T} \prod_{i=1}^{n-1} \frac{1}{2B(\sigma,\sigma)} \left(1 + w_{i,t} - x_{i,t}\beta\right)^{\sigma-1} \left(1 - w_{i,t} + x_{i,t}\beta\right)^{\sigma-1}$$

where $w_{i,t}$ is the fraction of faction i in period t and $x_{i,t}$ is the vector of ministerial allocation that faction i obtains in period t.

When the Shapley-Shubik index is used, the likelihood function is given by

$$L(\beta, \sigma | w_{i,t}, x_{i,t}) = \prod_{t=1}^{T} \prod_{i=1}^{n-1} \frac{1}{2B(\sigma,\sigma)} \left(1 + \phi_{i,t} - x_{i,t}\beta\right)^{\sigma-1} \left(1 - \phi_{i,t} + x_{i,t}\beta\right)^{\sigma-1}$$

where $\phi_{i,t}$ is the Shapley-Shubik index that faction i obtains in period t.

4 Empirical Results

4.1 Parameter Estimates

The estimation results are presented in Table 1. The estimates in column "AW (2008)" are the same as those reported in Adachi and Watanabe (2008). Remember that our cooperative formulation does not take into account the scale effect (α) or the time discount factor (δ). As we discuss below, the estimates of the ministerial ranking are similar in the three models. However, the estimate of the relative weight of the Prime Minister (25.2%) is as double as those in the cooperative models (8.1% in the Nash and 10.5% in the Shapley-Shubik). This difference arises presumably because while Adachi and Watanabe (2008) use the information on who the proposer is (i.e., the Prime Minister's faction) to estimate the relative ministerial weights, both of our cooperative bargaining models do not use that information. Thus, Adachi and Watanabe's (2008) non-cooperative bargaining model captures "formateur advantage".

	AW (2008)	Nash	Shapley-Shubik
Log Likelihood	−636.9577	−622.9177	−610.648
σ	377.9132 (26.7977)	244.8706 (27.5180)	229.1669 (23.2149)
α	0.0004 (0.4862)	-	-
δ	0.8361 (0.0491)	-	-
Prime Minister	0.2519 (0.0524)	0.0806 (0.0227)	0.1045 (0.0255)
Foreign Affairs	0.0301 (0.0069)	0.0341 (0.0080)	0.0238 (0.0077)
Home Affairs	0.0338 (0.0116)	0.0427 (0.0137)	0.043 (0.0131)
Finance	0.0443 (0.0085)	0.0572 (0.0088)	0.0604 (0.0083)
Justice	0.0335 (0.0060)	0.0289 (0.0080)	0.0367 (0.0073)
Education	0.0353 (0.0074)	0.0413 (0.0087)	0.0444 (0.0081)
Health and Welfare	0.0389 (0.0071)	0.0496 (0.0080)	0.0549 (0.0078)
Agriculture	0.0510 (0.0057)	0.0618 (0.0066)	0.0599 (0.0055)
International Trade and Industry	0.0391 (0.0077)	0.051 (0.0093)	0.0481 (0.0096)
Transport	0.0567 (0.0073)	0.0744 (0.0074)	0.0712 (0.0068)
Posts and Telecommunications	0.0343 (0.0063)	0.0386 (0.0078)	0.0351 (0.0084)
Labor	0.0451 (0.0062)	0.0532 (0.0079)	0.055 (0.0066)
Construction	0.0552 (0.0098)	0.0728 (0.0112)	0.0694 (0.0110)
Management and Coordination	0.0372 (0.0073)	0.0458 (0.0081)	0.0476 (0.0088)
Economic Planning	0.0530 (0.0087)	0.0662 (0.0102)	0.0649 (0.0091)
Hokkaido Development	0.0201 (0.0073)	0.0261 (0.0086)	0.0211 (0.0091)
National Public Safety	0.0154 (0.0116)	0.0183 (0.0139)	0.0162 (0.0131)
Defence	0.0433 (0.0071)	0.0572 (0.0084)	0.0504 (0.0092)
Science and Technology	0.0388 (0.0080)	0.0471 (0.0092)	0.039 (0.0095)
Cabinet Secretary	0.0430 (0.0212)	0.0531 (0.0213)	0.0544 (0.0247)

Table 1: Estimates and standard errors. Column (1) corresponds to the estimates reported in Adachi and Watanabe (2008), and column (2) to the corrected ones. Standard errors are in parentheses.

Rank	AW (2008)	Nash	Shapley-Shubik
1	Prime Minister	Prime Minister	Prime Minister
2	Transport	Transport	Transport
3	Construction	Construction	Construction
4	Economic Planning	Economic Planning	Economic Planning
5	Agriculture	Agriculture	Finance
6	Labor	Finance	Agriculture
7	Finance	Defence	Labor
8	Defence	Labor	Health and Welfare
9	Cabinet Secretary	Cabinet Secretary	Cabinet Secretary
10	International Trade and Industry	International Trade and Industry	Defence
11	Health and Welfare	Health and Welfare	International Trade and Industry
12	Science and Technology	Science and Technology	Management and Coordination
13	Management and Coordination	Management and Coordination	Education
14	Education	Home Affairs	Home Affairs
15	Posts and Telecommunications	Education	Science and Technology
16	Home Affairs	Posts and Telecommunications	Justice
17	Justice	Foreign Affairs	Posts and Telecommunications
18	Foreign Affairs	Justice	Foreign Affairs
19	Hokkaido Development	Hokkaido Development	Hokkaido Development
20	National Public Safety	National Public Safety	National Public Safety

Table 2: Comparison of the ministerial ranking reported in Adachi and Watanabe (2008), and the Nash and the Shapley-Shubik.

Table 2 ranks the ministers in higher to lower order. Notice that the ranking of the four highest ministers is common for all the three models (Prime Minister, Transport, Construction, and Economic Planning). Thus, one of the main findings in Adachi and Watanabe (2008) that pork-related posts such as the Ministers of Construction and Transport had high values is quite robust to the change in the formulation of bargaining. In addition, the estimated relative rankings of the Ministers of Foreign Affairs and of Justice, which were considered as prestigious positions for senior politicians, are low as in Adachi and Watanabe (2008).

4.2 Likelihood Ratio Test

Notice that the log likelihood is higher in both the Nash and the Shapley-Shubik solution concepts than in Adachi and Watanabe (2008). To formally compare the cooperative formulation with the non-cooperative formulation, we use the model specification test a là Vuong (1989). Let Adachi and Watanabe's (2008) non-cooperative model be called Model 0, the Nash model be called Model 1, and the Shapley-Shubik model be called Model 2. First, we compare Model 0 with Model 1 (Test 1). The null and the alternative hypotheses are:

$$\begin{aligned} H_0^1 &: \text{ Model 0 is as good as Model 1} \\ H_A^1 &: \text{ Model 1 is better than Model 0.} \end{aligned}$$

Vuong (1989) shows that when

$$LR^1 \equiv \text{Likelihood of Model 1} - \text{Likelihood of Model 0},$$

we have (1) under H_0^1, $2LR^1$ converges in distribution to the chi square distribution with freedom 1, and (2) under H_0^1, $2LR^1$ diverges to infinity almost surely. We have

$$2LR^1 = 2 \times \{-622.9177 - (-636.9577)\} = 28.08,$$

and this shows that H_0^1 is rejected at significance levels, 10%, 5% and 1% (see Table 3 below). Thus, the Nash cooperative model does a better job to explain the observed data than Adachi and Watanabe's (2008) non-cooperative bargaining model a là Baron and Ferejohn (1989).

Significance Level	10%	5%	1%
Chi Square	2.7055	3.8415	6.6349

Table 3: χ_α^2 at different significance levels

Next, we compare Model 1 with Model 2. Let the null and the alternative hypotheses be

$$\begin{aligned} H_0^2 &: \text{ Model 1 is as good as Model 2} \\ H_A^2 &: \text{ Model 2 is better than Model 1} \end{aligned}$$

respectively. It is then seen that H_0^2 is rejected at significance levels, 10%, 5% and 1%. Thus, the Shapley-Shubik cooperative model does the best job in the three models.

4.3 Comparison of Adachi and Watanabe (2008) and the Nash Formulation

Remember that in Adachi and Watanabe (2008), the payoff each faction obtains in a bargaining game is given by

$$v_i = 1 - \sum_{j \neq i} \delta \frac{w_j exp(\alpha w_j)}{\sum_{l=1}^{n} w_l exp(\alpha w_l)}$$

and

$$v_j = \delta \frac{w_j exp(\alpha w_j)}{\sum_{l=1}^{n} w_l exp(\alpha w_l)}$$

where proposer i's payoff is v_i and receiver j's payoff is v_j. On the other hand, in the Nash formulation, the equilibrium payoff is given by

$$v_i = \frac{w_i}{\sum_{l=1}^{n} w_l}$$

for each player i, which coincides with the equilibrium payoff in the if $\alpha = 0$ and $\delta = 1$. This similarity would be the main reason why the 12 (out of 20) ranks overlaps in Adachi and Watanabe (2008) and the Nash formulation (see Table 4).

		AW (2008)	Nash
1	Prime Minister	0.2519	0.0806
2	Transport	0.0567	0.0744
3	Construction	0.0552	0.0728
4	Economic Planning	0.053	0.0662
5	Agriculture	0.051	0.0618
9	Cabinet Secretary	0.043	0.0531
10	International Trade and Industry	0.0391	0.051
11	Health and Welfare	0.0389	0.0496
12	Science and Technology	0.0388	0.0471
13	Management and Coordination	0.0372	0.0458
19	Hokkaido Development	0.0201	0.0261
20	National Public Safety	0.0154	0.0183

Table 4: Comparison of Adachi and Watanabe (2008) and the Nash Formulation

Notice, however, that the Prime Minister does not have a high weight as in Adachi and Watanabe (2008). In the Nash formulation, both the Transport and the Construction have closer weights to the Prime Minister's. Considering the fact that the Nash has a better fit, the non-cooperative formulation in Adachi and Watanabe (2008), which necessarily creates non-asymmetry between the proposer and the non-proposers, may have less relevancy to model government formulation in the period of 1958 to 1993 in Japan.

4.4 Comparison of the Nash and the Shapley-Shubik Formulations

Next, we compare the Nash and the Shapley-Shubik formulations. Table 5 shows the estimated ministerial weights in both models.

	Nash		Shapley-Shubik	
1	Prime Minister	0.0806	Prime Minister	0.1045
2	Transport	0.0744	Transport	0.0712
3	Construction	0.0728	Construction	0.0694
4	Economic Planning	0.0662	Economic Planning	0.0649
5	Agriculture	0.0618	Finance	0.0604
6	Finance	0.0572	Agriculture	0.0599
7	Defence	0.0572	Labor	0.055
8	Labor	0.0532	Health and Welfare	0.0549
9	Cabinet Secretary	0.0531	Cabinet Secretary	0.0544
10	International Trade and Technology	0.051	Defence	0.0504
11	Health and Welfare	0.0496	International Trade and Technology	0.0481
12	Science and Technology	0.0471	Management and Coordination	0.0476
13	Management and Coordination	0.0458	Education	0.0444
14	Home Affairs	0.0427	Home Affairs	0.043
15	Education	0.0413	Science and Technology	0.039
16	Posts and Telecommunications	0.0386	Justice	0.0367
17	Foreign Affairs	0.0341	Posts and Telecommunications	0.0351
18	Justice	0.0289	Foreign Affairs	0.0238
19	Hokkaido Development	0.0261	Hokkaido Development	0.0211
20	National Public Safety	0.0183	National Public Safety	0.0162

Table 5: Comparison of Adachi and Watanabe (2008) and the Nash Formulation

Notice first that the estimated weight of the Prime Minister has a higher value in the Shapley-Shubik formulation. It is also seen that the Minister of Finance has a higher rank than the Minister of Agriculture.

4.5 The Modified Shapley-Shubik Index

So far, we have seen that the Shapley-Shubik performs the best in the Vuong (1989) specification test. In this subsection, we make a modification to the Shapley-Shubik concept. The formulation we have employed so far assumes that the consensus over government formation is made (i.e., the value of the characteristic function becomes 1) if the majority attains *within* the LDP members. Instead, we below assume that the consensus is made if the majority attains in all *parliamentary* members. Table 6 shows the results.

One can see that the log likelihood improves by the modification. To compare the modified Shapley-Shubik formulation with the original Shapley-Shubik formulation, let the null and the alternative hypotheses be

$$\begin{aligned} H_0^3 &: \text{ The original SS is as good as the modified SS} \\ H_A^3 &: \text{ The modified SS is better than the original SS.} \end{aligned}$$

Then, we have

$$2LR^3 = 2 \times \{-539.2502 - (-610.648)\} = 28.08,$$

as the statistic. Thus, H_0^3 is rejected at significance levels, 10%, 5% and 1%.

	Shapley-Shubik	Shapley-Shubik (Modified)
Log Likelihood	-610.648	-539.2502
σ	229.1669 (23.2149)	155.7914 (15.1616)
α	-	-
δ	-	-
Prime Minister	0.1045 (0.0255)	0.1259 (0.0404)
Foreign Affairs	0.0238 (0.0077)	0.0584 (0.0103)
Home Affairs	0.043 (0.0131)	0.0482 (0.0159)
Finance	0.0604 (0.0083)	0.0581 (0.0115)
Justice	0.0367 (0.0073)	0.0292 (0.0083)
Education	0.0444 (0.0081)	0.0412 (0.0091)
Health and Welfare	0.0549 (0.0078)	0.0395 (0.0103)
Agriculture	0.0599 (0.0055)	0.0598 (0.0089)
International Trade and Industry	0.0481 (0.0096)	0.0503 (0.0104)
Transport	0.0712 (0.0068)	0.0679 (0.0098)
Posts and Telecommunications	0.0351 (0.0084)	0.0344 (0.0098)
Labor	0.055 (0.0066)	0.0537 (0.0094)
Construction	0.0694 (0.0110)	0.0732 (0.0123)
Management and Coordination	0.0476 (0.0088)	0.0368 (0.0106)
Economic Planning	0.0649 (0.0091)	0.0681 (0.0129)
Hokkaido Development	0.0211 (0.0091)	0.0188 (0.0115)
National Public Safety	0.0162 (0.0131)	0.0052 (0.0138)
Defence	0.0504 (0.0092)	0.0519 (0.0112)
Science and Technology	0.039 (0.0095)	0.0484 (0.0119)
Cabinet Secretary	0.0544 (0.0247)	0.031 (0.0350)

Table 6: Estimates and standard errors. Column (1) corresponds to the estimates reported in Adachi and Watanabe (2008), and column (2) to the corrected ones. Standard errors are in parentheses.

	Nash	Shapley-Shubik	Shapley-Shubik (Modified)
1	Prime Minister	Prime Minister	Prime Minister
2	Transport	Transport	Construction
3	Construction	Construction	Economic Planning
4	Economic Planning	Economic Planning	Transport
5	Agriculture	Finance	Agriculture
6	Finance	Agriculture	Foreign Affairs
7	Defence	Labor	Finance
8	Labor	Health and Welfare	Labor
9	Cabinet Secretary	Cabinet Secretary	Defence
10	International Trade and Industry	Defence	International Trade and Industry
11	Health and Welfare	International Trade and Industry	Science and Technology
12	Science and Technology	Management and Coordination	Home Affairs
13	Management and Coordination	Education	Education
14	Home Affairs	Home Affairs	Health and Welfare
15	Education	Science and Technology	Management and Coordination
16	Posts and Telecommunications	Justice	Posts and Telecommunications
17	Foreign Affairs	Posts and Telecommunications	Cabinet Secretary
18	Justice	Foreign Affairs	Justice
19	Hokkaido Development	Hokkaido Development	Hokkaido Development
20	National Public Safety	National Public Safety	National Public Safety

Table 7: Comparison of the ministerial ranking reported in the Nash, the Shapley-Shubik, and the modified Shapley-Shubik

Table 7 shows that the rank of the Minister of Foreign Affairs goes up (from the 18th to the 6th). The reason for this would be probably that the Minister of Foreign Affairs was often selected from the pool of senior LDP politicians, and thus its selection may be less affected by the change in the distribution of bargaining power. Table 8 shows the correlation coefficients between the Nash solution and the Shapley-Shubik power index, between the Shapley-Shubik power index and the modified Shapley-Shubik power index, and between the Nash solution and the modified Shapley-Shubik power index. In particular, the modified Shapley-Shubik power index are seemingly much different in cabinets No.26-28 (Ohira) and No.33-35 (Nakasone).

Cabinet Number	Nash & SS	SS & SS (modified)	Nash & SS (modified)
No.1-4	0.993	0.980	0.991
No.5-8	0.998	0.988	0.994
No.9-14	0.998	0.988	0.993
No.15-17	0.997	0.960	0.966
No.18-19	0.999	0.996	0.995
No.20	0.997	0.982	0.969
No,21-25	0.997	0.964	0.959
No.26-28	0.996	0.396	0.439
No.29	0.992	0.891	0.885
No.30-32	0.992	0.891	0.885
No.33-35	0.975	0.269	0.336
No.36	0.949	0.777	0.910
No.37-40	0.947	0.707	0.882
No.41-44	0.960	0.834	0.955

Table 8: The correlation coefficients between two solutions concepts for each cabinet

5 Concluding Remarks

This paper structurally estimate different cooperative games of government formation. In contrast to the previous results by Adachi and Watanabe (2008) who formulate the problem as a non-cooperative multilateral sequential infinite-horizon bargaining game à la Baron and Ferejohn (1989), we consider the Nash solution concept, the Shapley-Shubik power index and its modified version. We obtain estimates of the relative ministerial weights in the period of 1958 to 1993 in Japan. Our Vuong (1989) test suggests that either of cooperative formulations has a better fit to the observed data than Adachi and Watanabe's (2008) non-cooperative formulation does. It is also verified that the relative weight for the Prime Minister is estimated lower based on our cooperative bargaining models, though the estimates of the ministerial ranking are similar in Adachi and Watanabe's (2008) non-cooperative model and the three cooperative models.

To see our results from a different angle, Table 9 shows that the estimated relative weights of selected ministers for each cooperative game when the value of the National Public Safety is normalized to be one.

	Nash	SS	SS (modified)
Prime Minister	4.4	6.5	24.2
Construction	4.0	4.3	14.1
Finance	3.1	3.7	11.2
Foreign Affairs	1.9	1.5	11.2
National Public Safety	1.0	1.0	1.0

Table 9: Relative weights of selected ministers relative to the National Public Safety

Notice that the weight of the Minister of Foreign Affairs is estimated high in the modified Shapley-Shubik, who has the best performance according to Vuong's (1989) specification test. Tables 10 to 12 compare the same selected ministers with each other for each of the three formulations. Table 10 shows that If the weight of the Minister of Foreign Affairs is normalized to be one, the relative weight of the Prime Minister is 2.4 and that of the Minister of Construction is 2.1. However, Table 11 shows that the relative weight of the Prime Minister is 1.5 with the Shapley-Shubik power index. By modifying the Shapley-Shubik power index, we have the weight of the Minister of Foreign Affairs close to that of the Minister of Finance, as Table 12 shows. The weight of the Minister of Construction is still high: 1.3 times as high as the weights of the Ministers of Foreign Affairs and of Finance. The Prime Minister has value 1.7 times as high as the Minister of Construction.

	PM	C	F	FA	NPS
Prime Minister	1.0	1.1	1.4	2.4	4.4
Construction	-	1.0	1.3	2.1	4.0
Finance	-	-	1.0	1.7	3.1
Foreign Affairs	-	-	-	1.0	1.9
National Public Safety	-	-	-	-	1.0

Table 10: Comparison of relative weights of selected ministers
(the Nash solution)

	PM	C	F	FA	NPS
Prime Minister	1.0	1.5	1.7	4.4	6.5
Construction	-	1.0	1.1	2.9	4.3
Finance	-	-	1.0	2.5	3.7
Foreign Affairs	-	-	-	1.0	1.5
National Public Safety	-	-	-	-	1.0

Table 11: Comparison of relative weights of selected ministers
(the Shapley-Shubik power index)

	PM	C	FA	F	NPS
Prime Minister	1.0	1.7	2.2	2.2	24.2
Construction	-	1.0	1.3	1.3	14.1
Foreign Affairs	-	-	1.0	1.0	11.2
Finance	-	-	-	1.0	11.2
National Public Safety	-	-	-	-	1.0

Table 12: Comparison of relative weights of selected ministers
(the modified Shapley-Shubik power index)

Lastly, the remaining issues include: applying other solution concepts such as the nucleolus to estimation, and using other data from other parliamentary democracies. These and other interesting issues on government formation are left for future research.

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Data Appendix

For the allocation data, see the dataset that is to be available online.

Faction	Number of Members	Nash	SS	SS (modified)
Sato->Tanaka	42	0.14094	0.13700	0.13294
Ikeda	38	0.12752	0.13056	0.12698
Ohno	44	0.14765	0.15198	0.15079
Ishida	21	0.07047	0.07698	0.07341
Kishi	56	0.18792	0.19841	0.18651
IB	15	0.05034	0.05198	0.03770
Kono	36	0.12081	0.11627	0.12698
Miki	35	0.11745	0.11270	0.12698
X	11	0.93691	0.02341	0.03771

Table A1: Cabinets 1 to 4

Faction	Number of Members	Nash	SS	SS (modified)
ST	46	0.15333	0.16071	0.15556
Ikeda	54	0.18000	0.19246	0.17341
Ohno	28	0.09333	0.09087	0.09603
Ishida	18	0.06000	0.05873	0.04643
Kishi	45	0.15000	0.15754	0.15556
FJ	34	0.11333	0.10952	0.11786
IB	5	0.01667	0.00913	0.02063
Kohno	34	0.11333	0.10952	0.11786
Miki	28	0.09333	0.09087	0.09603
X	8	0.02667	0.02063	0.02063

Table A2: Cabinets 5 to 8

Faction	Number of Members	Nash	SS	SS (modified)
Sato	46	0.15646	0.16190	0.15437
Ikeda	50	0.17007	0.17698	0.16825
Ohno	29	0.09864	0.09206	0.10079
Ishida	14	0.04762	0.04048	0.03929
Kishi	25	0.08503	0.08254	0.09286
KW	20	0.06803	0.06508	0.07500
FJ	20	0.06803	0.06508	0.07500
Kohno	46	0.15646	0.16190	0.15437
Miki	36	0.12245	0.12619	0.12262
X	8	0.02721	0.02778	0.01745

Table A3: Cabinets 9 to 14

Faction	Number of Members	Nash	SS	SS (modified)
Sato	53	0.18929	0.20692	0.19398
Ikeda	44	0.15714	0.16496	0.19398
Ohno	14	0.05000	0.04642	0.03893
MR	10	0.03571	0.03231	0.02933
Ishida	16	0.05714	0.05450	0.04095
Kishi	28	0.10000	0.09822	0.09398
KW	18	0.06429	0.06172	0.04347
FJ	16	0.05714	0.05450	0.04095
Kohno	24	0.08571	0.08029	0.08489
MO	14	0.05000	0.04642	0.03893
Miki	37	0.13214	0.13174	0.19398
MT	4	0.01429	0.01460	0.00559
X	2	0.00714	0.00739	0.00104

Table A4: Cabinets 15 to 17

Faction	Number of Members	Nash	SS	SS (modified)
ST	53	0.17667	0.19246	0.17833
IK	44	0.14667	0.15339	0.14550
Ohno	14	0.04667	0.04423	0.05087
MR	10	0.03333	0.02991	0.03320
Ishida	16	0.05333	0.04946	0.05939
Kishi	38	0.12667	0.12919	0.11621
KW	19	0.06333	0.05873	0.06999
FJ	6	0.02000	0.01829	0.02461
Kohno	35	0.11667	0.11703	0.10206
MO	13	0.04333	0.04012	0.04734
Miki	40	0.13333	0.13705	0.12732
MT	3	0.01000	0.00909	0.01577
X	7	0.02333	0.02103	0.02890

Table A5: Cabinets 18 to 19

Faction	Number of Members	Nash	SS	SS (modified)
Sato/Tanaka	44	0.14667	0.14935	0.13860
IK	43	0.14333	0.14776	0.13860
Ohno	10	0.03333	0.02951	0.02471
MR	16	0.05333	0.04935	0.05527
Ishida	12	0.04000	0.03506	0.03027
Kishi	65	0.21667	0.24141	0.27471
KW	17	0.05667	0.05173	0.05804
FJ	2	0.00667	0.00411	0.00725
Kohno	34	0.11333	0.11284	0.09138
Miki	38	0.12667	0.12157	0.11082
X	19	0.06333	0.05729	0.07035

Table A6: Cabinet 20

Faction	Number of Members	Nash	SS	SS (modified)
ST	48	0.16901	0.17763	0.20985
IK	45	0.15845	0.16136	0.20985
Ohno	9	0.03169	0.02565	0.02334
MR	13	0.04577	0.04390	0.03128
Ishida	9	0.03169	0.02565	0.02334
Kishi	56	0.19718	0.21255	0.20985
KW	18	0.06338	0.04787	0.03128
FJ	2	0.00704	0.00898	0.01818
Kohno	38	0.13380	0.13795	0.10985
Miki	37	0.13028	0.13279	0.10985
X	9	0.03169	0.02565	0.02333

Table A7: Cabinets 21 to 25

Faction	Number of Members	Nash	SS	SS (modified)
Sato/Tanaka	45	0.17308	0.18214	0.11111
Ikeda	39	0.15000	0.15754	0.11111
Ohno	8	0.03077	0.02421	0.11111
MR	11	0.04231	0.03373	0.11111
Ishida	4	0.01538	0.01230	0.00001
Kishi/Fukuda	53	0.20385	0.22421	0.11111
KW	11	0.04231	0.03373	0.11111
Kohno	39	0.15000	0.15754	0.11111
Miki	32	0.12308	0.12659	0.11111
X	18	0.06923	0.04802	0.11111

Table A8: Cabinets 26 to 28

Faction	Number of Members	Nash	SS	SS (modified)
Sato/Tanaka	52	0.20155	0.20667	0.10910
Ikeda/Ohira	50	0.19380	0.20072	0.10909
Ohno	4	0.01550	0.00786	0.10909
MR	5	0.01938	0.01144	0.10909
Ishida	2	0.00775	0.00390	0.00909
Kishi/Fukuda	49	0.18992	0.19834	0.10909
NG	10	0.03876	0.02215	0.10909
KW	2	0.00775	0.00390	0.00909
Kohno	41	0.15891	0.16144	0.10909
Miki	31	0.12016	0.16144	0.10909
X	12	0.04651	0.02215	0.10909

Table A9: Cabinet 29

Faction	Number of Members	Nash	SS	SS (modified)
Sato/Tanaka	64	0.22300	0.22381	0.19048
Ikeda/Ohira/Suzuki	63	0.21951	0.22381	0.19048
MR	3	0.01045	0.02857	0.00000
Kishi/Fukuda	46	0.16028	0.15714	0.19048
NG	11	0.03833	0.02857	0.02380
Kohno	47	0.16376	0.15714	0.19048
Miki	32	0.11150	0.11905	0.19048
X	21	0.07317	0.06190	0.02380

Table A10: Cabinets 30 to 32

Faction	Number of Members	Nash	SS	SS (modified)
Sato/Tanaka	68	0.25468	0.29286	0.12500
Ikeda/Ohira/Suzuki	52	0.19476	0.18095	0.12500
Kishi/Fukuda	43	0.16105	0.14048	0.12500
NG	6	0.02247	0.02143	0.12500
Kono/Nakasone	49	0.18352	0.17143	0.12500
Miki	21	0.07865	0.10000	0.12500
SJ	8	0.02996	0.02857	0.12500
X	13	0.04869	0.06428	0.12500

Table A11: Cabinets 33 to 35

Faction	Number of Members	Nash	SS	SS (modified)
Sato/Tanaka	87	0.28065	0.30000	0.25000
Ikeda/Ohira/Suzuki	59	0.19032	0.16667	0.25000
Kishi/Fukuda	56	0.18065	0.16667	0.25000
Kono/Nakasone	60	0.19355	0.16667	0.25000
Miki	28	0.09032	0.10000	0.00000
SJ	6	0.01935	0.00000	0.00000
X	14	0.04516	0.10000	0.00000

Table A12: Cabinet 36

Faction	Number of Members	Nash	SS	SS (modified)
Sato/Tanaka/Takeshita	87	0.28065	0.30000	0.25000
Ikeda/Ohira/Suzuki	59	0.19032	0.16667	0.25000
Kishi/Fukuda	56	0.18065	0.16667	0.25000
Kono/Nakasone	60	0.19355	0.16667	0.25000
Miki	28	0.09032	0.10000	0.00000
X	20	0.06452	0.10000	0.00000

Table A13: Cabinets 37 to 40

Faction	Number of Members	Nash	SS	SS (modified)
ST	69	0.24126	0.25714	0.23333
NK	4	0.01399	0.00714	0.00000
IK	62	0.21678	0.20714	0.23333
Kishi/Fukuda	61	0.21329	0.20714	0.23333
Kohno/Nakasone/Uno	48	0.16783	0.12381	0.23333
Miki/Kaifu	26	0.09091	0.12381	0.03334
X	16	0.05594	0.07381	0.03334

Table A14: Cabinets 41 to 44