The effect of different expenditure financing in the money-in-the-production-function model.

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Abstract

In this paper, we examine the effects of the different financing (an increase in the income tax rate and an increase in the nominal money growth rate) in the model where real money balances is an input in the production process. In contrast to the result in Palivos and Yip (1995) which examines the effects of the alternative financing under cash-in-advance constraints on consumption purchases and investment purchase, we show that the decrease in the growth rate is less under income tax financing than under money financing. Furthermore, we compare the welfare under each regime. We find that money financing is always more harmful to the welfare than income tax financing.

1 Introduction

Since Sidrauski's (1967) pioneering work, the analysis on monetary growth model has attracted many researchers. Recent development of endogenous growth theory, pioneered by Lucas (1988), Romer (1986) and Rebelo (1991), has invoked the study of the relationship between monetary policy and growth.

The effect of government policy has been also analyzed in many studies. Most of those studies, such as De Gregorio (1993), Jones and Manuelli (1995) , Pecorino (1995) and Mino (1997), has examined the relationship the long-run growth rate and money supply. They exclusively focus on the effect of monetary policy alone.

In general, there are several ways of government financing such as income tax, lump-sum tax, debt financing, money financing. The effect of alternative financing has been analyzed in several papers. Van der Ploeg and Alogoskous (1994) and examine the effects of lump-sum-tax-financed, money-financed and debt-financed

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increases in public spending under overlapping generations model in a Blanchard (1985)-type OLG model with a money-in-the-utility-function. Mino and Shibata (2000) compares the effect of income tax and money financing on welfare as well as economic growth in a Weil (1989)-type OLG model with a money-in-the-utility function. The effect of different ways of financing (income tax financing and money financing) under a cash-in-advance constraint is analyed by Palivos and Yip (1995). They find that the money financing is more favorable to economic growth than income tax financing for any given government size. The optimal financing in terms of welfare depends on the fraction of investment purchases that are subject to liquidity constraint. Gokan (2002) develops the stochastic monetary model to evaluate the influence of changes of public spending on the expected growth rate, the expected inflation rate and economic welfare. Palokangas (2003) examines the effect of seignorage and distorting tax in a model with both utility and productivity enhancing government expenditure. He introduces money as an intermediary good which reduces transaction costs. In this paper, we examine the effects of the alternative financing (an increase in the income tax rate and an increase in the nominal money growth rate) on economic growth and welfare in the model where real money balances is an input in the production process. The government expenditure has direct effect on neither household's utility nor productivity. That is, it is essentially useless to the economy (or it may be used for some foreign countries). The government sets the ratio of the real expenditure to the real output and finances it by two alternative methods.

Most of the theoretical monetary models are based of one of the following four models, the money-in-the-utility-function (M.I.U.) model, the cash-in-advance (C.I.A.) model, the shopping-time model, and the money-in-the-production-function (M.I.P.) model. Even the same policy has different effects in different monetary models.[1]

The money-in-the-production-function approach (or equivalently the transactions cost approach) assumes that economic agents (especially, firms) have to divert a part of employed production in costly activities, such as bargaining, setting prices, conducting wholesale operations. The more they produce, the more they suffer such costs. Money is introduced as an intermediate good which reduces transaction cost. Some researchers examine the dynamics of the economy , implications of monetary policy and government financing (For exmaple, Zhang (1996), Petrucci (1999) and Sailesh et al. (2002)). In this paper we compare the effect of income tax financing and money financing under the money-in-the-production-function model.

First, we can easily see an increase in government expenditures by each financing reduces economic growth, because it induces the higher rate of inflation, the higher opportunity cost of holding money and then, a decrease in holding money. Next, we find that the in the case the elasticity of intertemporal substi-

[1] Itaya (1998) and Kaneko and Matsuzaki (2009) find that neutrality of consumption tax depends on which monetary models we adopt.

tution is lower than 1, the decrease in the growth rate is less under income tax financing than under money financing and that that money financing is always more harmful to the welfare than income tax financing. This result is sharp contrast to the one in Palivos and Yip (1995). When the elasticity of intertemporal substitution is larger than 1, a numerical example shows that money financing is likely to deteriorate the growth rate and the welfare more than the income tax financing.

The remainder of the paper is organized as follows. Section 2 describes the economy. Section 3 examines the dynamic property of the economy and the effect of each financing on economic growth. In section 4 we compare the welfare effect of alternative financing. Section 5 summarizes our conclusion.

2 Model

2.1 Firms

We normalize a number of households and firms are equal to 1. We specify the technology as Cobb-Douglas form:

$$f(k(t), m(t)) = \Lambda k(t)^{\alpha} m(t)^{1-\alpha}, \quad 0 < \alpha < 1. \tag{1}$$

where $k(t)$ is physical capital and $m(t)$ is real money at time t.

More general specification of money-in-the-production-function with Ak technology can be as follows

$$\mathbf{y} = [1 - \phi(m/Ak)]\, Ak \tag{2}$$

where $\mathbf{y}$ is net output.[2] The function ϕ is decreasing function of m/Ak, which reflects the fact that frictions exist in goods markets transactions, and money is helpful in reducing those frictions.The function ϕ represents the pecuniary transaction cost.[3] If firms produce more, the amount of transactions increases and the firms lose more real output. Therefore, the partial derivative of the function ϕ with respect to Ak is assumed to be positive. If firms have more money, transactions run more smoothly. Hence, the partial derivative of the function ϕ with respect to m is assumed to be negative. We adopt a simple specification for function ϕ given in Shaw et al. (2005), such that, $\phi(m/Ak) = 1 - \phi_0(m/Ak)^{1-\alpha}$ with $0 < \phi_0 < 1$ and $0 < \alpha \leq 1$. Substituting this specification into the net production function yields production function which is equivalent to (1).

[2]Chen et al. (2008) use this general form of money-in-the-production-function to analyze the relashionship between interest-rate rule and both inflation and economic growht in an open economy.

[3]In Shaw et al. (2005), they consider the function as an indicator to the extent of financial development, since as financial system develops, the ratio of credit issued to GDP increases and it facilitates goods production.

Since $r(t)$ is the rental rate of physical capital, $R(t)$ is the rental rate of money or the cost of borrowing money and $\tau \in [0, 1)$ is income tax, the representative firm maximizes the following profit;

$$(1-\tau)f(k(t),\, m(t)) - r(t)k(t) - R(t)m(t).$$

The first-order conditions for profit maximization are as follows

$$(1-\tau)\alpha\Lambda(m(t)/k(t))^{1-\alpha} - r(t) = 0, \tag{3}$$
$$(1-\tau)(1-\alpha)\Lambda(m(t)/k(t))^{-\alpha} - R(t) = 0. \tag{4}$$

2.2 Households

A representative household gains utility from its amount of consumption and earns revenue from renting capital and lending money to firms. The households' real budget constraint is [4]

$$\dot{a}(t) = r(t)a(t) + (R(t) - \pi(t) - r(t))m(t) - c(t), \tag{5}$$

where $\pi(t)$ is the inflation rate of the commodity price $p(t)$. The lifetime-utility the households is as follows:

$$\int_0^\infty \frac{c(t)^{1-\sigma} - 1}{1-\sigma} \exp(-\rho t)dt, \tag{6}$$

where ρ is the rate of time preference and $1/\sigma$ is the elasticity of substitution for the utility function. The first-order conditions, the arbitrage condition and the transversality condition of this problem are

$$c(t)^{1-\sigma} - \lambda(t) = 0, \tag{7}$$
$$\dot{\lambda}(t) = (\rho - r(t))\,\lambda(t), \tag{8}$$
$$R(t) - \pi(t) - r(t) = 0, \tag{9}$$
$$\lim_{t\to\infty} \lambda(t)a(t)\exp(-\rho t) = 0. \tag{10}$$

where $\lambda(t)$ denotes the multiplier. From (7) and (8), we have the Euler equation as follows

$$\dot{c(t)}/c(t) = (r(t) - \rho)/\sigma. \tag{11}$$

Equation (9) implies the return of $k(t)$ must equal to the one of $m(t)$. Equation (10) is the transversality condition.

[4] The households' nominal budget constraint is

$$p(t)\dot{k}(t) + \dot{M}(t) = p(t)r(t)k(t) + p(t)R(t)m(t) - p(t)c(t).$$

where M is nominal money, p is the commodity price. Rearranging this equation gives (5)

2.3 Government

The government imposes an income tax or an inflation tax for nominal waste government consumption, $p(t)G(t)$. The government nominal budget constraint becomes

$$p(t)G(t) = \tau p(t) f(k(t), m(t)) + \mu M(t), \tag{12}$$

where μ is the growth rate of nominal money. In real terms,

$$G(t) = \tau f(k(t), m(t)) + \mu m(t). \tag{13}$$

2.4 Dynamics

For ease of exposition, we suppress the time index in what follows. Given a constant money growth rate μ, money market equilibrium requires that $\dot{m}/m = \mu - \pi$. Using (3), (4) and (9) to eliminate π, we have

$$\dot{m}/m = \mu + (1-\tau)\Lambda \left\{\alpha(m/k)^{1-\alpha} - (1-\alpha)(m/k)^{-\alpha}\right\}. \tag{14}$$

From (3), (4), (5), (13) and (14), the market equilibrium condition for a commodity is given by

$$\dot{k}/k = f/k - g - c/k, \tag{15}$$

where $g = G/k$. From (3) and (11), the growth rate of consumption is rewritten as

$$\dot{c}/c = \left\{(1-\tau)\Lambda\alpha(m/k)^{1-\alpha} - \rho\right\}/\sigma. \tag{16}$$

Defining c/k and m/k as χ (the consumption-to-capital ratio) and ω (the real money holdings-to-capital ratio), we have the following autonomous dynamic system.

$$\dot{\chi} = \chi\left[\left\{(1-\tau)\Lambda\alpha\omega^{1-\alpha} - \rho\right\}/\sigma - \Lambda\omega^{1-\alpha} + \chi + g\right], \tag{17}$$

$$\dot{\omega} = \omega\left[\mu + (1-\tau)\Lambda\left\{\alpha\omega^{1-\alpha} - (1-\alpha)\omega^{-\alpha}\right\} - \Lambda\omega^{1-\alpha} + \chi + g\right]. \tag{18}$$

3 The financing methods and growth

In this section, we compare the effect of each financing methods on the growth rate of the economy. In order to make the comparison clearly, we shall examine the effect of each method separately.[5]

[5] In Appendices 2.1 and 2.2, we confirm that there is a balanced growth equilibrium in each financing regime for small g.

3.1 Money financing

The government uses printing new money only to collect the certain value of waste government expenditure. Formally, from (13) the government set the money expansion rate as follows.

$$\mu = g/\omega. \tag{19}$$

Using (14), (16) and (19), the growth rate of consumption, θ^c_μ, and the growth rate of money, θ^m_μ, are expressed as

$$\theta^c_\mu = \left\{\Lambda\alpha\omega^{1-\alpha} - \rho\right\}/\sigma, \tag{20}$$
$$\theta^\omega_\mu = g/\omega + \Lambda\left\{\alpha\omega^{1-\alpha} - (1-\alpha)\omega^{-\alpha}\right\}. \tag{21}$$

The equilibrium value of ω, ω^*_μ, must satisfy the following equation:

$$\left(\frac{1}{\sigma} - 1\right)\Lambda\alpha\omega^{1-\alpha} + (1-\alpha)\Lambda\omega^{-\alpha} = \frac{\rho}{\sigma} + \frac{g}{\omega}. \tag{22}$$

From (22), we obtain

$$\frac{d\omega^*_\mu}{dg} = \frac{1/\omega}{\left[(1/\sigma - 1)\,\omega - 1\right]\Lambda\alpha(1-\alpha)\omega^{-\alpha-1} + g/\omega^2}. \tag{23}$$

From (20) and (23), we have

$$\frac{d\theta^c_\mu}{dg} = \frac{1/\sigma}{(1/\sigma - 1)\,\omega - 1 + g/(\Lambda\alpha(1-\alpha)\omega^{1-\alpha})}. \tag{24}$$

We show in the Appendix 3.1 that relative small level of g, the denominator in (24) always negative. Thus, the effect of money financing on growth is negative.

3.2 Income tax financing

In this case, the government imposes the income tax only to finance the government expenditure.

From (13), the income tax rate is

$$\tau = g/\Lambda\omega^{1-\alpha} \tag{25}$$

Using (14), (16) and (25), the growth rate of consumption, θ^c_τ, and the growth rate of real money, θ^m_τ, are as follows:

$$\theta^c_\tau = \left\{(1-\tau)\,\Lambda\alpha\omega^{1-\alpha} - \rho\right\}/\sigma, \tag{26}$$
$$\theta^\omega_\tau = (1-\tau)\,\Lambda\left\{\alpha\omega^{1-\alpha} - (1-\alpha)\omega^{-\alpha}\right\}. \tag{27}$$

It is obvious that income tax financing has direct distortionary effect on the economic growth because it reduces the rate of return on capital. The balanced growth equilibrium value of ω, ω_τ^*, must satisfy the following equation:

$$\left(\frac{1}{\sigma}-1\right)\Lambda\alpha\omega^{1-\alpha}+(1-\alpha)\Lambda\omega^{-\alpha}=\frac{\rho}{\sigma}+\frac{g}{\omega}+\frac{g}{\omega}\alpha\left[\left(\frac{1}{\sigma}-1\right)\omega-1\right], \tag{28}$$

From (28), we have

$$\frac{d\omega_\tau^*}{dg}=\frac{1/\omega+\left(\left(1/\sigma-1\right)\omega-1\right)\alpha/\omega}{\left[\left(1/\sigma-1\right)\omega-1\right]\Lambda\alpha(1-\alpha)\omega^{-\alpha-1}+(1-\alpha)g/\omega^2}. \tag{29}$$

From (26) and (29), we obtain

$$\frac{d\theta_\tau^c}{dg}=\frac{\left(1-g/(\Lambda\omega^{1-\alpha})\right)/\sigma}{\left(1/\sigma-1\right)\omega-1+g/(\Lambda\alpha\omega^{1-\alpha})}. \tag{30}$$

Since the denominator in (30) is negative (see Appendix 3.2), an increase in income tax financing lowers the growth rate.

3.3 Comparison of the effect of different financing on growth

In this subsection, we show that when $\sigma \geq 1$, the money financing is always harmful more than income tax financing regardless of the level of g. To derive the result, the following lemma would be useful.

lemma

The balanced equilibrium value of ω in income tax financing regime is always larger than the one in money financing regime. Namely

$$\omega_\mu^* < \omega_\tau^*. \tag{31}$$

proof

Let us denote the left-hand side of (22) as $F_\mu(\omega)$, the right-hand side as $G_\mu(\omega)$, the left-hand side of (28) as $F_\tau(\omega)$ and the right-hand side as $G_\tau(\omega)$. Differentiating $F_\mu(\omega)$ and $G_\mu(\omega)$ with respect to ω, we obtain

$$dF_\mu(\omega)/d\omega =[\left(1/\sigma-1\right)\omega-1]\Lambda\alpha(1-\alpha)\omega^{-\alpha-1}, \tag{32}$$

$$dG_\mu(\omega)/d\omega =-g/\omega^2. \tag{33}$$

Since $\sigma \geq 1$ and $g \geq 0$, (32) and (33) is negative. The denominator in (24) is negative implies that

$$dF_\mu(\omega)/d\omega - dG_\mu(\omega)/d\omega < 0. \tag{34}$$

Since $\frac{dG_\tau(\omega)}{d\omega} = -\frac{g}{\omega^2}+\frac{\alpha g}{\omega^2}$, it is always larger than $\frac{dG_\mu(\omega)}{d\omega}$. Using the fact that $F_\tau(\omega) = F_\mu(\omega)$ and (34), we can see that $\frac{dG_\tau(\omega)}{d\omega} > \frac{dF_\tau(\omega)}{d\omega}$ and $G_\tau(\omega) < G_\mu(\omega)$ for all ω easily. Thus the value of ω in equilibriums, ω_μ^* and ω_τ^*, are expressed as

$$\omega_\mu^* < \omega_\tau^*. \tag{35}$$

□

Using this lemma, we can derive the following proposition:

Proposition 1: When $\sigma \geq 1$, the money financing is always more harmful to economic growth than income tax financing regardless of the level of g.

proof

From (20) and (26), when the growth rate of income tax financing is higher than the one of money financing, the following condition must be true:

$$\left\{\Lambda\alpha\omega_{\mu}^{1-\alpha} - \rho\right\}/\sigma \leq \left\{\Lambda\alpha\omega_{\tau}^{1-\alpha} - \rho - \alpha g\right\}/\sigma. \tag{36}$$

This can be rewritten as

$$g/\Lambda \leq \omega_{\tau}^{1-\alpha} - \omega_{\mu}^{1-\alpha}. \tag{37}$$

When $g = 0$, the left-hand side and the right-hand side of (37) are equal to zero, because $\omega_\tau = \omega_\mu$. Thus, if the derivative value of the left-hand side with respect to g is larger than the one of the right-hand side with respect to g, that is,

$$1/\Lambda \leq (1-\alpha)\omega_{\tau}^{-\alpha} d\omega_\tau/dg - (1-\alpha)\omega_{\mu}^{-\alpha} d\omega_\mu/dg, \tag{38}$$

(36) holds. Using (23) and (29), (38) is rewritten as

$$\frac{\left[(1/\sigma - 1)\,\omega_\tau - 1\right]\alpha + g/(\Lambda\omega_{\tau}^{1-\alpha})}{\left[(1/\sigma - 1)\,\omega_\mu - 1\right]\alpha + g/(\Lambda(1-\alpha)\omega_{\mu}^{1-\alpha})} \geq 1 - \frac{g}{\Lambda\omega_{\tau}^{1-\alpha}}. \tag{39}$$

From (25), $0 \leq 1 - g/(\Lambda\omega_{\tau}^{1-\alpha}) \leq 1$ because $0 \leq \tau \leq 1$. Thus, if the following condition,

$$\frac{\left[(1/\sigma - 1)\,\omega_\tau - 1\right]\alpha + g/(\Lambda\omega_{\tau}^{1-\alpha})}{\left[(1/\sigma - 1)\,\omega_\mu - 1\right]\alpha + g/(\Lambda(1-\alpha)\omega_{\mu}^{1-\alpha})} \geq 1, \tag{40}$$

is true, (39) holds. The condition (40) can be rewritten as follows.

$$0 \leq \left(\frac{1}{\sigma} - 1\right)(\omega_\mu - \omega_\tau) + \frac{g}{\Lambda}\left(\frac{1}{(1-\alpha)\omega_{\mu}^{1-\alpha}} - \frac{1}{\omega_{\tau}^{1-\alpha}}\right). \tag{41}$$

From $\sigma \geq 1$ and (35), (41) is true. Thus, (39) holds. Therefore the growth rate in income tax financing is higher than the one in money financing when $\sigma \geq 1$.
□

In Palivos and Yip (1995), they obtain the result that for any given government size, the decrease in the growth rate is less under money financing than under income tax financing under a cash-in-advanced model.[6] Using M.I.P. model, we obtain the opposite result.

[6] Their analysis is done only under the condition $\sigma \geq 1$ which is a sufficient condition for the existence of the equilibrium.

Next, we show that even when $\sigma < 1$, money financing may be also more harmful than income tax financing by a numerical simulation. The crucial parameters are $1-\alpha$, the share of real money balances in production, and σ. We set the share of real money balances in production to 0.2, σ to 0.94. Under the set of parameters listed below, when $g = 0$, the growth rate of the economy is about 4.6%. As shown in Figure 1, the effect of an increase in the income tax is more harmful to the economic growth than the money financing.[7]

Λ	ρ	σ	α
0.1	0.03	0.94	0.8

One important difference between the C.I.A. model and the M.I.P. model is that when only consumption goods purchase is subject to the liquidity constraints, monetary policy is superneutral to economic growth. Though as the fraction of investment purchase subject to the C.I.A. constraint increases, the more monetary policy suppresses growth. However, the distortionary effect of monetary policy does not outweigh the one of income tax financing. On the other hand, in the M.I.P. model monetary policy is essentially non-neutrally since higher inflation rate, induced by the high rate of monetary expansion, increases the opportunity cost of holding money which deters the real activity.

4 Welfare comparison

In this section, we investigate the effect of the income tax financing and the money financing on welfare. As the economy is always on the balanced growth path, from (11) the level of consumption at time t is represented by

$$c(t) = c(0)e^{\frac{1}{\sigma}(r^* - \rho)t}. \tag{42}$$

Substituting it into (6) gives the life-time utility as follows:

$$U = -\frac{\sigma c(0)^{1-\sigma}}{(1-\sigma)((1-\sigma)(r^* - \rho) - \rho\sigma)} - \frac{1}{(1-\sigma)\rho}. \tag{43}$$

Since we foncus on the unstable equilibrium, $\dot{\chi} \equiv c/k$ jumps to its balanced equilibrium value immediately. In the money financing case from (17) and (19)

$$c_\mu(0) = \left\{(\sigma - \alpha)\Lambda\omega_\mu^{*1-\alpha} - \sigma g + \rho\right\} k(0)/\sigma. \tag{44}$$

Substituting this into (43), the level of the household can be expressed as

$$U_\mu = -\frac{\left\{(\sigma - \alpha)\Lambda\omega_\mu^{*1-\alpha} - \sigma g + \rho\right\}^{1-\sigma} \sigma^\sigma k(0)^{1-\sigma}}{(1-\sigma)((1-\sigma)(r_\mu^* - \rho) - \rho\sigma)} - \frac{1}{(1-\sigma)\rho}. \tag{45}$$

[7] In this numerical example, we have confirmed that the value of trace and determinant in each regime are positive for all g.

In the income tax financing case, from (17) and (25)

$$c_\tau(0) = \left\{(\sigma-\alpha)\Lambda\omega_\tau^{*1-\alpha} - (\sigma-\alpha)g + \rho\right\} k(0)/\sigma. \tag{46}$$

The level of the household utility in this case becomes

$$U_\tau = -\frac{\left\{(\sigma-\alpha)\Lambda\omega_\tau^{*1-\alpha} - (\sigma-\alpha)g + \rho\right\}^{1-\sigma} \sigma^\sigma k(0)^{1-\sigma}}{(1-\sigma)((1-\sigma)(r_\tau^* - \rho) - \rho\sigma)} - \frac{1}{(1-\sigma)\rho}. \tag{47}$$

Comparing (45) and (47), we can derive the following proposition:

Proposition 2:
When $\sigma \geq 1$ and $g > 0$, money financing is always more harmful to the welfare than income tax financing.

proof
Since $\sigma \geq 1$, from (47) and (45) the conditions for $U_\tau > U_\mu$ can be written as

$$\frac{\left\{(\sigma-\alpha)\Lambda\omega_\tau^{*1-\alpha} - (\sigma-\alpha)g + \rho\right\}^{1-\sigma}}{(1-\sigma)(r_\tau^* - \rho) - \rho\sigma} > \frac{\left\{(\sigma-\alpha)\Lambda\omega_\mu^{*1-\alpha} - \sigma g + \rho\right\}^{1-\sigma}}{(1-\sigma)(r_\mu^* - \rho) - \rho\sigma}. \tag{48}$$

Using (35), (44) and (46), we can derive the following condtion:

$$(\sigma-\alpha)\Lambda\omega_\tau^{*1-\alpha} - (\sigma-\alpha)g + \rho > (\sigma-\alpha)\Lambda\omega_\mu^{*1-\alpha} - \sigma g + \rho > 0. \tag{49}$$

Since $\sigma \geq 1$, (49) is rewritten as

$$0 < \left\{(\sigma-\alpha)\Lambda\omega_\tau^{*1-\alpha} - (\sigma-\alpha)g + \rho\right\}^{1-\sigma} < \left\{(\sigma-\alpha)\Lambda\omega_\mu^{*1-\alpha} - \sigma g + \rho\right\}^{1-\sigma}. \tag{50}$$

As we have proved that the growth rate under income tax financing is larger than the one under money financing, we obtain $0 < r_\mu^* < r_\tau^*$. Thus we have

$$\frac{(1-\sigma)(r_\tau^* - \rho) - \rho\sigma}{(1-\sigma)(r_\mu^* - \rho) - \rho\sigma} \geq 1. \tag{51}$$

Considering that $1 - \sigma \leq 0$, (48) can be written as

$$\frac{\left\{(\sigma-\alpha)\Lambda\omega_\tau^{*1-\alpha} - (\sigma-\alpha)g + \rho\right\}^{1-\sigma}}{\left\{(\sigma-\alpha)\Lambda\omega_\mu^{*1-\alpha} - \sigma g + \rho\right\}^{1-\sigma}} < \frac{(1-\sigma)(r_\tau^* - \rho) - \rho\sigma}{(1-\sigma)(r_\mu^* - \rho) - \rho\sigma}.$$

From (50) and (51), we can confirm that the welfare in income tax financing is higher than the one in money financing. □

As in the growth comparison we have demonstrated above, we can show that even when $\sigma < 1$, money financing may be also more harmful than income tax financing by a numerical simulation. Using the same numerical value, the welfare difference can be depicted as in Figure 2.

Life-time utility is generally determined by the rate of economic growth and the inflation rate. The higher inflation rate is, the higher the the nominal interest rate or the oppotunity cost of holding money is. It reduces welfare. From the money market equilibrium condition, $\dot{m}/m = \mu - \pi$, the inflation rate can be expressed as the monetary expansion rate and the balanced growth rate. As we have seen in Section 3, the growth rate of the economy in money financing is lower than the one in income tax financing. Only in the money financing regime, monetary expansion exists. Thus, the inflation rate in money financing is higher than the one in income tax financing. In money financing regime, the economic growth rate is lower and the inflation rate is higher, therefore welfare is worse.

5 Concluding remarks

In this paper, we examine the effects of the different financing in the M.I.P. model. In contrast to the result in Palivos and Yip (1995) which examines the effects of the alternative financing under cash-in-advance constraints on consumption purchases and investment purchase, we show that the decrease in the growth rate is less under income tax financing than under money financing when $\sigma \geq 1$. Furthermore, we compare the welfare under each regime. We find that money financing is always more harmful to the welfare than income tax financing when $\sigma \geq 1$. Even when $\sigma < 1$, a numerical examples shows that income tax financing is favaorable to the economic growth and the welfare.

There are various kinds of monetary economics, such as M.I.U. model, C.I.A. model, and some policy implication in each model is different. Our results suggest that the relative impact of the alternative tax change must be evaluated by using more integrated monetary model.

Several extensions may be fruitful for future research. First, we use a Cobb-Douglas form production function. It helps us to evaluate the effect of the different financing analytically in the case $\sigma \geq 1$, but obviously it is restrictive. More general form of money-in-the-production- function may yield more general results. Second, we can extend the model to a multi-sector model. In the multi-sector model, we can consider different share of real money balances in different sectors and the tax rate on each sector as well as overall tax burden would be important on growth and welfare. We can investigate not only the consequence of switching financing regime from one to another but also the one of changing tax burden on each sector.

Appendices

Appendix 1 On the transversality condition

From (5) and (9) in the balanced growth equilibrium, the household's budget constraint is rewritten as

$$\dot{a} = ra - c. \tag{52}$$

In order to satisfy the transversality condition, differentiating (10) with respect to t must be negative. Using (8) and (52), the condition gives

$$\frac{\dot{\lambda}}{\lambda} + \frac{\dot{a}}{a} - \rho = -\frac{c}{a}.$$

From (11), (52) and no-Ponzi game condition, we have

$$\frac{c}{a} = \frac{(\sigma - 1)r + \rho}{\sigma}.$$

Thus when $\sigma \geq 1$, the transversality condition is always satisfied.

Appendix 2 On the existence of the equilibrium

Here, we show that under relatively small government expenditure, there exists unique positive equilibrium level of ω which we can conduct comparative statics exercise.[8]

Appendix 2.1 In the case of money financing

The equilibrium value of ω must satisfy equation (22). Take the limit as $\omega \to 0$:

$$\lim_{\omega \to 0} F(\omega) = \infty, \quad \lim_{\omega \to 0} G(\omega) = \infty, \quad \lim_{\omega \to 0} F(\omega)/G(\omega) = 0. \tag{53}$$

Since in this case $F(\omega)$ becomes $\lim_{\omega \to \infty} F(\omega) = -\infty(\text{ or } 0 \text{ when } \sigma = 1)$, Taking into account condition (53), the graphs of $F(\omega)$ and $G(\omega)$ can be depicted as in Figure 5.

First, consider there is no government expenditure ($g = 0$). Graph of function G becomes flat line, and there is always unique equilibrium value of ω.

Next, define the value of g which satisfies two following conditions that $dF(\omega)/d\omega = dG(\omega)/d\omega$ and $F(\omega) = G(\omega)$ as $\bar{g}$. The condition that $dF(\omega)/d\omega = dG(\omega)/d\omega$ is rewritten as

$$g = (-(1/\sigma - 1)\omega + 1)\alpha(1 - \alpha)\Lambda\omega^{1-\alpha}. \tag{54}$$

[8] Suen and Yip (2005) examines the dynamic property of the MIP model without government expenditures.

Substituting (54) into equation $F(\omega) = G(\omega)$, we have that

$$(2-\alpha)(1/\sigma - 1)\Lambda\alpha\omega + (1-\alpha)^2\Lambda = \rho\omega^{\alpha}/\sigma.$$

The above equation gives a unique positive solution which we shall call ω_1. Introducing ω_1 into (54), we get a positive unique value of g, which we shall call $\bar{g}$. Since graph G moves up and to the right as g increases, we can conclude that when $0 < g < \bar{g}$, there is one equilibrium which satisfies $dF(\omega)/d\omega < dG(\omega)/d\omega$. On the other hand, when $\bar{g} < g$, there exists no equilibrium.

Appendix 2.2 In the case of income tax financing

The equilibrium value of ω must satisfy (28). Take the limit of $F_\tau(\omega)$ and $G_\tau(\omega)$ as $\omega \to 0$:

$$\lim_{\omega\to 0} F_\tau(\omega) = \infty, \quad \lim_{\omega\to 0} G_\tau(\omega) = \infty, \quad \lim_{\omega\to 0} F_\tau(\omega)/G_\tau(\omega) = 0.$$

Since $F_\tau(\omega)$ becomes $\lim_{\omega\to\infty} F(\omega) = -\infty$ (or 0 when $\sigma = 1$), graphs $F_\tau(\omega)$ and $G_\tau(\omega)$ can be depicted as in Figure 3.

As in money financing regime, it can be easily seen that when $g = 0$, there is a unique equilibrium value of ω.

To take a similar procedure as in the case of money financing, define the value of g which satisfy the two conditions that both $dF_\tau(\omega)/d\omega = dG_\tau(\omega)/d\omega$ and $F_\tau(\omega) = G_\tau(\omega)$ as $\tilde{g}$.

When

$$g = [-(1/\sigma - 1)\omega + 1]\alpha\Lambda\omega^{1-\alpha}, \tag{55}$$

$dF(\omega)_\tau/d\omega = dG(\omega)_\tau/d\omega$. Introducing (55) into $F_\tau(\omega) = G_\tau(\omega)$, we have that

$$[\alpha(1/\sigma - 1)\omega + 1 - \alpha]^2 = \rho\omega^{\alpha}/(\Lambda\sigma). \tag{56}$$

As indicated in Figure 4 ($\eta_\tau(\omega)$ and $\nu_\tau(\omega)$ denote the left-hand side and the right-hand side of (56) respectively in the figure.), (56) has two positive solutions. Introducing the smaller solution (which we call ω_2') into (55), we have $\tilde{g}$. Since $\sigma \geq 1$, $\tilde{g}$ is always positive. Moreover the fact that $\omega_2' \leq -(1-\alpha)/\alpha(\frac{1}{\sigma} - 1)$ ensures that

$$\frac{\partial G}{\partial g} = \frac{1}{\omega}(1-\alpha) + \alpha\left(\frac{1}{\sigma} - 1\right) > 0. \tag{57}$$

This implies that as g increases, the graph of G moves up and to the right as the arrows in Figure 5 indicates. In this sense, $\tilde{g}$ is the threshold value for the existence of equilibrium. We can conclude that when $0 < g < \tilde{g}$, there is one equilibrium which satisfies $dF(\omega)/d\omega < dG(\omega)/d\omega$. On the other hand, when $\tilde{g} < g$, there exists no equilibrium.

Appendix 3.1 Property of the balanced growth equilibrium in money financing

From (17), (18) and (19), the autonomous dynamic system becomes

$$\dot{\chi}_\mu = \chi_\mu \left[\frac{1}{\sigma} \left[\Lambda \alpha \omega_\mu^{1-\alpha} - \rho \right] - \Lambda \omega_\mu^{1-\alpha} + \chi_\mu + g \right], \tag{58}$$

$$\dot{\omega}_\mu = \omega_\mu \left[\frac{g}{\omega_\mu} + \Lambda \left\{ \alpha \omega_\mu^{1-\alpha} - (1-\alpha) \omega_\mu^{-\alpha} \right\} - \Lambda \omega_\mu^{1-\alpha} + \chi_\mu + g \right]. \tag{59}$$

Linearizing (58) and (59), we obtain

$$\begin{pmatrix} \dot{\chi_\mu} \\ \dot{\omega_\mu} \end{pmatrix} = \begin{pmatrix} \chi_\mu^* & \zeta_{12} \\ \omega_\mu^* & \zeta_{22} \end{pmatrix} \begin{pmatrix} \chi_\mu - \chi_\mu^* \\ \omega_\mu - \omega_\mu^* \end{pmatrix}, \tag{60}$$

where

$$\zeta_{12} \equiv \chi_\mu^* \Big[\left(\frac{1}{\sigma} \alpha - 1 \right) \Lambda (1-\alpha) \omega_\mu^{*-\alpha} \Big],$$

$$\zeta_{22} \equiv \omega_\mu^* \left[- \frac{g}{\omega_\mu^{*2}} - \Lambda (1-\alpha) \left\{ (1-\alpha) \omega_\mu^{*-\alpha} - \alpha \omega_\mu^{*-\alpha-1} \right\} \right].$$

The trace, T_1, and determinant, D_1, of the coefficient matrix (60) are given by

$$\begin{aligned} T_1 &= -\frac{1}{\sigma} \left[\Lambda \alpha \omega_\mu^{*1-\alpha} - \rho \right] + \Lambda \omega_\mu^{*1-\alpha} - g - \frac{g}{\omega_\mu^*} - \Lambda (1-\alpha) \left\{ (1-\alpha) \omega_\mu^{*1-\alpha} - \alpha \omega_\mu^{*-\alpha} \right\}, \\ &= \frac{(\sigma - 1) r^* + \rho}{\sigma} + \left[-\frac{g}{\omega_\mu^*} + \Lambda \alpha (1-\alpha) \omega_\mu^{*-\alpha} \right] (1 + \omega_\mu^*), \\ D_1 &= \chi_\mu^* \omega_\mu^* \left[-\frac{g}{\omega_\mu^{*2}} - \Lambda (1-\alpha) \left\{ (1-\alpha) \omega_\mu^{*-\alpha} - \alpha \omega_\mu^{*-\alpha-1} \right\} - \left(\frac{1}{\sigma} \alpha - 1 \right) \Lambda (1-\alpha) \omega_\mu^{*-\alpha} \right], \\ &= -\Lambda (1-\alpha) \omega_\mu^{*-\alpha-1} \chi_\mu^* \omega_\mu^* \left[\left(\frac{1}{\sigma} - 1 \right) \omega_\mu^* - 1 + \frac{g}{\Lambda (1-\alpha) \omega_\mu^{*1-\alpha}} \right]. \end{aligned}$$

Since both ω_μ and χ_μ are jumpable, when $T_1 > 0$ and $D_1 > 0$ and the coefficient matrix (60) has two positive eigenvalues, the economy immediately jumps to its balanced growth equilibrium. In this case, we obtain The condition for that $D_1 > 0$ is rewritten as

$$\left(\frac{1}{\sigma} - 1 \right) \omega_\mu^* - 1 + \frac{g}{\Lambda (1-\alpha) \omega_\mu^{*1-\alpha}} < 0. \tag{61}$$

Thus, when $\sigma \geq 1$ and the government expenditure is not so large, the conditions that $T_1 > 0$ and $D_1 > 0$ hold. Moreover, when (61) is true, the denominator in (24) is negative.

Appendix 3.2 Property of the balanced growth equilibrium in income tax financing

From (17), (18) and (25), the autonomous dynamic system becomes

$$\dot{\chi_\tau} = \chi_\tau \left[\frac{1}{\sigma} \left[\left(1 - \frac{g}{\Lambda \omega_\tau^{1-\alpha}} \right) \Lambda \alpha \omega_\tau^{1-\alpha} - \rho \right] - \Lambda \omega_\tau^{1-\alpha} + \chi_\tau + g \right], \tag{62}$$

$$\dot{\omega_\tau} = \omega_\tau \left[\left(1 - \frac{g}{\Lambda \omega_\tau^{1-\alpha}} \right) \Lambda \left\{ \alpha \omega_\tau^{1-\alpha} - (1-\alpha) \omega_\tau^{-\alpha} \right\} - \Lambda \omega_\tau^{1-\alpha} + \chi_\tau + g \right]. \tag{63}$$

Linearizing (62) and (63), we obtain

$$\begin{pmatrix} \dot{\chi_\tau} \\ \dot{\omega_\tau} \end{pmatrix} = \begin{pmatrix} \chi_\tau^* & \xi_{12} \\ \omega_\tau^* & \xi_{22} \end{pmatrix} \begin{pmatrix} \chi_\tau - \chi_\tau^* \\ \omega_\tau - \omega_\tau^* \end{pmatrix}, \tag{64}$$

where

$$\xi_{12} \equiv \chi_\tau^* \Big[\frac{1}{\sigma} \alpha (1-\alpha) \Lambda \omega_\tau^{*-\alpha} - (1-\alpha) \Lambda \omega_\tau^{*-\alpha} \Big],$$

$$\xi_{22} \equiv \omega_\tau^* \Big[\Lambda \alpha (1-\alpha) \omega_\tau^{*-\alpha} + \Lambda \alpha (1-\alpha) \omega_\tau^{*-\alpha-1} - (1-\alpha) \frac{g}{\omega_\tau^{*2}} - (1-\alpha) \Lambda \omega_\tau^{*-\alpha} \Big].$$

The trace, T_2, and determinant, D_2, of the coefficient matrix (64) are given by

$$T_2 = \chi_\tau^* + \omega_\tau^* \Big[\Lambda \alpha (1-\alpha) \omega_\tau^{*-\alpha} + \Lambda \alpha (1-\alpha) \omega_\tau^{*-\alpha-1} - (1-\alpha) \frac{g}{\omega_\tau^{*2}} - (1-\alpha) \Lambda \omega_\tau^{*-\alpha} \Big],$$

$$= \frac{(\sigma - 1) r^* + \rho}{\sigma} + \left[-\frac{g}{\omega_\tau^*} + \Lambda \alpha \omega_\tau^{*-\alpha} \right] (1 + \omega_\tau^*)(1-\alpha),$$

$$D_2 = -\chi_\tau^* \omega_\tau^* \Big[\left(\frac{1}{\sigma} - 1 \right) \alpha (1-\alpha) \Lambda \omega_\tau^{*-\alpha} - \Lambda \alpha (1-\alpha) \omega_\tau^{*-\alpha-1} + (1-\alpha) \frac{g}{\omega_\tau^{*2}} \Big],$$

$$= -\alpha (1-\alpha) \Lambda \omega_\tau^{*-\alpha-1} \chi_\tau^* \omega_\tau^* \Big[\left(\frac{1}{\sigma} - 1 \right) \omega_\tau^* - 1 + \frac{g}{\alpha \Lambda \omega_\tau^{*1-\alpha}} \Big].$$

Following the same argument in the above subsection, we can confirm that when $\sigma \geq 1$ and the government expenditure is not so large, the conditions that $T_2 > 0$ and $D_2 > 0$ hold and that the denominater in (30) is negative.

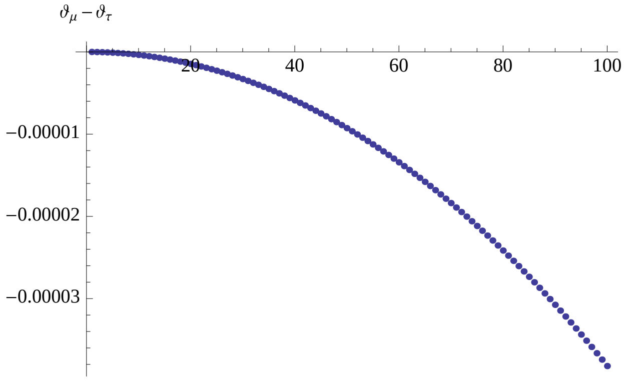

Figure 1: comparison of the growth rate when $\sigma < 1$ (the horizontal axis indicates $g \times 10^5$).

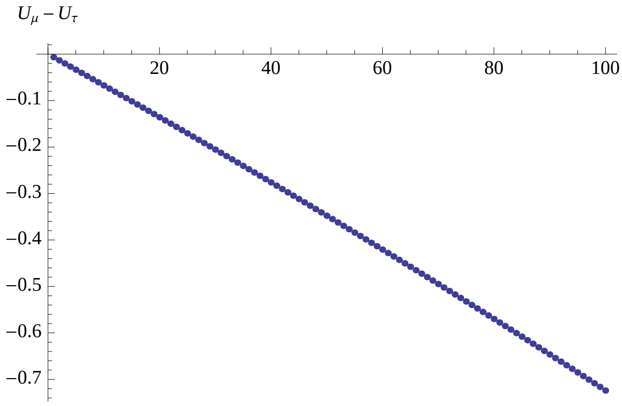

Figure 2: comparison of the welfare when $\sigma < 1$(the horizontal axis indicates $g \times 10^5$)

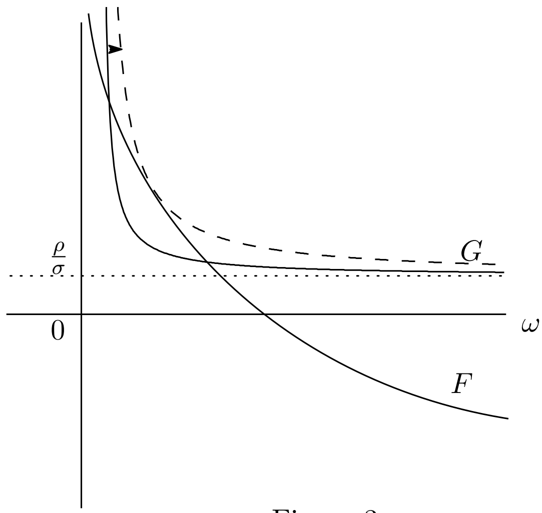

Figure 3

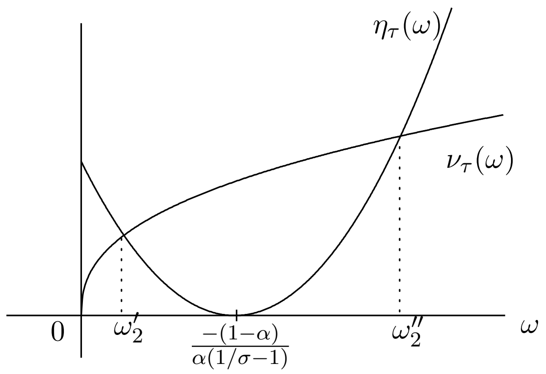

Figure 4

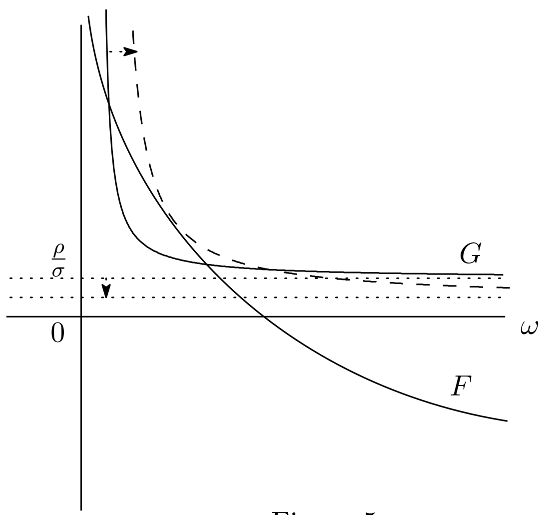

Figure 5

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