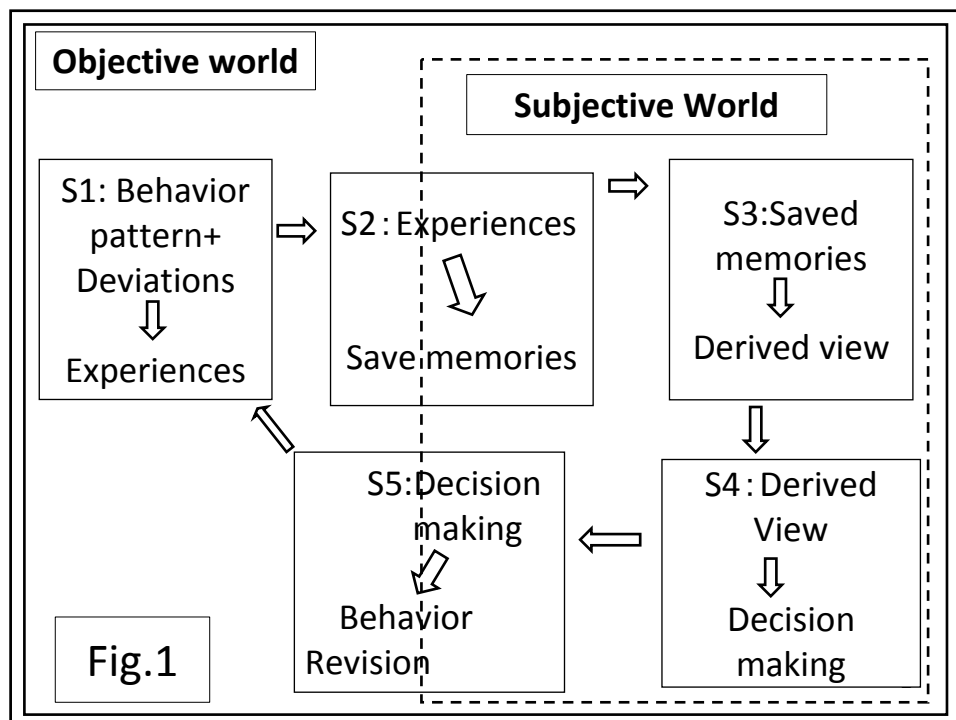


A Course on Inductive Game Theory: 2

by M. Kaneko (based on Kaneko-Kline papers)

- Basic Question: the origin/emergence of a player's beliefs/knowledge on the structure of a game situation?
- Experiential source:
 1. Experiences and their accumulation
 2. Inductive derivation of a subjective view from the accumulated experiences.
- Roles of a derived subjective view
 1. Understanding of the target social situation
 2. Use of his subjective view: decision making and behavior revision
- Checking of his view with experiences
 1. Comparisons between his view and new experiences
 2. Correct his view.

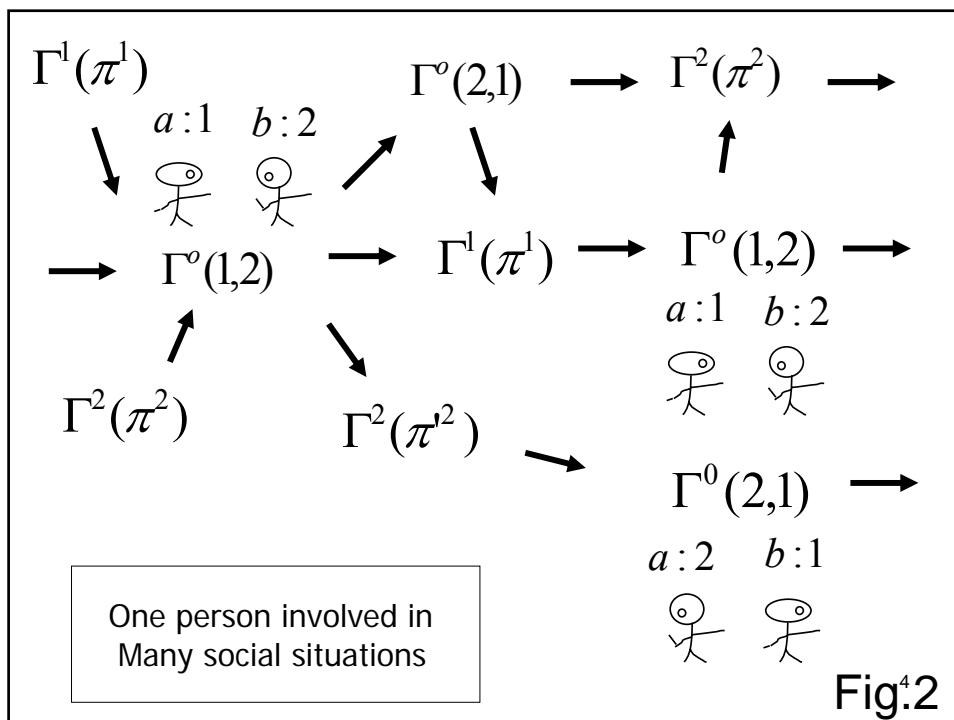
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● Immediate questions:

1. Time structure and learning?
2. Beliefs or knowledge (true belief)?
non-experiential vs. experiential
rejection of probabilistic beliefs!
3. Extensive game as a description of the objective situation?
Any problems?
4. Experiences? Information?
Any problem?
5. Accumulation of experiences? Mental structure?
6. Induction?
7. Extensive game as a description of a subjective view, even though a player may have very partial experiences?

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Components under Scrutiny

- Information:

Information sets from the objective point of view

Two roles

Information transmission

Memory capability

Information pieces

Our Theory

Memory function

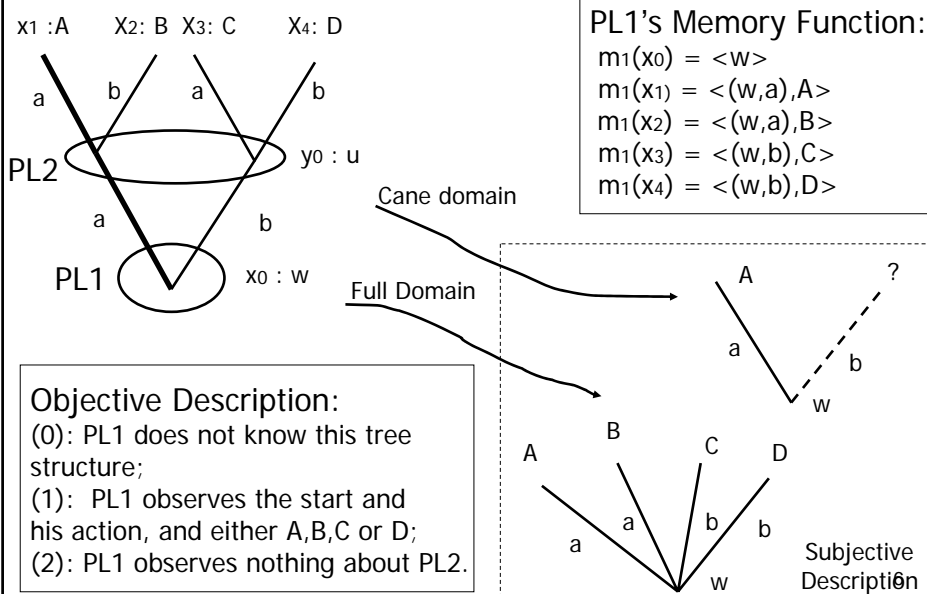
New Formulation of an Extensive Game (K-K'08a)

alternatively,

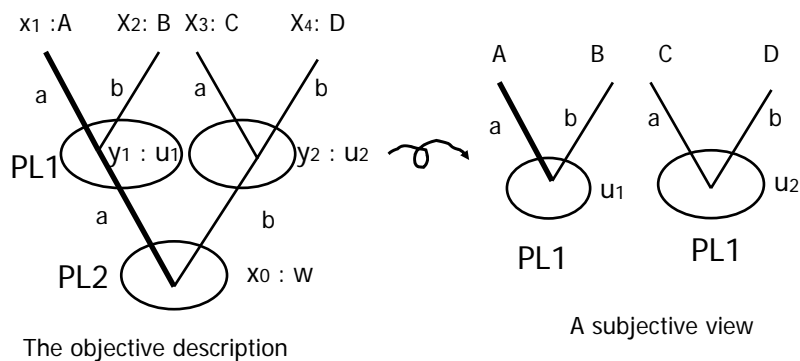
Direct Formulation: Info-memory protocol (K-K'08b)

5)

An Extensive Game as a Subjective View: Difficulties

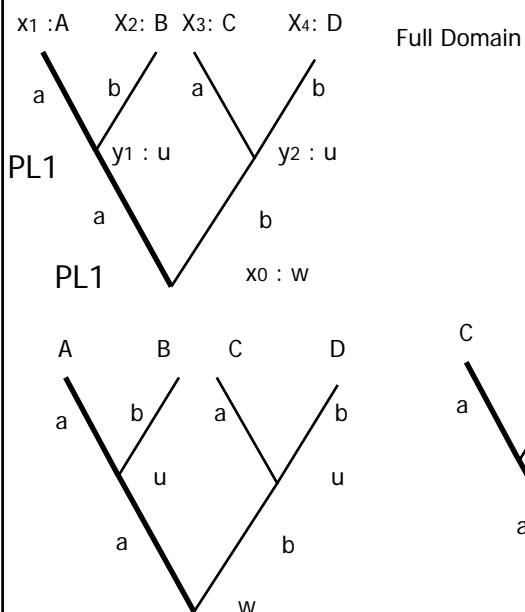


- **Partiality:** caused partial experiences
- **Ignorance of Resulting Outcomes:** an action is available but its outcome is totally unknown (e.g., cane domain)
- **Non-Determinism:** caused by the existence of an unknown player.
- **Violation of the Tree Structure:** if PL1 does know the existence of PL2 in the following game,



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Alternative Memory Function



Recall-1 Memory Function

$m_1(x_0) = \langle w \rangle$
 $m_1(y_1) = \langle (w,a), u \rangle$
 $m_1(y_2) = \langle (w,b), u \rangle$
 $m_1(x_1) = \langle (u,a), A \rangle$
 $m_1(x_2) = \langle (u,b), B \rangle$
 ...

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Extensive Game vs. Information Protocol

Extensive Game (Kaneko-Kline '08a)

- Substructure
 - a): tree structure;
 - b): causality from an action to the next node.
- Superstructure built on the substructure
 - a): information, available actions, payoffs
 - b): those are relevant for game theory.

Basic Difficulty in the subjective use:

Nodes and Branches – Hypothetical elements!

Information Protocol (kaneko-Kline '08b)

- a): consisting of only tangible basic elements;
- b): (convenient) axiomatic approach and axioms characterize violations of the above mentioned difficulties.

Comparisons between e.g. and i.p. (K-K08b).

An extensive game is given as $\Gamma = ((X, \prec), (\lambda, W), \{(\varphi_x, A_x)\}_{x \in X}, (\pi, N), h)$

with the following properties:

K1: (Game Tree): (X, \prec) is a finite forest (a tree by K14);

K11: X is a finite set of nodes, and \prec is a partial ordering over X ;

K12: $\{x \in X : x \prec y\}$ is totally ordered with \prec for all $y \in X$;

K13: $X^D = \{x \in X : x \prec y \text{ for some } y \in X\}$ and $X^E = X - X^D$;

K14(Root): (X, \prec) has the smallest element;

K2: (Information function): W is a finite set of information pieces, and

$\lambda : X \rightarrow W$ is a surjection with $\lambda(x) \neq \lambda(x')$ for any $x \in X^D$ and $x' \in X^E$;

K3: (Available action sets): A_x is a finite set of available actions for each $x \in X$;

K31: $A_x = \emptyset$ for all $x \in X^E$;

K32: $A_x = A_{x'}$ if $\lambda(x) = \lambda(x')$;

K33: for any $x \in X$, φ_x is a bijection from the set of immediate successors of x to A_x ;

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K4 : (Player Assignment) : N is a finite set of players, and $\pi : W \rightarrow 2^N$ is a player assignment with

K41 : $|\pi(w)| = 1$ for all $w \in \{\lambda(x) : x \in X^D\}$ and $\pi(w) = N$ if $w \in \{\lambda(x) : x \in X^E\}$;

K42 : for all $j \in N$, $j \in \pi(w)$ for some $w \in \{\lambda(x) : x \in X^D\}$;

K5 : (Payoff function) : $h = \{h_i\}_{i \in N}$, where $h_i : \{\lambda(x) : x \in X^E\} \rightarrow R$ is a payoff function for player $i \in N$.

• Possible Weakenings:

1. Elimination of K14(Root);

2. K33 can be weakened into: for any x in X ,

K33f: φ_x is a function from the set of immediate successors to A_x ;

K33i: φ_x is an injection;

K33s: φ_x is a surjection.

An information protocol is given as $(W, A, <)$ (+ player assignment + payoff functions) :

1): W is a finite set of information pieces;

2): A is a finite set of available actions;

3): $<$ is a finite set of $\bigcup_{m=0}^{\infty} [(W \times A)^m \times W]$.

$\langle \xi, w \rangle = \langle (w_1, a_1), \dots, (w_m, a_m), w_{m+1} \rangle$ is a (partial) history.

Basic Axiom B1 (subsequence-closed):

If $\langle \xi, w \rangle$ is in $<$, then any subsequence $\langle \eta, v \rangle$ of $\langle \xi, w \rangle$ is also in $<$.

Basic Axiom B2 (Basic Extension):

If $\langle \xi, w \rangle$ and $\langle (w, a), u \rangle$ are in $<$, then $\langle \xi, (w, a), v \rangle$ is in $<$ for some $v \in W$.

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Non-Basic Axioms

- A sequence $\langle \xi, w \rangle$ in $<$ is called a position iff it is an initial sequence of some maximal sequence in $<$.

Axiom N1 (Root):

There is a w_0 in W such every position $\langle \xi, w \rangle$ starts with w_0 .

Axiom N2 (Determination):

Let $\langle \xi, w \rangle, \langle \eta, v \rangle$ be positions.

If $\xi = \eta$ and ξ is nonempty, then $w = v$.

Axiom N3 (Independent Extension):

Let $\langle \xi, w \rangle$ be a position and

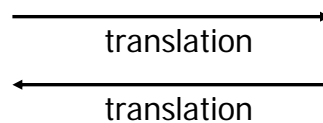
let $\langle (w, a), v \rangle$ be in $<$. Then $\langle \xi, (w, a), u \rangle$ is a position for some $u \in W$.

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Correspondence Theorem

KK'08b

- an e.g. with K33f but not K14 \iff an i.p. with B1 & B2.

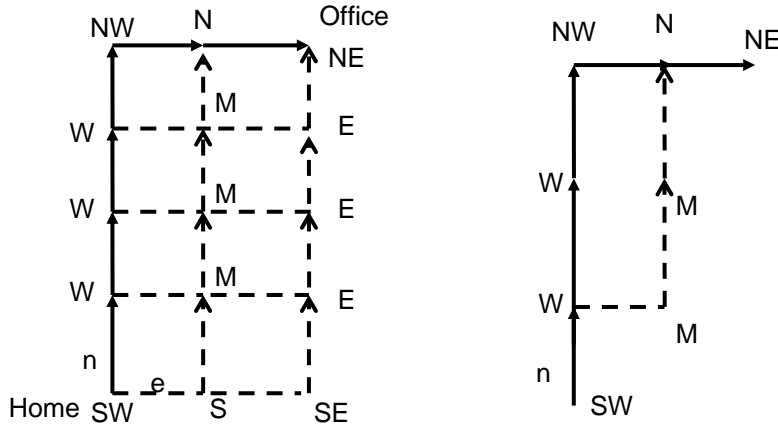


- K14 (root) \iff Axiom N1 (root)
- K33i (injection) \iff Axiom N2 (determination)
- K33s (surjection) \iff Axiom N3 (independent extension)

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Partiality in Experiences and Memories: Mike's Bike Commuting

- Recall -1 memory function : $m^{R1}(x) = \langle (u, a), w \rangle$
- Cane Domain indicated by the bold arrows
- The vertical length may not be correctly found.



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The memory kit T_{D_i} is defined by $T_{D_i} = \{m_i^o \langle \xi, w \rangle : \langle \xi, w \rangle \in D_i\}$.

Inductively Derived View

An i.d.view for i from T_{D_i} is a personal view (Π^i, m^i) iff

ID1: $W^i = \{w \in W^o : w \text{ occurs in } T_{D_i}\}; W^{iD} \subseteq W^{oD} \text{ and } W^{iE} \subseteq W^{oE};$

ID2(Actions): $A_w^i \subseteq A_w^o$ for each $w \in W^i$;

ID3(Feasible Sequences): $\Delta T_{D_i} \subseteq \prec^i$;

Other three conditions (definitions).

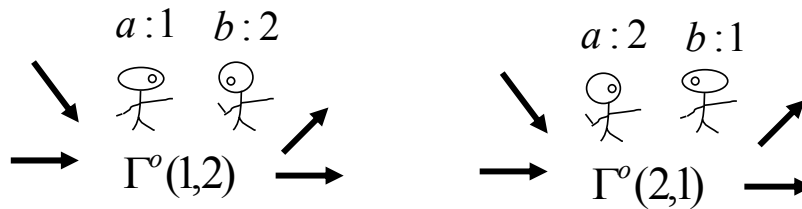
- In this formulation, we consider only player's own experiences, but do not think about a player's subjective thinking of other people.
- What is the experiential source for other people's thinking?

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Experiential Foundations of Other People's Beliefs

- Social roles: Persons vs. Players (roles)

man 1 : breakfast-maker a woman 2 : breakfast maker a
 woman 2 : dinner-maker b man 1 : dinner maker b



- Asymmetric Situation: Father vs. Son
 Father: boy & father; and Son: boy.

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Experiential Foundations of Other People's Beliefs

"As You like it" by W. Shakespeare

*All the world's a stage,
 And all the men and women merely players;
 They have their exits and entrances,
 And one man in his time plays many parts,
 His Acts being seven ages.*

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Other Problems

- Status of the Epistemic Logic with Shallow Depths
 - *Ex Ante* Decision in the Derived Subjective View
- Complexity of Interpersonal Thinking;
- Complexity of Intrapersonal Thinking.

- Checking of his Subjective View with new Experiences
 1. Doxastic Decisions;
 2. Errors of the 1st type and 2nd type.

- Communication, Education, etc.

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