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Indeterminacy in an R&D-based Endogenous Growth Model with Nominal Wage Stickiness

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Indeterminacy in an R&D-based Endogenous Growth Model with Nominal Wage Stickiness

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Abstract

This study proposes a monetary growth model involving a price stickiness and endogenous R&D by introducing nominal wage stickiness and money growth into a R&D-based growth model. The main purpose of this study is to examine how money growth affects long-run economic growth and determinacy property of equilibrium paths. We find that faster money growth causes faster balanced growth, although the balanced growth path is more likely to be indeterminate. As a result, policy-makers may face a trade-off between growth enhancing policies and economic stabilization policies.

JEL classification: O11, O42, E12, E31

keywords: endogenous growth, indeterminacy, new Keynesian Phillips curve, nominal rigidities, monetary growth model

1 Introduction

This study proposes a new monetary growth model involving a price stickiness and endogenous R&D.[1] The price stickiness is usually considered in short-run models of macroeconomics, as in new Keynesian models. In this study, we introduce nominal wage stickiness into an long-run growth model based on R&D, and investigate how the money growth affects long-run economic growth and the determinacy property of the steady states.

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[1] For details on the monetary growth theory, see Zhang (2010).

The dynamics of our model are based on the new Keynesian Phillips curve (NKPC), under which money is not superneutral, even in the long run.[2] Inoue and Tsuzuki (2011) and Tsuzuki and Inoue (2010) proposed the Dynamic General Equilibrium (DGE) model with the NKPC and technological change. In their model, money was not superneutral in the long run, and the long-run output gap existed when the money growth rate was lower than that of technological change.[3] However, their analyses assumed exogenous technological change, as did the Solow model.

This study provides the DGE model on the basis of Inoue and Tsuzuki (2011) with endogenous technological change, rather than exogenous growth, by introducing explicit R&D activities.[4] That is, in this study, the new Keynesian theory that represents the short-run theory is integrated with the endogenous growth theory that represents the long-run theory.[5]

First, we focus on the steady-state economic growth and employment. For sufficiently high money growth rates, there is a unique balanced growth path, and the economy exhibits sustained growth based on sustained R&D. Faster money growth causes greater employment and faster economic growth along the balanced growth path. Furthermore, under some parameter restrictions, there is no balanced growth path for low rates of money growth, and the economy is trapped in a steady state without long-run growth. These results suggest that money growth may be an important factor for long-run economic growth. That is, financial authorities are required to maintain high money growth rates to achieve sustained and faster economic

[2] That is, the long-run Phillips curve derived from our NKPC is downward sloping as the traditional Keynesian's Phillips curve. On the contrary, the other type of NKPC, which inherits the property of Friedman's expectations-augmented Phillips curve, is conceivable. Under such a NKPC, the long-run Phillips curve is vertical at the natural rate of unemployment, i.e., the natural rate hypothesis holds. Akerlof, Dickens, and Perry (2000, 1996) have proposed a long-run Phillips curve that is vertical for comparatively high inflation rates and downward sloping for lesser inflation rates. That is, their long-run Phillips curve is downward sloping in the low inflationary and deflationary economy as Japan in the 1990s-2000s. This study focuses on such a situation. For other empirical evidence that justifies the downward slope of the long-run Phillips curve, see Graham and Snower (2008, Sec.1).

[3] Some studies such as Christiano, Motto, and Rostagno (2003) also proposed new Keynesian models that introduced an exogenous technological trend. However, they did not analyze the long-run output gap.

[4] Annicchiarico, Pelloni, and Rossi (2011), Kühn (2010), and Rannenberg (2009) proposed a new Keynesian model in which sustained growth becomes endogenous through learning-by-doing or simple externality. Tsuzuki and Inoue (2011) also proposed a new Keynesian endogenous growth model introducing human capital accumulations, as in Lucas (1988).

[5] For details on the new Keynesian theory, see Woodford (2003) and Gali (2008).

growth.[6]

Most of the preceding theoretical studies on money and endogenous growth concluded that a higher money growth is associated with a lower rate of long-run growth, which is contrary to the conclusion of this study. See for example Marquis and Reffett (1995, 1991), Jones and Manuelli (1995), Pecorino (1995), and Mino (1997). In contrast, Mino and Shibata (2000, 1995) and Suen and Yip (2005) demonstrated the positive relationship between a monetary expansion and long-run growth. Our study provides new explanation following the line of these studies.[7]

Second issue of this study is with respect to the determinacy property of the steady state. By investigating the local dynamics within the neighborhoods of the steady states, we show that changes in money growth rates have an influence on determinacy of the equilibrium path. Under the specific parameters, whether the balanced growth path is determinate or indeterminate varies depending upon the money growth rate; therefore, policy-makers can eliminate volatility in the economy through their decisions. However, faster money growth causes faster balanced growth, although the balanced growth path is more likely to be indeterminate; that is, the policy trade-off may exist between growth promotion and economic stabilization.

Many studies analyzed indeterminate equilibria in the context of the monetary endogenous growth theory, such as Itaya and Mino (2007, 2003), Mino and Itaya (2004), and Suen and Yip (2005); however, sustained long-run growth in these studies does not stem from R&D, and does not include any rigidity. This study proposes a new channel attributed to nominal rigidities and endogenous R&D through which money growth influences determi-

[6]These results are similar to those in Shinagawa and Inoue (2011). However, this study differs from our preceding study in the following respects. First, we introduce capital accumulation into the model (note that labor had been the only production factor in our preceding model). In response to this addition, the specification of R&D is changed from knowledge-driven specification to lab equipment specification. For the details on the differences between these specifications, please refer to Rivera-Batiz and Romer (1991), Matsuyama (1995), and Gancia and Zilibotti (2005). Second, we assume a finite-lived patent instead of the infinitely-lived patent. Furthermore, we adopt the discrete-time version of the dynamic model because it works well with the assumption of temporary patent protection. These modified assumptions more appropriately explain the actual economy.

[7]Empirically, both of cross-country and panel regressions explaining economic growth often obtain a negative effect from inflation (Fischer, 1993). However, Levine and Zervos (1993) and Ericsson, Irons, and Tryon (2001) found that the negative correlation between inflation and growth obtained by cross-country regression is not robust. Bruno and Easterly (1998) concluded that growth and inflation are negatively related only in the extremely high inflationary economy.

nacy property of equilibria.[8]

The rest of this paper is organized as follows. The next section presents the model used in our theoretical investigation. Section 3 derives the law of motion and the steady states, which characterize the equilibrium path of the economy. Section 4 examines the local determinacy of the steady states. Section 5 concludes the paper.

2 Model

We consider a discrete-time dynamic model. Time is indexed by $t = 0, 1, 2, \ldots$. The economy is inhabited by many infinitely-lived households under monopolistic competition in the labor market, and there are rigidities of nominal wage. There is a single final good taken as a numeraire, which is produced using intermediate goods and labor. It is supplied competitively and can be consumed and invested. A new variety of intermediate goods is invented by allocating capital for R&D activities. Inventors are able to enjoy a one-period monopoly through temporary patent protection. The available intermediate goods are produced by multiple intermediate firms using capital. As a monetary policy rule, we use the k-percent rule under which financial authorities expand money supply at a constant rate.

2.1 Employment agency

The manufacturing and R&D sectors regard each household's labor as an imperfect substitute for any other household's labor. To simplify the analysis, we assume that an employment agency combines differentiated labor forces into a composite labor force according to the Dixit-Stiglitz function:

$$L_t = \left[\int_0^1 L_{j,t}^{\zeta} dj \right]^{\frac{1}{\zeta}}, \quad \zeta \in (0, 1),$$

where $L_{j,t}$ denotes differentiated labor supplied by household $j \in [0, 1]$, and L_t denotes the composite labor force. The number of households is normalized to 1. $1/(1 - \zeta)(> 1)$ is the elasticity of substitution between each pair of differentiated labor inputs.

[8] A number of studies, such as Benhabib, Perli, and Xie (1994), Evans, Honkapohja, and Romer (1998), Haruyama and Itaya (2006), Furukawa (2007a,b), Arnold and Kornprobst (2008), and Haruyama (2009), investigated the issue of indeterminacy in R&D-based growth models.

Cost minimization of the employment agency yields the following demand functions for differentiated labor j:

$$L_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{-\frac{1}{1-\zeta}} L_t,$$

where $W_{j,t}$ denotes the nominal wage rate of labor force j, and W_t denotes the nominal wage rate of the composite labor force, which is given by

$$W_t = \left[\int_0^1 W_{j,t}^{-\frac{\beta}{1-\beta}} dj\right]^{-\frac{1-\zeta}{\zeta}}.$$

2.2 Final goods producer

The final goods firm produces the quantity y_t according to the Dixit-Stiglitz function as follows:

$$y_t = L_t^{1-\alpha} \int_0^{N_t} x_{i,t}^{\alpha} di, \quad \alpha \in (0,1), \tag{2.1}$$

where L_t is the amount of composite labor, $x_{i,t}$ is the quantity of intermediate goods indexed by $i \in [0, N_t]$, and $1/(1-\alpha)$ (> 1) represents the elasticity of substitution between every pair of intermediate goods. N_t is the number of available intermediate goods in period t that represents the technology level of the economy.[9]

The nominal profit of the representative final goods firm is given by

$$P_t y_t - L_t W_t - \int_0^{N_t} p_{i,t} x_{i,t} di,$$

where P_t is the price of the final goods, and $p_{i,t}$ is the price of the intermediate goods i.

Profit maximization yields the following equations:

$$W_t = P_t(1-\alpha)\frac{y_t}{L_t}, \tag{2.2}$$

$$\frac{p_{i,t}}{P_t} = \alpha L_t^{1-\alpha} x_{i,t}^{\alpha-1}. \tag{2.3}$$

(2.3) is the inverse demand function for each intermediate goods firm i.

[9]BilBiie, Ghironi, and Melitz (2008) and Fujiwara (2007) have provided dynamic new Keynesian models with product-variety framework and endogenous entry based on Melitz (2003). However, no endogenous long-run growth occurs in their models.

2.3 Intermediate goods firms

Each intermediate good is produced using one unit of capital. The nominal profit of the intermediate goods firm i is given by

$$\Pi_{i,t} = (p_{i,t} - R_{t-1})x_t,$$

where R_{t-1} is the nominal price of capital.

Because of the temporary patent protection, the "old" intermediate goods, $[0, N_{t-1})$, are competitively supplied, whereas the "new" intermediate goods, which are invented in period t, $(N_{t-1}, N_t]$, are monopolistically supplied. Therefore, the price of the intermediate goods i is derived as

$$p_{i,t} = \begin{cases} R_{t-1}, & \text{for } i \in (0, N_{t-1}], \\ \frac{1}{\alpha}R_{t-1}, & \text{for } i \in (N_{t-1}, N_t]. \end{cases}$$

The monopoly profit earned by the intermediate firm $i \in (N_{t-1}, N_t]$ is $\Pi_t = \frac{1-\alpha}{\alpha}R_{t-1}x_{mt}$. All intermediate goods enter symmetrically into the production of the final goods, i.e., $x_{i,t} = x_{ct}$ for $i \in [0, N_{t-1}]$ and $x_{i,t} = x_{mt}$ for $i \in (N_{t-1}, N_t]$. By using (2.3), we can easily show $x_{ct} = \alpha^{-\frac{1}{1-\alpha}}x_{mt}$.

2.4 R&D

A new variety of intermediate goods is invented by allocating $1/\eta$ units of capital for R&D activities. Each inventor enjoys a one-period monopoly and earns a profit of Π_t. Therefore, in equilibrium, the following free-entry condition must be satisfied:

$$\Pi_t \leq R_{t-1}/\eta, \quad \text{with an equality whenever } N_t > N_{t-1}.$$

The breakeven point of x_{mt} is derived as

$$\bar{x}_{mt} \equiv \frac{\alpha}{1-\alpha}\eta^{-1}. \tag{2.4}$$

Finally, the capital market clears when

$$k_{t-1} = (x_{mt} + \eta^{-1})(N_t - N_{t-1}) + x_{ct}N_{t-1}, \tag{2.5}$$

where k_{t-1} denotes the amount of capital accumulated in period $t-1$ and available in period t. All capital is depreciated in one period. The available capital is utilized by R&D, producing monopolistic intermediate goods and competitive intermediate goods, as shown on the right-hand side of (2.5).

2.5 Households

Household j possesses nominal money balances, $M_{j,t+1}$, and the capital stock, $P_t k_{j,t}$. The capital stock $P_t k_{j,t}$ yields returns at rate R_t. Thus, the budget constraint in nominal terms of household j is given by

$$A_{j,t} = P_t k_{j,t} + M_{j,t+1} = M_{j,t} + R_{t-1} P_{t-1} k_{j,t-1} + W_{j,t} L_{j,t} - P_t c_{j,t} + P_t \tau_t,$$

where $A_{j,t}$ represents the nominal assets to household j, $L_{j,t}$ represents labor supplied elastically by household j, and $c_{j,t}$ represents consumption of household j. $P_t \tau_t$ is nominal transfer income from the financial authorities in a lump-sum fashion. Clearing the final goods market requires

$$y_t = k_t + c_t, \quad \text{where } c_t \equiv \int_0^1 c_{j,t} dj. \tag{2.6}$$

We can rewrite the budget constraint in real terms as

$$a_{j,t} = r_{t-1} a_{j,t-1} - (R_{t-1} - 1) m_{j,t} + \frac{W_{j,t}}{P_t} L_{j,t} - c_{j,t} + \tau_t,$$

where $\pi_t = (P_t/P_{t-1}) - 1$ represents the inflation rate, $r_{t-1} \equiv R_{t-1}/(1 + \pi_t)$ represents the real interest rate, $m_{j,t} \equiv M_{j,t}/P_t$ represents real money balances, and $a_{j,t} \equiv A_{j,t}/P_t$ represents the stock of assets in real terms.

Household j obtains utility from consumption, $c_{j,t}$, and real money balances $M_{j,t+1}/p_t$, and it encounters disutility from the labor supply, $L_{j,t}$, and wage negotiations.[10] Therefore, the instantaneous utility function of household j is given by

$$u\left(c_{j,t}, \frac{M_{j,t+1}}{p_t}, L_{j,t}, \omega_{j,t}\right) = \log c_{j,t} + \delta_m \log \frac{M_{j,t+1}}{p_t} - \delta_L \frac{L_{j,t}^{1+\psi}}{1+\psi} - \frac{\gamma}{2}\omega_{j,t}^2,$$

where $\psi > 0$ is the elasticity of the marginal disutility of labor supply.[11] $\gamma > 0$ denotes the scale of the nominal wage adjustment cost from wage negotiations and $\omega_{j,t} \equiv (W_{j,t}/W_{j,t-1}) - 1$. If $\gamma = 0$, the nominal wage is

[10] We specify the adjustment cost function as a quadratic expression following Rotemberg (1982). The adjustment cost can be defined as $\gamma\frac{(\omega_{j,t}-\omega^*)^2}{2}$ instead of $\gamma\frac{\omega_{j,t}^2}{2}$, where ω^* is the steady-state value of $\omega_{j,t}$. If we choose such an expression, wage stickiness will vanish in the long run and the natural rate hypothesis will be valid.

[11] In this study, we assume the so-called cash-when-I'm-done (CWID) timing, which supposes that the money balances held by a household at the end of period t (beginning of period $t+1$) enter the utility function in period t.

flexible; however, if $\gamma > 0$, the nominal wage is sticky. $\delta_m(> 0)$ and $\delta_L(> 0)$ are scale parameters.

Summarizing the above, household j faces the following dynamical optimization problem:

$$\max_{c_{j,t},m_{j,t},\omega_{j,t}} \sum_{t=0}^{\infty} \beta^t \left[\log c_{j,t} + \delta_m \log \frac{M_{j,t+1}}{P_t} - \delta_L \frac{L_{j,t}^{1+\psi}}{1+\psi} - \frac{\gamma}{2}\omega_{j,t}^2 \right],$$

$$\begin{aligned} \text{subject to} \quad & a_{j,t} = \tau_t + r_{t-1}a_{j,t-1} - (R_{t-1}-1)m_{j,t} + \frac{W_{j,t}}{P_t}L_{j,t} - c_{j,t}, \\ & W_{j,t} = (1+\omega_{j,t})W_{j,t-1}, \\ & L_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{-\frac{1}{1-\zeta}} L_t, \end{aligned} \tag{2.7}$$

where $\beta \in (0, 1)$ is the discount factor. Since all households behave symmetrically according to the same equations, $W_{j,t} = W_t$, $c_{j,t} = c_t$, $L_{j,t} = L_t$, and $m_{j,t} = m_t$ hold. When $\gamma > 0$, the solution to the optimization problem above is characterized by the Euler equations and the wage versions of the NKPC as follows:

$$\frac{c_{t+1}}{c_t} = \beta r_t, \tag{2.8}$$

$$\delta_m \frac{c_t}{m_t} = \beta(R_{t-1} - 1), \tag{2.9}$$

$$\Omega_{t+1} = \frac{1}{\beta}\Omega_t + \frac{\zeta}{1-\zeta} L_t \frac{w_t}{c_t} - \delta_L L_t^{1+\psi} \frac{1}{1-\zeta}, \tag{2.10}$$

where $m_t \equiv \int_0^1 m_{j,t}dj$ is real money balances for the entire economy, and $\Omega_t \equiv \beta\gamma\omega_t(1 + \omega_t)$. The transversality condition for the households is given by

$$\lim_{T\to\infty} \beta^T \frac{a_{T+1}}{c_T} = 0. \tag{2.11}$$

2.6 Money growth

We assume that financial authorities expand money supply M at a constant rate of $\theta \geq 0$; that is, the monetary policy is given by $(M_{t+1}/M_t) - 1 = \theta$. Therefore, we obtain the following equation:

$$\frac{m_{t+1}}{m_t} = \frac{1+\theta}{1+\pi_{t+1}}. \tag{2.12}$$

All seigniorage is transferred to households; that is, $P_t\tau_t = M_{t+1} - M_t$ holds.

2.7 Equilibrium

By using (2.5), we obtain the following equation:

$$\frac{N_t - N_{t-1}}{N_{t-1}} = \max\{0, \alpha^{-\frac{\alpha}{1-\alpha}}(\kappa_{t-1} - 1)\}, \tag{2.13}$$

where κ_{t-1} is defined as

$$\kappa_{t-1} \equiv \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)\eta \frac{k_{t-1}}{N_{t-1}}.$$

The positive amount of capital is allocated for R&D and technological progress occurs if and only if $\kappa_{t-1} > 1$; that is, the economy has a sufficient stock of capital relative to its technological level.

By using (2.1), (2.4), (2.13), and $x_{ct} = \alpha^{-\frac{1}{1-\alpha}} x_{mt}$, we obtain the total output as

$$\frac{y_t}{k_{t-1}} = \ell_t^{1-\alpha} \xi(\kappa_{t-1})^{-(1-\alpha)}, \tag{2.14}$$

where $\xi(\kappa) \equiv \min\{1, \kappa\}$ and $\ell_t \equiv \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)\eta L_t$.

3 Equilibrium paths

3.1 Law of motion

When the nominal wage is sticky ($\gamma > 0$), the equilibrium path is characterized by the transversality condition (2.11) and the following equations:[12]

$$R_t = \frac{1+\theta}{1+\theta-\beta(R_{t-1}-1)}, \tag{3.1}$$

$$\kappa_t = \begin{cases} (1-\alpha\beta\chi_{t-1})\frac{r_{t-1}}{\alpha}\kappa_{t-1}, & \text{if } \kappa_{t-1} \leq 1, \\ \frac{1-\alpha\beta\chi_{t-1}}{1+\alpha^{-\frac{\alpha}{1-\alpha}}(\kappa_{t-1}-1)} \frac{r_{t-1}}{\alpha}\kappa_{t-1}, & \text{if } \kappa_{t-1} > 1, \end{cases} \tag{3.2}$$

$$\chi_t = \frac{\alpha\beta\chi_{t-1}}{1-\alpha\beta\chi_{t-1}}, \tag{3.3}$$

$$\Omega(\omega_{t+1}) = \frac{1}{\beta}\Omega(\omega_t) + \frac{\zeta(1-\alpha)}{(1-\zeta)\alpha\beta}\frac{1}{\chi_{t-1}} - \Gamma_2 r_{t-1}^{\frac{1+\psi}{1-\alpha}} \xi(\kappa_{t-1})^{1+\psi}, \tag{3.4}$$

$$r_t = \left[\frac{R_t}{1+\omega_{t+1}} \frac{1-\alpha\beta\chi_{t-1}}{\alpha} \frac{\xi(\kappa_{t-1})}{\xi(\kappa_t)}\right]^{1-\alpha} r_{t-1}, \tag{3.5}$$

[12]Full derivations are given in Appendix A.

where $\chi_t \equiv c_t/k_t$, $\Omega(\omega_t) \equiv \beta\gamma\omega_t(1+\omega_t)$, and

$$\Gamma_2 \equiv \frac{\delta_\ell}{(1-\zeta)[\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)\eta]^{1+\psi}} > 0.$$

However, the non-predetermined variables, R and χ, satisfy the following equations for any $t \geq 0$: [13]

$$R_t = R^* \equiv \frac{1+\theta}{\beta}, \quad \chi_t = \chi^* \equiv \frac{1-\alpha\beta}{\alpha\beta}.$$

Therefore, the law of motion, (3.1) through (3.5), can be simplified as follows:

$$\begin{aligned}
\kappa_t &= \begin{cases} \beta r_{t-1}\kappa_{t-1}, & \text{if } \kappa_{t-1} \leq 1, \\ \frac{\beta r_{t-1}}{1+\alpha^{-\frac{\alpha}{1-\alpha}}(\kappa_{t-1}-1)}\kappa_{t-1}, & \text{if } \kappa_{t-1} > 1, \end{cases} \\
\Omega(\omega_{t+1}) &= \frac{1}{\beta}\Omega(\omega_t) + \Gamma_1 - \Gamma_2 r_{t-1}^{\frac{1+\psi}{1-\alpha}}\xi(\kappa_{t-1})^{1+\psi}, \\
r_t &= \left[\frac{1+\theta}{1+\omega_{t+1}}\frac{\xi(\kappa_{t-1})}{\xi(\kappa_t)}\right]^{1-\alpha} r_{t-1},
\end{aligned} \tag{3.6}$$

where

$$\Gamma_1 \equiv \frac{\zeta(1-\alpha)}{(1-\zeta)(1-\alpha\beta)} > 0.$$

3.2 Steady states

3.2.1 Balanced growth path

When the parameters satisfy $\Omega(\theta) > \frac{\beta}{1-\beta}\left[\Gamma_2\left(\frac{1}{\beta}\right)^{\frac{1+\psi}{1-\alpha}} - \Gamma_1\right]$, the law of motion, (3.6), has the following fixed point:[14]

$$\omega^* = \theta, \quad \kappa^* = 1 + \alpha^{\frac{\alpha}{1-\alpha}}(\beta r^* - 1), \quad r^* = \left[\frac{\left(\frac{1}{\beta}-1\right)\Omega(\theta) + \Gamma_1}{\Gamma_2}\right]^{\frac{1-\alpha}{1+\psi}}.$$

[13] See Appendix A.1.

[14] This condition is equivalent to $\beta r^* > 1$.

At this fixed point, y, N, c, and k continue to grow at a constant rate, $g^* \equiv \beta r^* - 1 > 0$. We shall define this steady state as the *balanced growth path* (BGP).

The inflation rate along the BGP is given by

$$1 + \pi^* = \frac{R^*}{r^*} = \frac{1+\theta}{\beta r^*} = \frac{1+\theta}{1+g^*}.$$

The amount of employment is $L^* = \frac{(r^*)^{\frac{1}{1-\alpha}}}{(1-\alpha)\eta\alpha^{\frac{1+\alpha}{1-\alpha}}}$.

3.2.2 No-growth steady state

In contrast, when $\Omega(\theta) \leq \frac{\beta}{1-\beta}\left[\Gamma_2\left(\frac{1}{\beta}\right)^{\frac{1+\psi}{1-\alpha}} - \Gamma_1\right]$ holds, the law of motion, (3.6), has the following fixed point:[15]

$$\omega^0 = \theta, \quad \kappa^0 = \left[\left(1 - \frac{1}{\beta}\right)\Omega(\theta) - \Gamma_1 + \Gamma_2\left(\frac{1}{\beta}\right)^{\frac{1+\psi}{1-\alpha}}\right]^{\frac{1}{1+\psi}}, \quad r^0 = \frac{1}{\beta}.$$

At this fixed point, R&D never occurs, and the economy does not grow. We shall refer to the fixed point, $(\kappa^0, r^0, \omega^0)$, as the *no-growth steady state.* The inflation rate at the no-growth steady state is given by $\pi = \theta$, and the amount of employment is given by $L^0 = \frac{\kappa^0}{\beta^{\frac{1}{1-\alpha}}\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)\eta}$.

Because we assume that the money growth rate, θ, is non-negative, $\Omega(\theta) = \beta\gamma\theta(1+\theta) > 0$ and $\Omega'(\theta) = \beta\gamma(2\theta+1) > 0$ hold. Therefore, we can summarize the above results in the following way:

Proposition 1 Let the rate of money growth, θ, be non-negative.

(a) When $\bar{\Gamma} \equiv \Gamma_2\left(\frac{1}{\beta}\right)^{\frac{1+\psi}{1-\alpha}} \leq \Gamma_1$ holds, the BGP, $(\kappa^*, \omega^*, r^*)$, uniquely exists for any positive values of θ.

(b) Let $\bar{\Gamma} > \Gamma_1$ hold. The BGP, $(\kappa^*, \omega^*, r^*)$, uniquely exists for $\theta > \theta_1$, whereas for $\theta \leq \theta_1$, the BGP does not exist, and the no-growth steady state, $(\kappa^0, \omega^0, r^0)$, is a unique steady state. θ_1 is the positive root of the following quadratic equation:

$$\Omega(\theta_1) - \frac{\beta}{1-\beta}\left(\bar{\Gamma} - \Gamma_1\right) = 0, \tag{3.7}$$

[15]This condition is derived from $\kappa^0 \leq 1$.

which uniquely exists as long as $\bar{\Gamma} > \Gamma_1$.

Proposition 1 establishes that the economy has a BGP for sufficiently high rates of money growth. Once the equilibrium path reaches the BGP, the economy will be able to sustain long-run positive growth. In contrast, when $\bar{\Gamma} > \Gamma_1$ holds, for low rates of money growth, the BGP does not exist, and the economy cannot sustain growth. In such case, it is trapped in a no-growth steady state in the long run as shown in the next section.

When the BGP exists, the following proposition can be verified.

Proposition 2 Let $\theta > \max\{\theta_1, 0\}$ hold; that is, a unique BGP exists. In response to a permanent increase in the money growth rate, θ, the economy experiences greater employment and faster economic growth along the BGP.[16]

It is easy to prove this proposition by using $\partial L^*/\partial r^* > 0$, $\partial g^*/\partial r^* > 0$ and $\partial r^*/\partial \theta > 0$. In this model, nominal wage stickiness remains at the steady state, and money is not superneutral, even in the long run. Faster money growth causes greater employment and faster economic growth along the BGP.[17].

4 Dynamics

4.1 Determinacy of no-growth steady states

With regard to local determinacy of the no-growth steady state, we can verify the following proposition.

Proposition 3 The no-growth steady state is locally indeterminate if it exists.

proof. See Appendix B. □

The trajectories converge toward the no-growth steady state for the initial conditions with κ_0 that belong to the neighborhoods within the no-growth steady state. However, the equilibrium paths which converge toward

[16]It is more realistic to assume the upper limit of labor supply, as in Inoue, Shinagawa, and Tsuzuki (2011). This study focuses on the situation in which employment does not reach the upper limit of labor supply.

[17]Note that even if the financial authorities add 1% to the money growth rate, the rise of long-run inflation rate is smaller than 1% because of the rise of the long-run growth rate, g^*.

the steady state exist continuously. Our model has no mechanism to choose between them, and thus the equilibrium path is indeterminate.

4.2 Determinacy of balanced growth paths

Local determinacy property of the BGP is investigated in the following way.

Proposition 4 Let $\bar{\bar{\Gamma}} \equiv \Gamma_2 \left(\frac{\alpha^{-\frac{\alpha}{1-\alpha}} - 1}{\beta} \right)^{\frac{1+\psi}{1-\alpha}} > \max\{\bar{\Gamma}, \Gamma_1\}$ hold.[18] For $\theta \in (\max\{0, \theta_1\}, \theta_2)$, the BGP is locally determinate. In contrast, for $\theta > \theta_2$, the BGP is locally indeterminate.[19] θ_2 is a root of the following quadratic equation:

$$\Omega(\theta_2) - \frac{\beta}{1-\beta}\left(\bar{\bar{\Gamma}} - \Gamma_1\right) = 0,$$

which uniquely exists and is larger than $\max\{0, \theta_1\}$.

proof. See Appendix C. □

Proposition 4 establishes that the money growth rate influences not only economic growth but also the determinacy property of the BGP. If the condition of Proposition 4 is satisfied, adjusting the money growth rate to the appropriate interval makes the determinate BGP possible.[20] However, we should note that faster money growth brings a higher balanced growth rate, whereas it makes the BGP indeterminate and the economy volatile. In other words, policy-makers may face a trade-off between implementing growth enhancing policies and economic stabilization policies.[21]

These effects on money growth are purely attributed to nominal wage stickiness. A small value of γ diminishes the impact of money growth on economic growth and determinacy property. In a flexible-price economy,

[18]The conditions $\bar{\bar{\Gamma}} > \bar{\Gamma}$ is necessary and sufficient to hold $\theta_2 > \theta_1$. The condition $\bar{\bar{\Gamma}} > \Gamma_1$ is necessary and sufficient to hold $\theta_2 > 0$. $\bar{\bar{\Gamma}} > \bar{\Gamma}$ is satisfied if and only if $\alpha > 1/2$, which is an adequate value. $\bar{\bar{\Gamma}} > \Gamma_1$ is more likely to satisfy for smaller values of η, β, and ζ, and larger values of δ_ℓ. When $\bar{\bar{\Gamma}} > \max\{\bar{\Gamma}, \Gamma_1\}$ does not hold, the BGP is locally indeterminate if it exists.

[19]When $\theta > \theta_2$, $\beta r^* > \alpha^{-\frac{\alpha}{1-\alpha}} - 1$ holds.

[20]Matsuyama (1990) have shown the opposite results; that is, indeterminacy is more likely to arise for low rates of money growth.

[21]The efficient rate of money growth is defined as the money growth rate that maximizes households' utility along the BGP. For the plausible range of parameter values, we can numerically verify that both cases in which the BGP is determinate or indeterminate may arise when the financial authorities apply the efficient rate of money growth.

a change in the money growth rate has no effect on economic growth and determinacy property.

5 Conclusions

This study has developed an R&D-based endogenous growth model by introducing exogenous money growth and nominal wage stickiness and investigated how money growth affects long-run economic growth and determinacy property of the steady state. In our model, money is not superneutral in the long run, and its growth has influences on both long-run growth rates and determinacy of the steady states.

When the money growth rate is sufficiently high, a unique balanced growth path exists, along which the economy can continue to grow in the long run based on sustained R&D. Furthermore, faster money growth results in faster balanced growth. In contrast, under some restricted parameters, when the money growth rate is sufficiently low, balanced growth path does not exist, and the economy is trapped in a no-growth steady state.

We analyzed the local determinacy of each steady state. The no-growth steady state is locally indeterminate without depending on money growth rate as long as it exists. On the other hand, the determinacy of the balanced growth path depends on the money growth rate. For low rates of money growth, the balanced growth path is locally determinate; however, for high rates of money growth, it becomes locally indeterminate. Summarizing the above results, we conclude that a policy trade-off may exist between growth promotion and economic stabilization.

A Derivation of the law of motion

Derivation of (3.1) Combining (2.8), (2.9), and (2.12) gives

$$\frac{R_t - 1}{R_{t-1} - 1} = \beta \frac{R_t}{1+\theta},$$

which is equivalent to (3.1).

Derivation of (3.2) In equilibrium, $P_t y_t = R_{t-1} P_{t-1} k_{t-1} + W_t L_t$ holds. Combining with (2.2), we obtain

$$\frac{y_t}{k_{t-1}} = \frac{R_{t-1}}{\alpha(1+\pi_t)} = \frac{r_{t-1}}{\alpha}. \tag{A.1}$$

Substituting (2.8) and (A.1) into the clearing condition of the final goods market (2.6) yields

$$k_t = \frac{r_{t-1}}{\alpha}(1 - \alpha\beta\chi_{t-1})k_{t-1}. \tag{A.2}$$

Multiplying both sides by $\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)\eta\frac{1}{N_t}$ and using (2.13) yield (3.2).

Derivation of (3.3) Dividing both sides of the Euler equation (2.8) by k_t yields

$$\frac{c_t}{k_t} = \beta r_{t-1}\frac{c_{t-1}}{k_{t-1}}\frac{k_{t-1}}{k_t}.$$

Substituting (A.2) into the above equation, we obtain (3.3).

Derivation of (3.4) Substituting (2.2), (2.8), and (A.1) into NKPC (2.10) yields

$$\Omega_{t+1} = \frac{1}{\beta}\Omega_t + \frac{\zeta}{1-\zeta}\frac{1-\alpha}{\alpha\beta\chi_{t-1}} - \frac{\delta_L}{(1-\zeta)[\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)\eta]^{1+\psi}}\ell_t^{1+\psi}. \tag{A.3}$$

From (2.14) and (A.1), we get

$$\ell_t = \left(\frac{r_{t-1}}{\alpha}\right)^{\frac{1}{1-\alpha}}\xi(\kappa_{t-1}). \tag{A.4}$$

Substituting (A.4) into (A.3) gives (3.4).

Derivation of (3.5) From (2.2), (A.1), and (A.2),[22]

$$1 + \omega_{t+1} = \frac{W_{t+1}}{W_t} = \frac{L_t}{L_{t+1}}R_t\frac{1-\alpha\beta\chi_{t-1}}{\alpha}.$$

On the other hand, from (A.4),

$$\frac{L_t}{L_{t+1}} = \frac{\ell_t}{\ell_{t+1}} = \left(\frac{r_t}{r_{t-1}}\right)^{\frac{1}{1-\alpha}}\frac{\xi(\kappa_t)}{\xi(\kappa_{t-1})}.$$

Summarizing the above equations yields (3.5).

[22] $\frac{y_{t+1}}{y_t} = \frac{r_t k_t}{r_{t-1}k_{t-1}} = \frac{1+\pi_t}{1+\pi_{t+1}}\frac{R_t}{R_{t-1}}\frac{k_t}{k_{t-1}}$.

A.1 Simplification of the law of motion

Rewriting (3.1) gives

$$\frac{1}{R_t} - \frac{\beta}{1+\theta} = R_{t-1}\left(\frac{1}{R_{t-1}} - \frac{\beta}{1+\theta}\right).$$

Because R_{t-1} is larger than 1, R diverges to infinity if $R_t \neq (1+\theta)/\beta$. Therefore, $R_t = R^* \equiv (1+\theta)/\beta$ must hold for any $t \geq 0$.

Similarly, rewriting (3.3) yields

$$\frac{1}{\chi_t} - \frac{\alpha\beta}{1-\alpha\beta} = \frac{1}{\alpha\beta}\left(\frac{1}{\chi_{t-1}} - \frac{\alpha\beta}{1-\alpha\beta}\right).$$

Because $1/\alpha\beta$ is larger than 1, χ diverges to infinity if $\chi_t \neq (1-\alpha\beta)/\alpha\beta$. Therefore, $\chi_t = \chi^* \equiv (1-\alpha\beta)/\alpha\beta$ must hold for any $t \geq 0$.

B Proof of Proposition 3

To prove local indeterminacy, we linearize the system (3.6) around the fixed point, $(\kappa^0, \omega^0, r^0)$:

$$\begin{bmatrix} \kappa_t - \kappa^0 \\ \omega_{t+1} - \omega^0 \\ r_t - r^0 \end{bmatrix} = \boldsymbol{J}^0 \begin{bmatrix} \kappa_{t-1} - \kappa^0 \\ \omega_t - \omega^0 \\ r_{t-1} - r^0 \end{bmatrix}.$$

$\boldsymbol{J}^0$ is the Jacobian matrix. Let us define the following implicit function:

$$f(\omega_{t+1}, \omega_t, r_{t-1}, \kappa_{t-1}) \equiv \Omega(\omega_{t+1}) - \frac{1}{\beta}\Omega(\omega_t) - \Gamma_1 + \Gamma_2 r_{t-1}^{\frac{1+\psi}{1-\alpha}} \xi(\kappa_{t-1})^{1+\psi}.$$

Applying an implicit function theorem, we obtain

$$\begin{aligned} &\frac{\partial \omega_{t+1}}{\partial \kappa_{t-1}} = -\frac{f_{\kappa_{t-1}}}{f_{\omega_{t+1}}} = -\frac{\Gamma_2 r_{t-1}^{\frac{1+\psi}{1-\alpha}} \xi'(\kappa_{t-1})}{\Omega'(\omega_{t+1})}, \quad \frac{\partial \omega_{t+1}}{\partial \omega_t} = -\frac{f_{\omega_t}}{f_{\omega_{t+1}}} = \frac{\frac{1}{\beta}\Omega'(\omega_t)}{\Omega'(\omega_{t+1})}, \\ &\frac{\partial \omega_{t+1}}{\partial r_{t-1}} = -\frac{f_{r_{t-1}}}{f_{\omega_{t+1}}} = -\frac{\frac{1+\psi}{1-\alpha}\Gamma_2 r_{t-1}^{\frac{1+\psi}{1-\alpha}-1}}{\Omega'(\omega_{t+1})}. \end{aligned} \tag{B.1}$$

By combining (B.1) and $\xi'(\kappa^0) = 1$ for $\kappa^0 < 1$, the Jacobian matrix is derived as

$$\boldsymbol{J}^0 = \begin{bmatrix} 1 & 0 & \beta\kappa^0 \\ j_{21}^0 & 1/\beta & j_{23}^0 \\ -\chi j_{21}^0 & -\chi/\beta & 1 - \chi j_{23}^0 \end{bmatrix},$$

where

$$j_{21}^0 \equiv -\frac{\Gamma_2 \beta^{-\frac{1+\psi}{1-\alpha}}}{\beta\gamma(1+2\theta)} < 0, \quad j_{23}^0 \equiv -\frac{\frac{1+\psi}{1-\alpha}\Gamma_2 \beta^{-\frac{1+\psi}{1-\alpha}+1}}{\beta\gamma(1+2\theta)}, < 0, \quad \chi \equiv \frac{1-\alpha}{(1+\theta)\beta} > 0.$$

The eigenvalues of $\boldsymbol{J}^0$, denoted as $\lambda_n^0, n = \{1, 2, 3\}$, are obtained by solving the following characteristic equation:

$$\begin{aligned} P^0(\lambda^0) \equiv & |\boldsymbol{J}^0 - \lambda^0 \boldsymbol{I}| \\ = & -(\lambda^0)^3 + \left(2 - \chi j_{23}^0 + \frac{1}{\beta}\right)(\lambda^0)^2 \\ & + \left[-1 - \frac{2}{\beta} + \chi(j_{23}^0 - \beta\kappa^0 j_{21}^0)\right]\lambda^0 + \frac{1}{\beta} = 0. \end{aligned}$$

The three-dimensional system, (3.6), has one predetermined variable, κ, and two non-predetermined variables, ω and r. If both roots have a modulus of less than 1, then the no-growth steady state is locally indeterminate. From $\lim_{\lambda\to\infty} P^0(\lambda^0) = -\infty$ and $P(1/\beta) = -[(1-\beta)/\beta^2]\chi j_{23}^0 - \kappa^0 \chi j_{21}^0 > 0$, there is at least one real root that is larger than $1/\beta$. We define this real root as λ_3^0. As for the other two roots, we will consider the following two cases.

Case of complex roots If the characteristic equation, $P^0(\lambda^0) = 0$, has complex roots, $\lambda_1^0 \equiv a + bi$ and $\lambda_2^0 \equiv a - bi$, where a and b are non-negative real numbers, they would satisfy the following equation:

$$\prod_{n=1}^{3} \lambda_n^0 = (a^2 + b^2)\lambda_3^0 = \frac{1}{\beta}.$$

Since λ_3^0 is larger than $1/\beta$, $a^2 + b^2$ is smaller than 1. Therefore, the complex roots have a modulus of less than one, and thus, the no-growth steady state is locally indeterminate.

Case of real roots Some algebra shows that[23]

$$\begin{aligned} (P^0)'(0) &= -1 - \frac{2}{\beta} + \chi(j_{23}^0 - \beta\kappa^0 j_{21}^0) < 0, \\ (P^0)'(1) &= -\chi j_{23}^0 - \chi\beta\kappa^0 j_{21}^0 > 0, \\ \lim_{\lambda^0\to\infty} (P^0)'(\lambda^0) &= -\infty < 0. \end{aligned}$$

[23]Since $\kappa^0 < 1$, $j_{23}^0 - \beta\kappa^0 j_{21}^0 < j_{23}^0 - \beta j_{21}^0 = \beta j_{21}^0(\frac{1+\psi}{1-\alpha} - 1) < 0$.

That is, the cubic function, $P^0(\lambda^0)$, has a local minimum point in $(0,1)$ and a local maximum point in $(1,\infty)$. Taking into account $P(1) = -\chi\beta\kappa^0 j_{21}^0 > 0$, we can verify that if the characteristic equation has three real roots, two of these belong to $(0,1)$. $\square$

C Proof of Proposition 4

Similar to the previous section, we linearize the system (3.6) around the fixed point, $(\kappa^*, \omega^*, r^*)$:

$$\begin{bmatrix} \kappa_t - \kappa^* \\ \omega_{t+1} - \omega^* \\ r_t - r^* \end{bmatrix} = \boldsymbol{J}^* \begin{bmatrix} \kappa_{t-1} - \kappa^* \\ \omega_t - \omega^* \\ r_{t-1} - r^* \end{bmatrix}.$$

Using (B.1) and $\xi'(\kappa^*) = 0$, we obtain the Jacobian matrix as follows:

$$\boldsymbol{J}^* = \begin{bmatrix} -(\alpha^{-\frac{\alpha}{1-\alpha}} - 1)/(\beta r^*) & 0 & \kappa^*/r^* \\ 0 & 1/\beta & j_{23}^* \\ 0 & -\chi r^* & 1 - \chi\beta r^* j_{23}^* \end{bmatrix},$$

where

$$j_{23}^* \equiv -\frac{\frac{1+\psi}{1-\alpha}\Gamma_2(r^*)^{\frac{1+\psi}{1-\alpha}-1}}{\beta\gamma(2\theta+1)} < 0.$$

One of the three eigenvalues of the Jacobian matrix, $\boldsymbol{J}^*$, is given by $\lambda_3^* \equiv -(\alpha^{-\frac{\alpha}{1-\alpha}} - 1)/(\beta r^*)$; the other two eigenvalues are equal to those of the following sub matrix:

$$\hat{\boldsymbol{J}}^* \equiv \begin{bmatrix} 1/\beta & j_{23}^* \\ -\chi r^* & 1 - \chi\beta r^* j_{23}^* \end{bmatrix}.$$

$\operatorname{tr}\hat{\boldsymbol{J}}^*$ and $\det\hat{\boldsymbol{J}}^*$ are derived as

$$\operatorname{tr}\hat{\boldsymbol{J}}^* = \frac{1}{\beta} + 1 - \chi\beta r^* j_{23}^*, \quad \det\hat{\boldsymbol{J}}^* = \frac{1}{\beta}.$$

Because j_{23}^* is negative, $1 < \det\hat{\boldsymbol{J}}^* < \operatorname{tr}\hat{\boldsymbol{J}}^* - 1 = 1/\beta - \chi\beta r^* j_{23}^*$ holds. Therefore, $\hat{\boldsymbol{J}}^*$ has real eigenvalues $\lambda_1^* \in (0,1)$ and $\lambda_2^* \in (1,\infty)$.[24]

The three-dimensional system, (3.6), has one predetermine variable, κ, and two non-predetermined variables, ω and r. Local determinacy of the BGP, $(\kappa^*, \omega^*, r^*)$, depends on the absolute value of λ_3^*. If $\alpha^{-\frac{\alpha}{1-\alpha}} - 1 > \beta r^*$ holds, $\lambda_3^* < -1$ and the BGP is locally determinate. On the other hand, if $\alpha^{-\frac{\alpha}{1-\alpha}} - 1 < \beta r^*$, $\lambda_3^* \in (-1,0)$ and the BGP is locally indeterminate. $\square$

[24] See Azariadis (1993, Chap.6) for further details.

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