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Models and Mathematics:

Pigou and Macroeconomic Modeling in the 1930s and 1940s

Norikazu Takami

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Norikazu Takami[1]

Abstract

The paper investigates the impact of macroeconomic models on the controversy that took place between Arthur Pigou and John Maynard Keynes in the late 1930s. This debate was concluded when one form of model analysis replaced another, specifically, when mathematical analysis replaced verbal logical analysis. In the first instance, Pigou used a simple model to oppose Keynes's assertion on the neutrality of money wage reduction to employment in certain conditions. Pigou (and Keynes too) applied verbal logical analysis to the model to derive his conclusions. Submitting a paper to the journal, Kaldor analyzed Pigou's model in mathematical terms and asserted that Pigou derived inconsistent conclusions from his model. Kaldor's method eventually convinced Pigou, Keynes, and Dennis Robertson (who participated in the debate in correspondence). This study provides a case study to the first category of Mary Morgan's two functions of economic modeling – models as an object to inquire into and models as an object to inquire with.

JEL classifications: B22, B40, E12

I. Introduction

The paper aims to investigate the impact of the growing use of macroeconomic models (as they would later come to be called) on the controversy that took place between Arthur Pigou and John Maynard Keynes in the late 1930s. To this end, I will discuss how models were used and analyzed by the participants of the debate and will highlight how these different approaches to model analysis played out in contributing to the final result of the controversy – an apparent concession on the part of Pigou. The discussion as a whole serves to establish the thesis that this debate was concluded when one form of model analysis replaced another, specifically, when mathematical analysis replaced verbal logical analysis.

In the introductory chapter of *The World in Model* (2012, forthcoming; but available online prior to publication), Mary Morgan summarizes the recent literature on the role of models in the history of economics (e.g. Morrison and Morgan 1999, Boumans 2005). She paints a picture of an increasing reliance upon case-based reasoning using specific models, which has replaced the verbal expression of general economic laws. To illuminate the accompanying epistemological shift, she introduces a distinction between models as objects to enquire into and models as objects to enquire with; that is, a distinction between, on the one hand, the investigation of the behavior of a model itself and, on the other, the use of a model to draw inferences about reality. Morgan argues that, with the increasing dominance of case-based model reasoning since the mid-twentieth century, research activity is no longer concerned with verbal general laws, and has turned to the manipulation of models and the drawing of inferences from the results. Thus, models are more properly to be compared with laboratory experiments than with postulation and proof. Just as laboratory scientists cannot guarantee the applicability of particular experiment results to other phenomena, economists realize that their models are only hypothetical representations of the world.

Morgan's first function of model reasoning – the model as an object to inquire into – is of particular relevance for the current paper. This is because, as I will argue below, the underlying crucial question in the controversy between Keynes and Pigou was not the substantial one of whether or not the behavior of a model corresponds with the real world, but rather a more preliminary issue concerning how the abstract world embodied in a model should behave. Put another way, the debate did not hinge on the significance of a particular experiment on the understanding of some more general phenomena; it turned rather on the right method of conducting an experiment in the first place. What settled the debate was neither theoretical insight (in the sense of a new insight not hitherto noticed) nor empirical accuracy (in the sense of correspondence with the world perceived statistically or some other

way). What settled the debate was rather the establishment of agreement on how to analyze the model at hand.

The surface issue in the controversy between Pigou and Keynes concerned the question of whether changes in money wages can mitigate unemployment in times of recession. The background to this particular issue can be traced back well before the great crash of 1929. As a loyal disciple of Marshall, Pigou was committed to the flexibility of money wages. After WWI, however, this commitment came into tension with an observed rigidity of money wages in the face of an unemployment rate that stood at around 10% in Britain throughout the 1920s. Commenting on this phenomena (Pigou 1927), Pigou did not mention the General Strike of 1926, although there can be little doubt that this unprecedented event in trade union activism would have made a strong impression on all economists of the day. Pigou did, however, attribute the intensified resistance on the side of the workers to the strengthening of their bargaining power as a consequence of social legislation such as state unemployment insurance, which was expanded after the First World War. Nevertheless, he did not call for a return to pre-war economic and political conditions in order to recover a more flexible labor market. Thus, when interviewed by the Macmillan Committee in 1930, the terms of reference of which was the analysis of and policy advice in the face of the developing economic depression, he insisted that "I would not be prepared to scrap unemployment insurance" (Pigou 1931, 49). Pigou's point was that money wage rigidity in the twenties was a new and unexpected phenomenon. Indeed, in the early twenties he had evidently not anticipated such an anomaly, endorsing the return to the gold standard thus: "I do not deny… that dear money is unpleasant or that it adversely affects the immediate interest of the Government as a borrower, of industries and businesses, and even of wage-earners. But in the situation in which we are, these things must be endured" (Pigou 1920, 10). For Pigou, the high unemployment of the 1920s was not caused by the monetary factors he had earlier

anticipated; it was rather the consequence of a money wage rigidity that thwarted the expected course of long-run conversion to low unemployment.[2] Subsequently, Pigou was particularly concerned with the specific issue of how money wage adjustment could have contained unemployment in the 1920s. *The Theory of Unemployment* (1933) is the most notable example of this new interest. Here he applied sophisticated mathematical formula to make a quantitative estimate for the elasticity of aggregate labor demand. He put it substantially above unity, suggesting that money wage reductions would have been highly effective in the twenties.

Keynes's castigation of Pigou in *The General Theory* (1936) should be read in this context. In a letter he wrote to Dennis Robertson after reading Pigou's book, Keynes criticized the latter's single-mindedness. Pigou, he wrote, "arbitrarily takes two items, namely employment and real wages, out of a complex, but presumably determinate system and then treats them, without proof or enquiry, as being analytic functions of one another. But they are not independent variables."[3] Keynes's discussion of money wages in *The General Theory* reflected his opposition to Pigou's treatment. In Chapter 16, Keynes presented the modern economic equivalent to "the fate of Midas." In a competitive monetary economy interest rates cannot fall below a certain minimum level and the capital stock multiplies rapidly to the extent that the marginal efficiency of capital remains constantly below the level of the interest rate. Consequently, firms find the current level of employment unprofitable and cut down the number of workers employed. This downward movement continues until peoples' incomes decline sufficiently to bring savings to zero. This was not merely a theoretical possibility for Keynes. He viewed the post-war economic stagnation as partly a reflection of this long-term downward trend.[4] A corresponding argument was also made in Chapter 19 of *The General Theory*, where Keynes discusses the effects of money wage changes upon employment and proposes what is now called the "Keynes effect," that is, the decreased liquidity preference

and lower interest rate that follow from a decline in money wages. Keynes here argued further that this is the only channel through which money wage adjustment can affect employment, thus implying that a money wage reduction is neutral to the level of employment under the condition where the interest rate is already on the lower bound, or the liquidity trap. With this argument, Keynes made a serious challenge to the view that the competitive economy would automatically achieve full employment.[5] The following year Pigou struck back with a new theoretical argument, which is the starting point of the main narrative of this paper.

In the remainder of this paper I will proceed as follows. Section 2 will discuss Pigou's 1937 *Economic Journal* article, focusing particularly on his macroeconomic model. Section 3 will analyze critical comment drawn from three primary directions: Keynes (as an opponent in the debate), Nicholas Kaldor (as an outside critic with a sharper analytical skill), and Robertson (as a neutral observer with regard to this particular theoretical point). I will point out the difference between Keynes's initial response and Kaldor's analytical criticism, noting that it was the latter which came to dominate the views of all four participants in the debate (including even Pigou). Section 4 will turn to the interesting fact that, subsequent to this debate Pigou came to rely extensively on multiple equation models, and I will offer some evidence that substantiates this apparent link between the earlier debate and his later research practice in macroeconomics. Section 5 will present the so-called 'Pigou effect' as arising out of this earlier debate. In conclusion, I will highlight the importance of mathematical model analysis for the assessment of the entire episode.

II. Macroeconomic Model: Pigou's 1937 Article

While not explicitly mentioning Keynes,[6] one of the claims advanced in Pigou's 1937 article was obviously intended to counter Keynes's challenge of the effectiveness of money wage changes in recessions when not accompanied by interest rate reductions. Pigou insisted that "a money wage cut is not simply a piece of ritual that enables the real cause of employment expansion – a fall in the rate of money interest to take effect" (Pigou 1937, 411). This claim was supported by what he called "a simplified model," explaining that "no advance in this field can be made without one [i.e., a model]" (Pigou 1937, 406). Only a macroeconomic model, Pigou was suggesting, allowed the economist to 'observe' the interaction between the interest rate and money wages in a controllable system that represented the relevant part of an economy. But Pigou was careful to note that the "results reached in this article are, of course, only demonstrated for the model in relation to which I have discussed them, not for the actual world" (Pigou 1937, 422). In other words, he recognized that a model is a self-contained object and that there is no guarantee of its correspondence with the real world.

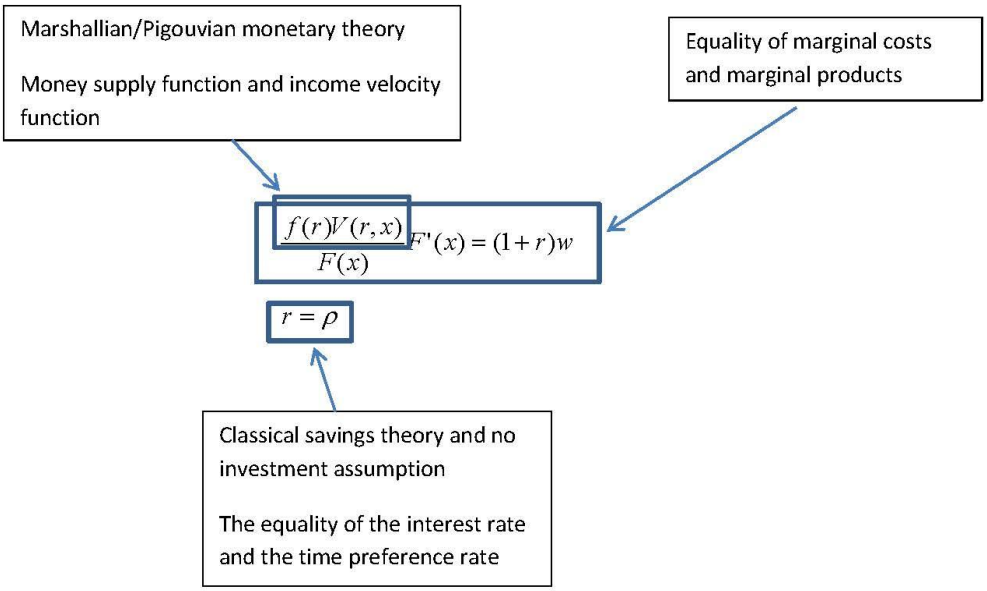
Pigou in this 1937 article comes across as experienced and confident in the use of models. So far as I am aware, this was only his second published use of the word 'model', the first appearing in the preface to *The Theory of Unemployment*. In the 1937 article, however, the idea of a model was extended to include the interest rate among the variables of the model to examine the interaction of the interest rate, money wages, and employment. In this sense, this *Economic Journal* article constituted a departure from Pigou's earlier theoretical repertoire. What historical circumstances might have contributed to his taking this step?

Robertson's letter to Keynes of December 1936 may well be relevant here, because it suggests that Pigou was aware of other early macroeconomic models. Robertson tells Keynes that he has discussed Roy Harrod's "*Econometrica* article"[7] with Pigou. Harrod's January 1937 article in *Econometrica* is now considered one of the earliest attempts to impose a simple mathematical formula on Keynes's argument in *The General Theory*.[8] We may

therefore surmise that Pigou was aware of the contemporary trend towards models within economics and recognized the need to catch up with it. Nevertheless, it should be noted that the model set out in Pigou's 1937 article consisted of equations that are not akin to the early versions of the IS-LM model.

What did Pigou's 1937 model consist of? As with the generality of economic models (as pointed out by Marcel Boumans, 2005), Pigou's model amounted to a collection of ideas culled from different sources, held together by mathematical formula (Boumans likens models to cake, which blends initially separate ingredients into a form in which these materials are no longer individually distinguishable). Three distinct theoretical elements can be identified in Pigou's model. The first theory is straightforward marginalism. The first equation of the two-equation system signifies the equality of marginal costs, $(1 + \mathrm{r})\mathrm{w}$; where r is the interest rate and w money wages, and the money value of marginal products, $\frac{I}{F(x)} F'(x)$; where I is money income, F(x) real income, and x employment. The second idea blended into this model is the Marshallian/Pigouvian monetary theory as presented in Pigou's (1917) account of the Cambridge monetary theory. The interest rate is here a barometer of overall economic activity: the higher the rate, the more active the economy. In addition, money supply depends on the willingness of the banking system to lend money, and the higher the interest rate, the more willing they are to do so. Therefore, money income is defined as positively correlated with the interest rate; namely, in the equation $\mathrm{I} = \mathrm{f(r)V(r,x)}$, where f is the money supply function and V the income velocity function, both $\frac{df}{dr}$ and $\frac{\partial V}{\partial r}$ are positive.[9] The third theory, embodied in the second equation of the model, was a classical savings theory, in which the time preference rate determines the level of interest rate, such that $\mathrm{r} = \rho$ —ρ is the time preference rate. Added to this is the assumption that there is no new investment taking place. In sum, this model as a whole constituted a new method for Pigou in

that it involved the interest rate among its variables but, on the other hand, the theories behind it were very conventional and did not contain the early IS-LM model builders' formulations.



Although the above two equations were not as clearly stated in Pigou's article as I have presented them above, there is no doubt that he discussed these equations as part of his argument, and that he did so with verbal logic rather than with mathematics. Pigou went out of his way to use argument by contradiction in order to derive the conclusion that money wage reductions will increase employment. If money wages are cut and employment does not change, he argued, the latter will leave money income unchanged; however, the same level of money income will keep prices at the old level; therefore, real wages must decline and employment must increase.[10] What was important for him was that this conclusion does not require an interest rate reduction. However, Pigou seems to have supposed that the model cannot by itself determine the level of the interest rate: "What will happen to the rate of interest and the volume of money income depend, of course, on the detailed circumstances" (Pigou 1937, 410). Therefore, he relied on a separate argument, claiming that the interest rate

will go through a complex movement after money wages are cut.[11] Pigou set out a model and used it, but he did so with intricate verbal logic and he abandoned it when he turned to what he really wanted to argue.

One aspect of Pigou's 1937 article is particularly crucial to my interpretation of his controversy with Keynes, and some scholars, such as Gerhard Ambrosi, offer a different interpretation of it. The key question is whether Pigou supposed the time preference rate to be a constant or a variable dependent on real income (or employment, monotonically positively correlates with real income in his model). Ambrosi (2003) argues that Pigou assumed that the time preference rate was a constant; an interpretation that is indeed tempting because it renders Pigou's conclusion as to the neutrality of the interest rate to employment compatible with his model. Nevertheless, certain sentences in Pigou's article suggest quite the opposite; for example: "But neither, so long as employment, and, therefore, real income is unaltered, can ρ be any different" (Pigou 1937, 409, where $\rho =$ time preference rate); which could be naturally translated thus: if real income is altered, the time preference rate will be different. In addition, further evidence suggests Pigou would have considered the time preference rate as a variable dependent on real income instead of a constant. To begin with, he had argued in his earlier *Economics of Stationary States* (1935) that an increase in real income would cause savings to rise – in fact, he made a stronger argument than this, that is, that such an increase would cause the proportion of savings to income to increase, and not only the absolute amount of savings. Furthermore, in a letter sent to Keynes after the publication of the 1937 *Economic Journal* article, Pigou explained that in the article: "I don't assume or make any assumption which implies that money income is fixed" (Moggridge 1973b, 256). In his model, money income depends on the interest rate, and the interest rate in turn is determined by the time preference rate; a constant time preference would be exactly an assumption that implies fixed money income. Finally, Pigou had no scruple in admitting, in his eventual

concession in the 1938 article, that savings depend on real income, and he did so without much discussion – as we will see below.[12] Therefore, there appears to be more reasonable grounds for the interpretation that Pigou supposed savings to be dependent on real income. When we return to this issue in the discussion of Robertson's assessment of Pigou's article it will be more cogent to suppose that Pigou did not realize the behavior of his model crucially turned on the assumption on the time preference rate.

In any case, and as we shall now see, Nicholas Kaldor quickly intervened to show that there was an incompatibility between an assumption that Kaldor supposed Pigou had made – savings being dependent on real income – and Pigou's conclusion. Kaldor's analytical argument successfully changed the way the participants of this debate perceived Pigou's argument.

III. Responses by Keynes, Kaldor, and Robertson

Keynes's and Kaldor's separate articles appeared in response to Pigou's 1937 article in the next issue of the *Economic Journal.* The story behind these publications has been well documented by Moggridge (1973b, 234-268). Keynes had read Pigou's paper and prepared a response to it already by the time he wrote to his assistant editor at the *Economic Journal,* Austin Robinson, on August 7, 1937. At this point, Keynes's criticism concerned the characteristics of the money supply (the quantity of money) function in Pigou's model. He asserted that Pigou at one point stated that money supply was a function of the interest rate only but at another point abandoned this idea and assumed money supply to be dependent on money income only.[13] Referring to Bertrand Russell's dictum that "from two inconsistent propositions any proposition can be made to follow" (Moggridge 1973b, 235), Keynes described Pigou's conclusion as logically derived from two inconsistent assumptions and

hence invalid. Just as had Pigou in his 1937 article, Keynes relied on verbal logic to interpret the model.

Pigou's article had appeared in the September issue of the *Economic Journal*, and by the end of that month Kaldor had submitted his criticism to Keynes in his capacity of editor. Kaldor, then a lecturer at the London School of Economics and aware of Hicks's IS-LM diagram,[14] centered his criticism on the saving function of Pigou's model. Kaldor turned Pigou's second equation, $r = \rho$ (r is the interest rate, ρ is the time preference rate), which he called the "old-fashioned savings-function in disguise," into what, from a macroeconomic perspective, was a more straightforward form: $S = \psi(r,x) = 0$ (x is employment, monotonically positively correlated with real income). Kaldor then identified the conditions required for Pigou's conclusion that a money wage reduction involves an increase in employment without accompanying a reduction in the interest rate; and one such condition was that $\frac{\partial S}{\partial x}$ be zero, i.e. savings remain constant even with a change of real income. On the other hand, if $\frac{\partial S}{\partial x}$ is positive (i.e. savings increase as real income rises or vice versa), then Pigou's conclusion no longer holds. Kaldor took it that Pigou was assuming the second case, thus claiming that Pigou's conclusion was incompatible with his assumptions. A money wage cut leads to an increase in employment only in so far as it entails an interest rate reduction; a money wage cut, Kaldor wrote, "is indeed such a piece of ritual" (Kaldor 1937, 753).

Kaldor's article elicited the immediate approval of two economists, Keynes and Robertson. In the course of the correspondence that followed Kaldor's submission, Keynes told Kaldor that he believed that Pigou was assuming that savings do not depend on real income: "My belief is that the assumption that Pigou is fundamentally making is that the whole of yesterday's income will be spent today. . . [Pigou] is tacitly denying, as you [i.e., Kaldor] point out, that saving is a function of real income" (Moggridge 1973b, 241). Whether he arrived at this

belief as a result of reading Kaldor's article is, of course, not clear; but it should be noted that Keynes had not discussed Pigou's assumptions about savings or the time preference rate in the earlier version of his criticisms. In any case, it is certain that Keynes supported the basic thrust of Kaldor's criticism.

After Keynes had consulted with him, Robertson sent notes to both Keynes and Pigou. In the former note he stated his agreement with Kaldor's claim that the interest rate must be smaller when money wages are lower and employment higher in the new position if savings are partly a function of real income. Robertson also noted that the assumption that savings are partly a positive function of real income is reasonable and in line with classical views. In his view, Pigou would not deny this assumption, and Pigou probably did not discuss the interest rate in connection with this interaction of savings, the interest rate, and real income: "I think he has not explicitly recognised its [i.e. of savings being a function of real income] consequences in this context" (Moggridge 1973b, 253). Indeed, Pigou told Keynes that his 1937 article had originally contained "a good deal" of separate argument on the interest rate (Moggridge 1973b, 257), and it is therefore likely that Pigou did not originally intend his model to be of primary importance as a means of examining the movement of the interest rate.

The final version of Keynes's article, which appeared in the December issue of the journal, contained the point concerning the saving function but also dealt with the issue related to the money supply function, which had been the main topic of the earlier version. Keynes met with opposition from Kaldor, Robertson, and Pigou who each separately told Keynes that this criticism was based on a misrepresentation of Pigou's argument.[15] Even so, Keynes insisted on maintaining this criticism in his article. The correspondence with Pigou reveals one reason he thought this point was so important. Keynes wrote Pigou, "I am concerned to dispute precisely what you re-affirm in your letter under reply. That is to say, I maintain that, if there is a cut in wages, unemployment being unchanged, there *is* a ground for a change in money

income" (Moggridge 1973b, 257, emphasis in original). In the previous letter, Pigou had reiterated a remark original made in his 1937 article, namely, that "*if* a cut in wages leaves employment unchanged, money income has no ground for change" (Moggridge 1973b, 256, emphasis in original). Obviously, Keynes was criticizing but one step in Pigou's explanation of the working of his entire model behavior; but on the other hand, Kaldor and Robertson, who rejected this criticism as founded on a misunderstanding, were concerned with the behavior of the model as a whole, of which many other step-by-step verbal explanations are possible.[16] In any case, as editor of the journal, Keynes had the final say on which article should be published: his two-page article was published in the December issue, together with Kaldor's nine-page article.[17]

IV. Professor's Retraction and Illumination

Kaldor's paper was analytical and assertive, and highlighted with symbols the inconsistency in Pigou's model. Apparently, this was enough to convince Keynes and Robertson. However, it was not so for Pigou, at least according to his letter to Keynes. After reading Kaldor's article his first response was to prepare a long paper intended to counter the criticism. It was only after David Champernowne, a former student of Keynes at Cambridge and lecturer at LSE at this time, approached Pigou and read his draft that he changed his mind. Richard Kahn in fact wrote to Keynes that he had "been keeping Champ. carefully briefed" on the affair (Moggridge 1973b, 265), and this leads Aslanbeigui and Oakes (2007) to suggest that Champernowne approached Pigou on Richard Kahn's request. An early pioneer of the modeling of *The General Theory*, Champernowne was mathematically inclined and well-disposed toward Keynes's work. His much neglected 1936 article in the *Review of Economic Studies* set out a diagrammatic treatment essentially similar to Hick's IS-LM model. However, as opposed to the single diagram of Hicks's 1937 article, Champernowne set out three

diagrams, representing respectively the labor market, the commodity market (savings and investment), and the money market. In this article, Champernowne also pointed to the divergence between Keynesian and classical theory other than the short-period case, later described as 'the liquidity trap': "in this case the monetary authority will not be able to prevent a constantly falling cost of living and there will necessarily be monetary unemployment constantly. It would be quite inappropriate to make use of the classical analysis in such a case" (Champernowne 1936, 216). Thus, Champernowne was perfectly capable of discerning the problem at the center of the dispute over Pigou's 1937 article, and of expressing it in mathematical terms. Following his approach, Pigou submitted a relatively short article to Keynes, which was published in the March issue of the *Economic Journal*, and in which he acknowledged Champernowne's assistance in helping him to understand Kaldor's article.[18]

It is important that we carefully examine what Pigou wrote in this reply because it is the most likely place to find some justification (implicit or otherwise) of his change of position. To begin with it is notable that here Pigou relied more overtly on mathematics than had Kaldor in his article. Pigou admitted that if employment were to increase, the interest rate needs to fall at the same time. Then, in order to show that a money wage cut involves a fall in the interest rate, Pigou differentiated one equation of the model and derived the sign of the derivative from the derivatives of functions whose signs are known.[19] This extensive use of differential calculus contrasts even with Kaldor's analytical argument in his 1937 article. Pigou thus appears to justify his change of position by showing the transparency of the argument he has found it necessary to yield to.

The practice of so analyzing models with differential calculus would become dominant in Pigou's later book, *Employment and Equilibrium* (1941). Beyond this surface similarity, archival evidence also suggests a link between the controversy in 1937-38 and the 1941 book.

In the Robertson papers archived in Trinity College, Cambridge, UK, several undated letters between Robertson and Pigou are filed with a title sheet "Exchanges between ACP and DHR about 'Employment and Equilibrium'" (Robertson Papers C7/1). One of the topics they dealt with in this exchange was as follows. In the first letter in the bundle, Robertson complained about Pigou's claim that the interest rate is necessarily lower when money wages are lower, and wrote, "I was ready to accept this conclusion for the world of your interchange with Kaldor, in which 'investment' was ruled out: I have a strong resistance to accepting it for a world in which 'investment' is possible." Robertson added that this conclusion would not hold under the additional assumption that investment is partly a function of current consumption, not a sole function of the interest rate, as Pigou assumed in that work. In the second letter in the folder, Pigou admitted this possibility but noted that it might entail unstable equilibrium. Thus, Robertson implied that this book was connected with the exchange with Kaldor in 1937-38; while Pigou was here concerned with the purely mathematical issue of stable equilibrium.

In fact, *Employment and Equilibrium* offers several models based on different assumptions, and the tables in the appendix meticulously show each sign of the derivative. This mechanical method of economic research enabled Pigou to pass on an important part of his work to his assistant: "The tables in the Appendix have been worked out and very carefully checked by Mrs. Glauert" (Pigou 1941, vii). According to Champernowne's letter to a current Pigou scholar (Collard 2002, xxx, n1), Mrs. Glauert was Pigou's typist and had a good command of mathematics. This clearly shows that the mathematical analysis applied in this book could be handled by someone not deeply versed in the economic theory behind it. This confirms an additional virtue of the laboratory experiment metaphor for models: people with different skill sets can work together by performing different parts of the work – in this case, theory and model analysis.

The reviewers of the 1941 work caught the importance of the new method. In his review of the work, Kaldor praised Pigou's method, which he had partly helped him to develop:

> [Pigou's] technique . . . enables anyone who has once mastered it to pass easily from assumptions to results and to reduce differences in results to differences in assumptions; and [it] makes possible such a choice of assumptions that they can easily be judged on empirical grounds. (Kaldor 1941, 459)

The young Samuelson, reviewing the book in the *American Economic Review*, was more concise: "With respect to methodology, it is almost ideal" (Samuelson 1941, 545). These two reviews clearly show the positive opinion of contemporary economic theoreticians. Especially Kaldor's review specified the virtue of Pigou's method, namely, its transparency in the processing from assumptions to conclusions.

What made Pigou change his mind and accept Kaldor's criticism? The evidence discussed in this section appears to suggest that mathematical analysis of his model convinced Pigou of the right model behavior and the connection between money wages, the interest rate, and employment in the model. This view makes Pigou's extensive and almost mechanical use of differential calculus in his later work more understandable because this can be explained by the strong impact Kaldor's paper would have had upon Pigou. Even if the connection between the controversy and Pigou's later work was not as direct as I am inclined to believe, it was certainly true that Pigou was much more certain about the behavior of variables in his model in his 1938 concession article and that Pigou's *Employment and Equilibrium* is within that trajectory of his theoretical development. In any case, Pigou's acceptance of Kaldor's criticism and acknowledgement of Champernowne's help would clearly indicate what reason was behind his decision to retract his earlier conclusion: namely, a reason that he was convinced of their mathematical analysis.

V. The Pigou Effect: Evolution with Presentation

So much for our discussion of the controversy; I now turn to one of the later ramifications of this debate. This ramification concerns the so-called 'Pigou effect', the final destination in Pigou's quest for a theory that approves the effectiveness of money wage flexibility under any circumstances. Here, too, I will note the importance of models, in this case by highlighting unequal effects of different presentations of the same theory.

In one chapter of *Employment and Equilibrium*, Pigou discussed the theory for which Don Patinkin later coined the term 'Pigou effect'. In the setting of this chapter, people save money not only for future consumption but also for the sake of savings itself. The latter motive was referred to as an "amenity"; more specifically, they do so because of the "sense of power, sense of security and so on" (Pigou 1941, 126). Peoples' savings thus depend on this amenity value of savings as well as the time preference rate. Pigou's idea here was that if the asset value in real terms increases in times of price decline, this amenity value of savings will decrease and people become less inclined to save. Therefore, even if the interest rate is already on the lower bound, price declines can still activate self-correction of the economy by stimulating consumption.

In the first edition of *Employment and Equilibrium*, Pigou was rather cautious as to how strong this effect might be. What he described as Keynes's scenario of a "vision of the Day of Judgment" was, he admitted, an alternative possibility because it cannot be definitely claimed that the amenity value will decrease with a price decline to a sufficient degree to bring the economy back to full employment. Interestingly, Pigou later became more confident as to his own theory. He concluded an article of 1943 in the *Economic Journal* article by asserting that, "provided that wage-earners adopt a competitive wage policy," a stationary state with full employment "is always possible; indeed it is the goal to which, granted this proviso, the economic system necessarily tends" (Pigou 1943, 350).

Pigou's change of attitude here can be explained simply in terms of the consequence of a different way of formulating his theory. In this 1943 article he simply presented a modified saving function, rather than offering an intricate theoretical argument with many arbitrary hypotheses, as he had done in *Employment and Equilibrium*.[20] The new saving function was: $S = f(C, x, r, T)$, where C is capital stock, T the real value of money stock, and $\frac{\partial f}{\partial T} < 0$, so that a price decline causes T to increase and savings to decrease; and he added, "$f(C, x, r, T)$ can assume a nil value, if T is sufficiently large, for no matter what values of C and x and r" (Pigou 1943, 350). A new way of theorization provided a heuristic benefit. With the earlier reasoning with arbitrary assumptions, he had not been certain whether the amenity value of savings would decrease sufficiently to activate the effect. But by the time of the 1943 article, he was able to draw on an internal argument that if the real value of money stock becomes sufficiently large, savings will necessarily be brought to nil.

This new theorization would later be adopted by Patinkin (1948, 547). According to Rubin (2005), Patinkin constructed his 1948 article in the *American Economic Review*, which gave currency to the term 'Pigou effect', by way of discussion with Milton Friedman and the British economist, Alexander Henderson. These two economists thought that the effect Pigou noted in 1941 and 1943 was strong enough to bring the economy out of the liquidity trap. Henderson wrote Patinkin, "It cannot be true of any net cash holder that there is any limit short of bliss to his consumption as all prices fall towards zero" (quoted in Rubin 2005, 52). These economists thus subscribed to Pigou's argument in exactly the same way Pigou himself did, namely, by supposing that there is no saturation in consumption due to the expansion of real assets. Patinkin, who was Keynesian in his general orientation, attempted to exert control over this argument by asserting that the effect is not strong or quick enough in

the short run. Thus, Pigou's 1943 article had a stronger impact on later generations than his 1941 book, as indeed it also did on Pigou himself.

VI. Conclusion

In this paper I have discussed an important undercurrent running through the 1937-38 controversy between Pigou and Keynes: the proper way to interpret a model. Kaldor's mathematical analysis won the de facto approval of the participants in the controversy for, as a matter of fact, they all subsequently came to adopt such analysis in order to discuss macroeconomic theory. Although they did not explicitly say so, Keynes, Robertson, and Pigou all eventually subscribed to this way of analyzing a macroeconomic model.

Morgan's distinction, as discussed in the introduction, between models as objects to enquire into and models as objects to enquire with, is indeed illuminating. The current study provides a well-documented case study of her first category – the analysis of the behavior of a model in itself. Our investigation of this episode suggests at least some of the reasons why, during the 1930s, economists came to prefer modeling over the verbal expression of economic theories. In the first instance, Pigou sought to represent the interaction between two endogenous variables (employment and the interest rate) within a single system of equations, whereas before he had only attempted to so study one such variable (employment). Although it can be doubted that Pigou intended this formula as the whole ground for his conclusion, the formula nevertheless became a conspicuous target for both Keynes and Kaldor. There is no doubt that Kaldor only attacked Pigou in terms of his model; as indeed did Keynes, albeit by a different method. It was in this way that these economists collectively but somewhat accidentally chose to discuss the model, and such argument over the model also served as a convenient common ground upon which to settle their differences.

It is also interesting to see Keynes's response to Pigou, especially the way it shifted between different drafts. After reading Kaldor's paper, Keynes added a criticism concerning the reality of Pigou's assumption on savings, and in order to do so he assigned to Pigou an opposite assumption to the one that Kaldor had done. Keynes thus contradicted part of Kaldor's criticisms in order to attack Pigou on a substantial ontological issue. Put another way, Keynes here moved out of Morgan's first function of model reasoning and returned to a more traditional form of economic controversy – an argument about the correspondence between a verbal statement and reality. But what is particularly fascinating here is that it is clear that for the other three economists Keynes's argument was not convincing. By the 1930s the use of model reasoning as a way to persuade others was steadily securing its position.

Simply laying out a system of equations, however, did not achieve the full potential of model reasoning. Without Kaldor's intervention, Pigou's original use of a model would have taken the debate on a totally different course. For Keynes would have examined Pigou's model nonetheless; and members of Keynes's inner circle who would very likely have participated in the debate more publicly might well have chosen that model as the main target of their criticisms. Thus, even had the model been a sole subject of the debate, nevertheless without the intervention of Kaldor's mathematical analysis it would not have accomplished the same ends. Kaldor's intervention made the behavior of the model, rather than just the structure of the model, transparent and indisputable in the eyes of other scholars. In other words, Kaldor's mathematical analysis of the model turned Pigou's model into Bruno Latour's 'immutable mobile' in the full sense of that notion; that is, it enabled the knowledge of the behavior of the model to transfer from the mind of one economist to the mind of another without changing its form.[21] Thus one can argue that, self-evident as it may sound, model reasoning as a scientific tool or as a way to convince other scholars was able to reach a higher capability with the proper use of mathematics.

Finally, I return to the question I posed in the very first sentence of this paper: the impact of macroeconomic model builders upon this controversy. Three influences are in order. First, Harrod's *Econometrica* paper was read by Robertson and Pigou, and it is likely that this paper led Pigou to realize the necessity of using a two-equation system. Second, as Warren Young has pointed out, Kaldor was aware of Hicks's paper on "Mr. Keynes and the Classics," and he actually used Hick's diagram to present his point in his paper. Third, David Champernowne, a much neglected precursor of Harrod and Hicks, privately instructed Pigou in the analysis of a model. Thus, I conclude that this controversy was part of the historical process whereby the IS-LM model was being created and achieving circulation.

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[1] Email: takami.jp@gmail.com. Waseda University. Research for this paper was conducted with the support of the Japanese Society for the Promotion of Sciences.

[2] Takami (2011) points to circumstantial evidence to suggest this. Pigou (1927) estimated that five percentage points of unemployment was attributable to the wage rigidity in the twenties. This estimate corresponds with another estimate on the elasticity of aggregate labor demand and the extent of the money wage decline that occurred during the 1870s and 1880s.

[3] The Robertson Papers, Trinity College, Cambridge. C2/3 folio 51, dated Sep 5, 1933. Also quoted in Moggridge (1973a, 312).

[4] "The post-war experiences of Great Britain and the United States are, indeed, actual examples of how an accumulation of wealth, so large that its marginal efficiency has fallen more rapidly than the rate of interest can fall in the face of the prevailing institutional and psychological factors, can interfere, in conditions mainly of laissez-faire, with a reasonable level of employment and with the standard of life which the technical conditions of production are capable of furnishing" (Keynes 1936, 219). The thesis that there is an underlying trend of the decreasing marginal efficiency of capital was to be later developed by Alvin Hansen (1939). Pigou specifically addressed Hansen in his *Economic Journal* article of 1943.

[5] Pigou published a critical review of *The General Theory* in *Economica* (Pigou 1936). However, here he seems to miss Keynes's point that money wage adjustment will not work under the liquidity trap. In this review article, Pigou mentions that money wage adjustment could halt the long-run downward movement that Keynes stressed in his *The General Theory*, but without noting Keynes's further claim of the ineffectiveness of wage movements in light of the liquidity trap.

[6] Pigou later told Keynes that he had been afraid that his 1937 article would disturb Keynes, who was then convalescing from a heart attack (Moggridge 1973b, 257).

[7] The Robertson Papers. C2/7 folio 7. Also quoted in Moggridge (1973b, 99). Robertson told Keynes, "After reading Harrod's *Econometrica* article and discussing it with Pigou, I should now be prepared to rewrite my section 6 more positively." Keynes conjectured that Pigou did not read *Econometrica*, (Young 1987, 38); but even if this is true, he was in constant communication with Robertson, who would no doubt inform him of ongoing trends in economics research.

[8] Warren Young (1987) has highlighted a joint seminar by Harrod, James Mead, and John Hicks on *The General Theory*, conducted at the Econometric Society at Oxford in September 1936. Those three would each subsequently publish on this subject, but Harrod's was the first to appear in print.

[9] The income velocity partly depends on income distribution, expressed as $\frac{xF'(x)}{F(x)}$. If the shape of the function $F(\cdot)$ remains the same, the amount of employment solely determines income distribution. Hence, x in V(r,x).

[10] "[W]hen w is reduced in conditions where x is fixed, price cannot be reduced, but marginal prime cost is reduced. In the new situation, therefore, marginal prime cost is not equal to, but is necessarily less than price" (Pigou 1937, 410).

[11] Obviously Pigou implicitly applied a separate theory on how a money wage reduction affects the interest rate. Pigou's view on money wages can be found in pp. 100-102 of *The Theory of Unemployment*, in which he claimed that a cut in money wages involves a decline of general prices only in a smaller proportion, thus resulting in a decrease in real wages. Robertson backed him on this point in a letter to Keynes (Moggridge 1973a, 319). Pigou told Keynes that he had a good deal on the interest rate in his earlier draft of this article.

[12] Readers might object to this argument because it is possible that he admitted the dependence of savings upon real income over the course of the debate with Keynes and Kaldor. But this is unlikely because a criticism that Pigou eventually accepted came from Kaldor, and Kaldor's criticism did not concern the plausibility of Pigou's

assumption on the time preference rate or savings but the consistency between such an assumption and his conclusion.

[13] However, it is fairer to say that Pigou meant that variable A (the quantity of money) is a function of X (the interest rate) and A holds a certain relation with another variable B (money income). Keynes thus imposed unfairly tight logic on Pigou's model. Nor did he mention Pigou's mathematical formula of his 1937 article in this early version of his response.

[14] According to Young (1987, 107-113), Hicks himself showed that diagram to Kaldor. Interestingly, in the interview with Young, Kaldor seems to imply that Pigou's saving function was essentially the same as Keynes's, as opposed to what Keynes claimed.

[15] Kaldor told Keynes, "I do not think Pigou assumed that the amount of money which the public want to hold at a given rate of interest… is irrespective of money wages and of money income in general" (Moggridge 1973b, 243; also see p. 249). Robertson told Keynes, "Pigou does not assume that the amount of money which the public want to hold at a given rate of interest depends entirely on their rates of time-preference, and is irrespective of money wages and of money income in general" (Moggridge 1973b, 253). Pigou wrote Keynes, "My impression is that your note is based on a misunderstanding of what I was trying to say" (Moggridge 1973b, 256). Robertson wrote Kaldor that he could not "make anything of Keynes' note" (quoted in Young 1987, 111).

[16] In parallel with his correspondence with Kaldor and Robertson, Keynes also turned to Kahn for advice, but he did not receive any substantive response from him. Kahn told Keynes that he had not read Kaldor's article and noted only that the determination of the interest rate by the rate of time preference was Pigou's fundamental error. Keynes replied that this assumption was reasonable in Pigou's context.

[17] In this respect, I agree with Aslanbeigui and Oakes (2007) who see the controversy as turning upon the unequal footings of Keynes as a journal editor and Pigou as a single contributor.

[18] Young (1987, 82-86, 95-97) discusses Champernowne's role in the movement toward the modeling of Keynes's idea. Champernowne's paper may well have been read by Hicks when submitted to the *Review of Economic Studies*, for Hicks was then joint editor. Champernowne told Young that his intention was not to build a simple tool to interpret Keynes's argument, but to highlight the importance of general expectations among businesspeople in Keynes's system. Pigou continued to work with Champernowne even after this contact, and the latter wrote an obituary of the former (Champernowne 1959).

[19] The equation was the first of the two equations system mentioned in the main text above: $(1 + \mathrm{r})\mathrm{w} = \frac{f(r)V(r,x)}{F(x)}F'(x)$, where f(r) is money supply function dependent on the interest rate r, F(x) real income function dependent on employment x. Pigou performed the calculus on it and obtained the following equation: $\frac{d}{dr}(1 + r)w = \frac{dx}{dr}\frac{d}{dx}\left(\frac{F'}{F}\right) \cdot f \cdot V + \frac{F'}{F} \cdot f' \cdot V + \frac{F'}{F} \cdot f \cdot \frac{\partial V}{\partial r}$ by assuming $\frac{\partial V}{\partial x}$ is negligible (Pigou 1938, 137). Since the right hand side is positive and $\frac{dw}{dr}$ is not likely to take the different sign from $\frac{d}{dr}(1 + r)w$, $\frac{dw}{dr}$ is positive.

[20] Pigou used the assumptions of constant incomes and the same proportionate saving in subsequent periods in *Employment and Equilibrium* (Pigou 1941, 104).

[21] See, for instance, Latour (1986). Immutable mobiles are representational objects that can be transported from one place to another without changing its original form, such as maps and numbers produced in laboratories. These objects can leave where they were produced and be accumulated in one place along with those obtained in different locations. Latour supposes that this process has enabled science and technology to develop much more globally and persistently in the modern period.