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Education Inequality among Different Social Groups

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Abstract

In this paper, we examine an education-planning problem by using a mechanism design approach. We consider a model where agents have different abilities in acquiring education and belong to different social groups (for instance, races or genders). Under the information constraint that the abilities of agents are observable and group memberships are unobservable, the social planner constructs direct mechanisms to determine the education levels of agents and to distribute income transfers and schooling help. We compare two sets of education policies derived under *Rawlsian* and *utilitarian* social welfare functions. Our main findings are as follows. First, under education policies obtained from the utilitarian social welfare function, agents with the same ability are equally treated, regardless of group membership. In contrast, education policies obtained from the Rawlsian social welfare function lead to a form of *reverse discrimination* in the sense that agents in a more disadvantaged social group achieve higher levels of education.

Keywords: Education; Mechanism design; Inequality; Different social groups

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1 Introduction

Education inequality is considered as a crucial cause of differences in wage and status, and other social inequalities. Inequalities of education are persistent across different social groups distinguished by, for instance, genders, races or parent's jobs in the forms of gaps in test scores and in school achievements. Some past studies conclude that inequalities in education among social groups arise from differences in their circumstances. For example, Fryer and Levitt (2004) argue that one cause of the observed test score gaps between blacks and whites in the U.S. may be systematically lower quality schools for blacks relative to whites. Filmer (2008) insists that, in many countries, inequalities in education persist due to gender stereotyping. In these cases, an individual in a disadvantaged group may achieve a lower level of education than another individual in an advantaged group, even if both individuals exert the same level of effort in acquiring education. On this basis, Roemer (1998), among others, insists that agents should not be held accountable for differences in backgrounds and circumstances, and that social inequalities arising from these differences should be corrected through policy. In this paper, we examine optimal government policy in distributing education resources across different social groups. We consider two social welfare functions, *utilitarian* and *Rawlsian*, and analyze implications of the education policies derived from both social welfare functions for different social groups

In our model, agents differ in two respects. First, agents have different abilities relevant to their costs of acquiring education. Second, we assume that each agent belongs to one of two social groups, either A (advantaged) or D (disadvantaged). We consider the situation where *because of socioeconomic and environmental factors*, agents in the socially disadvantaged group tend to pay larger costs of obtaining education than those in the advantaged group. [1] Therefore, we assume that the relative proportion of agents with higher abilities in the disadvantaged group is smaller than in the advantaged group.[2]

[1]See, for instance, Roemer (1998, pp. 5–12) for further discussion on this point.

[2]Note that our intention is not to suggest that the asymmetry of the distribution functions reflects variation in inherent ability across different social groups.

Moreover, we consider two kinds of benefit from education. First, each agent benefits from his or her own level of education. Examples of this kind of education benefit include the enhancement of human capital and the accumulation of knowledge. Second, each agent also benefits from the average education level of society as a whole as an externality. Examples of this kind of education benefit potentially include a reduction in crime, the development of new technologies and knowledge, and an increase in political awareness. Although there are a variety of forms of education externalities, we introduce education externality in a simple way in which the utility of agents depends on the average education level of society. [3] Note that the impact of education externalities differs across countries. For instance, some researches argue that education externalities are larger in developing countries than in developed countries.[4] Accordingly, in this paper, we analyze how the impact of the education externality affects the distribution of resources on education across different social groups.

In our analysis, the social planner's education policies are to distribute *income transfers* and *in-kind transfers on education* (referred to as "schooling help"), and to determine the education level of agents. These policies are determined to maximize a given social welfare function. We compare two sets of education policies derived from two especially well-known social welfare functions, the *maximin* and the *utilitarian*. The maximin social welfare function is also called the *Rawlsian* social welfare function as it is similar to the difference principle first proposed by Rawls (1971).

Our main findings are as follows. First, when we employ the utilitarian social welfare function, there is no difference in education policy for the different groups, i.e., the optimal allocation of the level of education and schooling help depends only on the abilities of agents. In contrast, when we adopt a Rawlsian social welfare function, the result is that given the same ability level:

(i) agents in the disadvantaged group achieve a higher education level than those in

[3]De Fraja (2002) and Green and Sheshinski (1975), among others, introduce an education externality in this form.

[4]See Hanushek (2002) for a discussion. Hanushek (2002) also argues that reducing social inequalities and the presence of education externalities provide major justifications for government intervention in education.

the advantaged group; and

(ii) agents in the disadvantaged group receive more (less, respectively) schooling help than those in the advantaged group if the impact of the education externality is sufficiently large (small, respectively).

These results imply that a Rawlsian education policy leads to a form of "discrimination" between social groups. In particular, we can interpret result (i) as a form of reverse discrimination in the sense that preferential treatment is given to more disadvantaged agents. Affirmative action is a specific example of such policies. In the area of political philosophy, there are some arguments on the relationship between affirmative action and Rawls' (1971) theory of justice.[5] Our results provide new evidence on the relationship between affirmative action and Rawls' (1971) difference principle. That is, given information asymmetry about the abilities of agents, Rawls' (1971) difference principle would derive an affirmative action policy for education in the sense that agents in the disadvantaged group achieve higher levels of education than those in the advantaged group.

1.1 Related Literature

There are many studies concerning the distribution of public expenditure on education. For example, Arrow (1971) considered the utilitarian approach to the problem of distributing public spending, while Ulph (1977) and Hare and Ulph (1979) analyzed situations where both education and income redistribution policies are performed simultaneously with a distribution problem under asymmetric information where agents' talents are unobservable. Both Ulph (1977) and Hare and Ulph (1979) employed the optimal income taxation framework constructed by Mirrlees (1971).

[5] For instance, Nagel (2003) discusses a relevance between Rawls' theory of justice and affirmative action. Taylor (2009) argues that most affirmative action policies are incompatible with Rawls' theory of justice, while Valls (2010) responds that Taylor (2009) misinterprets the implications of Rawls's theory for affirmative action policies and that most forms of affirmative action are compatible with Rawls' theory.

The model used in this paper mostly follows Fleurbaey et al. (2002). However, our model is an extension to the case where agents may belong to different social groups and an education externality exists. Further, Fleurbaey et al. (2002) investigated the distribution of public spending on education (schooling help) and income transfers where the planner has a social welfare function with constant elasticity of substitution (CES)-type functions displaying various degrees of inequality aversion. They argue that in cases of interior inequality aversions and continuous ability distributions, it is difficult to obtain a clear characterization of the second-best solutions. In contrast, we focus on two polar cases, i.e., the Rawlsian and the utilitarian, two of the best-known social welfare functions in welfare economics. By focusing on these polar cases, we clearly contrast the second-best policies derived from these alternative objective functions.

Several existing studies also consider externalities in education. For example, Green and Sheshinski (1975) extended Arrow's (1971) model to the case where all agents benefit from the average level of education as well as their own level of education. They showed that if an education externality exists, the utilitarian social planner shifts public expenditures to high-ability individuals that can use these resources most efficiently. In other work, De Fraja (2002) considered a similar type of education externality as Green and Sheshinski (1975) in constructing a mechanism for optimal education policies where incomes and the unobservable talents of children differ across households. In addition, De Fraja (2002) employed a utilitarian social welfare function and his model allowed for a private education sector.

The structure of the remainder of the paper is as follows. Section 2 introduces the model. In Section 3, we analyze the case where agents' talents are observable as a benchmark. In Section 4, we examine the distribution problem under incomplete information. Section 5 provides some concluding remarks.

2 Model

We consider an economy where the whole population is divided into two groups, A and D. For each $j = A, D$, abilities are distributed according to the distribution function $F_j : [\underline{\theta}, \overline{\theta}] \to [0, 1]$ with its density function f_j such that $f_j(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. We assume that, for $j = A, D$, f_j/F_j is decreasing in θ. This is a standard assumption in the mechanism design literature. Let p_j be the proportion of group j and $p_A + p_D = 1$. In the rest of the paper, we refer to an agent with ability $\theta \in [\underline{\theta}, \overline{\theta}]$ and $j \in \{A, D\}$ as type θj.

We assume that group A is more advantaged than group D in the following way: the proportion of agents with higher abilities in group A is larger than in group D in the sense of reverse hazard rate dominance. That is, for all $\theta \in (\underline{\theta}, \overline{\theta}]$:

$$\frac{f_D(\theta)}{F_D(\theta)} > \frac{f_A(\theta)}{F_A(\theta)}.$$ [6]

Each agent chooses an effort level $y \geq 0$ to acquire education. Schooling help, s, is an in-kind transfer provided by the government. We assume that an education production function g gives the educational achievement of an individual as an increasing function of effort and help. Let $e = g(y, s)$ be the education level when exerting effort y and receiving help s.

Each agent's gain from education is given by $e + aE$, where e is the agent's own education level, E is the average education level of society and $a > 0$ represents the magnitude of the education externality:

$$E \equiv \sum_{j=A,D} p_j \int_{\underline{\theta}}^{\overline{\theta}} e_j(\theta) f_j(\theta) d\theta,$$

where $e_j(\theta)$ is type θj's education level. Note that each agent benefits not only from his or her own education level, but also from the social education level as an externality.

[6]We can obtain the same results if there are finitely many social groups. In the case of m social groups, we assume that, for all $i, j \in \{1, ..., m\}$ such that $i > j$, and for all $\theta \in (\underline{\theta}, \overline{\theta}]$:

$$\frac{f_i(\theta)}{F_i(\theta)} > \frac{f_j(\theta)}{F_j(\theta)}.$$

In this case, group m is the most disadvantaged.

The total effort expenditure of an agent with ability θ is θy. The minimal amount of effort needed to achieve the education level while receiving s is denoted $y = C(e, s)$. By definition, $e \equiv g[C(e, s), s]$. Following Fleurbaey et al. (2002), we make assumptions directly on the cost function C and its derivatives.

Assumption. The mapping C is twice continuously differentiable, with partial derivatives satisfying:

(1) $C_e > 0,\ C_s < 0,\ C_{es} < 0$;

(2) $C_e \to 0$ as $e \to 0$ for all s; $C_s \to 0$ as $s \to +\infty$, and $C_s \to -\infty$ as $s \to 0$ for all e;

(3) C is strictly convex;

(4) $C_{ee} > |C_{es}|$ and $C_{ss} > |C_{es}|$.

Together, Assumptions (1)–(4) are almost the same as Assumption 1 in Fleurbaey et al. (2002, p. 121). Assumption (1) implies that additional education increases cost, additional help decreases cost, and additional help decreases the marginal cost of education. Assumption (2) is a condition to focus on the interior solution to ensure the simplicity of the analysis. Assumption (3) states that the returns to scale in the production of an individual level of education are strictly decreasing. Assumption (4) means that the cross second-order derivative C_{es} is sufficiently small in absolute terms. Each agent receives a monetary transfer $t \in \mathbb{R}$ from the public sector. We assume type θj's utility, is given as:

$$u_j(\theta) = t_j(\theta) + e_j(\theta) + aE - \theta C(e_j(\theta), s_j(\theta)),$$

where $t_j(\theta)$ and $s_j(\theta)$ are type θj's transfer and schooling help, respectively.

We introduce the two social welfare functions (SWF hereafter). The *utilitarian* SWF is given as:

$$\sum_{j=A,D} p_j \int_{\underline{\theta}}^{\overline{\theta}} u_j(\theta) f_j(\theta) d\theta.$$

Conversely, the *Rawlsian* SWF is defined as:

$$\min_{j=A,D} \min_{\theta \in [\underline{\theta}, \overline{\theta}]} u_j(\theta).$$

In the following sections, we compare the two kinds of education policies derived from these alternative SWFs.

3 The First-Best Problem

As a benchmark, in this section we consider the case where agents' abilities are observable. We then compare the first-best allocations with respect to the utilitarian and Rawlsian social welfare functions. The problems are:

$$\max_{e,s,t} \sum_{j=A,D} p_j \int_{\underline{\theta}}^{\overline{\theta}} u_j(\theta) f_j(\theta) d\theta, \text{ and } \max_{e,s,t} \min_{j=A,D} \min_{\theta \in [\underline{\theta},\overline{\theta}]} u_j(\theta), \tag{1}$$

subject to the balanced budget constraint (BB):

$$\sum_{j=A,D} p_j \int_{\underline{\theta}}^{\overline{\theta}} (t_j(\theta) + s_j(\theta)) f_j(\theta) d\theta = M, \tag{2}$$

where M is an exogenously given amount of money.

Let (e^R, s^R, t^R) and (e^U, s^U, t^U) be the first-best solutions of the Rawlsian and utilitarian SWFs, respectively. The optimality conditions are:

$$1 + a = \theta C_e(e_j^k(\theta), s_j^k(\theta)), \tag{3}$$

$$-1 = \theta C_s(e_j^k(\theta), s_j^k(\theta)), \tag{4}$$

$$\sum_{j=A,D} p_j \int_{\underline{\theta}}^{\overline{\theta}} [t_j^k(\theta) + s_j^k(\theta)] d\theta = M, \tag{5}$$

where $k = U, R$, and $j = A, D$. Notice that in the case of the Rawlsian SWF, all types achieve the same utility level:

$$u_A(\theta) = u_D(\theta) = u_A(\theta') = u_D(\theta') \ \forall \theta, \theta' \in [\underline{\theta}, \overline{\theta}].$$

Given the optimality conditions (3) and (4) are identical for both SWFs, the first-best solutions are the same. Therefore, we can put $(e^C, s^C) \equiv (e^R, s^R) = (e^U, s^U)$.

Let $(\theta, a) \mapsto (e^C(\theta, a), s^C(\theta, a))$ be the solution mappings of the above problem. We obtain the following result. [7]

[7] The result is almost identical to Proposition 1 in Fleurbaey et al. (2002) with the exception that we allow for the education externality.

Proposition 1. Suppose that Assumptions (1)–(4) hold.

(i) $e^C(\theta, a)$ is decreasing in θ.

(ii) $s^C(\theta, a)$ is increasing (nonincreasing, respectively) in θ if $a < -(C_{ee}+C_{es})/C_{es}$ ($a \geq -(C_{ee}+C_{es})/C_{es}$, respectively).

Proof. Differentiating (3) and (4) by θ and solving the system of the equations, we obtain:

$$e^C_\theta(\theta, a) = \frac{-1}{d(C'')}[(1+a)C_{ss} + C_{se}]\frac{1}{\theta^2}, \tag{6}$$

$$s^C_\theta(\theta, a) = \frac{1}{d(C'')}[(1+a)C_{es} + C_{ee}]\frac{1}{\theta^2}, \tag{7}$$

where $d(C'') = C_{ee}C_{ss} - (C_{es})^2 > 0$ and $j = 1, 2$. From equation (6) and Assumption (4), $e^C_\theta(\theta, a) < 0$.

Moreover, from equation (7) and $C_{es} < 0$, $s^C_\theta(\theta, a) > 0$ if $(1+a)C_{es} + C_{ee} > 0$, and $s^C_\theta(\theta, a) \leq 0$ if $(1+a)C_{es} + C_{ee} \leq 0$. □

Proposition 1 states that (i) the higher the ability of an agent, the higher the level of education he/she achieves; and (ii) if the impact of the externality a is sufficiently large (small, respectively), agents with higher abilities receive higher (lower, respectively) levels of help. An intuition behind the result (ii) is that if the impact of the externality is sufficiently large, then the planner has an incentive to reduce the costs of high-ability agents and so allow them to acquire more education to increase the externality.

Next, note that the left-hand sides of equations (3) and (4) are common for the two social groups. Thus, we obtain the following result.

Proposition 2. Under complete information, the allocation of education and help is the same for the two groups A and D. That is, $(e^C_A(\theta), s^C_A(\theta)) = (e^C_D(\theta), s^C_D(\theta))$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$.

Therefore, when the planner can observe the agents' abilities, there is *no discrimination* with respect to education and schooling between the two social groups.

4 The Second-Best Problem

In this section, we analyze the case where the planner cannot observe the agents' abilities. We introduce *Incentive Compatibility* (IC). We assume that each agent cannot affect the level of externality, since the population is infinitely large and hence his/her influence is negligible.
(IC): For all $\theta, \theta' \in [\underline{\theta}, \overline{\theta}]$,

$$u_j(\theta) \geq t_j(\theta') + e_j(\theta') + aE - \theta C(e_j(\theta'), s_j(\theta')) \quad (j = A, D).$$

This constraint means that any agent has no incentive to misreport his or her ability. Note that agents cannot lie about their group as the planner can observe each agent's group membership. Our problem is then to maximize the two social welfare functions subject to (BB) and (IC).

We derive the following condition (IC_f) from (IC).[8]
(IC_f): For all $j \in \{A, D\}$ and all $\theta \in [\underline{\theta}, \overline{\theta}]$:

$$u_j(\theta) = u_j(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} C(e_j(x), s_j(x))dx.$$

(IC) implies the following first-order condition: For each j and $\theta \in [\underline{\theta}, \overline{\theta}]$,

$$(IC') \quad t_j'(\theta) + e_j'(\theta) - \theta\Big[C_e(e_j(\theta), s_j(\theta))e_j'(\theta) + C_s(e_j(\theta), s_j(\theta))s_j'(\theta)\Big] = 0.$$

This equation implies:

$$t_j'(\theta) = -e_j'(\theta) + \theta\Big[C_e(e_j(\theta), s_j(\theta))e_j'(\theta) + C_s(e_j(\theta), s_j(\theta))s_j'(\theta)\Big].$$

By integrating this equation, we have:

$$t_j(\overline{\theta}) - t_j(\theta) = [-e_j(x)]_{\theta}^{\overline{\theta}} + \int_{\theta}^{\overline{\theta}} \theta\Big[C_e(e_j(\theta), s_j(\theta))e_j'(\theta) + C_s(e_j(\theta), s_j(\theta))s_j'(\theta)\Big]dx.$$

Applying integration by parts to the second term of the right-hand side, we have:

$$t_j(\overline{\theta}) - t_j(\theta) = [xC(e_j(x), s_j(x)) - e_j(x)]_{\theta}^{\overline{\theta}} + \int_{\theta}^{\overline{\theta}} C(e_j(x), s_j(x))dx.$$

[8] The proof is essentially from Fleurbaey (2002, Section 6).

Then, given $u_j(\theta) = t_j(\theta) + e_j(\theta) + aE - \theta C(e_j(\theta), s_j(\theta))$, we obtain:

$$u_j(\theta) = u_j(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} C(e_j(x), s_j(x))dx.$$

Next, we discuss the second-order condition of (IC). Differentiating

$$t_j(\theta') + e_j(\theta') + aE - \theta C(e_j(\theta'), s_j(\theta'))$$

twice with respect to θ' (with evaluating $\theta' = \theta$), we have the second-order condition as follows.

$$t_j''(\theta) + e_j''(\theta) - \theta \frac{d}{d\theta}\Big[C_e\big(e_j(\theta), s_j(\theta)\big)e_j'(\theta) + C_s(e_j(\theta), s_j(\theta)\big)s_j'(\theta)\Big] \leq 0.$$

Note that (IC') holds for all θ. Then, differentiating (IC') by θ,

$$\begin{aligned} & t_j''(\theta) + e_j''(\theta) - \theta \frac{d}{d\theta}\Big[C_e\big(e_j(\theta), s_j(\theta)\big)e_j'(\theta) + C_s\big(e_j(\theta), s_j(\theta)\big)s_j'(\theta)\Big] \\ & = \Big[C_e\big(e_j(\theta), s_j(\theta)\big)e_j'(\theta) + C_s\big(e_j(\theta), s_j(\theta)\big)s_j'(\theta)\Big]. \end{aligned}$$

By the two equations above, we obtain the following condition.
(IC_s): For all j all $\theta \in [\underline{\theta}, \overline{\theta}]$,

$$C_e\big(e_j(\theta), s_j(\theta)\big)e_j'(\theta) + C_s\big(e_j(\theta), s_j(\theta)\big)s_j'(\theta) \leq 0.$$

We also rewrite (BB). From

$$\sum_{j=A,D} p_j \int_{\underline{\theta}}^{\overline{\theta}} [t_j(\theta) + s_j(\theta)]f_j(\theta)d\theta = M,$$

and

$$u_j(\theta) = t_j(\theta) + e_j(\theta) + aE - \theta C(e_j(\theta), s_j(\theta)),$$

we obtain:

$$(BB') \quad M = \sum_{j=A,D} p_j \Big[\int_{\underline{\theta}}^{\overline{\theta}} \Big(s_j(\theta) - (1+a)e_j(\theta) + \theta C(e_j(\theta), s_j(\theta)) + u_j(\theta)\Big) f_j(\theta)d\theta\Big].$$

In the remainder of our analysis, we solve the utilitarian and the Rawlsian second-best problems under the new constraints (BB'), (IC_f) and (IC_s), instead of (BB) and (IC). In the next two subsections, we solve the problems by ignoring (IC_s). In the Appendix, we check that the solutions of the utilitarian and the Rawlsian second-best problems satisfy (IC_s).

4.1 The Utilitarian Solution

Now we analyze the second-best utilitarian education policies. We solve

$$\max_{(u,e,s)} \sum_{j=A,D} p_j \int_{\underline{\theta}}^{\overline{\theta}} u_j(\theta) f_j(\theta) d\theta,$$

subject to (BB') and (IC_f).

To solve the problem, we rewrite (BB') and (IC_f) as:

$$m'(\theta) = -\sum_{j=A,D} p_j \Big[s_j(\theta) - (1+a)e_j(\theta) + \theta C\big(e_j(\theta), s_j(\theta)\big) + u_j(\theta) \Big] f_j(\theta),$$
$$m(\underline{\theta}) = 0,\ m(\overline{\theta}) = -M,$$

and

$$u_j'(\theta) = -C\big(e_j(\theta), s_j(\theta)\big).$$

Then, the utilitarian planner's optimization problem becomes a standard optimal control problem.[9] We introduce two costate variables, denoted λ and μ_j ($j = A, D$), associated with the state variables m and u_j, respectively. The Lagrangian of the problem is defined as:

$$\mathcal{L} = \sum_{j=A,D} \Big[p_j u_j f_j - \lambda p_j \Big(s_j - (1+a)e_j + \theta C(e_j, s_j) + u_j \Big) f_j - \mu_j C(e_j, s_j) \Big].$$

Applying Pontryagin's maximum principle, we obtain the following conditions:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial e_j} &= -\lambda p_j [-(1+a) + \theta C_e] f_j - \mu_j C_e = 0, \\
\frac{\partial \mathcal{L}}{\partial s_j} &= -\lambda p_j [1 + \theta C_s] f_j - \mu_j C_s = 0, \\
\frac{\partial \mathcal{L}}{\partial u_j} &= p_j f_j - \lambda p_j f_j = -\mu_j', \quad \mu_j(\underline{\theta}) = \mu_j(\overline{\theta}) = 0, \\
\frac{\partial \mathcal{L}}{\partial m} &= 0 = -\lambda'.
\end{aligned}$$

[9] See, for instance, Leonard and Van Long (1995, Theorem 6.3.1).

From these equations, we obtain:

$$\frac{1+a}{\theta} = C_e(e_j^u(\theta), s_j^u(\theta)), \tag{8}$$

$$-\frac{1}{\theta} = C_s(e_j^u(\theta), s_j^u(\theta)), \tag{9}$$

where (e^u, s^u) are the second-best utilitarian education policies.

It is obvious that equations (8) and (9) are the same as the optimality conditions under complete information, i.e., equations (3) and (4). Therefore, $(e^u, s^u) = (e^C, s^C)$. Then, given that the first-best allocation of education and help (e^C, s^C) does not distinguish agents by their group membership, the same result holds for the allocation of the utilitarian education policies under incomplete information. We summarize the result as follows.

Proposition 3. Suppose that Assumptions (1)–(4) hold. Under incomplete information, utilitarian education policies do not discriminate between the different social groups in the following sense: For all $\theta \in [\underline{\theta}, \overline{\theta}]$:

$$e_A^u(\theta) = e_D^u(\theta),\ s_A^u(\theta) = s_D^u(\theta).$$

Thus, when the planner adopts the utilitarian SWF, agents with the same ability level receive the same level of education and help, regardless of their group membership.

As $(e^u, s^u) = (e^C, s^C)$, we can see that (IC) does not bind. Hence, any utility levels can be achieved as long as (IC) and (BB') are satisfied.

4.2 The Rawlsian Solution

We consider the second-best Rawlsian education policies. The maximization problem is:

$$\max_{(u,e,s)} \min_{\theta \in [\underline{\theta}, \overline{\theta}]} \min_{j=A,D} u_j(\theta),$$

subject to (BB') and (IC_f). From (BB') and (IC_f), we obtain:

$$\begin{aligned} M &= \sum_{j=A,D} p_j \int_{\underline{\theta}}^{\overline{\theta}} \Big(s_j(\theta) - (1+a)e_j(\theta) + \theta C(e_j(\theta), s_j(\theta)) + u_j(\theta) \Big) f_j(\theta) d\theta \\ &= \sum_{j=A,D} p_j \int_{\underline{\theta}}^{\overline{\theta}} \Big(s_j(\theta) - (1+a)e_j(\theta) + \theta C\big(e_j(\theta), s_j(\theta)\big) + u_j(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} C\big(e_j(x), s_j(x)\big) dx \Big) f_j(\theta) d\theta \end{aligned}$$

Then, computing the double integral $\int_{\underline{\theta}}^{\overline{\theta}} \int_{\theta}^{\overline{\theta}} C\big(e_j(x), s_j(x)\big) dx f_j(\theta) d\theta$, we have

$$M = \sum_{j=A,D} p_j \Big[\int_{\underline{\theta}}^{\overline{\theta}} \Big(s_j(\theta) - (1+a)e_j(\theta) + \big(\theta + \frac{F_j(\theta)}{f_j(\theta)}\big) C(e_j(\theta), s_j(\theta)) \Big) f_j(\theta) d\theta + u_j(\overline{\theta}) \Big].$$

$\cdots (BB'')$

Since the group memberships are observable, it is obvious that $u_A(\overline{\theta}) = u_D(\overline{\theta})$. From this observation, we can put the lowest utility level as $u(\overline{\theta})$. Then, (BB'') can be rewritten as:

$$u(\overline{\theta}) = M - \sum_j p_j \int_{\underline{\theta}}^{\overline{\theta}} \big[s_j(\theta) - (1+a)e_j(\theta) + \big(\theta + \frac{F_j(\theta)}{f_j(\theta)}\big) C(e_j(\theta), s_j(\theta)) \big] f_j(\theta) d\theta.$$

Then the second-best Rawlsian solution can be obtained by maximizing the right-hand side of the above equation. Let (e^r, s^r) be the second-best allocation concerning education. The first-order conditions are as follows. For $j = A, D$:

$$\frac{1+a}{\theta + \frac{F_j(\theta)}{f_j(\theta)}} = C_e\big(e_j^r(\theta), s_j^r(\theta)\big), \tag{10}$$

$$-\frac{1}{\theta + \frac{F_j(\theta)}{f_j(\theta)}} = C_s\big(e_j^r(\theta), s_j^r(\theta)\big). \tag{11}$$

Compared with equations (3) and (4), equations (10) and (11) show that the agent with ability $\theta \in [\underline{\theta}, \overline{\theta}]$ is treated as if his or her ability were $\theta + \frac{F_j(\theta)}{f_j(\theta)} > \theta$. That is, for all $j = A, D$ and all $\theta \in [\underline{\theta}, \overline{\theta}]$,

$$e_j^r(\theta, a) = e^C(\theta + \frac{F_j(\theta)}{f_j(\theta)}, a), \;\; s_j^r(\theta, a) = s^C(\theta + \frac{F_j(\theta)}{f_j(\theta)}, a).$$

The optimality conditions (10) and (11) imply that the education and help levels of all types except $\underline{\theta}A$ and $\underline{\theta}D$ are distorted. In contrast, as $F_j(\underline{\theta}) = 0$ ($j = A, D$), the education and help levels of agents with the highest ability are at the first-best levels.

From Proposition 1, it is obvious that, for all θ and $j = A, D$:
(i) $e_j^r(\theta, a) < e^C(\theta, a)$; and
(ii) $s_j^r(\theta, a) >$ ($\leq$, respectively) $s^C(\theta, a)$ if a is sufficiently small (large, respectively).
Moreover, if F_j/f_j is increasing in θ, then e_j^r is decreasing in θ, and s_j^r is increasing (nonincreasing, respectively) in θ when a is sufficiently small (large, respectively).

Now we consider the differences in the levels of education and schooling help for the two groups. Remember the assumption that for all $\theta \in (\underline{\theta}, \overline{\theta}]$:

$$\frac{f_D(\theta)}{F_D(\theta)} > \frac{f_A(\theta)}{F_A(\theta).}$$

By this assumption and Proposition 1-(i):

$$e^r_D(\theta) = e^C(\theta + \frac{F_D(\theta)}{f_D(\theta)}, a) > e^C(\theta + \frac{F_A(\theta)}{f_A(\theta)}, a) = e^r_A(\theta).$$

Similarly, by Proposition 1-(ii):

$$s^r_D(\theta) = s^C(\theta + \frac{F_D(\theta)}{f_D(\theta)}, a) < (\geq \text{ , respectively}) \ s^C(\theta + \frac{F_A(\theta)}{f_A(\theta)}, a) = s^r_A(\theta),$$

if $a <$ ($\geq$, respectively) $-(C_{ee} + C_{es})/C_{es}$.

In sum, we have obtained the following result.

Proposition 4. Suppose that Assumptions (1)–(4) hold. Then the second-best Rawlsian solution regarding education has the following properties.

(i) $e^r_D(\underline{\theta}) = e^r_A(\underline{\theta})$, $s^r_D(\underline{\theta}) = s^r_A(\underline{\theta})$.

(ii) For all $\theta \in (\underline{\theta}, \overline{\theta}]$, $e^r_D(\theta) > e^r_A(\theta)$.

(iii) For all $\theta \in (\underline{\theta}, \overline{\theta}]$, $s^r_D(\theta) < s^r_A(\theta)$ ($s^r_D(\theta) \geq s^r_A(\theta)$, respectively) if $a < -(C_{ee} + C_{es})/C_{es}$ ($a \geq -(C_{ee} + C_{es})/C_{es}$, respectively).

(i) implies that agents with the highest ability across the two groups are not discriminated against in education. That is, they obtain the same levels of education and help.

(ii) means that all agents except the highest-ability agents in the disadvantaged group achieve a higher education level than agents in the advantaged group. This could be interpreted as a form of reverse discrimination in that the more disadvantaged group is given more preferential treatment than the advantaged group.

(iii) suggests that all agents except the highest-ability agents in the disadvantaged group receive higher (lower, respectively) levels of schooling help than agents in the advantaged group whenever the degree of externality is sufficiently small (large, respectively). The magnitude of the externality affects the difference in the level of schooling help across the two groups.

Next, we have the following corollary to Proposition 3.

Corollary: Let $u_j^r(\theta)$ be the utility level of type θj under the Rawlsian second-best solution.

(i) $u_A^r(\overline{\theta}) = u_D^r(\overline{\theta})$.

(ii) If $a < -(C_{ee} + C_{es})/C_{es}$, $u_A^r(\theta) < u_D^r(\theta)$ for $\theta \in [\underline{\theta}, \overline{\theta})$.

(iii) If $a \geq -(C_{ee} + C_{es})/C_{es}$, $u_A^r(\theta) > u_D^r(\theta)$ for $\theta \in [\underline{\theta}, \overline{\theta})$.

Proof : (i) As shown above, it is obvious that $u_A(\overline{\theta}) = u_D(\overline{\theta})$.

(ii) If $a < -(C_{ee} + C_{es})/C_{es}$, then by Assumption (1) and Proposition 4, for all $\theta \in (\underline{\theta}, \overline{\theta}]$,

$$\begin{aligned} C\big(e_A^r(\theta), s_A^r(\theta)\big) - C\big(e_D^r(\theta), s_D^r(\theta)\big) &< C\big(e_D^r(\theta), s_A^r(\theta)\big) - C\big(e_D^r(\theta), s_D^r(\theta)\big) \\ &< C\big(e_D^r(\theta), s_D^r(\theta)\big) - C\big(e_D^r(\theta), s_D^r(\theta)\big) = 0. \end{aligned}$$

Then, from (IC_f) and (i), for $\theta \in [\underline{\theta}, \overline{\theta})$,

$$u_A^r(\theta) - u_D^r(\theta) = \int_\theta^{\overline{\theta}} \Big[C\big(e_A^r(x), s_A^r(x)\big) - C\big(e_D^r(x), s_D^r(x)\big)\Big] dx < 0.$$

Thus, we have $u_A^r(\theta) < u_D^r(\theta)$.

(iii) If $a \geq -(C_{ee} + C_{es})/C_{es}$, then by Assumption (1) and Proposition 4, for all $\theta \in (\underline{\theta}, \overline{\theta}]$,

$$\begin{aligned} C\big(e_A^r(\theta), s_A^r(\theta)\big) - C\big(e_D^r(\theta), s_D^r(\theta)\big) &> C\big(e_A^r(\theta), s_A^r(\theta)\big) - C\big(e_A^r(\theta), s_D^r(\theta)\big) \\ &\geq C\big(e_A^r(\theta), s_D^r(\theta)\big) - C\big(e_A^r(\theta), s_D^r(\theta)\big) = 0. \end{aligned}$$

Then, from (IC_f) and (i), for $\theta \in [\underline{\theta}, \overline{\theta})$,

$$u_A^r(\theta) - u_D^r(\theta) = \int_\theta^{\overline{\theta}} \Big[C\big(e_A^r(x), s_A^r(x)\big) - C\big(e_D^r(x), s_D^r(x)\big)\Big] dx > 0.$$

Hence, we obtain $u_A^r(\theta) < u_D^r(\theta)$. $\square$

The corollary means that the differences in utility levels between the social groups depend on the impact of externality a. If a is sufficiently large (small, respectively), the planner gives larger (smaller, respectively) schooling help to type θD than to type θA. Then, type θD obtains higher (lower, respectively) information rent than type θA.

5 Concluding Remarks

In this paper, we have studied second-best optimal education policies where agents differ in ability and group membership. We have obtained the following results. First, the Rawlsian education policy leads to a form of "reverse discrimination" in the following sense: agents in the advantaged group achieve a lower level of education than agents in the disadvantaged group, and the former receive less schooling help than the latter if the impact of the externality is sufficiently small (Proposition 4). Second, the education policies derived from the utilitarian social welfare function do not distinguish agents by their group membership (Proposition 3).

The differences in the policies arise through the incentive compatibility constraints. On the one hand, when using the utilitarian social welfare function, the constraints do not bind. Thus, the allocation coincides with the first-best allocation that does not discriminate between agents by their group. On the other hand, when we adopt the Rawlsian social welfare function, the incentive constraints bind. The social planner would then require that higher-ability agents achieve higher levels of education and so transfer income from agents with higher abilities to those with lower abilities. Given asymmetric information, under the incentive compatibility constraints, the planner must provide information rent $\int_{\theta}^{\overline{\theta}} C\big(e_j(x), s_j(x)\big)dx$ to higher-ability agents to make them exert a higher level of effort. To reduce the information rent, the planner would lower the cost of the lower-ability agents. To reduce this cost, the planner requires lower-ability agents to exert a lower effort, and thus to achieve a lower education level than the first-best level. Note that in the second-best Rawlsian solution, type θj is treated as if his or her ability were $\theta + \frac{F_j(\theta)}{f_j(\theta)}$. Then, type θA's virtual ability is lower than type θD's:

$$\theta + \frac{F_A(\theta)}{f_A(\theta)} > \theta + \frac{F_D(\theta)}{f_D(\theta)}.$$

Hence, the education level of type θA is lower than that of type θD. An intuition is that, as the relative proportion of agents with higher abilities in the group A is larger than in group D, the planner makes the education level of θA lower than that of θD to decrease the information rent.

Proposition 4 may be interpreted as a justification for an affirmative action policy on education. As discussed in the introduction, the result would show a relationship between affirmative action and Rawls' (1971) difference principle.

6 Appendix

In this appendix, we show that the (IC_s) constraint holds for both the utilitarian and Rawlsian second-best solutions. Given for all $j = A, D$ and all $\theta \in [\underline{\theta}, \overline{\theta}]$:

$$e_j^u(\theta) = e^C(\theta),\ e_j^r(\theta) = e^C\Big(\theta + \frac{F_j(\theta)}{f_j(\theta)}\Big),$$
$$s_j^u(\theta) = s^C(\theta),\ s_j^r(\theta) = s^C\Big(\theta + \frac{F_j(\theta)}{f_j(\theta)}\Big),$$

and $\theta + F_j/f_j$ is increasing in θ, it is sufficient to check that the solution (e^C, s^C) satisfies (IC_s).

From equations (8) and (9), we have:

$$C_e\big(e^C(\theta), s^C(\theta)\big) + C_s\big(e^C(\theta), s^C(\theta)\big) = \frac{a}{\theta}.$$

By use of this equation:

$$\begin{aligned}
&C_e\big(e^C(\theta), s^C(\theta)\big)e_\theta^C(\theta) + C_s\big(e^C(\theta), s^C(\theta)\big)s_\theta^C(\theta)\\
&\quad= \Big(\frac{a}{\theta} - C_s\big(e^C(\theta), s^C(\theta)\big)\Big)e_\theta^C(\theta) + C_s\big(e^C(\theta), s^C(\theta)\big)s_\theta^C(\theta)\\
&\quad= \frac{a}{\theta}e_\theta^C(\theta) + C_s\frac{1}{d(C'')\theta^2}\Big[(1+a)\Big(C_{ss}\big(e^C(\theta), s^C(\theta)\big) + C_{es}\big(e^C(\theta), s^C(\theta)\big)\Big)\\
&\quad+ \Big(C_{es}\big(e^C(\theta), s^C(\theta)\big) + C_{ee}\big(e^C(\theta), s^C(\theta)\big)\Big)\Big],
\end{aligned}$$

where the last equality is from equations (6) and (7) in Section 3. Recall that $d(C'') = C_{ee}C_{ss} - (C_{es})^2 > 0$, $C_s < 0$, and $e_\theta^C < 0$. Hence, by Assumption (4), we can conclude that:

$$C_e\big(e^C(\theta), s^C(\theta)\big)e_\theta^C(\theta) + C_s\big(e^C(\theta), s^C(\theta)\big)s_\theta^C(\theta) < 0.$$

Thus, we have proved that (IC_s) holds.

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