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Explicit evaluations of infinite utility streams with domain restrictions

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Explicit evaluations of infinite utility streams with domain restrictions*

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Abstract

We show that there are explicit expressions of efficient and/or impartial social welfare functions that satisfy Restricted Non-Substitution, when each generation's utilities lie in the set of reciprocals of positive and negative integers.

1 Introduction

Some recent studies on the problem of aggregating infinite utility streams with a social welfare function (SWF) boosted by Basu and Mitra (2003) suppose that the set of feasible utilities is a discrete set because human perception or cognition is not endlessly fine (Basu and Mitra, 2007; Alcantud and García-Sanz, 2010a, 2010b). Furthermore, some impossibility results for the existence of an efficient SWF are reversed on the supposition that the feasible utilities are discrete.

Basu and Mitra (2007) proved that there exist (implicit) SWFs satisfying Partial Pareto and Anonymity when each generation's utilities lie in $\mathbb{N}\cup\{0\}$, whereas they proved that there is no anonymous and weakly Paretian SWF if the possible utilities contain $[0,1]$. Furthermore, Alcantud and García-Sanz (2010a) proved that there are (implicit) SWFs that satisfy Restricted Non-Substitution, Anonymity, and Partial Pareto when each generation's utilities lie in $\mathbb{N}\cup\{0\}$, while Banerjee (2006) proved that there is no weakly dominant SWF satisfying Hammond Equity for the Future if the feasible utilities are $[0,1]$.

However, the possible policy use of Basu and Mitra's (2007) and Alcantud García-Sanz's (2010a) possibility results is limited since their proofs use the axiom of choice. On the contrary, Alcantud and García-Sanz (2010a) proved that there are explicit SWFs that satisfy Restricted Non-Substitution and Strong Pareto when each generation's utilities lie in $\mathbb{N}\cup\{0\}$, whereas Alcantud and García-Sanz (2010b) proved that there are explicit SWFs that satisfy Anonymity, Restricted Non-Substitution, and weak forms of efficiency if the possible utilities are $\mathbb{N}\cup\{0\}$.

On the other hand, Dubey and Mitra (2011) provided a complete characterization of domains (of the one period utilities) on which an anonymous and weakly Paretian aggregation is possible. By way

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of example, they proved that there exists an anonymous and weakly Paretian SWF when the feasible utilities are $\{1/n\}_{n\in\mathbb{N}}$ or $\{-1/n\}_{n\in\mathbb{N}} \cup \{1/n\}_{n\in\mathbb{N}}$, whereas Basu and Mitra (2007) proved that there is no anonymous and weakly Paretian SWF if the possible utilities contain $[0, 1]$.

The objective of the work is to consider whether we obtain explicit SWFs that satisfy Restricted Non-Substitution, Anonymity and/or some specifications of efficiency when each generation's utilities lie in $\{-1/n\}_{n\in\mathbb{N}} \cup \{1/n\}_{n\in\mathbb{N}}$.

The structure of the article is as follows. Mathematical preliminaries and axioms are introduced in Section 2. In Section 3, we will show that Restricted Non-Substitution and Strong Pareto can be jointly combined by means of explicit evaluations when the feasible utilities are $\{-1/n\}_{n\in\mathbb{N}} \cup \{1/n\}_{n\in\mathbb{N}}$. In Section 4, we show that Anonymity and Restricted Non-Substitution can be combined into explicit expressions.

2 Preliminaries

Let $\boldsymbol{X}$ denote a subset of $\mathbb{R}^{\mathbb{N}}$, which represents a domain of utility streams. We adopt the usual notation for such utility streams: $\boldsymbol{x} = (x_1, \dots, x_n, \dots) \in \boldsymbol{X}$. By $(y)_{\text{con}}$, we mean the constant sequence $(y, y, \dots)$, $(x, (y)_{\text{con}})$ holds for $(x, y, y, \dots)$, and $(x_1, \dots, x_k, (y)_{\text{con}}) = (x_1, \dots, x_k, y, y, \dots)$ means an eventually constant (to y) sequence. We write $\boldsymbol{x} \geq \boldsymbol{y}$ if $x_i \geq y_i$ for each $i = 1, 2, \dots$, and $\boldsymbol{x} > \boldsymbol{y}$ if $\boldsymbol{x} \geq \boldsymbol{y}$ and $\boldsymbol{x} \neq \boldsymbol{y}$.

An SWF is a function $W : \boldsymbol{X} \to \mathbb{R}$. Lauwers (1997a) introduced the next variation of Hammond Equity.

Hammond Equity -Lauwers' version-. If $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$ are such that $x_j \geq y_j \geq y_k \geq x_k$ for some $j, k \in \mathbb{N}$, and $x_t = y_t$ when for all $j \neq t \neq k$, then $W(\boldsymbol{y}) \geq W(\boldsymbol{x})$.

Asheim and Tungodden (2004) introduced the following one-sided equity condition.

Hammond Equity for the Future. If $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$ are such that $\boldsymbol{x} = (x_1, (x)_{\text{con}})$ and $\boldsymbol{y} = (y_1, (y)_{\text{con}})$ $(x_1 > y_1 > y > x)$, then $W(\boldsymbol{y}) \geq W(\boldsymbol{x})$.

Alcantud and García-Sanz (2010a, 2010b) introduced the next reinforcement of Hammond Equity for the Future.

Restricted Non-Substitution. If $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$ are such that $\boldsymbol{x} = (x_1, \dots, x_k, (x)_{\text{con}})$ and $\boldsymbol{y} = (y_1, \dots, y_l, (y)_{\text{con}})$ with $y > x$, then $W(\boldsymbol{y}) > W(\boldsymbol{x})$.[*1]

The follwoing axiom is the classical way to avoid discrimination among generations à la Sidgwick, which-by contrast to Hammond Equity for the Future and Restricted Non-Substitution does not incorporate any preference for egalitarian distribution of utilities among them.

[*1] Non-substitution axiom originates with Lauwers (1997b).

Anonymity. A finite permutation of a utility stream produces a utility stream with the same social utility.

Useful evaluations must account for some form of efficiency too. The literature usually invokes the next axioms:

Strong Pareto. If $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$ and $\boldsymbol{x} > \boldsymbol{y}$, then $W(\boldsymbol{x}) > W(\boldsymbol{y})$.

Weak Dominance. If $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$ and there is $j \in \mathbb{N}$ such that $x_j > y_j$ and $x_i = y_i$ for all $i \neq j$, then $W(\boldsymbol{x}) > W(\boldsymbol{y})$.

Alcantud and García-Sanz (2010b) imposed sensitivity properties like the following.

Sensitivity. $W(y, (x)_{\text{con}}) > W((x)_{\text{con}})$ for each $y > x$.

Lower Sensitivity. $W((x)_{\text{con}}) > W(y, (x)_{\text{con}})$ for each $y < x$.

3 Strongly Paretian SWFs

Alcantud and García-Sanz (2010a), Proposition 1 states that Restricted Non-Substitution and Strong Pareto can be jointly combined by means of explicit evaluations when the possible utilities are the non-negative integers. We show that those axioms can also be combined into explicit expressions when the feasible utilities are the inverses of positive and negative integers (Dubey and Mitra, 2011, Example 3).

Proposition 1. *There are explicit SWFs on* $\boldsymbol{X} = Y^{\mathbb{N}}$, *where* $Y = \{-1, -1/2, -1/3, \dots\} \cup \{1, 1/2, 1/3, \dots\}$, *that satisfy Restricted Non-Substitution and Strong Pareto.*

Proof. For any $\boldsymbol{x} = (x_1, x_2, x_3, \dots) \in \boldsymbol{X}$, let

$$W(\boldsymbol{x}) = \begin{cases} 1 + \dfrac{3 + \sum_{i=1}^{\infty} \frac{x_i}{2^i}}{2} \cdot 2^{-1/\limsup_n (x_n)} & \text{if } \limsup_n (x_n) \text{ is positive} \\ -1 + \dfrac{3 - \sum_{i=1}^{\infty} \frac{x_i}{2^i}}{2} \cdot 2^{1/\limsup_n (x_n)} & \text{if } \limsup_n (x_n) \text{ is negative} \\ \displaystyle\sum_{i=1}^{\infty} \frac{x_i}{2^i} & \text{otherwise.} \end{cases}$$

In order to prove that W satisfies Restricted Non-Substitution, let $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$ be such that $\boldsymbol{x} = (x_1, \dots, x_k, (x)_{\text{con}})$ and $\boldsymbol{y} = (y_1, \dots, y_l, (y)_{\text{con}})$ with $x > y$. We need to ensure $W(\boldsymbol{x}) > W(\boldsymbol{y})$. If $\limsup_n (y_n) = y$ is positive, then $\limsup_n (x_n) = x > y = \limsup_n (y_n) > 0$ and $2^{-1/\limsup_n (x_n)} \geq 2 \cdot 2^{-1/\limsup_n (y_n)}$. Because $\sum_{i=1}^{\infty} \frac{x_i}{2^i}$ takes values in $(-1, 1]$ and $\sum_{i=1}^{\infty} \frac{y_i}{2^i}$ takes values in $(-1, 1)$, we deduce

$$\begin{aligned} \frac{3 + \sum_{i=1}^{\infty} \frac{x_i}{2^i}}{2} \cdot 2^{-1/\limsup_n (x_n)} &\geq 2 \cdot \frac{3 + \sum_{i=1}^{\infty} \frac{x_i}{2^i}}{2} \cdot 2^{-1/\limsup_n (y_n)} \\ &> 2 \cdot 2^{-1/\limsup_n (y_n)} \\ &> \frac{3 + \sum_{i=1}^{\infty} \frac{y_i}{2^i}}{2} \cdot 2^{-1/\limsup_n (y_n)} \end{aligned}$$

thus $W(\boldsymbol{x}) > W(\boldsymbol{y})$. If $\limsup_n(y_n) = y = 0$ but $\limsup_n(x_n) = x > y = 0$, then $W(\boldsymbol{x}) > 1 > W(\boldsymbol{y})$. If $\limsup_n(y_n) = y < 0$ but $\limsup_n(x_n) = x \geq 0 > y$, then $W(\boldsymbol{x}) > -1 > W(\boldsymbol{y})$. If $\limsup_n(x_n) = x$ is negative, then $\limsup_n(y_n) = y < x = \limsup_n(x_n) < 0$ and $2^{1/\limsup_n(x_n)} \leq 2 \cdot 2^{1/\limsup_n(y_n)}$. Because $\sum_{i=1}^{\infty} \frac{x_i}{2^i}$ takes values in $(-1,1)$ and $\sum_{i=1}^{\infty} \frac{y_i}{2^i}$ takes values in $[-1,1)$, we deduce

$$\begin{aligned}\frac{3 - \sum_{i=1}^{\infty} \frac{x_i}{2^i}}{2} \cdot 2^{1/\limsup_n(x_n)} &\geq 2 \cdot \frac{3 - \sum_{i=1}^{\infty} \frac{x_i}{2^i}}{2} \cdot 2^{1/\limsup_n(y_n)} \\ &> 2 \cdot 2^{1/\limsup_n(y_n)} \\ &\geq \frac{3 - \sum_{i=1}^{\infty} \frac{y_i}{2^i}}{2} \cdot 2^{1/\limsup_n(y_n)}\end{aligned}$$

thus $W(\boldsymbol{x}) > W(\boldsymbol{y})$. The above cases exhaust all possibilities, and therefore our claim is established.

In order to prove that W satisfies Strong Pareto, suppose that $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$ and $\boldsymbol{x} > \boldsymbol{y}$. By construction, $\sum_{i=1}^{\infty} \frac{x_i}{2^i} > \sum_{i=1}^{\infty} \frac{y_i}{2^i}$. If $\limsup_n(y_n)$ is positive, then so is $\limsup_n(x_n)$, and because $\limsup_n(x_n) \geq \limsup_n(y_n)$, we obtain $W(\boldsymbol{x}) > W(\boldsymbol{y})$. If $\limsup_n(y_n)$ is zero but $\limsup_n(x_n)$ is positive, then $W(\boldsymbol{x}) > 1 > W(\boldsymbol{y})$. If both $\limsup_n(y_n)$ and $\limsup_n(x_n)$ are zero, then $W(\boldsymbol{x}) = \sum_{i=1}^{\infty} \frac{x_i}{2^i} > \sum_{i=1}^{\infty} \frac{y_i}{2^i} = W(\boldsymbol{y})$. If $\limsup_n(y_n)$ is negative but $\limsup_n(x_n)$ is positive or zero, then $W(\boldsymbol{x}) > -1 > W(\boldsymbol{y})$. If both $\limsup_n(x_n)$ and $\limsup_n(y_n)$ are negative, then $W(\boldsymbol{x}) > W(\boldsymbol{y})$ again. The above cases exhaust all logical possibilities, and therefore our claim is established. □

Remark 1. Our (explicit) SWF W also satisfies Restricted Non-Substitution and Strong Pareto when the possible utilities are the reciprocals of positive integers (Dubey and Mitra, 2011, Example 2) (which captures such a possibility that the poorer one generation is, the finer the perception is).

4 Anonymous and sensitive SWFs

Alcantud and García-Sanz (2010b), Proposition 3 states that Anonymity and Restricted Non-Substitution can be combined into explicit expressions under weaker forms of efficiency when the feasible utilities are the non-negative integers. We show that those axioms can also be jointly combined by means of explicit evaluations when the possible utilities are the inverses of positive and negative integers.

Proposition 2. *Let $\boldsymbol{X} = Y^{\mathbb{N}}$, where $Y = \{-1, -1/2, -1/3, \dots\} \cup \{1, 1/2, 1/3, \dots\}$.*

(a) There are explicit SWFs on $\boldsymbol{X}$ that satisfy Anonymity, Sensitivity, and Restricted Non-Substitution.

(b) There are explicit SWFs on $\boldsymbol{X}$ that satisfy Anonymity, Hammond Equity -Lauwers' version-, Lower Sensitivity, and Restricted Non-Substitution.

Proof. (a) For any $\boldsymbol{x} = (x_1, x_2, x_3, \dots) \in \boldsymbol{X}$, let

$$W_U(\boldsymbol{x}) = \begin{cases} 1 & \text{if } \liminf_n(x_n) = 1 \\ \liminf_n(x_n) + \frac{\sup_n(x_n)}{2} \cdot \frac{(\liminf_n(x_n))^2}{1 - \liminf_n(x_n)} & \text{otherwise.} \end{cases}$$

This is an anonymous SWF on $\boldsymbol{X}$.

In order to prove that W_U satisfies Sensitivity, suppose that $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$, $\boldsymbol{x} = (x)_{\text{con}}$, and $\boldsymbol{y} = (y, (x)_{\text{con}})$ with $y > x$. We need to ensure $W_U(\boldsymbol{y}) > W_U(\boldsymbol{x})$. Because $\liminf_n(x_n) = \liminf_n(y_n) = x < y \leq 1$ and

$x^2/(1-x) > 0$, we deduce

$$\liminf_n(y_n)+\frac{\sup_n(y_n)}{2}\cdot\frac{(\liminf_n(y_n))^2}{1-\liminf_n(y_n)} \geq x+\frac{y}{2}\cdot\frac{x^2}{1-x} > x+\frac{x}{2}\cdot\frac{x^2}{1-x} = \liminf_n(x_n)+\frac{\sup_n(x_n)}{2}\cdot\frac{(\liminf_n(x_n))^2}{1-\liminf_n(x_n)}$$

thus $W_U(\boldsymbol{y}) > W_U(\boldsymbol{x})$.

In order to prove that W_U satisfies Restricted Non-Substitution, let $\boldsymbol{x},\boldsymbol{y} \in \boldsymbol{X}$ be such that $\boldsymbol{x} = (x_1,\ldots,x_k,(x)_{\text{con}})$ and $\boldsymbol{y} = (y_1,\ldots,y_l,(y)_{\text{con}})$ with $x > y$. We need to ensure $W_U(\boldsymbol{x}) > W_U(\boldsymbol{y})$. If $\liminf_n(x_n) = x = 1$ and $\liminf_n(y_n) = y \leq 1/2$, then we deduce

$$\liminf_n(y_n) + \frac{\sup_n(y_n)}{2}\cdot\frac{(\liminf_n(y_n))^2}{1-\liminf_n(y_n)} \leq y + \frac{1}{2}\cdot\frac{y^2}{1-y} \leq \frac{3}{4} < 1$$

thus $W_U(\boldsymbol{x}) > W_U(\boldsymbol{y})$. If $\liminf_n(x_n) = x \leq 1/2$ and $\liminf_n(y_n) = y \leq 1/3$, then we deduce $\liminf_n(x_n)+\frac{\sup_n(x_n)}{2}\cdot\frac{(\liminf_n(x_n))^2}{1-\liminf_n(x_n)} \geq x+\frac{x}{2}\cdot\frac{x^2}{1-x}$ and $\liminf_n(y_n)+\frac{\sup_n(y_n)}{2}\cdot\frac{(\liminf_n(y_n))^2}{1-\liminf_n(y_n)} \leq y+\frac{1}{2}\cdot\frac{y^2}{1-y}$, where

$$\begin{aligned}
\frac{1}{2}\cdot\frac{y^2}{1-y} - \frac{x}{2}\cdot\frac{x^2}{1-x} &\leq \frac{1}{2}\cdot\left(\frac{y^2}{1-y} - \frac{y^3}{(1-y)^2(1-2y)}\right) \\
&= \frac{1}{2}\cdot\frac{y^2}{1-y}\cdot\left(1-\frac{y}{(1-y)(1-2y)}\right) \\
&\leq \frac{7}{12}\cdot\frac{y^2}{1-y} \\
&< \frac{y^2}{1-y} \\
&= \frac{y}{1-y} - y \\
&\leq x - y
\end{aligned}$$

because $x \geq y/(1-y)$ and $y^2/(1-y) > 0$. Thus $W_U(\boldsymbol{x}) > W_U(\boldsymbol{y})$.

(b) For any $\boldsymbol{x} = (x_1,x_2,x_3,\ldots) \in \boldsymbol{X}$, let

$$W_L(\boldsymbol{x}) = \begin{cases} -1 & \text{if } \liminf_n(x_n) = -1 \\ \liminf_n(x_n) + \frac{\inf_n(x_n)}{2}\cdot\frac{(\liminf_n(x_n))^2}{1+\liminf_n(x_n)} & \text{otherwise.} \end{cases}$$

This is an anonymous SWF on $\boldsymbol{X}$.

In order to prove that W_L satisfies Hammond Equity -Lauwers' version-, let $\boldsymbol{x},\boldsymbol{y} \in \boldsymbol{X}$ be such that $x_j \geq y_j \geq y_k \geq x_k$ for some $j,k \in \mathbb{N}$, and $x_t = y_t$ when $j \neq t \neq k$. We need to ensure $W_L(\boldsymbol{y}) \geq W_L(\boldsymbol{x})$. If $\liminf_n(x_n) = \liminf_n(y_n) = -1$, then we deduce $W_L(\boldsymbol{x}) = W_L(\boldsymbol{y}) = -1$. If $\liminf_n(x_n) = \liminf_n(y_n) \geq -1/2$, then we deduce

$$\liminf_n(x_n) + \frac{\inf_n(x_n)}{2}\cdot\frac{(\liminf_n(x_n))^2}{1+\liminf_n(x_n)} \leq \liminf_n(y_n) + \frac{\inf_n(y_n)}{2}\cdot\frac{(\liminf_n(y_n))^2}{1+\liminf_n(y_n)}$$

because $\inf_n(x_n) \leq \inf_n(y_n)$ and $(\liminf_n(x_n))^2/(1+\liminf_n(x_n)) > 0$. Thus $W_L(\boldsymbol{x}) \leq W_L(\boldsymbol{y})$.

In order to prove that W_L satisfies Lower Sensitivity, suppose that $\boldsymbol{x},\boldsymbol{y} \in \boldsymbol{X}$, $\boldsymbol{x} = (x)_{\text{con}}$, and $\boldsymbol{y} = (y,(x)_{\text{con}})$ with $y < x$. We need to ensure $W_L(\boldsymbol{x}) > W_L(\boldsymbol{y})$. Because $\liminf_n(x_n) = \liminf_n(y_n) = x > y \geq -1$ and $x^2/(1+x) > 0$, we deduce

$$\liminf_n(x_n)+\frac{\inf_n(x_n)}{2}\cdot\frac{(\liminf_n(x_n))^2}{1+\liminf_n(x_n)} \geq x+\frac{x}{2}\cdot\frac{x^2}{1+x} > x+\frac{y}{2}\cdot\frac{x^2}{1+x} = \liminf_n(y_n)+\frac{\inf_n(y_n)}{2}\cdot\frac{(\liminf_n(y_n))^2}{1+\liminf_n(y_n)}$$

thus $W_L(\boldsymbol{x}) > W_L(\boldsymbol{y})$.

In order to prove that W_L satisfies Restricted Non-Substitution, let $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$ be such that $\boldsymbol{x} = (x_1, \ldots, x_k, (x)_{\text{con}})$ and $\boldsymbol{y} = (y_1, \ldots, y_l, (y)_{\text{con}})$ with $x > y$. We need to ensure $W_L(\boldsymbol{x}) > W_L(\boldsymbol{y})$. If $\liminf_n(y_n) = y = -1$ and $\liminf_n(x_n) = x \geq -1/2$, then we deduce

$$\liminf_n(x_n) + \frac{\inf_n(x_n)}{2} \cdot \frac{(\liminf_n(x_n))^2}{1 + \liminf_n(x_n)} \geq x + \frac{-1}{2} \cdot \frac{x^2}{1+x} \geq -\frac{3}{4} > -1$$

thus $W_L(\boldsymbol{x}) > W_L(\boldsymbol{y})$. If $\liminf_n(y_n) = y \geq -1/2$ and $\liminf_n(x_n) = x \geq -1/3$, then we deduce $\liminf_n(x_n) + \frac{\inf_n(x_n)}{2} \cdot \frac{(\liminf_n(x_n))^2}{1+\liminf_n(x_n)} \geq x + \frac{-1}{2} \cdot \frac{x^2}{1+x}$ and $\liminf_n(y_n) + \frac{\inf_n(y_n)}{2} \cdot \frac{(\liminf_n(y_n))^2}{1+\liminf_n(y_n)} \leq y + \frac{y}{2} \cdot \frac{y^2}{1+y}$, where

$$\begin{aligned}
\frac{y}{2} \cdot \frac{y^2}{1+y} - \frac{-1}{2} \cdot \frac{x^2}{1+x} &\leq \frac{1}{2} \cdot \left(\frac{x^3}{(1+x)^2(1+2x)} + \frac{x^2}{1+x} \right) \\
&= \frac{1}{2} \cdot \frac{x^2}{1+x} \cdot \left(\frac{x}{(1+x)(1+2x)} + 1 \right) \\
&\leq \frac{7}{12} \cdot \frac{x^2}{1+x} \\
&< \frac{x^2}{1+x} \\
&= x - \frac{x}{1+x} \\
&\leq x - y
\end{aligned}$$

because $y \leq x/(1+x)$ and $x^2/(1+x) > 0$. Thus $W_L(\boldsymbol{x}) > W_L(\boldsymbol{y})$. □

Remark 2. Our SWF W_U (resp. W_L) also satisfies Anonymity, Sensitivity (resp. Hammond Equity -Lauwers' version- and Lower Sensitivity), and Restricted Non-Substitution when the feasible utilities are the reciprocals of positive integers.

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