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Abstract

We develop a monetary growth model incorporating wage stickiness. The steady-state analysis provides three notable conclusions that are instructive to policy-makers: First, if the inflation rate is higher than a certain level, the long-run Phillips curve will be vertical, and the unemployment rate will be minimum. Second, if the inflation rate is lower than the level, the long-run Phillips curve will be diagonal, and lower inflation rate will bring higher unemployment rate. Finally, the money growth rate of less than the technological change rate brings long-run deflation and negative employment gap.

1 Introduction

Since Sidrauski (1967) was published, the studies of the monetary growth theory using the intertemporal optimizing model have discussed the long-run superneutraliy of money, which means that the monetary growth does not influence real variables in the steady state.

The models of these studies include capital stock or technological change, which are ordinarily analyzed by long-run macroeconomic theory, but exclude price stickiness, which are ordinarily analyzed by short-run macroeconomic theory.

Our study aims to combine long-run and short-run models, and develop a kind of integrated model. Specifically, we introduce the New Keynesian Phillips curve (NKPC) into a Sidrauski-type monetary growth model with capital stock and technological change.

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Not the price of goods but the price of labor, i.e., nominal wage is assumed to be sticky in the model, because the nominal wage is stickier than the price of goods in the actual economy.

In general, there are three basic types of the NKPC: Calvo, Taylor, and Rotemberg types. Moreover, each NKPC has two variants based on whether the natural rate hypothesis is valid or not.

For example, the Rotemberg-type NKPC requires the assumption of the adjustment cost such as $\gamma\frac{(\omega-\omega^*)^2}{2}$ or $\gamma\frac{\omega^2}{2}$. ω is the change rate of nominal wage, ω^* is the steady-state change rate of nominal wage, and γ is parameter. If $\gamma\frac{(\omega-\omega^*)^2}{2}$ is assumed as adjustment cost, the wage stickiness will vanish in the steady state, and the natural rate hypothesis will be valid.

We adopt $\gamma\frac{\omega^2}{2}$, which is inconsistent with the hypothesis, in order to consider the long-run effect of wage stickiness. Note that the money growth rate might not influence the steady-state unemployment rate even in this case, because the long-run Phillips curve can stand vertically at a rate except the natural rate.

We are concerned with the long-run superneutrality of money, then examine whether monetary growth influences the real rate of interest, capital stock, and unemployment rate in the steady state.

In particular, we focus on the verticality of the long-run Phillips curve. If the curve is not vertical, it will be an issue how high the money growth rate brings the natural unemployment rate. We also resolve the issue.

2 Model

Suppose that there is an economy consisting of four types of agent: households, firms, retailers, and employment agencies. Each is continuously distributed, and the total number of each of these agents is normalized at unity.

In the economy, there are four markets. Markets of differentiated goods and differentiated labor forces are monopolistically competitive. Markets of composite goods and composite labor forces are perfectly competitive.

Firm i produces differentiated goods i ($i \in [0, 1]$), and sells retailers them. A retailer assembles differentiated goods into composite goods, and sells households them. A household consumes composite goods or accumulates them as capital stock.

Household j supplies differentiated labor forces j ($j \in [0, 1]$) to employment agencies.

An employment agency assembles differentiated labor forces into composite labor forces, and sells firms them.

Retailers

Each retailer assembles differentiated goods into composite goods on the basis of the Dixit-Stiglitz function[1]. The quantity of composite goods y is given by

$$y = \left[\int_0^1 y_i^{\frac{\phi-1}{\phi}} di \right]^{\frac{\phi}{\phi-1}}, \tag{2.1}$$

where y_i is the quantity of differentiated goods i. ϕ is the elasticity of substitution among differentiated goods. We assume that ϕ is constant over time, and $\phi > 1$.

Let p_i denote the price of goods i set by firm i. When the retailer minimizes its cost,

$$y_i = \left(\frac{p_i}{p} \right)^{-\phi} y \tag{2.2}$$

holds, where the price of composite goods p is given by

$$p = \left[\int_0^1 p_i^{1-\phi} di \right]^{\frac{1}{1-\phi}}. \tag{2.3}$$

All retailers sell composite goods at this price.

Employment Agencies

Each employment agency assembles differentiated labor forces into composite labor forces on the basis of the Dixit-Stiglitz function. The quantity of composite labor forces h is given by

$$h = \left[\int_0^1 h_j^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}},$$

where h_j is the quantity of differentiated labor forces j. η is the elasticity of substitution among differentiated labor forces. We assume that η is constant over time, and $\eta > 1$.

Let W_j denote the nominal wage rate of labor forces j set by household j. When the employment agency minimizes its cost,

$$h_j = \left(\frac{W_j}{W} \right)^{-\eta} h \tag{2.4}$$

[1] See Dixit and Stiglitz (1977) and Blanchard and Kiyotaki (1987).

holds, where the nominal wage rate of composite labor forces W is given by

$$W = \left[\int_0^1 W_j^{1-\eta} dj \right]^{\frac{1}{1-\eta}}. \tag{2.5}$$

All employment agencies sell composite labor forces at this wage rate.

In addition, h_j is given by $h_j = zl_j$, where l_j is a number of employed workers in household j and z is the technological level of a worker, which grows at a constant rate $g(\equiv \dot{z}/z)$.

Firms

The production function of firm i is given by

$$y_i = k_i^{\alpha} h_i^{1-\alpha} \quad (0 < \alpha < 1), \tag{2.6}$$

where k_i is the capital stock and h_i is the labor forces employed by firm i. For the sake of simplicity, we assume that capital stock does not depreciate.

Solving the intertemporal optimization problem of firm i:

$$\begin{aligned} &\max \Pi_i = \frac{p_i}{p} y_i - wh_i - rk_i, \\ &\text{subject to } y_i = \left(\frac{p_i}{p} \right)^{-\phi} y, \quad y_i = k_i^{\alpha} h_i^{1-\alpha}, \end{aligned} \tag{2.7}$$

yields

$$\frac{w}{\beta_\phi} = (1-\alpha)\hat{h}^{-\alpha}, \tag{2.8}$$

$$\frac{r}{\beta_\phi} = \alpha \hat{h}^{1-\alpha}, \tag{2.9}$$

where $w(\equiv W/p)$ is the real wage rate of composite labor forces, $\hat{h}(\equiv h/k)$ is capital-labor ratio, $k(\equiv \int_0^1 k_i di)$ is the aggregate capital stock, β_ϕ is defined as $\beta_\phi \equiv \frac{\phi-1}{\phi}$, and $1/\beta_\phi$ represents the markup rate.

Households

In the economy, working hours per capita is constant, and a lottery determines whether a worker becomes employed or unemployed. These assumptions are based on the concept of the indivisible labor presented by Hansen (1985).

However, there is a continuum of workers from 0 to 1 in each household, and each household institutes a lottery system unlike with Hansen (1985). This means that some are employed workers and the others are unemployed workers in each household.

$l_j(\leq 1)$ denotes the number of employed workers (employment rate) and $\upsilon_j(\equiv 1-l_j \leq 1)$ denotes the number of unemployed workers (unemployment rate) in household j.

The endowment of time that is normalized to be one, and WORK (<1) denotes the proportion of employed worker's working hours.

Household j gains utility also from consumption c_j, real cash balances m_j, and leisure, then the instantaneous utility of household j is

$$\ln c_j + \ln m_j + l_j\Psi - \frac{\gamma}{2}\omega_j^2, \qquad \Psi < 0, \tag{2.10}$$

where Ψ is defined as $\Psi \equiv \psi \ln(1-\text{WORK})$ and ψ is the utility weight on leisure [2]. Since WORK is a constant, Ψ is also a constant.

$\omega_j(\equiv \dot{W}_j/W_j)$ is the change rate of nominal wage, $\frac{\gamma}{2}\omega_j^2$ represents the cost of wage adjustment [3], and the parameter γ measures the degree to which a household dislikes the wage change. $\gamma \to 0$ implies flexible wage and $\gamma > 0$ implies sticky wage.

Each household's assets consist of money and equities. Thus, the nominal asset balances of the household A is $A = M + Q$, where M is nominal money balances, Q is the nominal equity price, and the quantity of equities is normalized at unity. Household's budget constraint is given by $\dot{A} = \dot{Q} + D + wpl - pc$, where D is nominal dividends, and c is real consumption. Rewriting this equation gives $\dot{a} = ra + wl - c - Rm$, where $a(\equiv A/p)$ is real assets, $R(\equiv \frac{\dot{Q}+D}{Q})$ is the nominal interest rate, $r(\equiv R/\pi)$ is the real interest rate, and $m(\equiv M/p)$ is real money balances.

[2]The instantaneous utility that household j gains from the leisure is

$$l_j\psi \ln(1-\text{WORK}) + (1-l_j)\psi \ln 1, \tag{2.11}$$

where $\psi \ln(1-\text{WORK})$ represents the utility of the employed worker and $\psi \ln 1$ represents the utility of the unemployed worker. Since $\ln 1 = 0$, (2.11) can be rewritten as $l_j\psi \ln(1-\text{WORK})$.

[3]See Rotemberg (1982).

Solving the intertemporal optimization problem of household j:

$$\begin{aligned}
&\max_{c_j, m_j, \omega_j} \int_0^\infty \left[\ln c_j + \ln m_j + l_j \Psi - \frac{\gamma}{2}\omega_j^2\right] e^{-\rho t} dt, \\
\text{subject to} \quad & \dot{a} = r a_j + w_j h_j - c_j - R m_j, \\
& \dot{W}_j = \omega_j W_j, \\
& h_i = \left(\frac{W_j}{W}\right)^{-\phi} h, \\
& h_j = z l_j, \\
& l_j \leq 1,
\end{aligned}$$

yields

$$\frac{\dot{c}}{c} + \rho + \pi = R = \frac{c}{m}, \tag{2.12}$$

$$\frac{\dot{\omega}}{\omega} \leq \rho + \left[(\eta - 1)\frac{hw}{c} + \Psi \eta l\right] \frac{1}{\gamma \omega} \quad \text{with equality when } l < 1, \tag{2.13}$$

where $\rho(> 0)$ is the subjective discount rate. (2.13) represents the wage version of the New Keynesian Phillips curve.

Monetary Growth

Financial authorities are assumed to expand money supply M by a constant rate θ. Therefore, the financial policy rule is given by $\dot{M}/M = \theta$. Since real money balances m is defined as $m = M/p$, $\dot{m}/m = \theta - \pi$ holds.

3 Steady-state analysis

Case of $l < 1$

We assume $l < 1$. Thus, by considering the clearing condition of the composite goods market $y = c + I$ where $I \equiv \dot{k}$, we obtain the system of differential equations:

$$
\begin{aligned}
\frac{\dot{R}}{R} &= R - (\theta + \rho), && (3.1)\\
\frac{\dot{\hat{c}}}{\hat{c}} &= \alpha\beta_{\phi}\hat{h}^{1-\alpha} - \rho - \hat{h}^{1-\alpha} + \hat{c}, && (3.2)\\
\frac{\dot{\hat{h}}}{\hat{h}} &= \frac{1}{\alpha}(R - \alpha\beta_{\phi}\hat{h}^{1-\alpha} - \omega), && (3.3)\\
\frac{\dot{l}}{l} &= \frac{\dot{\hat{h}}}{\hat{h}} + \hat{h}^{1-\alpha} - \hat{c} - g, \\
&= \frac{1}{\alpha}(R - \alpha\beta_{\phi}\hat{h}^{1-\alpha} - \omega) + \hat{h}^{1-\alpha} - \hat{c} - g, && (3.4)\\
\frac{\dot{\omega}}{\omega} &= \rho + \left[(\eta - 1)\frac{(1-\alpha)\beta_{\phi}\hat{h}^{1-\alpha}}{\hat{c}} + \Psi\eta l\right]\frac{1}{\gamma\omega}, && (3.5)
\end{aligned}
$$

where $\hat{c} \equiv c/k$.

The variables evaluated in the nontrivial steady state are

$$
\begin{aligned}
R^* &= \theta + \rho, && (3.6)\\
\hat{c}^* &= \frac{\rho + g}{\alpha\beta_{\phi}} - g, && (3.7)\\
\hat{h}^* &= \left(\frac{\rho + g}{\alpha\beta_{\phi}}\right)^{\frac{1}{1-\alpha}}, && (3.8)\\
l^* &= -\frac{1}{\Psi}\left[\frac{\gamma\omega\rho}{\eta} + \beta_{\eta}\beta_{\phi}\frac{(1-\alpha)(\rho+g)}{\rho + g - g\alpha\beta_{\phi}}\right], && (3.9)\\
\omega^* &= \pi^* = \theta - g, && (3.10)
\end{aligned}
$$

where variables with * denote the steady-state values.

Case of $l = 1$

In the case of $l = 1$, since $\dot{l}/l = 0$ holds, we obtain the system of differential equations:

$$\frac{\dot{R}}{R} = R - (\theta + \rho), \tag{3.11}$$

$$\frac{\dot{\hat{c}}}{\hat{c}} = \alpha\beta_\phi \hat{h}^{1-\alpha} - \rho - \hat{h}^{1-\alpha} + \hat{c}, \tag{3.12}$$

$$\frac{\dot{\hat{h}}}{\hat{h}} = -\hat{h}^{1-\alpha} + \hat{c} + g, \tag{3.13}$$

The variables evaluated in the nontrivial steady state are

$$R^* = \theta + \rho, \tag{3.14}$$

$$\hat{c}^* = \frac{\rho + g}{\alpha\beta_\phi} - g, \tag{3.15}$$

$$\hat{h}^* = \left(\frac{\rho + g}{\alpha\beta_\phi}\right)^{\frac{1}{1-\alpha}}. \tag{3.16}$$

l^*, ω^*, and π^* are

$$l^* = 1, \tag{3.17}$$

$$\omega^* = \pi^* = \theta - g. \tag{3.18}$$

Supernuetrality of money

(3.10) and (3.18) give the following proposition.

Proposition 1 The steady-state inflation rate π^* is equivalent to the difference between the money growth rate θ and the technological change rate g.

Measuring in terms of steady-state unemplyment rate $\upsilon^*(\equiv 1 - l^*)$, from (3.9) and (3.17) we obtain

$$\upsilon^* = \begin{cases} 1 + \dfrac{1}{\Psi}\left[\dfrac{\gamma\omega\rho}{\eta} + \beta_\eta\beta_\phi\dfrac{(1-\alpha)(\rho+g)}{\rho + g - g\alpha\beta_\phi}\right] & \text{if } \pi \leq \bar{\pi}, \\ \upsilon_m(=0) & \text{if } \pi > \bar{\pi}, \end{cases}$$

where υ_m denotes the minimum unemployment rate, and $\bar{\pi}$ is a constant, which is defined as

$$\bar{\pi} \equiv -\frac{\eta}{\gamma\rho}\left[\Psi + \beta_\eta\beta_\phi\frac{(1-\alpha)(\rho+g)}{\rho + g - g\alpha\beta_\phi}\right].$$

This equation gives the following proposition.

Proposition 2 If $\pi^* < \bar{\pi}$, higher steady-state inflation rate π^* will bring lower steady-state unemployment rate υ^*. If $\pi^* > \bar{\pi}$, the steady state unemployment rate υ^* will remain at the minimum rate $\upsilon_m(=0)$, irrespective of the inflation rate π^*.

r^*, $k^*$are

$$
\begin{aligned}
r^* &= g + \rho &(3.19)\\
k^* &= \frac{zl^*}{\hat{h}^*} &(3.20)
\end{aligned}
$$

Therefore, we obtain the following proposition about the superneutrality of money.

Proposition 3 If $\pi > \bar{\pi}$, the steady-state capital stock k^* and the unemployment rate υ^* will not depend on the money growth rate θ. If $\pi < \bar{\pi}$, the steady-state capital stock k^* and the unemployment rate υ^* will depend on the money growth rate θ. The steady-state real rate of interest r^* and capital-labor ratio $\hat{h}^*$ do not depend on money growth rate θ in any case.

Natural unemployment rate

Next, we would like to discuss the natural rate υ_n, which is the unemployment rate in the flexible-wage economy ($\gamma \to 0$). υ_n is given by

$$\upsilon_n = 1 + \frac{1}{\Psi}\left[\beta_\phi\beta_\eta\frac{(1-\alpha)(\rho+g)}{\rho+g-g\alpha\beta_\phi}\right]. \tag{3.21}$$

(3.9) and (3.21) give that if $\bar{\pi} \geq 0$, $\pi = 0$ brings $\upsilon^* = \upsilon_n$. If $\bar{\pi} < 0$, $\upsilon_n < \upsilon_m(=0)$ holds, and then $\upsilon^* = \upsilon_n$ is unachievable. $\Psi = \bar{\Psi}$ brings $\bar{\pi} = 0$, where $\bar{\Psi}$ is defined as

$$\bar{\Psi} = \beta_\phi\beta_\eta\frac{(1-\alpha)(\rho+g)}{\rho+g-g\alpha\beta_\phi}. \tag{3.22}$$

Therefore, we obtain the two following propositions.

Proposition 4 Let $\Psi \geq \bar{\Psi}$. Zero inflation rate makes the steady-state unemployment rate attain the natural rate ($\upsilon^* = \upsilon_n$). Deflation brings underemployment, namely negative employment gap ($\upsilon^* - \upsilon_n < 0$). Inflation brings excess employment, namely positive employment gap ($\upsilon^* - \upsilon_n > 0$).

Proposition 5 Let $\Psi < \bar{\Psi}$. The natural rate υ_n becomes less than the minimum rate υ_m. Thus, the steady-state unemployment rate υ^* is unable to attain the natural rate υ_n.

Long-run Phillips curve

If $\Psi \geq \bar{\Psi}$, the long-run Phillips curve is described as Figure 1. The vertical axis represents the steady-state inflation rate, the horizontal axis represents the steady-state unemployment rate, and the red line represents the long-run relation between the inflation rate and the unemployment rate.

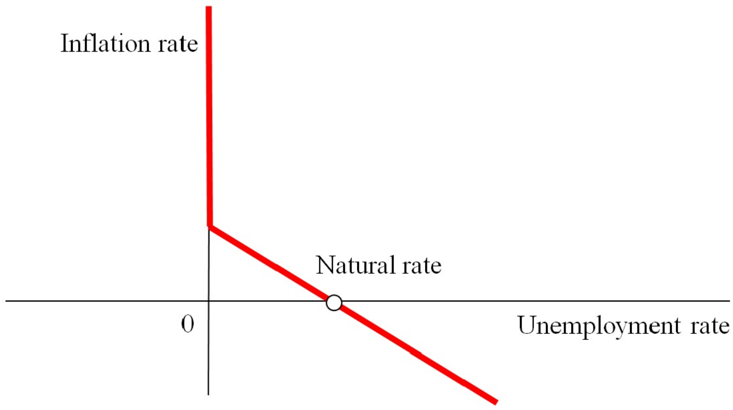

Figure 1: Long-run Phillips curve 1

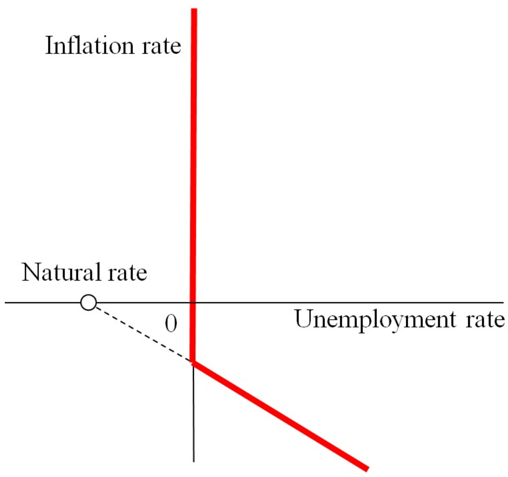

Figure 2: Long-run Phillips curve 2

If $\Psi < \bar{\Psi}$, the long-run Phillips curve is described as Figure 2. A kink point is below the horizontal axis on the graph, and then unemployment rate cannot attain the natural rate.

There exists a kink in any case. Let us term such as these the kinked version of long-run Phillips curve.

Calibrating the parameters with WORK$= 0.33$, $\psi = 2$, $\phi = 6$, $\eta = 21$, $\rho = 0.01$, $g = 0.01$, $\psi = 1$, $\alpha = 0.4$ gives $\Psi = 0.83$, $\bar{\Psi} = 0.000031$, and $\Psi > \bar{\Psi}$ [4] . Therefore, we argue that not Figure 2 but Figure 1 has realistic relevance and unemployment rate can attain the natural rate.

4 Conclusion

We have developed a monetary growth model incorporating wage stickiness. The steady-state analysis provides three notable conclusions that are instructive to policy-makers: First, if the inflation rate is higher than a certain level, the long-run Phillips curve will be vertical, and the unemployment rate will be minimum. Second, if the inflation rate is lower than the level, the long-run Phillips curve will be slanted, and lower inflation rate will bring higher unemployment rate. Finally, the money growth rate of less than the technological change rate brings long-run deflation and negative employment gap.

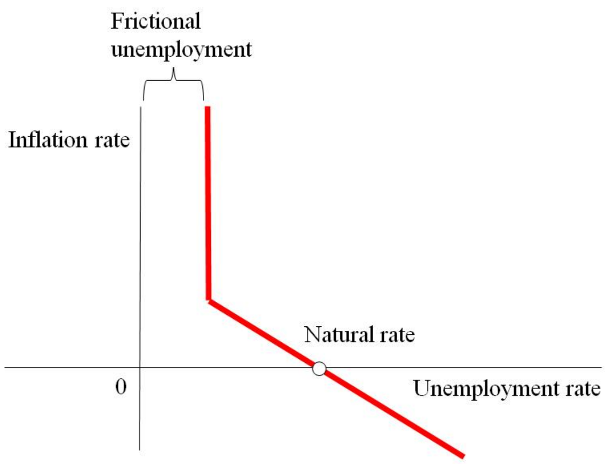

Figure 3: Long-run Phillips curve 3

[4]These values are used by Hansen (1985) or Fujiwara (2007)

While the minimum rate of unemployment is zero in this study, the rate is altered by considering the frictional unemployment, and then Figure 1 is rewritten as Figure 3, which is closer to reality.

This graph, which is similar to the long-run Phillips curve presented by Akerlof et al. (1996), implies that the long-run unemployment rate cannot be lower than a positive level, but can be higher than the level.

Anyway, we would like to emphasize the importance of such an asymmetry and the kink of the long-run Phillips curve. However, if someone demonstrates statistically based on data only from inflationary economies, he will obtain the result that the long-run Phillips curve is completely vertical.

Major country experienced relatively high inflation in the late 20th century, in which macroeconomics developed rapidly. Therefore, it is little wonder that most economists have supported the completely vertical version of long-run Phillips curve.

We put little emphasis on the difference between the minimum rate and natural rate of unemployment. Let us interpret the natural rate hypothesis as the meaning that policy-makers cannot maintain unemployment rate less and more than a certain level in the long run. Thus, the verticality of the long-run Phillips curve justifies the natural rate hypothesis.

However, we can now recognize that the natural rate hypothesis speaks only half true. The other half, to which advocators of the hypothesis close their eyes, are really important in disinflation or deflation times.

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