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Subsidization in Mixed Oligopoly with Managerial Delegation: Price Competition

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Abstract

This paper studies optimal production subsidy in a mixed duopoly with separation between ownership and management. Although many existing literature shows that the first best production allocation is achieved under several economic environments (so-called irrelevance result), we find that this result holds in the following two cases. (1) The owners simultaneously/sequentially decide their sales weight in the managerial delegation contracts. (2) The managers simultaneously/sequentially choose their quantity levels. Therefore, the irrelevance result is robust against the introduction of sales delegation with separation between ownership and management, even if the firms simultaneously/sequentially determine their sales delegation weights or quantity levels.

JEL Classification: D21, L13, L33

Keywords: Mixed Duopoly, Managerial Delegation, Price Compatition, Subsidization, Privatization

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INTRODUCTION

This paper presents a theoretical analysis of the optimal subsidization issue in a mixed duopolistic industry with Bertrand competition style – one welfare-maximizing public firm and one profit-maximizing private firm. In particular, we consider the situation wherein both the firms are managerial, *i.e.*, the owner of each firm enters into a managerial delegation contract with the firm's manager. Thus, we conduct an analysis where both the optimal production subsidy and each firm's delegation parameter are taken into account.[1] More precisely, we scrutinize the impact of production subsidy on the delegation contract and on the equilibrium market outcomes.

Many works have studied subsidization issue in a mixed oligopoly. White (1996), considered to be seminal work in this field, showed that the first best allocation with respect to social welfare is achieved in a simultaneous setting of a mixed oligopoly containing of one public firm and n private firms, if the government first commits to the production subsidy. Further, he found that such a first best allocation is observed again after the privatization of the public firm.[2] After White (1996), research on subsidization in mixed oligopoly has focused on confirming the robustness of White (1996)'s result against several types of changes in the economic environment: Poyago-Theotoky (2001) and Myles (2002) showed that White 1996's result holds in a Stackelberg competition where the public firm becomes the leader. Hashimzade, Khodavaisi and Myles (2007) showed that the irrelevance result extends to both quantity and price competitions with product differentiation.[3] However, the above papers assumed that both the public firm and the private firms are assumed to be entrepreneurial, *i.e.*, every managerial decision is made by the owner. Thus, in their models, the separation between ownership and management is ignored. In this paper, we consider subsidization in a mixed duopoly where each firm is managerial, *i.e.*, each firm's ownership and management are separable.

Past literature on managerial delegation challenged the traditional assumption that the objec-

[1]As in this paper, Tomaru, Nakamura and Saito (2009) tackled the optimal subsidization problem with managerial delegation in a quantity setting mixed oligopoly, and showed that the optimal subsidies yield the first-best allocation in Cournot mixed and private duopolies, and mixed duopolies with public output and delegation leadership.

[2]This result is referred to the "an irrelevance result" in Poyago-Theotoky (2001).

[3]Other works on subsidization in mixed oligopoly – Tomaru (2006), Kato and Tomaru (2007), and Tomaru and Saito (2009) – investigated the robustness of White (1996)'s result. More precisely, Tomaru (2006) showed that the irrelevance result holds under Matsumura (1998)'s partial privatization. Kato and Tomaru (2007) confirmed that a result similar to that in White (1996) is also satisfied even if the private firms maximize an objective function slightly different from their own profits: the weighted average of their profits and revenues (or negative cost). More recently, Tomaru and Saito (2009) showed that the irrelevance result also holds in mixed and private duopolies even if each firm's production timing is endogenized.

tive function of a (private) firm is to solely maximize its own profit, and introduced several types of delegation contracts which owners provide to managers in a principal-agent model, with the most well-known delegation regime being the *sales delegation* case presented in Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) ($FJSV$-fashioned contract case): the owner of each firm provides the manager with an incentive contract on the basis of the firm's profit and sales. This situation is studied in the context of various economic environments.[4] In particular, in the literature on the standard mixed oligopoly, Barros (1995) and White (2001) investigated the influence of the use of $FJSV$ delegation contract on social welfare before and after privatization. Nishimori and Ogawa (2005) analyzed the interaction between the length of $FJSV$ incentive contracts and market behavior. More recently, Heywood and Ye (2009) reconsidered the effect of the $FJSV$ delegation on welfare, and identified whether or not privatizing the public firm using an optimal incentive contract reduces welfare.[5] However, there is no prior literature on optimal production subsidy with managerial delegation in all firms. Ours is the first paper to study this problem in a price-setting mixed oligopoly.

The main purpose of this paper is to check the irrelevance result against the introduction of each firm's $FJSV$ contract in a mixed oligopoly. In this paper, we consider the following two cases. (*i*) *Delegation sequencing* case–the owners simultaneously or sequentially select the content of their delegation contracts which are provided to the managers, and the managers simultaneously determine their price levels. (*ii*) *Competition sequencing* case–the managers determine

[4]Many works on managerial delegation that study different delegation regimes other than the $FJSV$ contract, we can exemplify the following five papers. Salas Fumas (1992) and Miller and Pazgal (2001) considered the situation wherein the owner provides the manager with an incentive contract based on a weighted sum of the firm's own profit and the rival's profit (which they refer to as the *market share* case). Miller and Pazgal (2002) applied the relative performance regime in various competition types and found that managers' attitudes can be used as a strategic commitment device that can increase the firms' profits in certain environments. More recently, Jansen, Lier and van Witteloostuijn (2007) and Ritz (2008) together introduced the *market-share* case wherein the delegation contract is a combination of the firm's own profit and market share. On the other hand, the $FJSV$ contract has been used in many areas of applied economics. Zhang and Zhang (1997) analyzed the benefit of strategic delegation in an R&D setting with spillovers, and Kopel and Riegler (2006) reconsidered such a problem to clarify the correct influences of delegating the production and R&D decisions to managers on owners' benefits. Subsequently, Goering (2007) considered the implications of $FJSV$ contract for non-profit firms. Furthermore, in the context of international trade, Collie (1997) and Das (1997) explored the relation between sales delegation within a firm and strategic trade policy. Furthermore, Collie (2003), González-Maestre and López-Cuñat (2001), Straume (2006), Ziss (2001), etc., studied the some with regard to managers.

[5]Further, the following works study managerial delegation in the context of mixed duopoly: Bárcena-Ruiz (2009), Nakamura (2008), Nakamura and Inoue (2007), Nakamura and Inoue (2009), and Saha and Sensarma (2008).

their prices simultaneously or sequentially, while the owners simultaneously select the content of their delegation contracts. For case (*i*), we show that when the delegation contract is being determined, the sum of each firm's equilibrium production subsidy and delegation parameter, and the equilibrium market outcomes coincide in the simultaneous move case and the two types of sequential move cases (the Stackelberg leadership cases), resulting in the achievement of the first best production allocation. Furthermore, for case (*ii*), as in case (*i*), we find that the first best production allocation is achieved from a viewpoint of social welfare in the simultaneous move case and the two types of sequential move cases. On the other hand, we obtain the result that the equilibrium production subsidy of each firm differs for the simultaneous move and the two types of sequential move cases under both cases (*i*) and (*ii*). More precisely, in case (*i*), the government incurs a higher subsidy the owner of the public firm becomes a follower, while, in case (*ii*), it must pay the highest subsidy when the manager of the public firm is the leader. Thus, in mixed oligopolistic industries, where all firms are managerial, the government have to pay higher production subsidy in order to attain the first best level of social welfare, as compared to the subsidy paid in a standard mixed oligopoly without managerial delegation.

The remainder of this paper is organized as follows. In Section 2, we formulate the basic setting of the mixed duopolistic model considered in this paper. In Section 3, we investigate the delegation sequencing case where the two owners determine the content of their $FJSV$ contracts simultaneously or sequentially, while the manager simultaneously choose their prices. In Section 4, we presents the analysis of the competition sequencing case where the managers choose their price levels simultaneously or sequentially, while the owners determine the content of their $FJSV$ contracts. Section 5 provides the concluding remarks. A detailed description of each firm's delegation parameter in all cases (given the value of each firm's production subsidy as considered in sections 3 and 4) is relegated to the Appendix.

MODEL

We formulate a differentiated products mixed duopoly with linear demand and quadratic cost. The basic structure of the model follows Dixit (1979) and Singh and Vives (1984). We, thus, have an economy containing a monopolistic sector with a public firm and a private firm, and a competitive sector producing the numeraire good. The two firms produce differentiated goods in a monopolistic sector. Henceforth, we refer to the public firm as firm 0 (private firm as firm 1) and the owner of the public firm as owner 0 (owner of private firm as owner 1). On the demand side of the market, the representative consumer maximizes $U(q_0, q_1) - p_0 q_0 - p_1 q_1$, where $q_i \geq 0$

is the amount of the good i that firm i produces and p_i is its price ($i = 0, 1$). We assume that the function $U(q_0, q_1)$ is quadratic, strictly concave, and symmetric in q_0 and q_1:

$$U(q_0, q_1) = a(q_0 + q_1) - \frac{1}{2}\left(q_0^2 + 2bq_0q_1 + q_1^2\right), \quad a > 0, \quad b \in (0, 1), \tag{1}$$

where b represents the degree of product differentiation. The specification implies the following direct demand function:

$$q_i(p_0, p_1) = \frac{a(1-b) - p_i + bp_j}{1 - b^2}, \qquad i, j = 0, 1, \ i \neq j. \tag{2}$$

We assume that the technologies of both firms are specified by quadratic functions of their own outputs, *i.e.*, $C(q_i) = kq_i^2/2$. The profit function of the firm i is as follows:

$$\Pi_i = p_iq_i - \frac{1}{2}kq_i^2 + sq_i, \qquad i, j = 0, 1, \ i \neq j, \tag{3}$$

where s is the production subsidy given by the government and as such is assumed to be non-negative. Usually, social welfare, denoted by W, is measured as the sum of consumer surplus (CS) and producer surplus (PS):

$$W = CS + PS - s(q_0 + q_1), \tag{4}$$

where $PS = \Pi_1 + \Pi_2$, and CS is given by

$$CS = U(q_0, q_1) - p_0q_0 - p_1q_1 = \frac{a^2(1-b) + p_0^2 - 2bp_0p_1 + p_1^2 - 2a(1-b)(p_0 + p_1)}{2(1-b^2)}. \tag{5}$$

Owner 0 is assumed to be a welfare maximizer, while owner 1 is assumed to maximize her/his own profit.

In addition, this paper focuses on the managerial aspects of the firms. As such, we consider the situation where the firms' owners decide to delegate control to managers. To formalize managerial delegation, we assume that each firm is owned by a single agent, and follow Lambertini (2000), Nakamura (2008), and Nakamura and Inoue (2009). In each firm i, the owner provides the following type of of incentive contract $V_i(\Pi_i, q_i)$ to her/his manger:

$$V_i(\Pi_i(q_i), q_i) = \Pi_i(q_i) + \theta_iq_i, \quad \theta_i \in \mathbb{R}, \quad i = 0, 1, \tag{6}$$

where parameter θ_i measures the relevance of the sales of firm i, ($i = 0, 1$).[6] Alternatively, θ_i can be interpreted as a variant of the subsidies provided by the owner of each firm to the associated

[6]Question arises as to why the manager of firm i selects the price such that the following function is maximized:

$$V_i(\Pi_i(p_i), p_i) = \Pi_i(p_i) + \theta_ip_i, \quad \theta_i \in \mathbb{R}, \quad i = 0, 1. \tag{7}$$

manager. Note that this parameter is not a physical subsidy, that is, θ_i does not imply a direct transfer from the owner to the manager.

The manager of firm i can maximize her/his payoff by choosing a price p_i that maximizes U_i $(i = 0, 1)$. This can be supported by the assumption that the payoff to the manager of firm i is represented as $\lambda_i + \mu_i V_i$ for some real number λ_i and some positive number μ_i $(i = 0, 1)$. Similar to many existing works, we assume that the payoffs to the managers are negligible as compared to the profits because we emphasize the impact of managerial delegation on the equilibrium outcomes. In this model, we consider that each manager receives two types of subsidies from the two agents; the production subsidy by the government and the level of the delegation parameter by her/his owner. Therefore, in the following analysis, we define the *effective subsidy* as the sum of the production subsidy and the level of each firm's delegation parameter. We should note that V_i can be redefined using net profit and effective subsidy:

$$\widehat{V}_i(p_0, p_1, u_i) = \left(p_i q_i(p_0, p_1) - \frac{1}{2} k q_i(p_0, p_1)^2\right) + u_i q_i(p_0, p_1),$$

where $u_i = \theta_i + s$, $(i = 0, 1)$.

In the following two sections, we will present the analysis for the (i) *Delegation sequencing* case and (ii) *Competition sequencing* case, respectively.

DELEGATION SEQUENCING

In this section, we investigate the following three stage game. In the first stage, the government sets the subsidy level of each firm while trying maximize of social welfare. In the second stage, the owner i decides the delegation parameter θ_i of firm i $(i = 0, 1)$.[7] Then, we consider the following three cases: In the first case, denoted by the superscript dB, the two owners simultaneously decide the delegation parameters. In the second case, which is denoted by the superscript dL, owner 0 first decides her/his level of the delegation parameter and is then followed by owner 1. In the third case, which is denoted by the superscript dF, wherein the first mover is owner 1 and the second mover is the owner 0. Note that in virtuality, both owners' control variables can be regarded

However, we obtain the same equilibrium market outcomes for both cases (6) and (7), since each price has a one-to-one correspondence with each quantity under a given opponent's price as per the direct demand function as in (2). Thus, in what follows, as in Lambertini (2000) and Nakamura (2008) we only consider the case wherein the objective function of each manager of the firm i, $(i = 0, 1)$ is represented as (6). The above fact is also indicated in Nakamura and Inoue (2009).

[7]Note that in this paper, we identify the owner 0 with the government.

as effective subsidies, u_i, given the level of physical subsidy s. Hence, we presume that owners decide their effective subsidies in the above three cases; this enables us to analyze our model far more easily. In the final and third stage, the managers simultaneously choose their price levels.

We obtain the reaction functions of the owners in the second stage:

$$u_0 = R_0(u_1) = \frac{a(1-b^2)(-2+b+b^2-k)^2 + b\left[b^2-b^4+k(2+k)\right]u_1}{(1+k)(2+k)^2+b^4(3+k)-b^2(7+7k+2k^2)} \quad \text{and}$$

$$\begin{aligned} u_1 &= R_1(u_0, s), \\ &= \frac{-ab^2\left[2+b^3+b^4+3k+k^2-b(1+k)-b^2(3+2k)\right]+\left[-b^6+(2+k)^3+b^4(7+3k)-b^2(14+13k+3k^2)\right]s+b^3(1-b^2+k)u_0}{(2+k)^3+b^4(4+k)-2b^2(6+5k+k^2)}. \end{aligned}$$

Prameters u_i $(i = 0, 1)$ are strategic complement for both owner 0 and owner 1. Note that the reaction function of owner 0, R_0, is independent of s, whereas that of owner 1, R_1, is dependent. The government is not interested in income transfer through physical subsidies since these eventually return to the government. Hence, s does not affect the reaction of owner 0. On the other hand, an increase in subsidy s raises u_1 and thus shifts the reaction curve of the owner 1 outward. As s increases, owner 1 selects a higher u_1, which encourages his manager to produce more, in order to receive more benefits from the higher subsidy. Further, note that if the government owns both firms 0 and 1 and proposes managerial contracts to managers, the first best allocation can be attained. This is because it is possible for the government to adjust u_i such that both managers maximize social welfare.

Figure 1 demonstrates the attainment of the first best and the three cases (simultaneous move and the two Stackeleberg leaderships). Curves R_0R_0' and R_1R_1' are reaction curves of the owners 0 and 1, respectively. AA' depicts the output profile satisfying $u_0 = argmax_{u_0} W(p_0(u_0, u_1), p_1(u_0, u_1))$. R_0R_0' and AA' intersect at FB, which is the first best allocation. Further, R_0R_0 intersects R_1R_1' at S, which is the equilibrium point in the simultaneous move case. As described above, the subsidy shifts the private owner's reaction curve outwards. Thus, the government can achieve the first best allocation by adjusting subsidy rate s such that S overlaps FB. Likewise, in the case where the government is a Stackleberg leader, this allocation is achieved at the same rate, since the government can always choose S.

Unfortunately, this subsidy level does not yield the first best allocation in the case of private leadership, which is shown in Figure 1. The iso-profit curve $\Pi_1\Pi_1$, which touches the reaction curve of the government at F, is depicted in this figure. The figure indicates that the private owner 1 commits to a lower u_1. This stems from avoiding stiff competition through both owners' providing effective subsidies for their managers excessively. Accordingly, a higher s is required to tally F and FB.

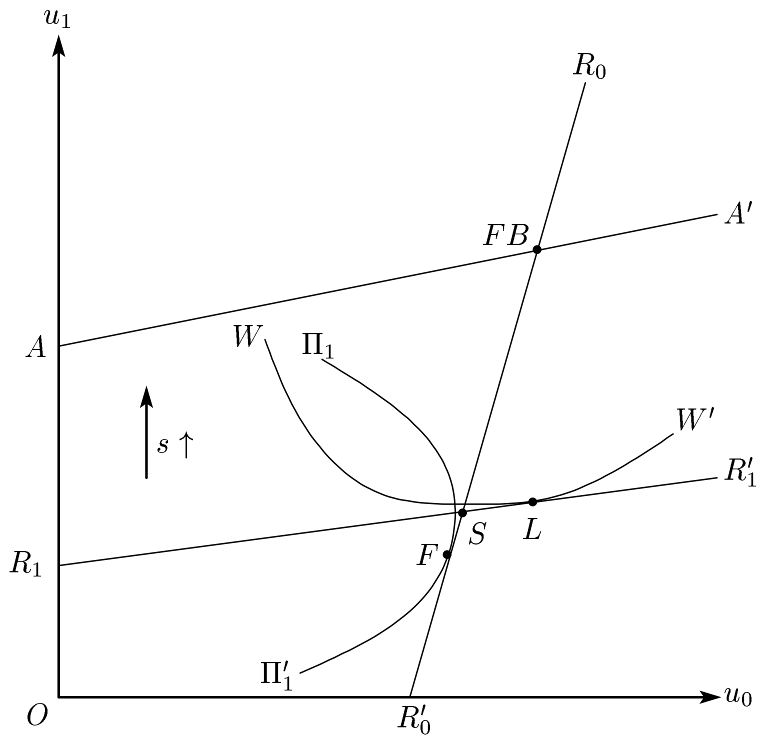

Figure 1: Delegation sequencing

These results are summarized in the following proposition.

Proposition 1. *When the government provides output subsidization, the first best allocations are achieved in the following three cases: the simultaneous move* (dB) *and the two sequential move cases* (dL *and* dF)*. Further, the effective subsidy, i.e., the sum of the equilibrium subsidy and the delegation parameter is the same for all the three cases:*

$$u_j^{di} := s^{di} + \theta_j^{di} = \frac{a\left(1-b^2\right)}{1+b+k}, \quad i = S, L, F, \quad j = 0, 1. \tag{8}$$

Moreover, the optimal subsidy rates and the relationships among them are as follows:

$$\textit{(i)}\ s^{dB} = s^{dL} = \frac{a\left(2-2b^2+k\right)}{\left(2-b^2+k\right)(1+b+k)} \quad \textit{and}$$

$$s^{dF} = \frac{a\left[2+b^4+3k+k^2-b^2(3+k)\right]}{(2+k)\left[-b^3+b(1+k)-b^2(1+k)+(1+k)^2\right]}, \quad \textit{and}$$

$$\textit{(ii)}\ s^{dB} = s^{dL} < s^{dF}.$$

This proposition states that even if there is separation between the ownership and management within each firm, and each owner enters into a $FJSV$-fashioned contract her/his manager, the "irrelevance result" still holds irrespective of whether the two firms move simultaneously or one acts as the leader and the other as the follower with regard to the decision of the delegation parameter. More precisely, in a mixed market with managerial delegation, since the effective subsidy, *i.e.*, the sum of the production subsidy and each firm's delegation parameter plays the same role as the sole production subsidy in the case where each firm is entrepreneurial (every managerial decision is made by the owner of the firm), the two market failures – under-production resulting from imperfect competition and the distortion of outputs (additionally, cost inefficiency) occurring because of firms having different objective functions – are resolved, and instead, cost efficiency is restored by forcing all firms to produce the same amount of output at equal cost.[8]

It is noteworthy that equilibrium delegation parameters in all the delegation sequencing regimes are negative.

$$\theta_i^{dB} = \theta_i^{dL} = -\frac{ab^2\left(1-b^2+k\right)}{2-b^3+3k+k^2-b^2(1+k)+b(2+k)} < 0 \quad \text{and}$$

$$\theta_i^{dF} = -\frac{ab^2\left[1+3k+k^2-b^2(1+k)\right]}{\left(1-b^2+k\right)(2+k)(1+b+k)} < 0, \quad i = 0, 1.$$

[8] The equilibrium value of the production subsidy in the entrepreneurial case coincides with that of the effective subsidy in the managerial case investigated in this paper.

Put differently, owners 0 and 1 impose a non-physical tax on their managers. Although these results are intriguing, they are not very surprising. The welfare-maximizing government has an incentive to provide physical subsidies to both firms excessively because it fears that loose competition prevails in the market. To refrain from increased production because of such excessive subsidies, both owners offer negative delegation parameters. We should also note that the equilibrium profits for dB and dL are equal and smaller than that for dF. In fact,

$$\Pi_i^{dB} = \Pi_i^{dL} = \frac{a^2\left[(2+k)^2 - b^2(4+k)\right]}{2(2-b^2+k)(1+b+k)^2}, \quad \Pi_i^{dF} = \frac{a^2\left[2b^4 + (1+k)(2+k)^2 - b^2\left(6+4k+k^2\right)\right]}{2(1-b^2+k)(2+k)(1+b+k)^2},$$

$$\text{and} \quad \Pi_i^{dF} - \Pi_i^{dB} = \frac{a^2b^2(\left[b^2 - b^4 + k(2+k)\right]}{(2+k)(1+b+k)^2\left[2+b^4+3k+k^2-b^2(3+2k)\right]} > 0, \quad i = 0, 1.$$

However, the net profit, $\Pi_i - sq_i$, is equal for all the delegation sequencing structures.

$$\Pi_j^{di} - s^{di}q_j^{di} = \frac{a^2k}{2(1+b+k)^2}, \quad i = B, L, F \quad j = 0, 1.$$

COMPETITION SEQUENCING

In this section, somewhat unlike in the delegation sequencing case, we investigate the following three stage game. In the first stage, the government sets the optimum production subsidy level for each firm in terms of social welfare. In the second stage, the owners simultaneously choose their delegation parameter values. In the third stage, we consider the following three cases. In the first case which is denoted by the superscript cB, the managers simultaneously decide the prices of their firms. In the second case, which is denoted by the superscript cL, first the maneger of firm 0 decides her/his price and is then followed by the manager of firm 1. In the third case which is denoted by the superscript cF, the first mover is the manager of firm 1 and the second mover is the manager of firm 0.[9]

As in the delegation case considered in the previous section, we obtain the result that the first best production allocation is achieved in all the three game structures.

Proposition 2. *When the government provides output subsidy, the first best allocations are achieved in the following three cases: the simultaneous move* (cB) *and the two sequential move cases*

[9]Obviously, the equilibrium market outcomes in case cB coincide with those in case dB since the game structure is the same in both the cases.

(cL *and* cF)*. Then, the effective subsidies, i.e., the sum of the equilibrium subsidies and the delegation parameters in all the three cases are given as follows:*

(i) $u_0^{cB} = u_1^{cB} = u_1^{cL} = u_0^{cF} = \dfrac{a\left(1-b^2\right)}{1+b+k}$, *and* $u_0^{cL} = u_1^{cF} = \dfrac{a\left(2-2b^2+k\right)}{(2-b^2+k)(1+b+k)}$, *and*

(ii) $u_0^{cB} = u_1^{cB} = u_1^{cL} = u_0^{cF} > u_0^{cL} = u_1^{cF}$,

where $u_i^{cj} = s^{cj} + \theta_i^{cj}$ $(i = 0, 1,\ j = B, L, F)$*. Moreover, the optimal subsidies and the relationships among them are as follows:*

(i) $s^{cL} = \dfrac{a\left[b^4 + (2+k)^2 - b^2(5+2k)\right]}{(1+b+k)\left[(2+k)^2 - b^2(3+k)\right]}$ *and* $s^{cB} = s^{cF} = \dfrac{a\left(2-2b^2+k\right)}{(2-b^2+k)(1+b+k)}$, *and*

(ii) $s^{cL} > s^{cB} = s^{cF}$.

It is relatively clear that the first best allocation is achievable in all the competition regimes, because two control variables, s and θ_0, are available to the government to correct distortion due to shortage in supplies and to correct the competition structures (or the leaderships of public and private firms). Thus, in what follows, we concentrate on explaining why the effective subsidies in equilibrium differ for the three competition regimes. For this purpose, we present the delegation parameters in equilibrium:

$$\theta_0^{cL} = -\frac{ab^4\left(1-b^2+k\right)}{b(2+k)^3+(1+k)(2+k)^3+b^5(3+k)+b^4\left(3+4k+k^2\right)-b^3\left(10+9k+2k^2\right)-b^2\left(10+19k+11k^2+2k^3\right)} < 0,$$

$$\theta_1^{cL} = -\frac{ab^2\left(1-b^2+k\right)(2+k)}{b(2+k)^2+(1+k)(2+k)^2-b^3(3+k)-b^2(3+4k+k^2)} < 0,$$

$$\theta_0^{cF} = \theta_i^{cB} = -\frac{ab^2\left(1-b^2+k\right)}{2-b^3+3k+k^2-b^2(1+k)+b(2+k)} < 0, \quad i = 0, 1, \quad \text{and} \quad \theta_1^{cF} = 0,$$
$$\forall b \in (0,1) \quad \text{and} \quad \forall k > 0.$$

First, we demonstrate the case of cF. In the case of cB, owner 1 sets θ_1 such that her/his manager exhibits the same behavior as the owner who is a leader in the price setting game without any separation between management and ownership. In the case of cF, however, the manager behaves as a leader without any delegation contracts; thus, owner 1 sets $\theta_1 = 0$ regardless of the delegation parameter of the public firm θ_0.[10] Nevertheless, since the first best allocation is achieved,

[10]Thus, in the case of cF, the objective function of owner 1, *i.e.*, her/his sole profit coincides with that of her/his corresponding manager.

the reaction curve of the public firm's manager should go through the first best allocation point and the iso-profit curve of the private firm should touch this curve at that point. These hold only when $u_0 = u_0^{cB}$, *i.e.*, when $u_0^{cF} = u_0^{cB}$. Moreover, taking into account the fact that the tangency between the reaction curve of the public firm's manager and the iso-profit curve at the first best allocation is owing to the profit maximization by the private owner under a given subsidy rate, we immediately find that $s^{cF} = s^{cB}$. This implies that u_1^{cF} is less than u_1^{cB} by θ_1^{cB}.

Next, let us proceed to the case of cL. Of course, the first-best allocation is attained in this case, too. As in the case of cF, the iso-payoff curve of the public firm's manager as a leader should be tangent to the reaction curve of the private firm's manager at the first best allocation point, and as such, $u_1^{cL} = u_1^{cB}$. Here, recall that the third stage is parallel in all the competition sequencing regimes, in that both the managers have identical objectives, *i.e.*, maximizing the sum of profit and effective subsidy. Further, recall that $u_1^{cB} = u_0^{cF}$. Thus, if $u_1^{cL} = u_0^{cF}$, then $u_0^{cL} = u_1^{cF}$ in order to ensure the first best allocation. However, the optimal subsidy s^{cL} is larger than the other ones $s^{cB} = s^{cF}$. In the case of cL, the government should encourage the managers to produce more, because not only does the private firm's manager behave less aggressively owing to the managerial contract offered by the private owner but also the public firm's manager, as a Stackelberg leader, is less aggressively.

Finally, we make some remarks. The first relates to the profits of the firms in competition sequencing that are as follows:

$$\Pi_i^{cL} = \frac{a^2\left[2b^4 + (2+k)^3 - b^2\left(10+7k+k^2\right)\right]}{2(1+b+k)^2\left[(2+k)^2 - b^2(3+k)\right]}, \quad \Pi_i^{cB} = \Pi_i^{cF} = \frac{a^2\left[(2+k)^2 - b^2(4+k)\right]}{2(2-b^2+k)(1+b+k)^2}, \quad \text{and}$$

$$\Pi_i^{cL} - \Pi_i^{cB} = \frac{a^2 b^4(1-b^2+k)}{(2-b^2+k)(1+b+k)^2\left[(2+k)^2 - b^2(3+k)\right]} > 0, \quad i = 0, 1.$$

As in the delegation sequencing case, in competition sequencing, the profit of the firm that receives the highest subsidy rate is the largest in all the three cases. Likewise, the net profits are identical:

$$\Pi_j^{ci} - s^{ci} q_j^{ci} = \frac{a^2 k}{2(1+b+k)^2}, \quad i = B, L, F, \quad j = 0, 1.$$

The second remark is on the importance of Proposition 2, which in our case is higher as compared to in previous literature on mixed oligopoly with managerial delegation. If the production subsidy is not taken into account, analogous to this paper's case where the production subsidy is considered, the equilibrium delegation parameter of firm 1, as in Nakamura (2008) and Nakamura and Inoue (2009), is negative for an arbitrary degree of production differentiation in both simultaneous and sequential move cases. Both Nakamura (2008) and Nakamura and Inoue (2009) consider a price setting mixed oligopoly with quadratic cost functions and constant marginal cost. However, in this paper, in both the delegation sequencing and competition sequencing cases, the delegation parameter of firm 0 is also negative for all $b \in (0, 1)$ and $k > 0$. This is quite unlike in Nakamura (2008) and Nakamura and Inoue (2009).

The final remark is on the relationship between our propositions and the results in the existing works on mixed oligopoly with subsidization. Hashimzade *et al.* (2007) showed that the first best allocation can be achieved by providing equal subsidy under both cases: the simultaneous move case and the case where there is public leadership in mixed oligopoly with price competition and no separation between the management and the ownership. One of the contributions of this paper is to examine the robustness of their results when managerial delegation is considered. Propositions 1 and 2 state that their results hold only in delegation sequencing. Surprisingly, we found that, by focusing on the effective subsidies, their results can be extended to the case where the public firm is a follower in delegation sequencing. Further, as indicated in Tomaru and Saito (2009) and Zikos (2007), a mixed oligopoly with both subsidization and private leadership requires a smaller subsidy in quantity competition and larger subsidy in price competition, as compared to the subsidy required in the simultaneous move case to achieve the first best allocation. Once we take into account the separation of the management and ownership, these results are altered.

CONCLUSION

This paper investigated the optimal subsidization problem in a mixed duopoly where the owners of both the public firm and the private firm enter into a sales delegation contract with their managers as in Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) (the so-called $FJSV$ contract). For this purpose, we formulated a game with the following three stages. In the first stage, the government sets each firm's production subsidy at the optimum level with respect to social welfare. In the second stage, the owner of each firm decides her/his delegation parameter for the sales delegation contract, and subsequently, in the third stage, each firm determines its price in the differentiated goods market. Then, we analyze the following two cases. (*i*) Delegation sequencing case: where the firms simultaneously/sequentially decide their delegation parameters in the second stage, and the managers simultaneously choose their prices in the third stage. (*ii*) Competition sequencing case: the case where the firms set their prices sequentially/simultaneously in the third stage, and the owners simultaneously decide their delegation parameters in the second stage.

In both the cases, for all types of moves, we showed that the first best production allocation with respect to social welfare is achieved, and the equilibrium delegation parameter of each firm is negative for any degree of production differentiation. The equilibrium delegation parameter of the public firm being negative was quite unexpected: on the other hand, the private firm's delegation parameter being negative was expected and in line with Nakamura (2008) and Nakamura and Inoue (2009) who consider a mixed duopolistic industry without subsidization. More concretely, in case (*i*), the effective subsidy of each firm, *i.e.*, the sum of production subsidy and the delegation parameter of each firm, coincide for all the three types of moves. Furthermore, when the owner of the public firm is the follower, in order to increase the equilibrium total output in the market, the government gives the highest production subsidy to both the firms in all the three cases because the total output of the market tends to stay low as the private firm (the market leader) is less aggressive. On the other hand, in case (*ii*), we find that the government provides excess subsidy, when the public firm is the leader. However, the first best allocation is achieved for all the three moves. Thus, we realize that the move that the government paying the highest production subsidy is different for cases (*i*) and (*ii*). This result implies that in a mixed oligopolistic industry of managerial firms, the government must set the value of each firm's production subsidy taking into account their moves. This leads to the equilibrium level of each firm's production subsidy being larger as compared to the equilibrium (optimal) production subsidy obtained in a standard mixed oligopoly without managerial delegation.

Consequently, the achievement of the first best allocation using the optimal production subsidy in mixed oligopoly, the so-called *irrelevance result* is robust against the introduction of sales delegation in price competition. This is similar to the quantity competition considered in Tomaru et al. (2009). In addition, even if the market contains two private firms whose objective functions are their profits and whose owners enter into a $FJSV$ delegation contract with their managers, *i.e.*, private duopoly with sales delegation, the first best allocation, as in the above two propositions in this paper, holds as long as the owners simultaneously decide their delegation parameters in the second stage, and their prices simultaneously in the third stage.[11]

We seek to extend our analysis of mixed duopoly to general mixed oligopoly with one public firm and $n\,(\geq 2)$ private firms in the future, and thus follow existing works on subsidization in a mixed oligopolistic industry, such as White (1996), Poyago-Theotoky (2001), and Kato and Tomaru (2007). However, we consider that the result obtained in this model holds in a general mixed oligopoly model as long as multiple private firms move simultaneously. Furthermore, although we addressed sales delegation as in Fershtman and Judd (1987) Sklivas (1987), and

[11]The equilibrium market outcomes for this case (case (dpB)) are presented in the Appendix.

Vickers (1985), as an incentive contract within a firm, we must clarify the robustness of our result against other delegation schemes, such as the market share delegation of Jansen *et al*. (2007) and the relative performance case of Miller and Pazgal (2001). The extensions are left for future research.

References

Bárcena-Ruiz JC. 2009. The decision to hire managers in mixed markets under Bertrand competition. *Japanese Economic Review* (forthcoming).

Barros F. 1995. Incentive schemes as strategic variables: An application to a mixed duopoly. *International Journal of Industrial Organization* **13**: 373-386.

Collie DR. 1997. Delegation and Strategic Trade Policy. *International Economic Journal* **11**: 35-46.

Collie DR. 2003. Mergers and Trade Policy under Oligopoly. *Review of International Economics* **11**: 55-71.

Das SP. 1997. Strategic Managerial Delegation and Trade Policy. *Journal of International Economics* **43**: 173-188.

Dixit A. 1979. A Model of Duopoly Suggesting a Theory of Entry Barriers. *Bell Journal of Economics* **10**: 20–32.

Fershtman C, Judd K. 1987. Equilibrium Incentives in Oligopoly. *American Economic Review* **77**: 927-940.

Goering GE. 1985. The Strategic Use of Managerial Incentives in a Non-Profit Firm Mixed Duopoly *Managerial and Decision Economics* **28**: 83-91.

González-Maestre M, López-Cuñat J. 2001. Delegation and mergers in oligopoly. *International Journal of Industrial Organization* **19**: 1263-1279.

Hashimzade N, Khodavaisi H, Myles G. 2007. An irrelevance result with differentiated goods. *Economics Bulletin* **8(2)**: 1-7.

Heywood JS. Ye G. 2009. Delegation in a Mixed Oligopoly: The Case of Multiple Private Firms. *Managerial and Decision Economics* **30**, 71-82.

Jansen T, van Lier A, van Witteloostruijn A. 2007. A Note on Strategic Delegation: The Market Share Case. *International Journal of Industrial Organization* **25**: 531-539.

Kato K, Tomaru, Y. 2007. Mixed Oligopoly, Privatization, Subsidization, and the Order of Firm's Move: Several Types of Objectives. *Economics Letters* **96**: 287–292.

Kopel M, Riegler C. 2006. R&D in a Strategic Delegation Game Revisited: A Note. *Managerial and Decision Economics* **27**: 605-612.

Lambertini L. 2000. Extended Games Played by Managerial Firms. *Japanese Economic Review* **51**: 274-283.

Matsumura T. 1998. Partial Privatization in Mixed Duopoly. *Journal of Public Economics* **70**: 473-483.

Miller N, Pazgal A. 2001. The Equivalence of Price and Quantity Competition with Delegation. *RAND Journal of Economics* **32**: 283-301.

Miller N, Pazgal A. 2002. Relative Performance as a Strategic Commitment Mechanism. *Managerial and Decision Economics* **23**, 51-68.

Myles G. 2002. Mixed Oligopoly, Subsidization and the Order of Firms' Moves: an Irrelevance Result for the General Case. *Economics Bulletin* **12(1)**: 1-6.

Nakamura Y. 2008. Endogenous Timing in a Mixed Duopoly with Maagerial Delegation: A Quadratic Cost Case. *Journal of Economic Research* **13**: 239-266.

Nakamura Y, Inoue T. 2007. Endogenous Timing in a Mixed Duopoly: The Managerial Delegation Case. *Economics Bulletin* **12(27)**: 1-7.

Nakamura Y, Inoue T. 2009. Endogenous Timing in a Mixed Duopoly: Price Competition with Managerial Delegation. *Managerial and Decision Economics* (forthcoming).

Nishimori A, Ogawa H. 2005. Long-term and Short-term Contract in a Mixed Market. *Australian Economic Papers* **44**: 275-289.

Poyago-Theotoky J. 2001. Mixed oligopoly, subsidization and the order of firms' moves: an irrelevance result. *Economics Bulletin* **12(3)**: 1-5.

Ritz RA. 2008. Strategic Incentives for Market Share. *International Journal of Industrial Organization* **26**: 586-597.

Saha B, Sensarma R. 2008. The Distributive Role of Managerial Incentives in a Mixed Duopoly. *Economics Bulletin* **12(27)**: 1-10.

Salas Fumas V. 1992. Relative Performance Evaluation of Management. *International Journal of Industrial Organization* **10**: 473-489.

Singh N, Vives X. 1984. Price and Quantity Competition in a Differentiated Duopoly. *Rand Journal of Economics* **15**: 546-554.

Sklivas, S. D. 1987. The Strategic Choice of Management Incentives. *Rand Journal of Economics* **18**: 452-458.

Straume OR. 2006. Managerial Delegation and Merger Incentives with Asymmetric Costs. *Journal of Institutional and Theoretical Economics* **162**: 450-469.

Tomaru Y. 2006. Mixed Oligopoly, Paatial Privatization and Subsidization. *Economics Bulletin* **12(5)**: 1-6.

Tomaru, Y. and M. Saito, 2009. Mixed duopoly, privatization and subsidization in an endogenous timing framework. *Manchester School* (forthcoming).

Tomaru Y, Nakamura Y, Saito M. 2009. Subsidization and Privatization in a Mixed Duopoly with Managerial Delegation. Mimeo.

van Witteloostuijn A, Jansen T, van Lier A. 2007. Barganing over Managerial Contracts in Delegation Games: Managerial Power, Contract Disclosure and Cartel Behavior. *Managerial and Decision Economics* **28**: 897-904.

Vickers J. 1985. Delagation and the Theory of the Firm. *Economic Journal* **95**: 138-147.

White MD. 1996. Mixed Oligopoly, Privatization and Subsidization. *Economics Letters* **53**: 189-195.

White MD. 2001. Managerial Incentives and the Decision to Hire Managers in Markets with Public and Private Firms. *European Journal of Political Economy* **17**: 877-896.

Zhang J, Zhang Z. 1997. R&D in a Strategic Delegation Game. *Managerial and Decision Economics* **18**: 391-398.

Zikos V. 2007. Stackelberg mixed oligopoly with asymmetric subsidies. *Economics Bulletin* **12(13)**: 1-5.

Ziss S. 2001. Horizontal Mergers and Delegation. *International Journal of Industrial Organization* **19**: 471-492.

Appendix

Private Firms in the simultaneous move case

Here, we present the equilibrium market outcomes in the case (case dpB) where both the firms are private and profit-maximizers, and simultaneously decide their delegation parameters, and price levels.

$$\theta_i^{dpB} = \frac{ab^2\left(1-b^2+k\right)}{2-b^3+3k+k^2-b^2(1+k)+b(2+k)}, \quad s^{dpB} = \frac{a\left(2-2b^2+k\right)}{2-b^3+3k+k^2-b^2(1+k)+b(2+k)},$$
$$p_i^{dpB} = \frac{ak}{1+b+k}, \quad q_i^{dpB} = \frac{a}{1+b+k}, \quad \text{and} \quad \Pi_i^{dpB} = \frac{a^2\left[(2+k)^2-b^2(4+k)\right]}{2(2-b^2+k)(1+b+k)^2}, \quad (i=0,1),$$
$$\text{and} \quad CS^{dpB} = \frac{a^2(1+b)}{(1+b+k)^2}, \quad W^{dpB} = \frac{a^2}{1+b+k}. \tag{9}$$