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Subsidization and Privatization in a Mixed Duopoly with Managerial Delegation

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Abstract

This paper provides a theoretical analysis related to the issue of optimal subsidization in a mixed duopolistic model that involves homogeneous goods and composes a public firm and a private firm where the owner within each firm provides his/her managerial delegation to a manager. Similar to existing literature, this study shows that optimal subsidies attain the first-best allocation in Cournot mixed and private duopolies and in mixed duopolies wherein the public firm acts as the leader with respect to the decision of the quantity level or the weight of each firm's output in the delegation contract. Further, this study shows that the optimal subsidy with respect to social welfare is lower in a mixed duopoly with public output leadership than in other situations, even though although the first-best production allocation is also achieved in this case. This result implies that in that case where a mixed oligopolistic industry composes managerial firms, the authority that provides unit output subsidies to firms must carefully consider to the order of firms' moves because the excessive subsidies may leads to needless welfare loss.

JEL Classification: D21, L13, L33
Keywords: Mixed Duopoly, Managerial Delegation, Subsidization, Privatization

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1 Introduction

This paper presents a theoretical analysis related to the situation wherein the government provides a unit output subsidy to both a public firm and a private firm in a mixed duopoly, while considering the separation between ownership and management. In other words, this paper deals with the issue of optimal subsidization in a mixed duopolistic model that composes firms wherein the owners provide managerial delegation to the managers. Under the above setting, we particularly focus on the content of each firm's equilibrium delegation parameter and the equilibrium subsidy level in accordance with the change in the order of firms' moves.

Although past literature on mixed oligopoly has considered many economic issues such as privatization and strategic trade policies, the issue of subsidization policy for the government is one of the main streams.[1] White (1996) found the surprising result that, in a standard mixed oligopoly that comprises one public firm and n private firms, if the government can provide an optimal output subsidy, each firm's equilibrium output, profit, and social welfare are identical before and after privatization. Further, the first-best production allocation with respect to social welfare is achieved in this case. Accordingly, the privatization of a public firm is not effective under the intervention of the government's subsidization policy. This is different from the result of DeFraja and Delbono (1989).[2] Poyago-Theotoky (2001) and Myles (2002) obtained the same result as that of White (1996) that such first-best allocation can be attained by the same subsidy even in sequential competition when a public firm becomes the leader in the market. Further, Hashimzade *et al*. (2007) showed that the irrelevance result still holds in a price-setting mixed oligopoly with differentiated goods.[3] This paper acts as as extension of the above works and

[1]The study of mixed oligopoly using the modern game theoretical approach can be traced back to the paper of DeFraja and Delbono (1989). Subsequently, this approach has gained prominence with regard to tractability of theoretical treatment and the importance of public firms in a real world economy, which involves not only developing and/or formerly communist countries, but also developed countries. The studies on mixed oligopoly that focus on topics excluding the subsidization issue, are as follows. Bárcena-Ruiz and Garzón (2006), Kato (2006), and Ohori (2006) analyzed environmental policies. Ishibashi and Matsumura (2006), Nishimori and Ogawa (2002), and Poyago-Theotoky (1998) investigated R&D competition between the public firm and the private firm. Corneo and Jeanne (1994), Fjell and Pal (1996), Fjell and Heywood (2002), Pal and White (1998), and Matsushima and Matsumura (2006) explored the issue of the international competition. In addition, see DeFraja and Delbono (1990) and Nett (1994) for comprehensive surveys related to theoretical mixed oligopoly research.

[2]This result has been referred to as an "*irrelevance result*" since Poyago-Theotoky (2001).

[3]With regard to other works on the subsidization issue in the context of mixed oligopoly, Kato and Tomaru (2007) showed that the irrelevance result still holds in the case where the objective function of the private firm is slightly different from the firm's sole profit, for example, the firm's revenue or negative cost. Moreover, Tomaru

examines whether or not their results still hold in our model.

On the other hand, for the analysis with respect to large corporations, although the separation between ownership and management has been considered in many works since classical and seminal works such as Baumol (1958), Simon (1964), and Williamson (1968), the works of Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) specifically introduced a multiple-stage game approach wherein the owner within a single firm provides an incentive contract to a manager (*strategic delegation*).[4] In particular, the papers of Fershtman and Judd and Sklivas mentioned above considered the following two-stage private duopoly setting where, in the first stage, the owners, whose objectives are to maximize their profits, choose the contents of incentive contracts for managers on the basis of a linear combination of profits and sales, and subsequently, in the second stage, the managers, aware of the contents of the contracts presented to them by the owners, choose the quantities. For tractability, managerial delegation à la Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) ($FJSV$ delegation contract) is applied in various contexts of theoretical economics.[5] In this paper, we also consider the effect of the $FJSV$ delegation contract on mixed duopolistic market outcomes with subsidization.[6]

(2006) confirmed the robustness of this irrelevance result in spite of considering partial privatization introduced by Matsumura (1998). More recently, Tomaru and Saito (2009) analyzed the subsidization policy of the government in a mixed duopoly in a situation where the production timing of each firm is endogenously determined and they found that the irrelevance result still holds in this case.

[4]Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) analyzed the strategic manipulation of firms' owners through incentive contracts that they provided to the managers on the basis of each firm's sales and profit, and these studies found that using such incentive contracts changes the strategic position of each firm in the market.

[5]There exist many works that have adopted the $FJSV$ contract for the analysis of strategic delegation. Szymanski (1994) and Bughin (1995) considered the issue of the wage bargaining by each firm's manager with the labor union. Further, Zhang and Zhang (1997) and Kopel and Riegler (2006) explored the effect of R&D spillovers on the optimal incentive contract of each firm. In the context of international economics, Collie (1997) and Das (1997) investigated the relationship between strategic trade policy and each firm's delegation contract. On the other hand, several papers introduced delegation regimes apart from those involving the FJSV delegation contract. For example, Salas Fumas (1992) and Miller and Pazgal (2001, 2002) considered the incentive scheme as the linear combination of the profit of a firm and its rival firm (*market share* delegation). Presenting an application of relative performance delegation on international competition, Miller and Pazgal (2005) showed that if firms' owners have sufficient control over their managers' behavior, then the optimal strategic trade policy does not depend on the mode of product-market competition, i.e., whether or not firms compete by setting prices or quantities. Moreover, Ritz (2008) and Jansen *et al*. (2007) investigated a case in which an owner within each firm provides the weighted sum of the firm's profit and market share to the manager (market share delegation).

[6]Tomaru *et al*. (2009) tackled a similar problem in a mixed duopoly that both the firms compete over prices with differentiated goods. They showed that in the price-setting mixed duopoly, the irrelevance result is robust against the introduction of managerial delegation à la Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985).

In the context of mixed oligopoly, managerial delegation has been extensively studied in recent years. Barros (1995) was a pioneering paper that addressed this issue. It examined the influence of managerial delegation within each firm on equilibrium market outcomes under asymmetric information. Subsequently, White (2001) reconsidered such a problem with complete information to strictly focus on the strategic benefits of managerial incentive contracts. Nishimori and Ogawa (2002) addressed the issue of intertemporal contract decisions in a mixed duopoly market and investigated how the contract of a private firm is altered when the competitor is a public firm. In particular, Heywood and Ye (2009) sought a more general condition regarding whether or not privatizing the public firm facing an optimal incentive contract reduces welfare in a mixed oligopolistic industry wherein all the firms are managerial, relative to the studies of Barros (1995) and White (2001).[7] In this paper, like Tomaru *et al*. (2009), we consider the optimal subsidization issue with respect to the separation between ownership and management in the manner of Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985).

The main purpose of this paper is to check the robustness of the so-called irrelevance result against the $FJSV$ delegation contract. As mentioned above, past literature on subsidization policies in mixed oligopoly has investigated not only simultaneous competition, but also sequential move competition. In this paper, we consider the following three types of competition structures: (*i*) *Cournot duopoly*, the owners of both the public firm and the private firm simultaneously determine their delegation parameters and the managers simultaneously choose the level of output; (*ii*) *Public output leadership*, the manager of the public firm chooses the level of the quantity in advance and the manager of the private is a follower. the owners of both firms simultaneously decide their delegation parameters; (*iii*) *Public delegation leadership*, wherein the owner of the public firm acts as the leader and that of the private firm is a follower, whereas the managers of both the firms simultaneously decide the levels of the quantities. Furthermore, in each of the three structures, we also consider the situation after privatization and compare the equilibrium market outcomes before and after privatization.

Similar to existing literature, in this paper, we show that the optimal subsidies yield the first-best allocation in Cournot mixed and private duopolies and in mixed duopolies with public output and delegation leadership. Thus, the irrelevance result is robust against the introduction of the

[7] Nakamura and Inoue (2008) and Nakamura and Inoue (2009) examined the impact of the introduction of $FJSV$ delegation on endogenous timing in a mixed duopolistic model for differentiated goods in the context of quantity competition and price competition, respectively. In particular, with respect to both domestic and foreign competition, Saha and Sensarma (2008) analyzed the distributive role of the $FJSV$ delegation contract in a mixed duopolistic industry where only a private firm is managerial.

$FJSV$ delegation contract.[8] Moreover, as an interesting and surprising result obtained in this paper, we showed that the optimal subsidy with respect to social welfare is lower in a mixed duopoly with public output leadership than that in any other situation, even though the first-best production allocation is also achieved in this case. This arises from the fact that the impact of the aggressive behavior of the manager of the public firm with regard to the choice of his/her quantity dominates the impact of less aggressive behavior of the public owner for any subsidy rate. Further, such a result has a very important economic implication: in a mixed duopoly with public output leadership, the privatization of the public firm further increases the competitiveness of the market, and consequently, a private duopoly with simultaneous moves regarding both the decision of each firm's delegation parameters and the output level is induced after such a privatization. As a result, the need for an increase in the subsidy may cause loss of welfare and/or resource wastes such as an objection by tax payers. Therefore, in the case of a mixed oligopolistic market composing managerial firms, the government carefully consider the order of the moves of both the owner and the manager within each firm. Otherwise, the resultant expensive physical subsidy may cause loss of welfare and/or resources.

The rest of this paper is organized as follows. In Section 2, we formulate the basic model considered in this paper. In Section 3, we present the three types of competition structures: (*i*) *Cournot mixed duopoly*, (*ii*) *Public output leadership*, and (*iii*) *Public delegation leadership*. Furthermore, we derive the equilibrium market outcome for each competition structure before and after privatization for a given level of output subsidy of each firm. In Section 4, we investigate whether or not a public firm should be privatized with respect to the three competition structures. In Section 5, we examine the optimal subsidization policy for the government in each structure before and after the privatization of the public firm. Finally, Section 6 concludes the paper with several remarks. The proofs of Proposition 1 are provided in the Appendix.

2 The model

There exist a private firm and a public firm denoted by the subscript 1 and 0, respectively, in a single market for a homogeneous good. The demand for the homogeneous good is assumed to be given as $P = P(Q) = a - Q$ with $Q = q_0 + q_1$, where q_0 is the output of the public firm and q_1 is the output of the private firm. These two firms have identical cost functions with increasing

[8]Tomaru *et al*. (2009) obtained a similar result to that of this paper stating that the irrelevance results still hold in a price-setting competition with differentiated goods for the three types competition structures considered in this paper.

marginal cost represented by the quadratic cost function $C(q_i) = \frac{1}{2}k(q_i)^2$.[9]

As assumed in DeFraja and Delbono (1989) which is a seminal paper in this field, and other studies on mixed oligopoly, we assume that the owners of private firms maximize their own profits and the government that owns a public firm maximizes social welfare. The profit of firm i is

$$\Pi_i(q_0, q_1, s) = P(Q)q_i - C(q_i) + sq_i,$$

where s is a production subsidy provided by the government. The social welfare is measured as the sum of consumers surplus (CS) and profits less payments for subsidies, given by

$$\begin{aligned} W(q_0, q_1) &= CS + \Pi_0(q_0, q_1, s) + \Pi_1(q_0, q_1, s) - sQ, \\ &= \int_0^Q P(z)dz - C_0(q_0) - C_1(q_1). \end{aligned}$$

The subsidies are included in the welfare expression as both a component of profits and a state expenditure. Hence, tile direct effect of the subsidy on welfare is zero.

As stated in the introduction, this paper aims to investigate the effect of subsidies on competition among private and public firms with managerial delegation. To envisage a situation wherein the owners of these firms delegate control to their managers, we employ the model used by Lambertini (2000), Lambertini and Trombetta (2002), Nakamura (2008), and Nakamura and Inoue (2009). In this model, the owners assess the performance of their managers according to two easily observable indicators (*i.e.*, the profit and output of the firm) and design incentive contracts to compensate their managers. These contracts are represented as follows: $\overline{V}_i = \Pi_i(q_0, q_1, s) + \theta_i q_i$, where parameter $\theta_i \geq 0$ identifies the weight attached to the value of sales. Parameter θ_i is a variant of subsidies provided by the owner of the firm to the manager. We should notice that θ_i is not a physical subsidy. In other words, if does not represent the transfer from the owner to the manager. To emphasize this aspect of θ_i as a subsidy, we redefine each owner's contract function as follows.

$$V_i(q_0, q_1, u_i) = P(Q)q_i - C(q_i) + u_i q_i,$$

where $u_i = s_i + \theta_i$, which represents the effective subsidy. Given the effective subsidy u_i, the manager selects q_i to maximize V_i. For the validity of the maximization of V_i by the manager, it

[9] Since DeFraja and Delbono (1989), many works on mixed oligopoly have assumed that the technology of each firm is represented as a cost function with respect to the firm's quantity level, for example, Bárcena-Ruiz and Garzón (2003), Fujiwara (2007), and Wang *et al*. (2009).

might be convenient to regard $M_i = A_i V_i + B_i$ with constants A_i and B_i as the remuneration for the manager. Such remuneration acts as an incentive for him to maximize V_i.[10]

3 Three types of competition structures for a given level of subsidy

To clarify the effects of unit production subsidies on mixed duopoly with managerial delegation, we consider the differences in market structures. As Fjell and Heywood (2002) and Matsumura (2003) pointed out, the roles of public firms (that is, as Cournot competitors or Stackelberg leaders) influence the market outcomes. Further, we observe not only mixed oligopolistic markets that exhibit Cournot competition, but also mixed markets with public leadership that feature Stackelberg competition. Thus, it is essential to explicitly analyze and compare Cournot competitive markets and markets led by a public firm. For this purpose, we posit three cases. In case 1, the public private owners simultaneously determine their delegation parameters, and then the managers of these firms also choose the outputs, simultaneously (*Cournot mixed duopoly*). In case 2, like case 1, owners determine the delegation parameters simultaneously. In this case, the difference from case 1 is that the public firm's manager chooses the output in advance, and after observing this, the private firm's manager chooses the output (*Public output leadership*). In case 3, we consider a situation wherein the public owner takes that lead in determining the delegation parameters and the managers set their outputs simultaneously after the delegation setting stage (*Public delegation leadership*). Furthermore, we consider the effect of privatization in each case. In this section, we specifically conduct an equilibrium analysis of the above three cases.

3.1 Case 1: Cournot duopoly

We explore the two-stage game as the level of subsidy is given. In the first stage, observing the level of subsidy provided by the government, an owner of private firms and the government as an owner of a public firm simultaneously determine their incentive contract parameter θ_i. In the second stage, each manager—being aware of his incentive scheme and that of the rivals—decides the quantity of the firm q_i. The superscripts *MC* and *PC* denote mixed and private duopolies where

[10] In this case, the payoff of the owner is $\Pi_i - M_i$, which suggests that the owner's optimization behavior be affected by that of his manager. To preclude such affection, the previous literature assumes that the remuneration M_i is equalized with the manager's reservation incomes or opportunity costs which are constant and exogenous, $\bar{K}$. See Basu (1995) and White (2001).

the owners and managers in both the firms simultaneously decide the content of the delegation parameter and the quantity levels, respectively.

3.1.1 Mixed duopoly

We solve this game using backward induction. A manager of firm i selects the output to maximize V_i. The first order condition is given by

$$\frac{\partial V_i}{\partial q_i} = a - Q - q_i - kq_i + u_i = 0.$$

Thus, the (quasi-) reaction function is

$$q_i = r_i(q_j, u_i) = \frac{a - q_j + u_i}{k+2}, \quad i = 0, 1, \ \ i \neq j, \tag{1}$$

which increases with the effective subsidy u_i and decreases with the output of its rival. These reaction functions yield equilibrium outputs in the second stage:

$$q_i^*(u_0, u_1) = \frac{(k+1)a + (k+2)u_i - u_j}{(k+1)(k+3)}.$$

An increase in an effective subsidy to one firm leads to an increase in the output of the firm and a decrease in that of the other firm. The latter is caused by strategic substitution. The equilibrium outputs can be rewritten as functions that have θ_i and s as independent variables;

$$\hat{q}_i(\theta_0, \theta_1, s) = \frac{(k+1)(a+s) + (k+2)\theta_i - \theta_j}{(k+1)(k+3)}, \tag{2}$$

Now, we proceed to the first stage. In this stage, the government as an owner of the public firm sets θ_0 to maximize the social welfare $\hat{W}(\theta_0, \theta_1, s) := W(\hat{q}_0, \hat{q}_1)$ and an owner of the private firm chooses θ_1 to maximize their profits $\hat{\Pi}_1(\theta_0, \theta_1, s) := \Pi_1(\hat{q}_0, \hat{q}_1, s)$. These optimization behaviors yield owners' reaction functions as follows:

$$\theta_0 = R_0(\theta_1, s) = \frac{a(1+k)^2 - (1+k)^2(2+k)s + (-1+2k+k^2)\theta_1}{1+7k+5k^2+k^3} \quad \text{and} \tag{3}$$

$$\theta_1 = R_1(\theta_0, s) = \frac{(1+k)(a+s) - \theta_0}{4+10k+6k^2+k^3} \tag{4}$$

It is easily seen that the private owner's delegation parameter θ_1 is a strategic substitute for that of the public firm. The reason for this strategic behavior is analogous to that for $\partial q_i^*/\partial u_j$. On the other hand, it is not obvious whether the government's delegation parameter θ_0 is a strategic

substitute or a complement. If k is higher, it is a strategic complement; Otherwise, it is a strategic substitute.

In contrast to the effects of delegation parameters, the subsidy rate s has clear effects on delegation parameters. The delegation parameters of the private firm is positively related to s, whereas that of the public firm is negatively related. An increase in s lowers firms' marginal costs, and thus, the private owner exhibits aggressive behavior (i.e., θ_1 becomes larger). The public owner, likely to be aggressive because of a welfare-maximizer, attempts to improve cost efficiency by substituting the output of the relatively efficient private firm for that of the public firm, so that the public owner pretends to be less aggressive (i.e., θ_0 becomes smaller). This is why an increase in s raises θ_1 and lowers θ_0.

Solving equations (4) and (3), we obtain the equilibrium delegation parameters, profits, and welfare. These outcomes are summarized by the following lemma.

Lemma 1. *In a subsidized Cournot mixed duopoly with managerial delegation, in equilibrium, the delegation parameters, profits, and welfare in equilibrium are*

$$\theta_0^{MC}(s) = \frac{a(1+5k+4k^2+k^3)-(3+9k+12k^2+6k^3+k^4)s}{1+12k+16k^2+7k^3+k^4},$$

$$\theta_1^{MC}(s) = \frac{ak(2+k)+(1+3k+k^2)s}{1+12k+16k^2+7k^3+k^4}, \quad \Pi_0^{MC}(s) = \frac{\left[a(1+6k+5k^2+k^3)-(2+k)s\right]A_1}{2(1+12k+16k^2+7k^3+k^4)^2},$$

$$\Pi_1^{MC}(s) = \frac{(2+k)(2+4k+k^2)\left[ak(2+k)+(1+3k+k^2)s\right]^2}{2(1+12k+16k^2+7k^3+k^4)^2}, \quad \textit{and}$$

$$W^{MC}(s) = \frac{A_2}{2(1+12k+16k^2+7k^3+k^4)^2},$$

where MC represents a Cournot mixed duopoly with managerial delegation, and

$$A_1 = ak(3+8k+5k^2+k^3)+(2+14k+23k^2+12k^3+2k^4)s \quad \textit{and}$$

$$A_2 = (1+23k+160k^2+308k^3+263k^4+113k^5+24k^6+2k^7)a^2 - k(2+k)^3(1+7k+5k^2+k^3)s^2 + 2k(2+17k+32k^2+24k^3+8k^4+k^5)sa.$$

3.1.2 Private duopoly

After the public firm is privatized, the privatized firm seeks its profit maximization as it has a private owner who aims to achieve this. In this case, the private owners of both private and privatized firms simultaneously determine their delegation parameters, and then, the managers of these firms select the outputs. Thus, we obtain the following results.

Lemma 2. *In a subsidized private duopoly where the manager of the public firm is the leader and the owners simultaneously decide their delegation parameters, the delegation parameters, profits, and welfare in equilibrium are*

$$\theta_0^{PC}(s) = \frac{a+s}{5+5k+k^2}, \qquad \theta_1^{PC}(s) = \frac{a+s}{5+5k+k^2},$$
$$\Pi_0^{PC}(s) = \frac{(2+k)(2+4k+k^2)(a+s)^2}{2(5+5k+k^2)^2}, \quad \Pi_1^{PC}(s) = \frac{(2+k)(2+4k+k^2)(a+s)^2}{2(5+5k+k^2)^2}, \quad \textit{and}$$
$$W^{PC}(s) = \frac{(2+k)(a+s)\left[a(6+6k+k^2)-(2+k)^2 s\right]}{(5+5k+k^2)^2}.$$

3.2 Case 2: Output leadership

We consider the following stage game. In the first stage, public and private owners decide their delegation parameters simultaneously. In the second stage, the manager of the public firm (or privatized firm) chooses the output. After observing his/her choice, the manager of the private firm chooses the output in the third stage. The superscripts *MO* and *PO* denote mixed and private duopolies, respectively, where the manager of firm 0 is the leader in output setting stage.

3.2.1 Mixed duopoly

Lemma 3. *In a subsidized mixed duopoly where the manager of the public firm is the leader and the owners simultaneously decide their delegation parameters, the delegation parameters, profits, and welfare in equilibrium are*

$$\theta_0^{MO}(s) = \frac{a(1+9k+18k^2+17k^3+7k^4+k^5)-(5+32k+73k^2+75k^3+39k^4+10k^5+k^6)s}{2+37k+96k^2+97k^3+47k^4+11k^5+k^6},$$
$$\theta_1^{MO}(s) = \frac{(2+k)\left[ak(2+k)+(1+3k+k^2)s\right]}{1+18k+39k^2+29k^3+9k^4+k^5}, \ \Pi_0^{MO}(s) = \frac{(1+k)\left[a(1+9k+6k^2+k^3)-(3+k)s\right]B_1}{2(1+18k+39k^2+29k^3+9k^4+k^5)^2},$$
$$\Pi_1^{MO}(s) = \frac{(1+k)(2+k)(3+k)(1+4k+k^2)\left[ak(2+k)+(1+3k+k^2)s\right]^2}{2(1+18k+39k^2+29k^3+9k^4+k^5)^2}, \quad \textit{and}$$
$$W^{MO}(s) = \frac{(1+k)B_2}{2(1+18k+39k^2+29k^3+9k^4+k^5)^2},$$

where

$$B_1 = ak(3+16k+17k^2+7k^3+k^4)+(2+21k+52k^2+45k^3+16k^4+2k^5)s \quad \textit{and}$$
$$B_2 = (1+34k+344k^2+900k^3+1004k^4+580k^5+183k^6+30k^7+2k^8)a^2$$
$$+2k(3+31k+91k^2+96k^3+47k^4+11k^5+k^6)as-k(3+k)^2(2+17k+32k^2+24k^3+8k^4+k^5)s^2$$

Note that $\theta_0^{MC}(s) > \theta_0^{MO}(s)$ if the level of subsidy s is low. In particular, $\theta_0^{MC}(0) > \theta_0^{MO}(0)$ always holds. The intuition behind this is as follows. The manager of the public firm with output leadership has an incentive to commit a higher output than that of the manager of the public firm as a Cournot competitor. To mitigate such excessive production, the public owner decreases the delegation parameter θ_0. As expected from strategic substitution (see equation (4)), $\theta_1^{MC}(s) < \theta_1^{MO}(s)$ holds if s is low.

3.2.2 Private duopoly

Lemma 4. *In a subsidized private duopoly where the manager of the public firm is the leader and the owners simultaneously decide their delegation parameters, the delegation parameters, profits, and welfare in equilibrium are*

$$\theta_0^{PO}(s) = 0, \quad \theta_1^{PO}(s) = \frac{(1+3k+k^2)(a+s)}{3+16k+20k^2+8k^3+k^4},$$

$$\Pi_0^{PO}(s) = \frac{(2+4k+k^2)(1+6k+5k^2+k^3)^2(a+s)^2}{2(2+k)(1+4k+k^2)^2(3+4k+k^2)^2}, \quad \Pi_1^{PO}(s) = \frac{(1+3k+k^2)^2(a+s)^2}{2(1+k)(2+k)(3+k)(1+4k+k^2)},$$

$$\textit{and} \quad W^{PO}(s) = \frac{(a+s)C}{2(1+k)(1+4k+k^2)^2(6+5k+k^2)^2},$$

where

$$\begin{aligned} C = {} & a(35+354k+1284k^2+2094k^3+1792k^4+864k^5+236k^6+34k^7+2k^8) \\ & -(25+248k+878k^2+1424k^3+1250k^4+636k^5+188k^6+30k^7+2k^8)s. \end{aligned}$$

As shown by Lambertini (2000), the owner of the privatized firm that takes the lead in the output setting stage decides that it is optimal not to allow for any output expansion, i.e., $\theta_0^{PO}(s) = 0$. This result remains unchanged even when subsidies are considered.

3.3 Case 3: Delegation leadership

In this subsection, we propose a three stage game. In the first stage, the government (or a private owner of a privatized firm) decides the delegation parameter.In the second stage, after observing this decision, the owner of private firm 1 chooses his/her delegation parameter, and in the final stage, the managers of both firms select outputs simultaneously. The superscripts *MD* and *PD* denote mixed and private duopoly, where the owner of firm 0 is the leader with respect to the delegation setting stage.

3.3.1 Mixed duopoly

Lemma 5. *In a subsidized mixed duopoly where the managers decide their delegation parameters simultaneously and the public firm owner is a leader, the delegation parameters, profits, and welfare in equilibrium are*

$$\theta_0^{MD}(s) = \frac{a(1+8k+12k^2+6k^3+k^4)-(3+14k+28k^2+23k^3+8k^4+k^5)s}{1+19k+39k^2+29k^3+9k^4+k^5},$$
$$\theta_1^{MD}(s) = \frac{ak(3+4k+k^2)+(1+6k+5k^2+k^3)s}{1+19k+39k^2+29k^3+9k^4+k^5},$$
$$\Pi_0^{MD}(s) = \frac{\Big[a(1+9k+6k^2+k^3)-(2+k)s\Big]D_1}{2(1+k)(1+18k+21k^2+8k^3+k^4)^2}, \quad \Pi_1^{MD}(s) = \frac{(2+k)(2+4k+k^2)D_2}{2(1+k)^2(1+18k+21k^2+8k^3+k^4)^2},$$
$$\textit{and} \quad W^{MD}(s) = \frac{a^2(1+18k+29k^2+14k^3+2k^4)+2ak(2+3k+k^2)s-k(2+k)^3s^2}{2(1+19k+39k^2+29k^3+9k^4+k^5)},$$

where

$$D_1 = ak(5+16k+17k^2+7k^3+k^4)+(2+20k+51k^2+45k^3+16k^4+2k^5)s \quad \textit{and}$$
$$D_2 = \Big[ak(3+4k+k^2)+(1+6k+5k^2+k^3)s\Big]^2.$$

Similar to the mixed duopoly of Case 2, the public firm owner that takes the lead in the delegation parameter setting stage sets a parameter that is lower than the parameter set simultaneously by both public and private owners (i.e., $\theta_0^{MC}(s) > \theta_0^{MD}(s)$) if the level of subsidy is low. This also stems from the public firm owner's expectation of an improvement in cost efficiencies.

3.3.2 Private duopoly

Lemma 6. *In a subsidized private duopoly where the managers decide their delegation parameters simultaneously and the public owner is the leader, the delegation parameters, profits, and*

welfare in equilibrium are

$$\theta_0^{PD}(s) = \frac{(1+3k+k^2)(a+s)}{3+16k+20k^2+8k^3+k^4}, \quad \theta_1^{PD}(s) = \frac{(1+6k+5k^2+k^3)(a+s)}{(1+k)(2+k)(3+k)(1+4k+k^2)},$$

$$\Pi_0^{PD}(s) = \frac{(1+3k+k^2)^2(a+s)^2}{2(1+k)(2+k)(3+k)(1+4k+k^2)}, \quad \Pi_1^{PD}(s) = \frac{\left(2+4k+k^2\right)\left(1+6k+5k^2+k^3\right)^2(a+s)^2}{2\,(2+k)\,(1+4k+k^2)^2\,(3+4k+k^2)^2},$$

$$\textit{and} \quad W^{PD}(s) = \frac{(a+s)E}{2(1+k)(1+4k+k^2)^2(6+5k+k^2)^2},$$

where

$$E = a\left(35+354k+1284k^2+2094k^3+1792k^4+864k^5+236k^6+34k^7+2k^8\right)$$
$$- s\left(25+248k+878k^2+1424k^3+1250k^4+636k^5+188k^6+30k^7+2k^8\right).$$

We should note that the allocation of production in the case of private output leadership is coincident with that in the case of private delegation leadership for any subsidy, i.e., $q_i^{PO}(s) = q_i^{PD}(s)$ $(i = 0, 1)$. This indicates that the owner of the privatized firm adjusts θ_0 so as to realize the allocation of production in the former case, $(q_0^{PO}(s), q_1^{PO}(s))$.

To close this section, we compare Lemmas 1, 3, and 5 with respect to mixed duopoly, and Lemmas 2, 4, and 6 with respect to private duopoly.

Proposition 1. *In a mixed and private duopoly, the following holds.*

(1) $W^{MD}(s) \geq W^{MC}(s)$,

(2) $W^{MC}(s) \gtreqless W^{MO}(s) \Leftrightarrow s \gtreqless \dfrac{a(2+34k+110k^2+135k^3+76k^4+20k^5+2k^6)}{10+159k+463k^2+581k^3+382k^4+138k^5+26k^6+2k^7}$,

(3) $W^{PO}(s) = W^{PD}(s)$, *and*

(4) $W^{PC}(s) \gtreqless W^{PO}(s) \Leftrightarrow s \gtreqless \dfrac{a(11+88k+313k^2+470k^3+347k^4+134k^5+26k^6+2k^7)}{49+442k+1369k^2+2014k^3+1633k^4+774k^5+214k^6+32k^7+2k^8}$.

Proof. See Appendix.

(1) and (3) in Proposition 1 are fairly obvious. With respect to (1), the government holding a delegation leadership can propose the delegation contract, which gives rise to higher welfare than that when both owners choose their contracts because it can, at least, select the delegation parameter in equilibrium of *MC*. Moreover, for (3), as stated above, given the level of subsidy, the allocation in private output leadership is equivalent to that in private delegation leadership. Thus, inevitably, the welfare levels in these leaderships are equalized. Further, cases (2) and (4) are highly intuitive. In both regimes of *MO* and *PO*, the manager of firm 0 is the leader.

The leading manager acts aggressively for any levels of subsidy, which results in serious cost inefficiency when the subsidy is fully provided by the government. Thus, for a higher subsidy rate, the equilibrium welfare is higher in the case where outputs are simultaneously chosen than that in the case where the manager of firm 0 acts as the leader.

In any case, this proposition suggests that in a mixed or private duopoly, with respect to existing subsidies, the authorities should amend the law that concerns the foundation of market structures and competition regimes so as to prevent the possibility that decreases the level of social welfare.

4 The decision on whether or not to conduct privatization for a given level of subsidy

In this section, given the level of subsidy s, we examine whether or not privatization improves social welfare in the following three market structures: (i) Cournot duopoly with managerial delegation, (ii) output leadership exhibited by firm 0, and (iii) firm 0's delegation leadership.

4.1 Case 1: Cournot duopoly

To examine whether the government should privatize its public firm, we define

$$\begin{aligned}\alpha(s) &=: W^{MC}(s) - W^{PC}(s),\\ &= \frac{(1+k)(1+10k+48k^2+66k^3+38k^4+10k^5+k^6)\left[a(1+k)-(2+k)^2 s\right]^2}{2(5+5k+k^2)^2(1+12k+16k^2+7k^3+k^4)^2}.\end{aligned}$$

The function α is nonegative for any k and s. Although a Cournot private duopoly with managerial delegation leads to output expansion of firms and to higher welfare as compared to that without managerial delegation, the above result demonstrates that the contribution to welfare by aggressive behavior of the public owner dominates the welfare improvement effect of such output expansion. Moreover, note that α is zero when $s = \frac{(k+1)a}{k+2}$, that is, privatization is indifferent from status quo for the government. This result is summarized as follows:

Proposition 2. *Under subsidized Cournot duopoly, the privatization of a public firm should deteriorate social welfare, except for the case of* $s = \frac{(k+1)a}{k+2}$.

4.2 Case 2: Output leadership

We define β as follows.

$$\begin{aligned}\beta(s) &:= W^{MO}(s) - W^{PO}(s),\\ &= \frac{\left(5+32k+73k^2+75k^3+39k^4+10k^5+k^6\right)\left(\begin{matrix}5+68k+763k^2+4393k^3+11954k^4+18152k^5+17101k^6\\ +10567k^7+4378k^8+1209k^9+214k^{10}+22k^{11}+k^{12}\end{matrix}\right)(s-\Omega_1)(s-\Omega_2)}{2(1+k)(6+29k+27k^2+9k^3+k^4)^2(1+18k+39k^2+29k^3+9k^4+k^5)^2}, \quad (5)\end{aligned}$$

where

$$\Omega_1 = \frac{a(1+9k+18k^2+17k^3+7k^4+k^5)}{5+32k+73k^2+75k^3+39k^4+10k^5+k^6} \quad \text{and}$$

$$\Omega_2 = \frac{a(1+21k+256k^2+1235k^3+3072k^4+4364k^5+3781k^6+2072k^7+723k^8+156k^9+19k^{10}+k^{11})}{5+68k+763k^2+4393k^3+11954k^4+18152k^5+17101k^6+10567k^7+4378k^8+1209k^9+214k^{10}+22k^{11}+k^{12}}.$$

The sign of $\beta(s)$ depends on the sign of the numerator in (5). Since simple calculation yields $\Omega_1 < \Omega_2$, it is clear that $\beta(s) < 0$ when $\Omega_1 < s < \Omega_2$ and $\beta(s) \geq 0$ when $s \geq \Omega_1$ or $s \leq \Omega_2$. Evidently, given θ_0, θ_1, and s, equilibrium outcomes in the output setting stage are the same, regardless of whether or not firm 0 is privatized. Thus, the sign of $\beta(s)$ is determined by the difference of the behaviors of owners of firm 0. The manager of firm 0 as an output leader acts aggressively, irrespective of pro-privatization and pre-privatization. In addition, in the case that firm 0 is the public firm, the owner tries to offset such aggressive behavior by setting the delegation parameter since the substitution of the private firm's output for the public firm's output by less aggressive behavior of the public firm improves social cost efficiency. Moreover, such offsetting behavior strengthens as s increases (see Lemma 3). On the other hand, the owner of the privatized firm allows the aggressiveness of his manager (see Lemma 4), which leads to output expansion of the firm. Thus, there emerge thresholds Ω_1 and Ω_2 at which both the effects of cost efficiency in pre-privatization and those of output expansion in pro-privatization are equated.

Proposition 3. *Suppose that there exists a duopoly where a public firm or privatized firm is the leader in the output setting stage. If the level of subsidy is considerably lower or higher, privatization should worsen social welfare. On the other hand, if the level is in the middle range, privatization should improve social welfare.*

4.3 Case 3: Delegation leadership

We define

$$\gamma(s) := W^{MD}(s) - W^{PD}(s),$$
$$= \frac{\left[a(1+6k+23k^2+22k^3+8k^4+k^5) - (5+41k+97k^2+97k^3+47k^4+11k^5+k^6)s\right]^2}{2(6+29k+27k^2+9k^3+k^4)^2(1+19k+39k^2+29k^3+9k^4+k^5)}.$$

This implies that for any subsidy level, privatization should deteriorate social welfare under a mixed duopoly where firm 0 is the leader of the delegation setting stage. The owner of the privatized firm is committed to seting a higher delegation parameter so that the firm maintains a large market share. As a result, the privatized firm becomes more aggressive, which further causes a considerable loss of welfare owing to cost inefficiency. This is the reason why $W^{MD}(s) \geq W^{PD}(s)$. The result is summarized as follows:

Proposition 4. *Under a duopoly where an owner of a public or privatized firm is the leader in the delegation setting stage, privatization should worsen social welfare for any subsidy level, except for* $s = (1+6k+23k^2+22k^3+8k^4+k^5)a/(5+41k+97k^2+97k^3+47k^4+11k^5+k^6)$.

5 Optimal subsidization policies

Having derived and compared pre- and pro-privatization in mixed duopoly for a given level of subsidy, we proceed to the analyses of optimal subsidization policies by the government. As described in detail in Section 1 (Introduction), numerous works have investigated the relationship between subsidization and privatization in mixed oligopoly without managerial delegation. White (1996) showed that the first-best allocation can be achieved by maintaining an identical level of subsidy before and after privatization under Cournot competition without any managerial delegation. Similarly, Poyago-Theotoky (2001) found that this result holds even when the public firm is a Stackelberg leader. Myles (2002) proved the result obtained in Poyago-Theotoky in a general setting. Tomaru and Saito (2009) showed that privatization with subsidization policies is irrelevant even if the production timings of public and private firms are endogenously determined. The series of these results is called *irrelevance results*.[11] In this section, we analyze whether irrelevance results hold in mixed and private duopoly with managerial delegation under quantity competition.

[11] For other works on irrelevance results, see Tomaru (2006), Hashimzade *et al*. (2007), and Kato and Tomaru (2007).

For the above purpose, we present the first-best allocation. When the welfare-maximizing government controls the production of two firms, the first-best allocation is realized, irrespective of whether managerial delegation exists or not. By simple calculation, we have

$$q_i^{FB} = \frac{a}{k+2}, \quad \Pi_i^{FB} = \frac{a^2 k}{2(2+k)^2}, \quad W^{FB} = \frac{a^2}{k+2}, \qquad i = 0, 1,$$

where *FB* denotes the first-best allocation.[12] White (1996) and Poyago-Theotoky (2001) showed that without managerial delegation, this allocation can be achieved by the optimal subsidy rate $s = \frac{a}{k+2}$ in Cournot mixed and private oligopolies and in Stackelberg competition with public leadership.

[**Table 1 around here**]

Table 1: Optimal Subsidies: Pre-privatization and Pro-privatization

[**Table 2 around here**]

Table 2: Social Welfare: Pre-privatization and Pro-privatization

Now, we examine the optimal subsidies and the resulting equilibrium welfare in all cases of mixed and private duopoly with managerial delegation. Tables 1 and 2 summarize the optimal subsidies and welfare for various cases. It is clear that the first-best allocation is attained in both Cournot mixed and private duopolies. This result is parallel to that of White (1996). Further, we find that allocations in the cases of public output and delegation leadership are the first-best as well. This is obvious from the fact that it is derived in Cournot competition and that the government has two variables, θ_0 and s, for enhancing welfare. However, the first-best allocation is not attainable in private output leadership and private delegation leadership. The uniform subsidy provided by the government cannot mitigate the aggressive behavior of the privatized firm as the leader and the reaction of the private firm so as to cause the privatized firm to produce q_i^{FB}. Discriminatory subsidies are required in order to attain the first-best allocation, . From the above investigation, we obtain one of the main results as follows.

Proposition 5. *The first-best allocation can be achieved by some unit production subsidies in three types of mixed duopolies: (i) Cournot mixed duopoly with managerial delegation, (ii) mixed*

[12]This fact is derived from the following first-order condition:

$$\frac{\partial W}{\partial q_i} = a - (k+1)\, q_i - q_j = 0, \quad \text{where} \quad i = 0, 1; \;\; i \neq j.$$

duopoly with public output leadership, and (iii) mixed duopoly with public delegation leadership and (iv) Cournot private duopoly.

Interestingly, although the welfare level is maximized in the cases of *MC*, *PC*, *MO*, and *MD*, only the optimal subsidies of *MO* are lower than those of others. The intuition behind this is as follows. As stated in the previous section, the public owner has an incentive to cause the public firm to be less aggressive, and therefore, it sets its delegation parameter at a lower level in *MC* and *MD*. However, in the case of *MO*, the manager of the public firm chooses its output aggressively. Since the manager directly controls the output, the impact of this aggressive behavior on the determination of its output dominates that of less aggressive behavior by the public owner for any subsidy rate. This failure in output control by the public owner leads to a tendency of excess production by the public firm. Thus, the government enables firms to produce q_i^{FB} with lower subsidies.

In sum, the equilibrium outcomes of *MO* are different from those of *MC*, *PC*, and *MD* in terms of the optimal subsidy. This implies that if privatization in the case of *MD* is accompanied with ensuing competitiveness between firms, *i.e.*, if *PC* prevails after privatization, then the need for an increase in the subsidy may cause the loss of welfare or resource wastes such as an objection by tax payers. The result is summarized as follows.

Proposition 6. *Privatization of a public firm with managerial delegation is irrelevant under subsidized Cournot duopoly. Furthermore, Cournot mixed duopoly is also irrelevant to mixed duopoly with public delegation leadership. However, mixed duopoly with public output leadership is not irrelevant to that with public delegation leadership and Cournot duopolies.*

[**Table 3 around here**]

Table 3: Effective Subsidies: Pre-privatization and Pro-privatization

Public and private firms (or their managers) receive not only physical subsidies s from the government but also non-physical subsidies θ_i from the firms' owners . For measuring the performances of these firms, it may be suitable to focus on not only the physical subsidy s, but also effective subsidies $u_i = s + \theta_i$. The effective subsidies for all cases presented in Table 3. This table indicates two important points. First, the effective subsidies in cases *MC*, *PC*, and *MD* are the same as the optimal subsidy in White (1996) and Poyago-Theotoky (2001). Hence, regarding the effective subsidies in our model as the optimal (physical) subsidy specified in White (1996) and Poyago-Theotoky (2001) shows that mixed and private duopolies with managerial delegation are irrelevant to those without it. Second, in the cases of *MO*, *PO*, and *PD*, although the physical

subsidy by the government is uniform, public and private firms virtually receive discriminatory subsidies as the sum of physical subsidies and non-subsidies.

6 Conclusion

This paper examined the relationships between subsidization and privatization policies in mixed duopolies with managerial delegation. For precise analysis, we focused on three market structures. First, we considered the case where owners of one public (or privatized) firm and one private firm simultaneously decide their managerial contracts, and then, the managers of these firms simultaneously determine the outputs. In the second case, owners' choices are simultaneous and the manager of the public (or privatized) firm leads the output choices. Finally, in the third case, the owner of the public firm (or privatized firm) leads the delegation choices and the managers' output choices are simultaneous. Furthermore, we derived the optimal subsidy for each market structure before and after privatization and compared them and the resulting levels of welfare for the above three cases.

In this paper, we showed that the optimal subsidies yield the first-best allocation in Cournot mixed and private duopolies (Cases *MC* and *PC*) and mixed duopolies with public output and delegation leadership (Cases *MO* and *MD*). In addition, the optimal subsidy in a mixed duopoly with public output leadership is lower than in other cases. Thus, *MC*, *PC*, and *MD* are irrelevant, and they are not irrelevant to *MO*. This implies that if privatization in case *MD* is accompanied with ensuing competitiveness between firms, *i.e.*, if *PC* entails after privatization, then the need for an increase in the subsidy may cause the loss of welfare and/or resource wastes such as an objection by tax payers. Therefore, the authority that provides physical subsidies specifically must pay attention to the following two facts: (1) Whether or not the separation between ownership and management is observed within each firm and (2) the order of firms's moves.[13]

Although we assumed the sales delegation contract type, in the manner of Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) to be each firm's managerial contract (the $FJSV$ delegation contract), the literature on strategic delegation has introduced other delegation regimes, for example, *relative performance* delegation presented in Salas Fumas (1992) and Miller and Pazgal (2001, 2002) and *market share* delegation presented in Ritz (2008) and Jansen *et al*. (2007). Thus, we must check the robustness of the result obtained in this paper, when such delegation contracts are regarded as each firm's managerial regimes. Moreover, although we assumed only

[13]In addition to (1) and (2), determining the type of contract provided by an owner within each firm to his/her manager is also an important issue for the authority.

the domestic private firm to be the competitor of the domestic public firm, past literature on mixed oligopoly has explored a situation wherein private firms in the market are foreign-owned, while considering the fact that the domestic private firm competes against foreign private firms.[14] Further, since the existence of the foreign private firm plays a crucial and less intuitive role on the equilibrium market outcomes, we must again check the robustness of the results obtained in this paper by conducting a similar analysis under the assumption that foreign private firms exist in the market. Extending the paper toward these directions is left for future research.

Appendix

Proof of Proposition 1

$$W^{MD}(s) - W^{MC}(s) = \frac{k^2\left[a(1+k)-(2+k)^2 s\right]^2}{2(1+12k+16k^2+7k^3+k^4)^2(1+19k+39k^2+29k^3+9k^4+k^5)} \geq 0 \quad \text{and}$$

$$W^{MO}(s) - W^{MC}(s) = -\frac{k(1+7k+5k^2+k^3)\left[ak(2+k)+(1+3k+k^2)s\right]\left(10+159k+463k^2+581k^3+382k^4+138k^5+26k^6+2k^7\right)(s-\Omega_3)}{2(1+12k+16k^2+7k^3+k^4)^2(1+18k+39k^2+29k^3+9k^4+k^5)^2},$$

where

$$\Omega_3 = \frac{a(2+34k+110k^2+135k^3+76k^4+20k^5+2k^6)}{10+159k+463k^2+581k^3+382k^4+138k^5+26k^6+2k^7}.$$

Thus, $W^{MC}(s) \geq W^{MO}(s)$ if $s \geq \Omega_3$. Similarly,

$$W^{PO}(s) - W^{PC}(s) = -\frac{(a+s)\left(49+442k+1369k^2+2014k^3+1633k^4+774k^5+214k^6+32k^7+2k^8\right)(s-\Omega_4)}{2(1+k)(5+5k+k^2)^2(6+29k+27k^2+9k^3+k^4)^2},$$

where

$$\Omega_4 = \frac{a(11+88k+313k^2+470k^3+347k^4+134k^5+26k^6+2k^7)}{49+442k+1369k^2+2014k^3+1633k^4+774k^5+214k^6+32k^7+2k^8}.$$

Thus, $W^{PC}(s) \geq W^{PO}(s)$ if $s \geq \Omega_4$. ■

[14] In the real world, public firms are large enterprises, and inevitably, mixed oligopolistic markets generally span across different nations. For example, Renault which is a French automobile enterprise and is partially privatized, competes against private automobile enterprises located worldwide, such as Ford, General Mortors, Nissan, and Toyota. Thus, many previous studies on mixed oligopoly have regarded foreign private firms as the competitors of the domestic public firm, for example, Fjell and Pal (1996), Fjell and Heywood (2002), and Pal and White (1998).

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	Pre-privatization	Pro-privatization
Cournot competition	$s^{MC} = \frac{(k+1)a}{(k+2)^2}$	$s^{PC} = \frac{(k+1)a}{(k+2)^2}$
Output leadership	$s^{MO} = \frac{(k^2+3k+1)a}{k^3+6k^2+11k+6}$	$s^{PO} = \frac{a(5+53k+203k^2+335k^3+271k^4+114k^5+24k^6+2k^7)}{25+248k+878k^2+1424k^3+1250k^4+636k^5+188k^6+30k^7+2k^8}$
Delegation leadership	$s^{MD} = \frac{(k+1)a}{(k+2)^2}$	$s^{PD} = \frac{a(5+53k+203k^2+335k^3+271k^4+114k^5+24k^6+2k^7)}{25+248k+878k^2+1424k^3+1250k^4+636k^5+188k^6+30k^7+2k^8}$

Table 1: Optimal Subsidies: Pre-privatization and Pro-privatization

	Pre-privatization	Pro-privatization
Cournot competition	$W^{MC}(s^{MC}) = \frac{a^2}{k+2}$	$W^{PC}(s^{PC}) = \frac{a^2}{k+2}$
Output leadership	$W^{MO}(s^{MO}) = \frac{a^2}{k+2}$	$W^{PO}(s^{PO}) = \frac{a^2(5+26k+32k^2+14k^3+2k^4)^2}{2(1+k)(25+248k+878k^2+1424k^3+1250k^4+636k^5+188k^6+30k^7+2k^8)}$
Delegation leadership	$W^{MD}(s^{MD}) = \frac{a^2}{k+2}$	$W^{PD}(s^{PD}) = \frac{a^2(5+26k+32k^2+14k^3+2k^4)^2}{2(1+k)(25+248k+878k^2+1424k^3+1250k^4+636k^5+188k^6+30k^7+2k^8)}$

Table 2: Social Welfare: Pre-privatization and Pro-privatization

	Pre-privatization	Pro-privatization
Cournot competition	$u_0^{MC}(s^{MC}) = \frac{a}{k+2}$ $u_1^{MC}(s^{MC}) = \frac{a}{k+2}$	$u_0^{PC}(s^{MC}) = \frac{a}{k+2}$ $u_1^{PC}(s^{MC}) = \frac{a}{k+2}$
Output leadership	$u_0^{MO}(s^{MO}) = \frac{(k+1)a}{(k+2)^2}$ $u_1^{MO}(s^{MO}) = \frac{a}{k+2}$	$u_0^{PO}(s^{PO}) = \frac{a(5+53k+203k^2+335k^3+271k^4+114k^5+24k^6+2k^7)}{25+248k+878k^2+1424k^3+1250k^4+636k^5+188k^6+30k^7+2k^8}$ $u_1^{MO}(s^{MO}) = \frac{a(15+145k+527k^2+925k^3+894k^4+501k^5+162k^6+28k^7+2k^8)}{(1+k)(25+248k+878k^2+1424k^3+1250k^4+636k^5+188k^6+30k^7+2k^8)}$
Delegation leadership	$u_0^{MD}(s^{MD}) = \frac{a}{k+2}$ $u_1^{MD}(s^{MD}) = \frac{a}{k+2}$	$u_0^{PD}(s^{PD}) = \frac{a(15+145k+527k^2+925k^3+894k^4+501k^5+162k^6+28k^7+2k^8)}{(1+k)(25+248k+878k^2+1424k^3+1250k^4+636k^5+188k^6+30k^7+2k^8)}$ $u_1^{PD}(s^{PD}) = \frac{a(10+114k+469k^2+879k^3+878k^4+499k^5+162k^6+28k^7+2k^8)}{(1+k)(25+248k+878k^2+1424k^3+1250k^4+636k^5+188k^6+30k^7+2k^8)}$

Table 3: Effective Subsidies: Pre-privatization and Pro-privatization