G-COE GLOPE II Working Paper Series

Intergenerational equity and consensus among generations

Kohei Kamaga and Yoshio Kamijo

Working Paper No.26

Intergenerational equity and consensus among generations*

Kohei Kamaga[†] Yoshio Kamijo[‡]

June 2, 2009

Abstract

This paper studies evaluation relations for infinite-horizon utility streams. We propose a new resolution concept for conflicts among the present and future generations, under the name Respect for Unanimous Gain for the Future (RUGF), which requires us not to conclude social strict preference against unanimous strict preferences of future generations. The basic infinite-horizon extensions of the utilitarian and leximin principles proposed by Basu and Mitra (BM) [J Econ Theory 133: 350-373] and Bossert, Sprumont and Suzumura (BSS) [J Econ Theory 135: 579-589] satisfy this condition. On the other hand, the utilitarian and leximin overtaking criteria violate the condition, although they alleviate incompleteness of the utilitarian and leximin principles of BM and BSS type. We formulate new evaluation relations, called the consensus leximin and the consensus utilitarian social welfare relations (SWRs), and show that our new SWRs satisfy RUGF and also alleviate incompleteness of the leximin and utilitarian principles of BM and BSS type. The axiomatic characterizations of these new SWRs are established.

Keywords: Intergenerational equity; Respect for Unanimous Gain for the Future; Leximin principle; Utilitarianism

JEL Classification Numbers: D63; D71

*We are grateful to Geir Asheim and Koichi Suga for helpful comments. Needless to say, the authors are solely responsible for any defects which may still remain.

[†]Graduate School of Economics, Waseda University, Tokyo, Japan;
e-mail: k-kmg@ruri.waseda.jp (K. Kamaga)

[‡]Faculty of Political Science and Economics, Waseda University, Tokyo, Japan;
e-mail: kami-jo@suou.waseda.jp (Y. Kamijo)

1 Introduction

This paper studies ranking principles for infinite utility streams (infinite-dimensional vectors of real numbers). Infinite utility streams are usually interpreted as utility distributions among an infinite number of generations or as period utilities of an infinitely-lived individual. In both cases, utilities are assumed to be generated by very long-term economic policies. Thus, ranking principles for infinite utility streams which we will work on can be used to evaluate relative goodness of alternative long-term economic policies underlying utility streams. Recent studies on intergenerational welfare evaluations provide axiomatic analyses of ranking principles based on the former interpretation of infinite utility streams. In this paper, we follow this approach and examine eligible evaluation relations for intergenerational utility distributions.

In the literature on intergenerational welfare evaluations, Basu and Mitra (2007) recently propose the infinite-horizon reformulation of utilitarianism, called the *utilitarian* social welfare relation (SWR). Its leximin counterpart, called the *leximin* SWR, is also formalized in Bossert, Sprumont and Suzumura (2007). Compared to the existing infinite-horizon reformulations of utilitarianism and the leximin principle, these two SWRs can be seen to be the most basic ones since their axiomatic characterizations are given (in terms of subrelation) by the infinite-horizon variants of the axioms characterizing the finite-horizon utilitarian and leximin social welfare orderings (SWOs). The utilitarian SWR is characterized (in terms of subrelation) with the standard axioms of efficiency and impartiality, Strong Pareto (**SP**) and Finite Anonymity (**FA**), and the informational invariance axiom called Partial Translation-Scale Invariance (**PTSI**) (Basu and Mitra 2007), and the leximin SWR is done by replacing **PTSI** with the equity axiom called Hammond Equity (**HE**) (Bossert, Sprumont and Suzumura 2007).[1] In view of their axiomatic foundations, it seems plausible to use the utilitarian and leximin SWRs in evaluating intergenerational utility distributions. However, these SWRs are incomplete since both of them rank only the streams whose tail-parts are Pareto comparable, and consequently, selectivity inherent in them remains in an unsatisfactory level.

Recent contributions by Lauwers (2006) and Zame (2007) confirm the conjecture by Fleurbaey and Michel (2003) that any infinite-horizon SWO satisfying **SP** and **FA** must involve the use of non-constructive mathematics, i.e., it cannot have an explicit description and is of no use for a practical purpose. This means that we cannot extend the utilitarian and the leximin SWRs into a complete SWR with explicit description. Hence, the task we should address now is to extend these SWRs to, though still incomplete, as selective ones as possible.[2] The *overtaking* SWR due to von Weizsäcker

[1] Partial Translation-Scale Invariance is called Partial Unit Comparability in Basu and Mitra (2007).

[2] Instead of laying down completeness, Fleurbaey and Michel (2003), Sakai (2009) and Kamaga (2009) examine another route by weakening transitivity to quasi-transitivity.

(1965) and its leximin counterpart, called the *W-leximin* SWR, proposed by Asheim and Tungodden (2004) do extend the utilitarian and the leximin SWRs respectively and alleviate incompleteness of the utilitarian and the leximin SWRs at some level. However, enhanced selectivity of the overtaking and the W-leximin SWRs comes at the cost of a certain sacrifice of future generations' interests. For example, consider the following streams $\boldsymbol{x}$ and $\boldsymbol{y}$:

$$\begin{cases} \boldsymbol{x} = \left(1, \frac{1}{8}, \frac{1}{32}, \dots, \frac{1}{2^{2n-1}}, \dots\right) \\ \boldsymbol{y} = \left(0, \frac{1}{4}, \frac{1}{16}, \dots, \frac{1}{2^{2(n-1)}}, \dots\right) \end{cases}$$

As we will observe with the precise definitions of the overtaking and the W-leximin SWRs in Sect. 3, both of these SWRs conclude that $\boldsymbol{x}$ is strictly preferable to $\boldsymbol{y}$. However, the reader may notice that every generation excepting the first generation attains higher level of utility in $\boldsymbol{y}$ than in $\boldsymbol{x}$. Thus, such an evaluation for $\boldsymbol{x}$ and $\boldsymbol{y}$ entails a sacrifice of interests of all the future generations and legitimacy of such an evaluation may be questionable.

The purpose of this paper is to extend the utilitarian and the leximin SWRs to more selective ones, but in a different way from the utilitarian and leximin overtaking criteria to avoid the aforementioned undesirable conclusion. To work on this task, we formulate a new resolution concept called Respect for Unanimous Gain for the Future (**RUGF**). **RUGF** asserts that in the situations involving the conflict between the present generations and all the future generations, typically described in the streams $\boldsymbol{x}$ and $\boldsymbol{y}$, we should not conclude social strict preference against unanimous strict preferences of all the future generations. Thus, for the streams $\boldsymbol{x}$ and $\boldsymbol{y}$ above, this axiom requires us not to conclude that $\boldsymbol{x}$ is strictly preferable to $\boldsymbol{y}$. In this paper, we explore new SWRs which extend the utilitarian and the leximin SWRs without compromising **RUGF**. As we observed in the example of the streams $\boldsymbol{x}$ and $\boldsymbol{y}$, the leximin and utilitarian overtaking criteria violate **RUGF**, although they alleviate incompleteness of the utilitarian and the leximin SWRs. Our first result (Proposition 1) shows that this impossibility is ascribed to incompatibility of **RUGF** and three axioms common to the leximin and utilitarian overtaking criteria: **SP**, **FA**, and Weak Preference Continuity (**WPC**).

As an alternative to the utilitarian and the leximin overtaking criteria, we formulate new extended utilitarian and leximin SWRs, called the *consensus utilitarian* and the *consensus leximin* SWRs. We show that both the consensus utilitarian and the consensus leximin SWRs satisfy **RUGF** and also achieve strictly higher level of selectivity than the utilitarian and the leximin SWRs. Furthermore, they satisfy the extended anonymity called Fixed-Step Anonymity (or $\mathcal{S}$-Anonymity) and also exhibit higher selectivity even than the well-established extensions of the utilitarian and the leximin

SWRs called $\mathcal{S}$-*utilitarian* and the $\mathcal{S}$-*leximin* SWRs proposed by Banerjee (2006) and Kamaga and Kojima (2009a). The axiomatic characterizations of the consensus leximin and the consensus utilitarian SWRs are established by adding a consistency axiom called Fixed-Step Coherence (**FC**) to the basic axioms corresponding to the leximin and the utilitarian SWRs.

We also clarify the relationship between the consensus leximin and the consensus utilitarian SWRs, on the one hand, and some existing extensions of the leximin and utilitarian overtaking criteria, on the other. Categorizing the consensus leximin and consensus utilitarian SWRs and the $\mathcal{S}$-leximin and $\mathcal{S}$-utilitarian SWRs as one group constituting one possible route of extension of the leximin and utilitarian SWRs, our first proposition tells that **RUGF** demarcates this route of extension and another well-established route of extension via the leximin and utilitarian overtaking criteria. Examples of the SWRs which belong to the latter route include the *fixed-step W-leximin* and the *fixed-step overtaking* SWRs proposed by Kamaga and Kojima (2009b).[3] We will show that in the former route of extension, the consensus leximin and the consensus utilitarian SWRs can be seen to be the counterparts of the fixed-step W-leximin and the fixed-step overtaking SWRs.

The rest of the paper is organized as follows. Sect. 2 introduces notation and definitions. In Sect. 3, we introduce **RUGF** and also present the impossibility result regarding this axiom. Sect. 4 introduces the consensus leximin and the consensus utilitarian SWRs, and we establish the axiomatic characterizations of them. Then, we clarify the relationship between the two consensus SWRs and some well-established SWRs. Sect. 5 concludes with a few remarks.

2 Notation and definitions

Let $\mathbb{R}$ be the set of all real numbers, $\mathbb{Q}$ the set of all rational numbers, and $\mathbb{N}$ the set of all positive integers. Throughout this paper excepting Proposition 1 in Sect. 3, we let $X = \mathbb{R}^{\mathbb{N}}$ be the set of all utility streams $\boldsymbol{x} = (x_1, x_2, \dots)$. For all $i \in \mathbb{N}$, x_i is interpreted as the utility level of the ith generation.

Negation of a statement is indicated by the symbol $\neg$. Our notation for vector inequalities on X is as follows: for all $\boldsymbol{x}, \boldsymbol{y} \in X$, $\boldsymbol{x} \geqslant \boldsymbol{y}$ if $x_i \geq y_i$ for all $i \in \mathbb{N}$, and $\boldsymbol{x} > \boldsymbol{y}$ if $\boldsymbol{x} \geqslant \boldsymbol{y}$ and $\boldsymbol{x} \neq \boldsymbol{y}$. We also use this notation for finite-horizon vectors. Given two sets A and B, we write $A \subseteq B$ to mean A is a subset of B and $A \subsetneq B$ to mean $A \subseteq B$ and $A \neq B$.

We present some symbols for (finite or infinite) utility streams. For all $\boldsymbol{x} \in X$ and

[3] Asheim and Banerjee (2009) also examine these SWRs in a generalized formulation.

all $m, n \in \mathbb{N}$ with $m < n$, we write $\boldsymbol{x}_{[m,n]}$ and $\boldsymbol{x}_{[m,\infty]}$ respectively as:

$$\boldsymbol{x}_{[m,n]} = (x_m, x_{m+1}, \ldots, x_n) \in \mathbb{R}^{n-m+1} \text{ and } \boldsymbol{x}_{[m,\infty]} = (x_m, x_{m+1}, \ldots) \in X.$$

For convenience, we also write $\boldsymbol{x}_{[1,1]}$ and $(\boldsymbol{x}_{[1,0]}, \boldsymbol{y})$ as, respectively, $\boldsymbol{x}_{[1,1]} = x_1$ and $(\boldsymbol{x}_{[1,0]}, \boldsymbol{y}) = \boldsymbol{y}$, for all $\boldsymbol{x}, \boldsymbol{y} \in X$. Thus, for all $\boldsymbol{x} \in X$ and all $n \in \mathbb{N}$, $\boldsymbol{x} = (\boldsymbol{x}_{[1,n]}, \boldsymbol{x}_{[n+1,\infty]})$. For all $\boldsymbol{x} \in X$ and all $k, n \in \mathbb{N}$, we denote the nth part of k-periodic segments of $\boldsymbol{x}$ by $\boldsymbol{x}_{(n;k)}$. Formally, $\boldsymbol{x}_{(n;k)}$ is defined by:

$$\boldsymbol{x}_{(n;k)} = (x_{(n-1)k+1}, \ldots, x_{(n-1)k+(k-1)}, x_{nk}) \in \mathbb{R}^k.$$

For each finite subset $N \subseteq \mathbb{N}$ and all $\boldsymbol{x}, \boldsymbol{y} \in X$, let $(\boldsymbol{x}_N, \boldsymbol{y}_{\mathbb{N}\setminus N})$ denote the stream in X whose ith element is x_i if $i \in N$ and is y_i otherwise. Thus, taking $\boldsymbol{z}$ as $\boldsymbol{z} \equiv (\boldsymbol{x}_N, \boldsymbol{y}_{\mathbb{N}\setminus N})$, it must be that $z_i = x_i$ for all $i \in N$ and $z_i = y_i$ for all $i \in \mathbb{N}\setminus N$. Along the line of this symbolization, for all $\boldsymbol{x}, \boldsymbol{y} \in X$ and all $k, n \in \mathbb{N}$, let $(\boldsymbol{x}_{(n;k)}, \boldsymbol{y}_{\mathbb{N}\setminus(n;k)})$ denote the stream in X whose ith element is x_i if $i \in \{(n-1)k+1, \ldots, nk\}$ and is y_i otherwise, i.e., nth part of k-periodic segments of $\boldsymbol{y}$ is replaced with that of $\boldsymbol{x}$. Similarly, for all $\boldsymbol{x}, \boldsymbol{y} \in X$, all $k \in \mathbb{N}$ and all finite subset $N \subset \mathbb{N}$, we write $(\boldsymbol{x}_{(N;k)}, \boldsymbol{y}_{\mathbb{N}\setminus(N;k)})$ to denote the stream in X whose ith element is x_i if $i \in \{(n-1)k+1, \ldots, nk\}$ for some $n \in N$ and is y_i otherwise. For all $x \in \mathbb{R}$, let $(x)_{\text{con}} = (x, x, \ldots) \in X$. For all $n \in \mathbb{N}$ and all $(x_1, \ldots, x_n) \in \mathbb{R}^n$, $(\tilde{x}_1, \ldots, \tilde{x}_n)$ denotes a rank-ordered permutation of $(x_1, \ldots, x_n)$ such that $\tilde{x}_1 \leq \cdots \leq \tilde{x}_n$, ties being broken arbitrarily.

A binary relation $\succsim$ on X is a subset of $X \times X$. For convenience, the fact that $(\boldsymbol{x}, \boldsymbol{y}) \in \succsim$ will be symbolized by $\boldsymbol{x} \succsim \boldsymbol{y}$. An asymmetric part of $\succsim$ is denoted by $\succ$ and a symmetric part by $\sim$, i.e. $\boldsymbol{x} \succ \boldsymbol{y}$ if and only if $\boldsymbol{x} \succsim \boldsymbol{y}$ and $\neg(\boldsymbol{y} \succsim \boldsymbol{x})$, and $\boldsymbol{x} \sim \boldsymbol{y}$ if and only if $\boldsymbol{x} \succsim \boldsymbol{y}$ and $\boldsymbol{y} \succsim \boldsymbol{x}$. A SWR is a reflexive and transitive binary relation on X, i.e. a quasi-ordering, and a SWO is a complete SWR.[4] A binary relation $\succsim_A$ is said to be a subrelation of a binary relation $\succsim_B$ if, for all $\boldsymbol{x}, \boldsymbol{y} \in X$, (i) $\boldsymbol{x} \sim_A \boldsymbol{y}$ implies $\boldsymbol{x} \sim_B \boldsymbol{y}$ and (ii) $\boldsymbol{x} \succ_A \boldsymbol{y}$ implies $\boldsymbol{x} \succ_B \boldsymbol{y}$. A SWO which includes a binary relation $\succsim$ as a subrelation is said to be an ordering extension of $\succsim$.

We represent any permutation on the set $\mathbb{N}$ by a permutation matrix. A permutation matrix is an infinite matrix $\boldsymbol{P} = (p_{ij})_{i,j \in \mathbb{N}}$ satisfying the following properties:

1. for each $i \in \mathbb{N}$, there exists $j(i) \in \mathbb{N}$ such that $p_{ij(i)} = 1$ and $p_{ij} = 0$ for all $j \neq j(i)$;

2. for each $j \in \mathbb{N}$, there exists $i(j) \in \mathbb{N}$ such that $p_{i(j)j} = 1$ and $p_{ij} = 0$ for all $i \neq i(j)$.

[4] A binary relation $\succsim$ on X is (i) reflexive if, for all $\boldsymbol{x} \in X$, $\boldsymbol{x} \succsim \boldsymbol{x}$; (ii) transitive if, for all $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in X$, $\boldsymbol{x} \succsim \boldsymbol{z}$ holds whenever $\boldsymbol{x} \succsim \boldsymbol{y}$ and $\boldsymbol{y} \succsim \boldsymbol{z}$; (iii) complete if, for all $\boldsymbol{x}, \boldsymbol{y} \in X$ with $\boldsymbol{x} \neq \boldsymbol{y}$, $\boldsymbol{x} \succsim \boldsymbol{y}$ or $\boldsymbol{y} \succsim \boldsymbol{x}$.

Let $\mathcal{P}$ be the set of all permutation matrices. Note that, for all $\boldsymbol{x} \in X$ and all $\boldsymbol{P} \in \mathcal{P}$, the product $\boldsymbol{Px} = (Px_1, Px_2, \ldots)$ belongs to X, where $Px_i = \sum_{k \in \mathbb{N}} p_{ik} x_k$ for all $i \in \mathbb{N}$. For any $\boldsymbol{P} \in \mathcal{P}$, let $\boldsymbol{P}'$ be the inverse of $\boldsymbol{P}$ satisfying $\boldsymbol{P}'\boldsymbol{P} = \boldsymbol{P}\boldsymbol{P}' = \boldsymbol{I}$, where $\boldsymbol{I}$ is the infinite identity matrix.[5] For all $\boldsymbol{P} = (p_{ij})_{i,j \in \mathbb{N}} \in \mathcal{P}$ and all $n \in \mathbb{N}$, let $\boldsymbol{P}(n)$ denote the $n \times n$ matrix $(p_{ij})_{i,j \in \{1,\ldots,n\}}$. A matrix $\boldsymbol{P} = (p_{ij})_{i,j \in \mathbb{N}} \in \mathcal{P}$ is a finite permutation matrix if there exists $n \in \mathbb{N}$ such that $p_{ii} = 1$ for all $i > n$. Let $\mathcal{F}$ be the set of all finite permutation matrices. We denote by $\mathcal{S}$ the following set of permutation matrices:

$$\mathcal{S} = \left\{ \boldsymbol{P} \in \mathcal{P} : \begin{array}{l} \text{there exists } k \in \mathbb{N} \text{ such that, for each } n \in \mathbb{N}, \\ \boldsymbol{P}(nk) \text{ is a finite-dimensional permutation matrix} \end{array} \right\}.$$

The class $\mathcal{S}$ is exactly the set of all fixed-step permutations which was first introduced by Lauwers (1997b).[6] It is easily checked that $\mathcal{F} \subsetneq \mathcal{S}$.

3 Respect for unanimous gain for the future

We introduce a new resolution concept which deals with conflicts between the present generation and all the future generations.

Respect for Unanimous Gain for the Future (RUGF): For all $\boldsymbol{x}, \boldsymbol{y} \in X$, if $x_1 > y_1$ and $y_n > x_n$ for all $n \in \mathbb{N} \backslash \{1\}$, then $\neg(\boldsymbol{x} \succ \boldsymbol{y})$.

RUGF asserts that if the present generation solely prefers a stream $\boldsymbol{x}$ to $\boldsymbol{y}$ while all the future generations unanimously show the opposite strict preference, then we should not conclude that $\boldsymbol{x}$ is strictly preferable to $\boldsymbol{y}$. **RUGF** and the following standard Paretian axiom together give a veto to the coalition of all the future generations:

Strong Pareto (SP): For all $\boldsymbol{x}, \boldsymbol{y} \in X$ with $\boldsymbol{x} > \boldsymbol{y}$, $\boldsymbol{x} \succ \boldsymbol{y}$.

Formally, **RUGF** and **SP** imply that for all $\boldsymbol{x}, \boldsymbol{y} \in X$, if $y_n > x_n$ for all $n \in \mathbb{N} \backslash \{1\}$, then $\neg(\boldsymbol{x} \succ \boldsymbol{y})$.

The axiom **RUGF** implies the one-sided equity axiom due to Asheim and Tungodden (2005), called *Hammond Equity for the Future*. Furthermore, in the presence of **SP**, it also implies the weak version of *Non-Substitution* originally introduced by Lauwers (1998):[7]

Hammond Equity for the Future (HEF): For all $x, y, z, w \in \mathbb{R}$, if $x > y > z > w$, then $\neg((x, (w)_{\text{con}}) \succ (y, (z)_{\text{con}}))$.

[5] For any $\boldsymbol{P}, \boldsymbol{Q} \in \mathcal{P}$, the product $\boldsymbol{PQ}$ is defined by $(r_{ij})_{i,j \in \mathbb{N}}$ with $r_{ij} = \sum_{k \in \mathbb{N}} p_{ik} q_{kj}$.
[6] See also Fleurbaey and Michel (2003), Lauwers (2006) and Mitra and Basu (2007).
[7] The axiom WNS is the version considered by Asheim, Mitra and Tungodden (2007).

Weak Non-Substitution (WNS): For all $x, y, z, w \in \mathbb{R}$, if $z > w$, then $\neg((x, (w)_{\text{con}}) \succ (y, (z)_{\text{con}}))$.

These three axioms can be used to assess reflected consideration for future generations' interests in the SWRs we use. **RUGF** gives the strongest priority to future generations' interests among the three.

To make clear differences among these three axioms, we introduce two types of infinite-horizon extensions of the finite-horizon utilitarian and leximin orderings. For each $n \in \mathbb{N}$, let $\succsim_U^n$ denote the finite-horizon utilitarian ordering defined on $\mathbb{R}^n$: for all $\boldsymbol{x}_{[1,n]}, \boldsymbol{y}_{[1,n]} \in \mathbb{R}^n$, $\boldsymbol{x}_{[1,n]} \succsim_U^n \boldsymbol{y}_{[1,n]}$ iff $\sum_{i=1}^n x_i \geq \sum_{i=1}^n y_i$. For each $n \in \mathbb{N}$, let $\succsim_L^n$ denote the finite-horizon leximin ordering defined on $\mathbb{R}^n$: for all $\boldsymbol{x}_{[1,n]}, \boldsymbol{y}_{[1,n]} \in \mathbb{R}^n$, $\boldsymbol{x}_{[1,n]} \succsim_L^n \boldsymbol{y}_{[1,n]}$ iff $(\tilde{x}_1, \ldots, \tilde{x}_n) = (\tilde{y}_1, \ldots, \tilde{y}_n)$ or there exists an integer $m < n$ such that $(\tilde{x}_1, \ldots, \tilde{x}_m) = (\tilde{y}_1, \ldots, \tilde{y}_m)$ and $\tilde{x}_{m+1} > \tilde{y}_{m+1}$.

The *utilitarian* and *leximin* SWRs, denoted $\succsim_U$ and $\succsim_L$ respectively, are defined as: for all $\boldsymbol{x}, \boldsymbol{y} \in X$,

$$\begin{aligned} \boldsymbol{x} \succsim_U \boldsymbol{y} &\text{ iff there exists } n \in \mathbb{N} \text{ such that} \\ &\boldsymbol{x}_{[1,n]} \succsim_U^n \boldsymbol{y}_{[1,n]} \text{ and } \boldsymbol{x}_{[n+1,\infty]} \geqslant \boldsymbol{y}_{[n+1,\infty]}; \quad (1) \\ \boldsymbol{x} \succsim_L \boldsymbol{y} &\text{ iff there exists } n \in \mathbb{N} \text{ such that} \\ &\boldsymbol{x}_{[1,n]} \succsim_L^n \boldsymbol{y}_{[1,n]} \text{ and } \boldsymbol{x}_{[n+1,\infty]} \geqslant \boldsymbol{y}_{[n+1,\infty]}. \quad (2) \end{aligned}$$

The *overtaking* and *W-leximin* SWRs, $\succsim_O$ and $\succsim_{Lw}$, are defined as: for all $\boldsymbol{x}, \boldsymbol{y} \in X$,

$$\boldsymbol{x} \succsim_O \boldsymbol{y} \text{ iff } \begin{cases} \text{there exists } \bar{n} \in \mathbb{N} \text{ such that } \boldsymbol{x}_{[1,n]} \succ_U^n \boldsymbol{y}_{[1,n]} \text{ for all } n \geq \bar{n} \\ \text{or there exists } \bar{n} \in \mathbb{N} \text{ such that } \boldsymbol{x}_{[1,n]} \sim_U^n \boldsymbol{y}_{[1,n]} \text{ for all } n \geq \bar{n}; \end{cases} \quad (3)$$

$$\boldsymbol{x} \succsim_{Lw} \boldsymbol{y} \text{ iff } \begin{cases} \text{there exists } \bar{n} \in \mathbb{N} \text{ such that } \boldsymbol{x}_{[1,n]} \succ_L^n \boldsymbol{y}_{[1,n]} \text{ for all } n \geq \bar{n} \\ \text{or there exists } \bar{n} \in \mathbb{N} \text{ such that } \boldsymbol{x}_{[1,n]} \sim_L^n \boldsymbol{y}_{[1,n]} \text{ for all } n \geq \bar{n}. \end{cases} \quad (4)$$

The first two SWRs, $\succsim_U$ and $\succsim_L$, are proposed by Basu and Mitra (2007) and Bossert, Sprumont and Suzumura (2007) respectively, and the last two, $\succsim_O$ and $\succsim_{Lw}$, are due to von Weizsäcker (1965) and Asheim and Tungodden (2004) respectively. While all of the four SWRs satisfy HEF, the W-leximin SWR $\succsim_{Lw}$ violates WNS. Hence, from the viewpoint of reflected consideration for future generations, WNS excludes $\succsim_{Lw}$ from permissible SWRs, whereas HEF does none of the four SWRs. Now recall that in comparing the following streams $\boldsymbol{x}$ and $\boldsymbol{y}$, both $\succsim_{Lw}$ and $\succsim_O$ conclude that $\boldsymbol{x}$ is strictly preferable to $\boldsymbol{y}$:

$$\boldsymbol{x} = \left(1, \frac{1}{8}, \frac{1}{32}, \ldots, \ \frac{1}{2^{2n-1}}, \ \ldots\right) \text{ and } \boldsymbol{y} = \left(0, \frac{1}{4}, \frac{1}{16}, \ldots, \frac{1}{2^{2(n-1)}}, \ldots\right).$$

Note that our new axiom, **RUGF**, requires us not to conclude $\boldsymbol{x}$ is strictly preferable to $\boldsymbol{y}$. Thus, **RUGF** sets the boundary between $\succsim_L$ and $\succsim_U$ which d'Aspremont (2007) generically calls *simplified criteria*, on the one hand, and the leximin and utilitarian *overtaking* criteria $\succsim_{Lw}$ and $\succsim_O$, on the other, asserting that only the first two SWRs are permissible among the four.[8]

As we will see in the proposition below, this observation regarding rejection by **RUGF** can be generalized to any SWR satisfying **SP** and the following two axioms that $\succsim_{Lw}$ and $\succsim_O$ commonly satisfy:

$\mathcal{F}$-Anonymity (FA): For all $\boldsymbol{x} \in X$ and all $\boldsymbol{P} \in \mathcal{F}$, $\boldsymbol{Px} \sim \boldsymbol{x}$.

Weak Preference Continuity (WPC): For all $\boldsymbol{x}, \boldsymbol{y} \in X$, if $(\boldsymbol{x}_{[1,n]}, \boldsymbol{y}_{[n+1,\infty]}) \succ \boldsymbol{y}$ for all $n \in \mathbb{N}$, then $\boldsymbol{x} \succ \boldsymbol{y}$.

The axiom **FA** is a standard requirement of impartial treatment of generations and is also called *Finite* (or *Weak*) *Anonymity* in the literature. The axiom **WPC** is the version recently proposed by Asheim and Banerjee (2009), and it basically asserts that our evaluation of infinite-horizon utility streams should be consistent with an infinite number of evaluation of their truncated streams.[9]

The following proposition tells that our observation regarding rejection by **RUGF** can be ascribed to the incompatibility of **RUGF** and three basic axioms, **SP**, **FA** and **WPC**.

Proposition 1. *Let $X \supseteq Y^{\mathbb{N}}$ with $Y = [0,1] \cap \mathbb{Q}$. Then, there is no SWR on X that satisfies **SP**, **FA**, **WPC** and **RUGF**.*

Proof. Consider the streams $\boldsymbol{x}$ and $\boldsymbol{y}$ we discussed above, i.e., $\boldsymbol{x} = (1, \frac{1}{8}, \ldots, \frac{1}{2^{2n-1}}, \ldots)$ and $\boldsymbol{y} = (0, \frac{1}{4}, \ldots, \frac{1}{2^{2(n-1)}}, \ldots)$. Clearly, $\boldsymbol{x}, \boldsymbol{y} \in X$. Let $\succsim$ be any SWR defined on X satisfying **SP**, **FA** and **WPC**. Then, consider a sequence of permutations $\{\boldsymbol{P}^n = (p^n_{ij})_{i,j \in \mathbb{N}}\}_{n \in \mathbb{N}} \subseteq \mathcal{F}$ such that $\boldsymbol{P}^1 = \boldsymbol{I}$ and, for all $n \in \mathbb{N} \backslash \{1\}$, $p^n_{21} = \ldots = p^n_{ji} = \ldots = p^n_{1n} = 1$ where $i \in \{2, \ldots, n-1\}$ and $j = i+1$, and $p^n_{jj} = 1$ for all $j > n$. Let

[8] Some may assert that following the spirit of utilitarianism, the ranking by $\succsim_O$ that $\boldsymbol{x}$ is strictly preferable to $\boldsymbol{y}$ is reasonable since the difference in utility sums taken for $\boldsymbol{x}$ and $\boldsymbol{y}$ converges to a positive number. However, we should note that in comparing the streams $\boldsymbol{w} = (\frac{3}{4})_{\text{cont}}$ and $\boldsymbol{z} = (\frac{1}{4}, 1, \frac{7}{8}, \ldots, (\frac{3}{4} + \frac{1}{2^n}), \ldots)$ for which the difference in utility sums converges to zero, $\succsim_O$ concludes, against the adherence to the utilitarian spirit and also the requirement of **RUGF**, that $\boldsymbol{w}$ is strictly preferable to $\boldsymbol{z}$. In view of this inconsistency in adherence to the utilitarian spirit, the assessment by **RUGF** that $\succsim_O$ is ineligible infinite-horizon utilitarian principle may get a certain level of support from the utilitarian point of view. The similar observation also holds for $\succsim_{Lw}$.

[9] In Asheim and Banerjee (2009), **WPC** is called *Weak Preference Continuity 1* (WPC1). This version of the axiom is the weakest among the similar axioms formalized in the same spirit in the literature. They establish the characterizations of $\succsim_O$ and $\succsim_{Lw}$ with WPC1 in a general form.

$\boldsymbol{w}^n = \boldsymbol{P}^n\boldsymbol{x}$ for all $n \in \mathbb{N}$. Note that $(\boldsymbol{w}^n_{[1,n]}, \boldsymbol{y}_{[n+1,\infty]}) > \boldsymbol{y}$ for all $n \in \mathbb{N}$. By **SP** and **FA**, $(\boldsymbol{w}^n_{[1,n]}, \boldsymbol{y}_{[n+1,\infty]}) \succ \boldsymbol{y}$ and $(\boldsymbol{x}_{[1,n]}, \boldsymbol{y}_{[n+1,\infty]}) \sim (\boldsymbol{w}^n_{[1,n]}, \boldsymbol{y}_{[n+1,\infty]})$ for all $n \in \mathbb{N}$. By transitivity, $(\boldsymbol{x}_{[1,n]}, \boldsymbol{y}_{[n+1,\infty]}) \succ \boldsymbol{y}$ for all $n \in \mathbb{N}$. By **WPC**, $\boldsymbol{x} \succ \boldsymbol{y}$, which means $\succsim$ violates **RUGF**. ■

Remark 1. The impossibility in Proposition 1 still holds even if we weaken (i) collective rationality from transitivity to *finite transitivity* and (ii) preference continuity from **WPC** to *Time-Invariant Preference Continuity*:[10]

Finite Transitivity: For all $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in X$ with $\boldsymbol{x}_{[n+1,\infty]} = \boldsymbol{y}_{[n+1,\infty]} = \boldsymbol{z}_{[n+1,\infty]}$ for some $n \in \mathbb{N}$, $\boldsymbol{x} \succsim \boldsymbol{z}$ holds whenever $\boldsymbol{x} \succsim \boldsymbol{y}$ and $\boldsymbol{y} \succsim \boldsymbol{z}$;

Time-Invariant Preference Continuity (IPC): For all $\boldsymbol{x}, \boldsymbol{y} \in X$, if there exists a finite set $M \subseteq \mathbb{N}$ such that, for all finite set N with $M \subseteq N$, $(\boldsymbol{x}_N, \boldsymbol{y}_{\mathbb{N}\setminus N}) \succ \boldsymbol{y}$, then $\boldsymbol{x} \succ \boldsymbol{y}$.

Finitely transitive relations are examined in Fleurbaey and Michel (2003) and Sakai (2009). IPC is first introduced by Asheim, d'Aspremont and Banerjee (2008). From Proposition 1 and Remark 1, violation of **RUGF** is observed for the existing relations satisfying those axioms: e.g., the *catching-up criterion* due to Atsumi (1965) and von Weizsäcker (1965) and its leximin counterpart by Asheim and Tungodden (2004); the leximin and utilitarian *time-invariant overtaking criteria* by Asheim, d'Aspremont and Banerjee (2008); and the leximin and utilitarian *fixed-step catching-up* by Asheim and Banerjee (2009); *Type 3* relation in Fleurbaey and Michel (2003); the *future agreement extension* of the leximin (and utilitarian) principle in Sakai (2009); and the four SWRs we will present in Sect. 4.2: the S*-W-leximin*, the S*-overtaking*, and the *fixed-step leximin* and the *fixed-step overtaking* SWRs by Kamaga and Kojima (2009b).[11]

As we observed above, both $\succsim_L$ and $\succsim_U$ pass the assessment set by **RUGF**. The key in this possibility result is the fact that these two SWRs apply the Pareto criterion to tail-parts of utility streams. However, due to the use of the Pareto criterion, they exhibit incompleteness at a serious level. In the leximin and utilitarian overtaking criteria, incompleteness is alleviated by replacing the simple use of the Pareto criterion with an infinite number of comparison of finite-horizon *truncated* streams, where *earlier generations* are permitted to retain their impact on the evaluation of utility streams at a certain level, and consequently, we are led to violation of **RUGF**.

Now, the task we should work on will be to explore extensions of $\succsim_L$ and $\succsim_U$ which alleviate incompleteness of these SWRs without compromising **RUGF**. The following proposition firmly tells us that exploration of such extensions will never prove fruitless.

[10] Note that to complete the proof of Proposition 1, it suffices to assume $\succsim$ satisfies finite transitivity. Furthermore, taking $M \subseteq \mathbb{N}$ as $M = \{1\}$, the impossibility in the case of IPC is obtained by using the streams $\boldsymbol{x}$ and $\boldsymbol{y}$ and applying basically the same argument.

[11] The future agreement extension of the utilitarian principle is exactly Type 3 relation by Fleurbaey and Michel (2003). Kamaga (2009) also analyzes finitely transitive relations, and he characterizes several relations satisfying **RUGF**, including the *future domination extensions* by Sakai (2009).

Furthermore, since $\succsim_L$ and $\succsim_U$ satisfies **SP** and **FA**, we obtain the direct corollary of it, which shows a positive consequence of dropping **WPC** in Proposition 1.

Proposition 2. *For each of $\succsim_L$ and $\succsim_U$, there exists an ordering extension of it on X satisfying **RUGF**.*

Proof. See Appendix. ■

Corollary 1. *There exists a SWO on X satisfying **SP**, **FA** and **RUGF**.*

As we mentioned in Introduction, the ordering extensions whose existence is ensured by Proposition 2 involve the use of non-constructive mathematics (Lauwers 2006; Zame 2007). However, Proposition 2 suggests that there would be (and as shown later, certainly exist) constructible SWRs which achieve higher level of selectivity than $\succsim_L$ and $\succsim_U$ without compromising **RUGF**. In the next section, we explore those SWRs.

4 Leximin and utilitarian consensus rules

4.1 Characterizations

We propose new infinite-horizon extensions of the leximin and utilitarian principles, which achieve higher selectivity than $\succsim_L$ or $\succsim_U$ without compromising **RUGF**. Let us introduce the following binary relations $\succsim_{CL}$ and $\succsim_{CU}$ defined on X: for all $\boldsymbol{x}, \boldsymbol{y} \in X$,

$$\boldsymbol{x} \succsim_{CL} \boldsymbol{y} \text{ iff there exists } k \in \mathbb{N} \text{ such that } \boldsymbol{x}_{(n;k)} \succsim_L^k \boldsymbol{y}_{(n;k)} \text{ for all } n \in \mathbb{N}; \quad (5)$$

$$\boldsymbol{x} \succsim_{CU} \boldsymbol{y} \text{ iff there exists } k \in \mathbb{N} \text{ such that } \boldsymbol{x}_{(n;k)} \succsim_U^k \boldsymbol{y}_{(n;k)} \text{ for all } n \in \mathbb{N}. \quad (6)$$

We call $\succsim_{CL}$ and $\succsim_{CU}$, respectively, *consensus leximin SWR* and *consensus utilitarian SWR* (cf. Remark 2). It is easily checked that $\succsim_{CL}$ and $\succsim_{CU}$ satisfy **RUGF**.

Remark 2. The relations $\succsim_{CL}$ and $\succsim_{CU}$ are reflexive and transitive on X, i.e., well-defined as a SWR on X. Furthermore, the asymmetric and symmetric parts of $\succsim_{CL}$ and $\succsim_{CU}$ are characterized as follows: for all $\boldsymbol{x}, \boldsymbol{y} \in X$,

$$\begin{cases} \boldsymbol{x} \succ_{CL} \boldsymbol{y} \text{ iff there exists } k \in \mathbb{N} \text{ such that } \boldsymbol{x}_{(n;k)} \succsim_L^k \boldsymbol{y}_{(n;k)} \text{ for all } n \in \mathbb{N} \\ \qquad \text{and there exists } \hat{n} \in \mathbb{N} \text{ such that } \boldsymbol{x}_{(\hat{n};k)} \succ_L^k \boldsymbol{y}_{(\hat{n};k)}, & \text{(7a)} \\ \boldsymbol{x} \sim_{CL} \boldsymbol{y} \text{ iff there exists } k \in \mathbb{N} \text{ such that } \boldsymbol{x}_{(n;k)} \sim_L^k \boldsymbol{y}_{(n;k)} \text{ for all } n \in \mathbb{N}; & \text{(7b)} \end{cases}$$

$$\begin{cases} \boldsymbol{x} \succ_{CU} \boldsymbol{y} \text{ iff there exists } k \in \mathbb{N} \text{ such that } \boldsymbol{x}_{(n;k)} \succsim_U^k \boldsymbol{y}_{(n;k)} \text{ for all } n \in \mathbb{N} \\ \qquad \text{and there exists } \hat{n} \in \mathbb{N} \text{ such that } \boldsymbol{x}_{(\hat{n};k)} \succ_U^k \boldsymbol{y}_{(\hat{n};k)}, & \text{(8a)} \\ \boldsymbol{x} \sim_{CU} \boldsymbol{y} \text{ iff there exists } k \in \mathbb{N} \text{ such that } \boldsymbol{x}_{(n;k)} \sim_U^k \boldsymbol{y}_{(n;k)} \text{ for all } n \in \mathbb{N}. & \text{(8b)} \end{cases}$$

This remark will be verified by Lemma 2 in Appendix which generalizes these results to the class of relations including $\succsim_{CL}$ and $\succsim_{CU}$ as special cases.

To verify properties inherent in the consensus SWRs in further detail and to make it easier to compare them with other existing SWRs, we provide axiomatic characterizations of $\succsim_{CL}$ and $\succsim_{CU}$. Let us introduce the following four axioms.

$\mathcal{S}$-Anonymity (SA): For all $\boldsymbol{x} \in X$ and all $\boldsymbol{P} \in \mathcal{S}$, $\boldsymbol{P}\boldsymbol{x} \sim \boldsymbol{x}$.

Hammond Equity (HE): For all $\boldsymbol{x}, \boldsymbol{y} \in X$, if there exist $i, j \in \mathbb{N}$ such that $y_i < x_i < x_j < y_j$ and $x_k = y_k$ for all $k \in \mathbb{N} \setminus \{i, j\}$, then $\boldsymbol{x} \succsim \boldsymbol{y}$.

Partial Translation Scale Invariance (PTSI): For all $\boldsymbol{x}, \boldsymbol{y} \in X$, all $\boldsymbol{a} \in \mathbb{R}^{\mathbb{N}}$ and all $n \in \mathbb{N}$, if $\boldsymbol{x}_{[n,\infty]} = \boldsymbol{y}_{[n,\infty]}$ and $\boldsymbol{x} \succsim \boldsymbol{y}$, then $\boldsymbol{x} + \boldsymbol{a} \succsim \boldsymbol{y} + \boldsymbol{a}$.

Fixed-Step Coherence (FC): For all $\boldsymbol{x}, \boldsymbol{y} \in X$, (i) if there exists $k \in \mathbb{N}$ such that $(\boldsymbol{x}_{(N;k)}, \boldsymbol{y}_{\mathbb{N}\setminus(N;k)}) \succsim \boldsymbol{y}$ for all finite subsets $N \subseteq \mathbb{N}$, then $\boldsymbol{x} \succsim \boldsymbol{y}$; (ii) if, in addition to this, there exists $\hat{n} \in \mathbb{N}$ such that $(\boldsymbol{x}_{(N;k)}, \boldsymbol{y}_{\mathbb{N}\setminus(N;k)}) \succ \boldsymbol{y}$ for all finite subsets $N \subseteq \mathbb{N}$ with $\{\hat{n}\} \subseteq N$, then $\boldsymbol{x} \succ \boldsymbol{y}$.

The axiom **SA** is also called *Fixed-Step Anonymity* and is stronger than **FA** since $\mathcal{F} \subsetneq \mathcal{S}$. **HE** is an infinite-horizon variant of the equity axiom due to Hammond (1976), which asserts that an order-preserving change which diminishes inequality of utilities between conflicting two generations is socially preferable. It formalizes stronger equity principle than HEF. **PTSI** postulates the invariance property corresponding to the assumption that utility differences of generations are comparable but utility levels are not.[12] **FC** is an axiom of consistency. Note that the antecedent in (i) is equivalent to the statement that "if there exists $k \in \mathbb{N}$ such that $(\boldsymbol{x}_{(N;k)}, \boldsymbol{y}_{\mathbb{N}\setminus(N;k)}) \succsim \boldsymbol{y}$ for all $n \in \mathbb{N}$ and all finite subsets $N \subseteq \mathbb{N}$ with $\{n\} \subseteq N$." Thus, this axiom postulates the consistency between an evaluation of infinite-horizon streams, on the one hand, and an infinite number of pairwise evaluations involving fictitious streams constructed not only by periodic replacement starting from the first k generations but also by all the k-periodic replacements of streams, on the other (cf. **WPC**).[13] In the presence of **FC**, **FA** and **SA** become equivalent.

We are ready to state the following characterizations of $\succsim_{CL}$ and $\succsim_{CU}$.

Theorem 1. *A SWR $\succsim$ on X satisfies **SP**, **FA**, **HE**, and **FC** if and only if $\succsim_{CL}$ is a subrelation of $\succsim$.*

Theorem 2. *A SWR $\succsim$ on X satisfies **SP**, **FA**, **PTSI**, and **FC** if and only if $\succsim_{CU}$ is a subrelation of $\succsim$.*

[12]For the detailed explanation of informational invariance axioms, we refer the reader to d'Aspremont and Gevers (2002) and Bossert and Weymark (2004).

[13]Thus, **FC** is weaker than the k-periodic extension of **WPC** introduced in Kamaga and Kojima (2009b).

Proof of Theorems 1 and 2. See Appendix, where the proof is done by using Lemma 3 which generalizes Theorems 1 and 2. ■

Theorem 1 (resp. 2) is interpreted as saying that $\succsim_{CL}$ (resp. $\succsim_{CU}$) is the least element (with respect to set inclusion) in the class of SWRs satisfying the axioms.[14] Theorems 1 and 2 can also be stated with **SA** in place of **FA**, but redundant.

4.2 Comparison with existing SWRs

We now clarify the relationship between the consensus SWRs and some well-established SWRs in the literature, including $\succsim_L$, $\succsim_U$, $\succsim_{Lw}$ and $\succsim_O$. Let us introduce six extended leximin and utilitarian SWRs.

The $\mathcal{S}$*-leximin* and $\mathcal{S}$*-utilitarian* SWRs, denoted by $\succsim_{SL}$ and $\succsim_{SU}$ respectively, are defined as: for all $\boldsymbol{x}, \boldsymbol{y} \in X$,

$$\boldsymbol{x} \succsim_{SL} \boldsymbol{y} \text{ iff there exists } \boldsymbol{P} \in \mathcal{S} \text{ such that } \boldsymbol{Px} \succsim_L \boldsymbol{y}; \tag{9}$$

$$\boldsymbol{x} \succsim_{SU} \boldsymbol{y} \text{ iff there exists } \boldsymbol{P} \in \mathcal{S} \text{ such that } \boldsymbol{Px} \succsim_U \boldsymbol{y}. \tag{10}$$

The $\mathcal{S}$*-W-leximin* and $\mathcal{S}$*-overtaking* SWRs, $\succsim_{SLw}$ and $\succsim_{SO}$, are defined by: for all $\boldsymbol{x}, \boldsymbol{y} \in X$,

$$\boldsymbol{x} \succsim_{SLw} \boldsymbol{y} \text{ iff there exist } \boldsymbol{P}, \boldsymbol{Q} \in \mathcal{S} \text{ such that } \boldsymbol{Px} \succsim_{Lw} \boldsymbol{Qy}; \tag{11}$$

$$\boldsymbol{x} \succsim_{SO} \boldsymbol{y} \text{ iff there exist } \boldsymbol{P}, \boldsymbol{Q} \in \mathcal{S} \text{ such that } \boldsymbol{Px} \succsim_O \boldsymbol{Qy}. \tag{12}$$

The *fixed-step leximin* and *fixed-step overtaking* SWRs, $\succsim_{FLw}$ and $\succsim_{FO}$, are defined as: for all $\boldsymbol{x}, \boldsymbol{y} \in X$,

$$\boldsymbol{x} \succsim_{FLw} \boldsymbol{y} \text{ iff } \begin{cases} \text{there exists } k \in \mathbb{N} \text{ such that } \boldsymbol{x}_{[1,nk]} \succ_L^{nk} \boldsymbol{y}_{[1,nk]} \text{ for all } n \in \mathbb{N} \\ \text{or there exists } k \in \mathbb{N} \text{ such that } \boldsymbol{x}_{[1,nk]} \sim_L^{nk} \boldsymbol{y}_{[1,nk]} \text{ for all } n \in \mathbb{N}; \end{cases} \tag{13}$$

$$\boldsymbol{x} \succsim_{FO} \boldsymbol{y} \text{ iff } \begin{cases} \text{there exists } k \in \mathbb{N} \text{ such that } \boldsymbol{x}_{[1,nk]} \succ_U^{nk} \boldsymbol{y}_{[1,nk]} \text{ for all } n \in \mathbb{N} \\ \text{or there exists } k \in \mathbb{N} \text{ such that } \boldsymbol{x}_{[1,nk]} \sim_U^{nk} \boldsymbol{y}_{[1,nk]} \text{ for all } n \in \mathbb{N}. \end{cases} \tag{14}$$

The SWRs $\succsim_{SU}$ and $\succsim_{SL}$ are proposed by Banerjee (2006) and Kamaga and Kojima (2009a) respectively. The other four SWRs are introduced by Kamaga and Kojima (2009b). The first four SWRs, $\succsim_{SL}$, $\succsim_{SU}$, $\succsim_{SLw}$ and $\succsim_{SO}$, are seen to be $\mathcal{S}$-anonymous extensions of $\succsim_L$, $\succsim_U$, $\succsim_{Lw}$ and $\succsim_O$ respectively. The SWRs $\succsim_{FLw}$ and

[14] For the formal explanation of this interpretation, see Banerjee (2006) and Basu and Mitra (2007). As usually discussed in the literature, we can conclude from Arrow's (1963) variant of Szpilrajn's (1930) Theorem that there exists a complete SWR, i.e. SWO, in these classes of SWRs (see Lemma 1 in Appendix). However, as we mentioned in Sect. 1, those SWOs can never be explicitly described (Lauwers 2006; Zame 2007).

$\succsim_{FO}$ are defined by applying the k-periodic extension method à la Lauwers (1997b) and Fleurbaey and Michel (2003) to $\succsim_{Lw}$ and $\succsim_O$. It is known that $\succsim_{SLw}$ and $\succsim_{FLw}$ are equivalent (Kamaga and Kojima 2009b).

Remark 3. The following relationship holds among the twelve SWRs we presented so far, where we write $\succsim_A \sqsubset \succsim_B$ to mean $\succsim_A$ is a subrelation of $\succsim_B$:

$$\begin{cases} \succsim_L & \sqsubset \succsim_{SL} & \sqsubset \succsim_{CL} \\ \sqcap & \sqcap & \sqcap \\ \succsim_{Lw} & \sqsubset \succsim_{SLw} & = \succsim_{FLw} \end{cases} \text{with} \begin{cases} \succ_L & \subsetneq \succ_{SL} & \subsetneq \succ_{CL} \\ \rotatebox[origin=c]{-90}{$\subsetneq$} & \rotatebox[origin=c]{-90}{$\subsetneq$} & \rotatebox[origin=c]{-90}{$\subsetneq$} \\ \succ_{Lw} & \subsetneq \succ_{SLw} & = \succ_{FLw} \end{cases} \text{and} \begin{cases} \sim_L & \subsetneq \sim_{SL} & = \sim_{CL} \\ \| & \| & \| \\ \sim_{Lw} & \subsetneq \sim_{SLw} & = \sim_{FLw} \end{cases} \tag{15}$$

$$\begin{cases} \succsim_U & \sqsubset \succsim_{SU} & \sqsubset \succsim_{CU} \\ \sqcap & \sqcap & \sqcap \\ \succsim_O & \sqsubset \succsim_{SO} & \sqsubset \succsim_{FO} \end{cases} \text{with} \begin{cases} \succ_U & \subsetneq \succ_{SU} & \subsetneq \succ_{CU} \\ \rotatebox[origin=c]{-90}{$\subsetneq$} & \rotatebox[origin=c]{-90}{$\subsetneq$} & \rotatebox[origin=c]{-90}{$\subsetneq$} \\ \succ_O & \subsetneq \succ_{SO} & = \succ_{FO} \end{cases} \text{and} \begin{cases} \sim_U & \subsetneq \sim_{SU} & \subsetneq \sim_{CU} \\ \| & \| & \| \\ \sim_O & \subsetneq \sim_{SO} & \subsetneq \sim_{FO} \end{cases} \tag{16}$$

Note that $\succsim_O \sqsubset \succsim_{SO} \sqsubset \succsim_{FO}$ and $\succsim_{CU} \sqsubset \succsim_{FO}$ together imply that for all $\succsim \in \{\succsim_O, \succsim_{SO}, \succsim_{FO}\}$ and all $\boldsymbol{x}, \boldsymbol{y} \in X$, neither (i) $\boldsymbol{x} \succ_{CU} \boldsymbol{y}$ and $\boldsymbol{y} \succsim \boldsymbol{x}$ nor (ii) $\boldsymbol{x} \succ \boldsymbol{y}$ and $\boldsymbol{y} \succsim_{CU} \boldsymbol{x}$ take place. The same observation follows for the leximin case as well.[15]

Theorems 1 and 2 and the existing characterization results together provide an axiomatic explanation about the relationship among the SWRs stated in Remark 3. Given **SP**, **FA** and **PTSI** (resp. **HE**), i.e., the axioms characterizing $\succsim_U$ (resp. $\succsim_L$) (Basu and Mitra 2007; Bossert, Sprumont and Suzumura 2007), if we additionally impose **WPC** then the permissible SWRs are restricted to only those SWRs which include $\succsim_O$ (resp. $\succsim_{Lw}$) as a subrelation (Asheim and Banerjee 2009; Asheim and Tungodden 2004; Basu and Mitra 2007), and we are led to violation of **RUGF** (Proposition 1). The difference between the upper and lower lines in (15) and (16) become much clear by introducing the following consistency axiom underlying the upper (and also lower) lines:

Coherence in Strict Preference (CP): For all $\boldsymbol{x}, \boldsymbol{y} \in X$, if $(\boldsymbol{x}_N, \boldsymbol{y}_{\mathbb{N}\setminus N}) \succsim \boldsymbol{y}$ for all finite subsets $N \subseteq \mathbb{N}$ and $(\boldsymbol{x}_N, \boldsymbol{y}_{\mathbb{N}\setminus N}) \succ \boldsymbol{y}$ for all finite subsets $N \subseteq \mathbb{N}$ with $\{1\} \subseteq N$, then $\boldsymbol{x} \succ \boldsymbol{y}$.

CP is the one-step counterpart of the requirement for strict preference in **FC**. Clearly, **WPC** (as well as **FC**) implies CP. The SWRs in the upper lines of (15) and (16) satisfy this one-step coherence property, although they violate **WPC**. Since CP is implied by **SP**, it has not been explicitly employed in the literature.[16] Now, Proposition 1 can be reinterpreted as saying that if we strengthen CP to **WPC** in a strongly Paretian and

[15] Set inclusion in (15) and (16) is easily verified by Remark 2 and the existing results by Banerjee (2006) and Kamaga and Kojima (2009a,b).

[16] One can easily verify that **SP** implies CP, noting that for any binary relation $\succsim$ satisfying **SP**, we have that for all $n \in \mathbb{N}$, $(\boldsymbol{x}_{\{n\}}, \boldsymbol{y}_{\mathbb{N}\setminus\{n\}}) \succsim \boldsymbol{y}$ (resp. $(\boldsymbol{x}_{\{n\}}, \boldsymbol{y}_{\mathbb{N}\setminus\{n\}}) \succ \boldsymbol{y}$) implies $x_n \geq y_n$ (resp. $x_n > y_n$).

finitely anonymous SWR, then we inevitably end up with violation of **RUGF**, and in this sense, consideration for future generations' interests is lost in our evaluation.

In view of CP, the status of $\succsim_{CL}$ and $\succsim_{CU}$ in the upper lines of (15) and (16) now becomes straightforward. These SWRs can be seen to be the fixed-step extensions of $\succsim_L$ and $\succsim_U$, satisfying the fixed-step variant of coherence property, **FC**. The following equivalent reformulations of $\succsim_L$ and $\succsim_U$ help make clear this point: for all $\boldsymbol{x}, \boldsymbol{y} \in X$,

$\boldsymbol{x} \succsim_L \boldsymbol{y}$ iff there exists $n \in \mathbb{N}$ such that

$$\boldsymbol{x}_{[1,n]} \succsim_L^n \boldsymbol{y}_{[1,n]} \text{ and } x_{n'} \succsim_L^1 y_{n'} \text{ for all } n' > n;$$

$\boldsymbol{x} \succsim_U \boldsymbol{y}$ iff there exists $n \in \mathbb{N}$ such that

$$\boldsymbol{x}_{[1,n]} \succsim_U^n \boldsymbol{y}_{[1,n]} \text{ and } x_{n'} \succsim_U^1 y_{n'} \text{ for all } n' > n.$$

Since $\succsim_L^n$ and $\succsim_U^n$ are strongly Paretian, evaluation by $\succsim_L$ and $\succsim_U$ of tail-parts of streams is equivalently represented by the Pareto criterion. Now, $\succsim_{CL}$ and $\succsim_{CU}$ seem to be natural infinite-horizon extensions of finite-horizon orderings in the sense that these SWRs uniformly apply a fixed-length finite-horizon ordering in evaluating streams.

We should also note that $\succsim_{CL}$ and $\succsim_{CU}$ are seen to be the counterparts of $\succsim_{FLw}$ and $\succsim_{FO}$ in the upper lines. This can be understood more clearly by introducing the following axioms:

Weak Fixed-Step Indifference Continuity (WFIC): For all $\boldsymbol{x}, \boldsymbol{y} \in X$, if there exists $k \in \mathbb{N}$ such that $(\boldsymbol{x}_{[1,nk]}, \boldsymbol{y}_{[nk+1,\infty]}) \sim \boldsymbol{y}$ for all $n \in \mathbb{N}$, then $\boldsymbol{x} \sim \boldsymbol{y}$.

Fixed-Step Coherence in Indifference Relation (FCI): For all $\boldsymbol{x}, \boldsymbol{y} \in X$, if there exists $k \in \mathbb{N}$ such that $(\boldsymbol{x}_{(N;k)}, \boldsymbol{y}_{\mathbb{N}\setminus(N;k)}) \sim \boldsymbol{y}$ for all finite subsets $N \subseteq \mathbb{N}$, then $\boldsymbol{x} \sim \boldsymbol{y}$.

The axiom WFIC is introduced by Asheim and Banerjee (2009),[17] and it implies FCI which is seen to be the variant of **FC** formalizing the fixed-step coherence property only for indifference relation. Asheim and Banerjee (2009) show that the consistency axiom used in the characterizations of $\succsim_{FLw}$ and $\succsim_{FO}$ by Kamaga and Kojima (2009b) can be weakened to the pairs **WPC** and WFIC. Thus, the status of $\succsim_{FLw}$ and $\succsim_{FO}$ in the lower lines is captured in terms of WFIC. On the other hand, we can easily verify that in the presence of the three basic axioms of $\succsim_L$ or $\succsim_U$, WFIC and FCI become equivalent and **FC** implies them.[18] Consequently, given **FC**, the difference between $\succsim_{CL}$ and $\succsim_{CU}$, on the one hand, and $\succsim_{FLw}$ and $\succsim_{FO}$, on the other, is explained only

[17] In Asheim and Banerjee (2009), WFIC is called *Weak Fixed-Step Indifference Continuity 1*.

[18] More precisely, for any SWR $\succsim$ including $\succsim_L$ or $\succsim_U$ as a subrelation, WFIC and FCI are equivalent and if $\succsim$ satisfies **FC** then it also does these two axioms. This can be checked by using Claim 4 in Appendix (the detailed proof is available upon request).

Table 1: Additional properties setting boundaries among permissible SWRs

			FC	**RUGF** Test
		SA	**SA**	
	$\succsim_L$ $\succsim_U$	$\succsim_{SL}$ $\succsim_{SU}$	$\succsim_{CL}$ $\succsim_{CU}$	pass
WPC	$\succsim_{Lw}$ $\succsim_O$	$(\succsim_{SLw})$ $\succsim_{SO}$	$\succsim_{FLw}$ $\succsim_{FO}$	fail

in terms of **WPC**. In Table 1, we summarize the discussion presented here.[19]

5 Concluding remarks

We established the characterizations of the new extended leximin and utilitarian SWRs: the consensus leximin and the consensus utilitarian SWRs. These new SWRs alleviate incompleteness of the leximin and utilitarian SWRs without compromising **RUGF**. To the best of our knowledge, the consensus leximin and utilitarian SWRs achieve the highest selectivity among those SWRs satisfying **RUGF** already proposed in the literature. However, they remain to be incomplete relations. As suggested by Proposition 2, there still be possible frontier where we can construct further extensions of the leximin and the utilitarian SWRs without compromising **RUGF**. The issue to be addressed next will be to explore and construct those extensions to realize further comparisons of utility streams. We leave this issue for future research.

Appendix

We prove Proposition 2 and Theorems 1 and 2 by applying generalized results we will present below.[20] Recall that $\succsim_L$, $\succsim_{CL}$, $\succsim_U$ and $\succsim_{CU}$ are defined by sequences of the finite-horizon leximin and utilitarian orderings $\{\succsim_L^n\}_{n\in\mathbb{N}}$ and $\{\succsim_U^n\}_{n\in\mathbb{N}}$. Both sequences satisfy the following two properties:[21]

P1: For all $n \in \mathbb{N}$ and all $\boldsymbol{x}_{[1,n]}, \boldsymbol{y}_{[1,n]} \in \mathbb{R}^n$, if $\boldsymbol{x}_{[1,n]} > \boldsymbol{y}_{[1,n]}$, then $\boldsymbol{x}_{[1,n]} \succ^n \boldsymbol{y}_{[1,n]}$;

P2: For all $n \in \mathbb{N}$ and all $\boldsymbol{x}_{[1,n]}, \boldsymbol{y}_{[1,n]} \in \mathbb{R}^n$, if $\boldsymbol{x}_{[1,n]}$ is a permutation of $\boldsymbol{y}_{[1,n]}$, then $\boldsymbol{x}_{[1,n]} \sim^n \boldsymbol{y}_{[1,n]}$;

[19] As for the results regarding $\succsim_{SL}$, $\succsim_{SU}$, $\succsim_{SLw}$ and $\succsim_{SO}$ in Table 1, we refer the reader to Kamaga and Kojima (2009a,b).

[20] The general argument we demonstrate here is initiated by d'Aspremont (2007) and is followed by Asheim, d'Aspremont and Banerjee (2008), Sakai (2009), Kamaga and Kojima (2009a,b), Asheim and Banerjee (2009), and Kamaga (2009).

[21] **P1** is the finite-horizon version of **SP**. **P2** is the standard anonymity axiom in a finite-horizon framework. **P3** is a kind of independence requirement similar to *Extended Independence of the Utilities of Unconcerned Individuals* introduced by Blackorby, Bossert and Donaldson (2002) in the variable population social choice, which requires social ranking to be independent of the existence of a utility-unconcerned generation.

P3: For all $n \in \mathbb{N}$, all $\boldsymbol{x}_{[1,n]}, \boldsymbol{y}_{[1,n]} \in \mathbb{R}^n$ and all $r \in \mathbb{R}$, $(\boldsymbol{x}_{[1,n]}, r) \succsim^{n+1} (\boldsymbol{y}_{[1,n]}, r)$ if and only if $\boldsymbol{x}_{[1,n]} \succsim^n \boldsymbol{y}_{[1,n]}$.

Proof of Proposition 2

In what follows, we introduce binary relations, with which we prove the existence of ordering extensions of $\succsim_L$ and $\succsim_U$ by using Arrow's (1963) variant of Szpilrajn's (1930) Theorem.

For any sequence of finite-horizon orderings $\{\succsim^n\}_{n\in\mathbb{N}}$, let us define the binary relation $\succsim^*$ on X generated by the sequence as follows: for all $\boldsymbol{x}, \boldsymbol{y} \in X$,

$$\boldsymbol{x} \succsim^* \boldsymbol{y} \text{ iff there exists } n \in \mathbb{N} \text{ such that } \boldsymbol{x}_{[1,n]} \succsim^n \boldsymbol{y}_{[1,n]} \text{ and } \boldsymbol{x}_{[n+1,\infty]} \geqslant \boldsymbol{y}_{[n+1,\infty]}. \tag{17}$$

This generalized relation $\succsim^*$ is first introduced by d'Aspremont (2007) under the name *simplified criterion*. For any sequence of finite-horizon orderings $\{\succsim^n\}_{n\in\mathbb{N}}$ satisfying **P1** and **P3**, the relation $\succsim^*$ generated by the sequence satisfies the following property: for all $\boldsymbol{x}, \boldsymbol{y} \in X$,

$$\begin{cases} \boldsymbol{x} \succ^* \boldsymbol{y} \text{ iff there exists } n \in \mathbb{N} \text{ such that } \boldsymbol{x}_{[1,n]} \succ^n \boldsymbol{y}_{[1,n]} \text{ and } \boldsymbol{x}_{[n+1,\infty]} \geqslant \boldsymbol{y}_{[n+1,\infty]}; & \text{(18a)} \\ \boldsymbol{x} \sim^* \boldsymbol{y} \text{ iff there exists } n \in \mathbb{N} \text{ such that } \boldsymbol{x}_{[1,n]} \sim^n \boldsymbol{y}_{[1,n]} \text{ and } \boldsymbol{x}_{[n+1,\infty]} = \boldsymbol{y}_{[n+1,\infty]}. & \text{(18b)} \end{cases}$$

The equivalence assertions in (18a) and (18b) can be proved by basically the same argument as in the proof of (20a) and (20b) below, thus we omit easy proof.

Using the relation $\succsim^*$, we define another binary relation on X, denoted by $\succsim^{**}$. Then, we will prove the existence of an ordering extension of it.

For any sequence of finite-horizon orderings $\{\succsim^n\}_{n\in\mathbb{N}}$, we define the binary relation $\succsim^{**}$ on X as follows: for all $\boldsymbol{x}, \boldsymbol{y} \in X$,

$$\boldsymbol{x} \succsim^{**} \boldsymbol{y} \text{ iff } \boldsymbol{x} \succsim^* \boldsymbol{y} \text{ or there exists } \bar{n} \in \mathbb{N} \text{ such that } x_n > y_n \text{ for all } n \geq \bar{n}. \tag{19}$$

Claim 1. *Let $\{\succsim^n\}_{n\in\mathbb{N}}$ be a sequence of finite-horizon orderings satisfying **P1** and **P3**. Then, for all $\boldsymbol{x}, \boldsymbol{y} \in X$,*

$$\begin{cases} \boldsymbol{x} \succ^{**} \boldsymbol{y} \textit{ iff } \boldsymbol{x} \succ^* \boldsymbol{y} \textit{ or there exists } \bar{n} \in \mathbb{N} \textit{ such that } x_n > y_n \textit{ for all } n \geq \bar{n}; & \text{(20a)} \\ \boldsymbol{x} \sim^{**} \boldsymbol{y} \textit{ iff } \boldsymbol{x} \sim^* \boldsymbol{y} & \text{(20b)} \end{cases}$$

The if-part of (20b) is straightforward from (19). We only verify (20a) and the only-if-part of (20b).

[If-part of (20a)] Let $\boldsymbol{x}, \boldsymbol{y} \in X$, and suppose that the following (i) or (ii) holds: (i) $\boldsymbol{x} \succ^* \boldsymbol{y}$ or (ii) there exists $\bar{n} \in \mathbb{N}$ such that $x_n > y_n$ for all $n \geq \bar{n}$. By (19), $\boldsymbol{x} \succsim^{**} \boldsymbol{y}$. We show, by contradiction, that $\neg(\boldsymbol{y} \succsim^{**} \boldsymbol{x})$. By way of contradiction, suppose that $\boldsymbol{y} \succsim^{**} \boldsymbol{x}$. Then, by (19), we have (iii) $\boldsymbol{y} \succsim^* \boldsymbol{x}$ or (iv) there exists $\hat{n} \in \mathbb{N}$ such that $y_n > x_n$ for all $n \geq \hat{n}$. Notice that (ii) and (iv) together immediately give a contradiction. Consider the case where (i) and (iii) hold. By (17) and (20a) together with **P1**, **P3** and transitivity of $\succsim^n$ ($\forall n \in \mathbb{N}$), we can find $n \in \mathbb{N}$ such that $\boldsymbol{x}_{[1,n]} \succ^n \boldsymbol{y}_{[1,n]}$ and $\boldsymbol{y}_{[1,n]} \succsim^n \boldsymbol{x}_{[1,n]}$, a contradiction. Next, consider the case where we have (ii) and (iii). By (17), (iii) implies that there exists $n \in \mathbb{N}$ such that $\boldsymbol{y}_{[n+1,\infty]} \geqslant \boldsymbol{x}_{[n+1,\infty]}$, which gives a contradiction to (ii). Similarly, we obtain a contradiction in the case where (i) and (iv) hold. Thus, $\neg(\boldsymbol{y} \succsim^{**} \boldsymbol{x})$, and we have $\boldsymbol{x} \succ^{**} \boldsymbol{y}$.

[Only-if-part of (20a)] Let $\boldsymbol{x}, \boldsymbol{y} \in X$, and suppose that $\boldsymbol{x} \succ^{**} \boldsymbol{y}$. By (19) and (17), we have (i) there exists $n \in \mathbb{N}$ such that $\boldsymbol{x}_{[1,n]} \succsim^n \boldsymbol{y}_{[1,n]}$ and $\boldsymbol{x}_{[n+1,\infty]} \geqslant \boldsymbol{y}_{[n+1,\infty]}$ or (ii) there exists $\bar{n} \in \mathbb{N}$ such that $x_{n'} > y_{n'}$ for all $n' > \bar{n}$. If $\boldsymbol{x}_{[1,n]} \sim^n \boldsymbol{y}_{[1,n]}$ and $\boldsymbol{x}_{[n+1,\infty]} = \boldsymbol{y}_{[n+1,\infty]}$ hold, then we have $\boldsymbol{y} \succsim^{**} \boldsymbol{x}$ by (17) and (19), a contradiction. Thus, it must be that (i-a) $\boldsymbol{x}_{[1,n]} \succ^n \boldsymbol{y}_{[1,n]}$ and $\boldsymbol{x}_{[n+1,\infty]} \geqslant \boldsymbol{y}_{[n+1,\infty]}$ or (i-b) $\boldsymbol{x}_{[1,n]} \succsim^n \boldsymbol{y}_{[1,n]}$ and $\boldsymbol{x}_{[n+1,\infty]} > \boldsymbol{y}_{[n+1,\infty]}$. In the case of (i-b), we have, by **P1**, **P3** and transitivity of $\succsim^n$ ($\forall n \in \mathbb{N}$), that there exists $n' \geq n$ such that $\boldsymbol{x}_{[1,n']} \succ^{n'} \boldsymbol{y}_{[1,n']}$ and $\boldsymbol{x}_{[n'+1,\infty]} \geqslant \boldsymbol{y}_{[n'+1,\infty]}$. Thus, the proof is completed.

[Only-if-part of (20b)] Let $\boldsymbol{x}, \boldsymbol{y} \in X$, and suppose that $\boldsymbol{x} \sim^{**} \boldsymbol{y}$. By (19), (i) there exists $n \in \mathbb{N}$ such that $\boldsymbol{x}_{[1,n]} \succsim^n \boldsymbol{y}_{[1,n]}$ and $\boldsymbol{x}_{[n+1,\infty]} \geqslant \boldsymbol{y}_{[n+1,\infty]}$ or (ii) there exists $\bar{n} \in \mathbb{N}$ such that $x_n > y_n$ for all $n > \bar{n}$. If (ii) holds, then by (20a), $\boldsymbol{x} \succ^{**} \boldsymbol{y}$, a contradiction. Thus, we must have (i). Furthermore, if $\boldsymbol{x}_{[1,n]} \succ^n \boldsymbol{y}_{[1,n]}$ or $\boldsymbol{x}_{[n+1,\infty]} > \boldsymbol{y}_{[n+1,\infty]}$ hold, then by (18a) and (20a) together with **P1**, **P3** and transitivity of $\succsim^n$ ($\forall n \in \mathbb{N}$), we have $\boldsymbol{x} \succ^{**} \boldsymbol{y}$, a contradiction. Thus, $\boldsymbol{x}_{[1,n]} \sim^n \boldsymbol{y}_{[1,n]}$ and $\boldsymbol{x}_{[n+1,\infty]} = \boldsymbol{y}_{[n+1,\infty]}$.

Claim 2. *Let $\{\succsim^n\}_{n\in\mathbb{N}}$ be a sequence of finite-horizon orderings satisfying **P1** and **P3**. Then, $\succsim^{**}$ is reflexive and transitive on X.*

Assuming **P1** and **P3**, $\succsim^*$ is reflexive and transitive on X (Claim 1 in Kamaga and Kojima 2009a). Thus, by (17) and (19), Claim 2 is easily verified, and we omit easy proof.

Claim 3. *Let $\{\succsim^n\}_{n\in\mathbb{N}}$ be a sequence of finite-horizon orderings satisfying **P1** and **P3**, and $\succsim$ be a binary relation on X. Suppose that $\succsim^{**}$ is a subrelation of $\succsim$. Then, $\succsim$ satisfies **RUGF** and $\succsim^*$ is a subrelation of $\succsim$.*

Claim 3 is verified as follows. By (18a), (18b), (20a) and (20b), it is straightforward that $\succsim^*$ is a subrelation of $\succsim^{**}$. Thus, $\succsim^*$ is also a subrelation of $\succsim$. To check **RUGF**, let $\boldsymbol{x}, \boldsymbol{y} \in X$, and suppose that $y_1 > x_1$ and $x_n > y_n$ for all $n \in \mathbb{N}\backslash\{1\}$. Then, by (20a), $\boldsymbol{x} \succ^{**} \boldsymbol{y}$. Since $\succsim^{**}$ is a subrelation of $\succsim$, $\boldsymbol{x} \succ \boldsymbol{y}$. Thus, $\neg(\boldsymbol{y} \succ \boldsymbol{x})$.

Proof of Proposition 2. By Claim 2, the relation $\succsim^{**}$ generated by $\{\succsim_L^n\}_{n\in\mathbb{N}}$ is reflexive and transitive on X. We now state the following lemma due to Szpilrajn (1930):

Lemma 1 (Szpilrajn 1930; Arrow 1963)**.** *For any SWR $\succsim$, there exists an ordering extension of $\succsim$.*

By Lemma 1, there exists an ordering extension $\succsim$ of the relation $\succsim^{**}$ generated by $\{\succsim_L^n\}_{n\in\mathbb{N}}$. By Claim 3, the ordering $\succsim$ is an ordering extension of $\succsim_L$ and satisfies **RUGF**. The same argument can be directly applied to the case of $\succsim_U$. ■

Proof of Theorems 1 and 2

We now generalize $\succsim_{CL}$ and $\succsim_{CU}$ as a relation generated by a sequence of finite-horizon orderings. We define the generalized consensus rule generated by a sequence of finite-horizon orderings $\{\succsim^n\}_{n\in\mathbb{N}}$ as the following binary relation $\succsim_C$ on X: for all $\boldsymbol{x}, \boldsymbol{y} \in X$,

$$\boldsymbol{x} \succsim_C \boldsymbol{y} \text{ iff there exists } k \in \mathbb{N} \text{ such that } \boldsymbol{x}_{(n;k)} \succsim^k \boldsymbol{y}_{(n;k)} \text{ for all } n \in \mathbb{N}. \tag{21}$$

The following lemma confirms Remark 2 as its special case.

Lemma 2. *Let $\{\succsim^n\}_{n\in\mathbb{N}}$ be a sequence of finite-horizon orderings satisfying **P2** and **P3**. Then, the relation $\succsim_C$ defined in (21) is reflexive and transitive on X. Furthermore, for all $\boldsymbol{x}, \boldsymbol{y} \in X$,*

$$\begin{cases} \boldsymbol{x} \succ_C \boldsymbol{y} \textit{ iff there exists } k \in \mathbb{N} \textit{ such that } \boldsymbol{x}_{(n;k)} \succsim^k \boldsymbol{y}_{(n;k)} \textit{ for all } n \in \mathbb{N} \\ \qquad\qquad \textit{and there exists } \hat{n} \in \mathbb{N} \textit{ such that } \boldsymbol{x}_{(\hat{n};k)} \succ^k \boldsymbol{y}_{(\hat{n};k)}, & \text{(22a)} \\ \boldsymbol{x} \sim_C \boldsymbol{y} \textit{ iff there exists } k \in \mathbb{N} \textit{ such that } \boldsymbol{x}_{(n;k)} \sim^k \boldsymbol{y}_{(n;k)} \textit{ for all } n \in \mathbb{N}; & \text{(22b)} \end{cases}$$

Proof. Since $\succsim^k$ is reflexive for all $k \in \mathbb{N}$, $\succsim_C$ is reflexive. To prove transitivity of $\succsim_C$, we begin by verifying the following claim.

Claim 4. (a) *For all $\boldsymbol{x}, \boldsymbol{y} \in X$ and all $k, n \in \mathbb{N}$, if $\boldsymbol{x}_{(n';k)} \succsim^k \boldsymbol{y}_{(n';k)}$ for all $n' \in \{1, \ldots, n\}$, then $\boldsymbol{x}_{[1,nk]} \succsim^{nk} \boldsymbol{y}_{[1,nk]}$;*
(b) *For all $\boldsymbol{x}, \boldsymbol{y} \in X$ and all $k, n \in \mathbb{N}$, if $\boldsymbol{x}_{(n';k)} \succsim^k \boldsymbol{y}_{(n';k)}$ for all $n' \in \{1, \ldots, n\}$ and $\boldsymbol{x}_{(\hat{n};k)} \succ^k \boldsymbol{y}_{(\hat{n};k)}$ for some $\hat{n} \in \{1, \ldots, n\}$, then $\boldsymbol{x}_{[1,nk]} \succ^{nk} \boldsymbol{y}_{[1,nk]}$.*

First, we prove (a). Let $\boldsymbol{x}, \boldsymbol{y} \in X$ and $k, n \in \mathbb{N}$, and suppose that $\boldsymbol{x}_{(n';k)} \succsim^k \boldsymbol{y}_{(n';k)}$ for all $n' \in \{1, \dots, n\}$. To prove $\boldsymbol{x}_{[1,nk]} \succsim^{nk} \boldsymbol{y}_{[1,nk]}$, consider the following $n+1$ vectors $\{\boldsymbol{w}^0, \dots, \boldsymbol{w}^n\}$ in $\mathbb{R}^{nk}$: $\boldsymbol{w}^0 = \boldsymbol{x}_{[1,nk]}$, $\boldsymbol{w}^1 = (\boldsymbol{y}_{(1;k)}, \boldsymbol{x}_{(2;k)}, \dots, \boldsymbol{x}_{(n;k)})$, $\cdots$, $\boldsymbol{w}^m = (\boldsymbol{y}_{(1;k)}, \dots, \boldsymbol{y}_{(m;k)}, \boldsymbol{x}_{(m+1;k)}, \dots, \boldsymbol{x}_{(n;k)})$, $\cdots$, $\boldsymbol{w}^n = \boldsymbol{y}_{[1,nk]}$. By **P3**, $\boldsymbol{w}^0 \succsim^{nk} \boldsymbol{w}^1$. Similarly, by **P2**, **P3**, and transitivity of $\succsim^{nk}$, we have $\boldsymbol{w}^{m-1} \succsim^{nk} \boldsymbol{w}^m$ for all $m \in \{2, \dots, n\}$. By transitivity, $\boldsymbol{w}^0 \succsim^{nk} \boldsymbol{w}^n$, i.e., $\boldsymbol{x}_{[1,nk]} \succsim^{nk} \boldsymbol{y}_{[1,nk]}$. The same argument can be applied to prove (b), noting that we obtain $\boldsymbol{w}^{\hat{n}-1} \succ^{nk} \boldsymbol{w}^{\hat{n}}$, and thus $\boldsymbol{x}_{[1,nk]} \succ^{nk} \boldsymbol{y}_{[1,nk]}$.

We now verify transitivity of $\succsim_C$. Let $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in X$ with $\boldsymbol{x} \succsim_C \boldsymbol{y}$ and $\boldsymbol{y} \succsim_C \boldsymbol{z}$. By (21), there exist $k, k' \in \mathbb{N}$ such that, for all $n \in \mathbb{N}$, $\boldsymbol{x}_{(n;k)} \succsim^k \boldsymbol{y}_{(n;k)}$ and $\boldsymbol{y}_{(n;k')} \succsim^{k'} \boldsymbol{z}_{(n;k')}$. Let $\bar{k}$ be $\bar{k} = k \cdot k'$. By Claim 4, we have that for all $n \in \mathbb{N}$, $\boldsymbol{x}_{(n;\bar{k})} \succsim^{\bar{k}} \boldsymbol{y}_{(n;\bar{k})}$ and $\boldsymbol{y}_{(n;\bar{k})} \succsim^{\bar{k}} \boldsymbol{z}_{(n;\bar{k})}$. By transitivity of $\succsim^{\bar{k}}$, $\boldsymbol{x}_{(n;\bar{k})} \succsim^{\bar{k}} \boldsymbol{z}_{(n;\bar{k})}$ for all $n \in \mathbb{N}$, and by (21), $\boldsymbol{x} \succsim_C \boldsymbol{z}$.

Next, we prove the equivalence assertions in (22a) and (22b). The if-part of (22b) is straightforward from (21).

[Only-if-part of (22a)] Let $\boldsymbol{x}, \boldsymbol{y} \in X$, and suppose that $\boldsymbol{x} \succ_C \boldsymbol{y}$. By (21), there exists $k \in \mathbb{N}$ such that $\boldsymbol{x}_{(n;k)} \succsim^k \boldsymbol{y}_{(n;k)}$ for all $n \in \mathbb{N}$. Now, suppose that there is no $\hat{n} \in \mathbb{N}$ such that $\boldsymbol{x}_{(\hat{n};k)} \succ^k \boldsymbol{y}_{(\hat{n};k)}$. Then, $\boldsymbol{x}_{(n;k)} \sim^k \boldsymbol{y}_{(n;k)}$ for all $n \in \mathbb{N}$. By (21), $\boldsymbol{y} \succsim_C \boldsymbol{x}$, a contradiction.

[If-part of (22a)] Let $\boldsymbol{x}, \boldsymbol{y} \in X$, and suppose that there exists $k \in \mathbb{N}$ such that $\boldsymbol{x}_{(n;k)} \succsim^k \boldsymbol{y}_{(n;k)}$ for all $n \in \mathbb{N}$, and that there exists $\hat{n} \in \mathbb{N}$ such that $\boldsymbol{x}_{(\hat{n};k)} \succ^k \boldsymbol{y}_{(\hat{n};k)}$. By (21), $\boldsymbol{x} \succsim_C \boldsymbol{y}$. We prove $\boldsymbol{x} \succ_C \boldsymbol{y}$ by contradiction. By way of contradiction, suppose that $\boldsymbol{y} \succsim_C \boldsymbol{x}$. By (21), there exists $k' \in \mathbb{N}$ such that $\boldsymbol{y}_{(n;k')} \succsim^{k'} \boldsymbol{x}_{(n;k')}$ for all $n \in \mathbb{N}$. By Claim 4, it must be that $\boldsymbol{y}_{[1,\bar{n}]} \succsim^{\bar{n}} \boldsymbol{x}_{[1,\bar{n}]}$, where $\bar{n} = (\hat{n} \cdot k) \cdot k'$. However, recall that it is now supposed that $\boldsymbol{x}_{(n;k)} \succsim^k \boldsymbol{y}_{(n;k)}$ for all $n \in \mathbb{N}$ and $\boldsymbol{x}_{(\hat{n};k)} \succ^k \boldsymbol{y}_{(\hat{n};k)}$ holds for $\hat{n}$. Thus, by Claim 4, we have $\boldsymbol{x}_{[1,\bar{n}]} \succ^{\bar{n}} \boldsymbol{y}_{[1,\bar{n}]}$, where $\bar{n} = (\hat{n} \cdot k') \cdot k$. A contradiction.

[Only-if-part of (22b)] Let $\boldsymbol{x}, \boldsymbol{y} \in X$, and suppose that $\boldsymbol{x} \sim_C \boldsymbol{y}$. By (21), there exists $k \in \mathbb{N}$ such that $\boldsymbol{x}_{(n;k)} \succsim^k \boldsymbol{y}_{(n;k)}$ for all $n \in \mathbb{N}$. If there exists $\hat{n} \in \mathbb{N}$ such that $\boldsymbol{x}_{(\hat{n};k)} \succ^k \boldsymbol{y}_{(\hat{n};k)}$, we have $\boldsymbol{x} \succ_C \boldsymbol{y}$ by (22a), a contradiction. Thus, $\boldsymbol{x}_{(n;k)} \sim^k \boldsymbol{y}_{(n;k)}$ for all $n \in \mathbb{N}$. ■

In Lemma 3 below, we generalize Theorems 1 and 2. Let us introduce the property called *strict extension*.[22] A binary relation $\succsim$ on X is said to be a strict extension of a sequence of finite-horizon orderings $\{\succsim^n\}_{n \in \mathbb{N}}$ if, for all $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in X$ and all $n \in \mathbb{N}$,

$$\boldsymbol{x}_{[1,n]} \succsim^n \boldsymbol{y}_{[1,n]} \text{ if and only if } (\boldsymbol{x}_{[1,n]}, \boldsymbol{z}_{[n+1,\infty]}) \succsim (\boldsymbol{y}_{[1,n]}, \boldsymbol{z}_{[n+1,\infty]}).$$

[22] Sakai (2009) is the first who formally employs the notion of strict extension to analyze relations $\succsim$ on X.

Lemma 3. *Let $\{\succsim^n\}_{n\in\mathbb{N}}$ be a sequence of finite-horizon orderings satisfying **P2** and **P3**. Then, a SWR $\succsim$ on X is a strict extension of the sequence $\{\succsim^n\}_{n\in\mathbb{N}}$ and satisfies **FC** if and only if $\succsim_C$ is a subrelation of $\succsim$.*

Proof. First, we prove the only-if-part. Let $\boldsymbol{x}, \boldsymbol{y} \in X$, and suppose that $\boldsymbol{x} \succ_C \boldsymbol{y}$. We will show that $\boldsymbol{x} \succ \boldsymbol{y}$. By (22a), there exists $k \in \mathbb{N}$ such that $\boldsymbol{x}_{(n;k)} \succsim^k \boldsymbol{y}_{(n;k)}$ for all $n \in \mathbb{N}$, and there exists $\hat{n} \in \mathbb{N}$ such that $\boldsymbol{x}_{(\hat{n};k)} \succ^k \boldsymbol{y}_{(\hat{n};k)}$. Choose any finite subset $N \subseteq \mathbb{N}$. By Claim 4, $(\boldsymbol{x}_{(N;k)}, \boldsymbol{y}_{\mathbb{N}\setminus(N;k)})_{[1,nk]} \succsim^{nk} \boldsymbol{y}_{[1,nk]}$ holds for $n \in \mathbb{N}$ with $n = \max\{n' : n' \in N\}$. Since $\succsim$ is a strict extension of $\{\succsim^n\}_{n\in\mathbb{N}}$, we obtain $(\boldsymbol{x}_{(N;k)}, \boldsymbol{y}_{\mathbb{N}\setminus(N;k)}) \succsim \boldsymbol{y}$. Similarly, by Claim 4 (b), for any finite subset $N \subseteq \mathbb{N}$ with $\{\hat{n}\} \subseteq N$, we have $(\boldsymbol{x}_{(N;k)}, \boldsymbol{y}_{\mathbb{N}\setminus(N;k)}) \succ \boldsymbol{y}$. Thus, by **FC**, $\boldsymbol{x} \succ \boldsymbol{y}$. The argument we demonstrate here can be directly applied to show that if $\boldsymbol{x} \succsim_C \boldsymbol{y}$ then $\boldsymbol{x} \succsim \boldsymbol{y}$.

Next, we prove the if-part. We begin with the following claim.

Claim 5. *$\succsim_C$ is a strict extension of the sequence $\{\succsim^n\}_{n\in\mathbb{N}}$.*

Take any $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in X$ and any $n \in \mathbb{N}$. Let $\boldsymbol{u} = (\boldsymbol{x}_{[1,n]}, \boldsymbol{z}_{[n+1,\infty]})$ and $\boldsymbol{v} = (\boldsymbol{y}_{[1,n]}, \boldsymbol{z}_{[n+1,\infty]})$. First, suppose that $\boldsymbol{u}_{[1,n]} \succsim^n \boldsymbol{v}_{[1,n]}$. By reflexivity of $\succsim^n$, $\boldsymbol{u}_{(n';n)} \succsim^n \boldsymbol{v}_{(n';n)}$ for all $n' > 1$. Thus, by (21), $\boldsymbol{u} \succsim_C \boldsymbol{v}$. Next, suppose that $\boldsymbol{u} \succsim_C \boldsymbol{v}$. By (21), there exists $k \in \mathbb{N}$ such that $\boldsymbol{u}_{(n';k)} \succsim^k \boldsymbol{v}_{(n';k)}$ for all $n' \in \mathbb{N}$. By Claim 4, we can assume $k \geq n$. Then, by **P3**, $\boldsymbol{u}_{(1;k)} \succsim^k \boldsymbol{v}_{(1;k)}$ implies $\boldsymbol{u}_{[1,n]} \succsim^n \boldsymbol{v}_{[1,n]}$. Thus, the claim is verified.

[Strict extension] Take any $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in X$ and any $n \in \mathbb{N}$. Let $\boldsymbol{u} = (\boldsymbol{x}_{[1,n]}, \boldsymbol{z}_{[n+1,\infty]})$ and $\boldsymbol{v} = (\boldsymbol{y}_{[1,n]}, \boldsymbol{z}_{[n+1,\infty]})$. By Claim 5, we have that $\boldsymbol{u}_{[1,n]} \succsim^n \boldsymbol{v}_{[1,n]}$ if and only if $\boldsymbol{u} \succsim_C \boldsymbol{v}$. We now show that $\boldsymbol{u} \succsim_C \boldsymbol{v}$ if and only if $\boldsymbol{u} \succsim \boldsymbol{v}$, then combining these equivalence assertions, the proof will be completed. Since $\succsim_C$ is a subrelation of $\succsim$, it is straightforward that $\boldsymbol{u} \succsim_C \boldsymbol{v}$ only if $\boldsymbol{u} \succsim \boldsymbol{v}$. We prove, by contradiction, that $\boldsymbol{u} \succsim_C \boldsymbol{v}$ if $\boldsymbol{u} \succsim \boldsymbol{v}$. Suppose that $\boldsymbol{u} \succsim \boldsymbol{v}$ but $\neg(\boldsymbol{u} \succsim_C \boldsymbol{v})$. By Claim 5 and completeness of $\succsim^n$, $\neg(\boldsymbol{u} \succsim_C \boldsymbol{v})$ implies $\boldsymbol{v}_{[1,n]} \succ^n \boldsymbol{u}_{[1,n]}$. Then, by reflexivity of $\succsim^n$ and (22a), we have $\boldsymbol{v} \succ_C \boldsymbol{u}$. Since $\succsim_C$ is a subrelation of $\succsim$, $\boldsymbol{v} \succ \boldsymbol{u}$ holds, but which gives a contradiction to $\boldsymbol{u} \succsim \boldsymbol{v}$. Thus, $\boldsymbol{u} \succsim_C \boldsymbol{v}$ must hold.

[**FC**] Let $\boldsymbol{x}, \boldsymbol{y} \in X$, and suppose that antecedent (i) in **FC** holds. Since $\succsim$ is a strict extension of $\{\succsim^n\}_{n\in\mathbb{N}}$, we have that for all $n \in \mathbb{N}$, $(\boldsymbol{y}_{[1,(n-1)k]}, \boldsymbol{x}_{(n;k)}) \succsim^{nk} \boldsymbol{y}_{[1,nk]}$. By **P2**, **P3** and transitivity of $\succsim^{nk}$, we have $\boldsymbol{x}_{(n;k)} \succsim^k \boldsymbol{y}_{(n;k)}$ for all $n \in \mathbb{N}$. By (21), $\boldsymbol{x} \succsim_C \boldsymbol{y}$. Since $\succsim_C$ is a subrelation of $\succsim$, we have $\boldsymbol{x} \succsim \boldsymbol{y}$. Next, suppose that antecedents (i) and (ii) in **FC** hold. Then, by the same argument as we just demonstrated above, $\boldsymbol{x}_{(n;k)} \succsim^k \boldsymbol{y}_{(n;k)}$ for all $n \in \mathbb{N}$, and furthermore, $\boldsymbol{x}_{(\hat{n};k)} \succ^k \boldsymbol{y}_{(\hat{n};k)}$. By (22a), $\boldsymbol{x} \succ_C \boldsymbol{y}$. Since $\succsim_C$ is a subrelation of $\succsim$, $\boldsymbol{x} \succ \boldsymbol{y}$. ■

Proof of Theorems 1 and 2. We can prove the only-if parts of Theorems 1 and 2 by using Lemma 3 together with the following lemma due to Sakai (2009), where the existing results regarding the finite-horizon leximin ordering (Hammond 1979; Asheim and

Tungodden 2004; Bossert, Sprumont and Suzumura 2007) and the finite-horizon utilitarian ordering (d'Aspremont and Gevers 1977; Asheim and Tungodden 2004; Basu and Mitra 2007) are restated in a useful format for the current analysis.[23]

Lemma 4 (Lemmas 5 and 6, Sakai 2009)**.** (i) *If a SWR* $\succsim$ *on* X *satisfies* ***SP**, **FA** and **HE**, then it is a strict extension of* $\{\succsim_L^n\}_{n\in\mathbb{N}}$*;* (ii) *If a SWR* $\succsim$ *on* X *satisfies* ***SP**, **FA** and **PTSI**, then it is a strict extension of* $\{\succsim_U^n\}_{n\in\mathbb{N}}$.

As for the if-parts of Theorems 1 and 2, Lemma 3 tells that $\succsim$ satisfies **FC**. It is easy to verify that $\succsim$ satisfies the other axioms, and we omit easy proof. ■

References

Arrow, K.J. (1963) Social choice and individual values (2nd edition). Wiley, New York

Asheim, G.B., d'Aspremont, C., Banerjee, K. (2008) Generalized time-invariant overtaking. Mimeo, University of Oslo

Asheim, G.B., Banerjee, K. (2009) Fixed-step anonymous overtaking and catching-up. Mimeo, University of Oslo, forthcoming in Int J Econ Theory

Asheim, G.B., Mitra, T., Tungodden, B. (2007) A new equity condition for infinite utility streams and the possibility of being Paretian. In: Roemer, J., Suzumura, K. (eds) Intergenerational equity and sustainability. Palgrave, London

Asheim, G.B., Tungodden, B. (2004) Resolving distributional conflicts between generations. Econ Theory 24: 221-230

Asheim, G.B., Tungodden, B. (2005) Do Koopmans' postulates lead to discounted utilitarianism? Mimeo, University of Oslo

d'Aspremont, C. (2007) Formal welfarism and intergenerational equity. In: Roemer, J., Suzumura, K. (eds) Intergenerational equity and sustainability. Palgrave, London

d'Aspremont, C., Gevers, L. (1977) Equity and the informational basis of collective choice. Rev Econ Stud 44: 199-209

d'Aspremont, C., Gevers, L. (2002) Social welfare functionals and interpersonal comparability. In: Arrow, K.J., Sen, A.K., Suzumura, K. (eds) Handbook of social choice and welfare vol. I. North-Holland, Amsterdam

[23] In the original statements of Lemmas 5 (corresponding to (ii)) and 6 (corresponding to (i)) in Sakai (2009), he assumes completeness and finite transitivity of $\succsim$. However, it is known that the characterizations of the finite-horizon leximin and utilitarian orderings which Sakai (2009) uses to complete his proofs of Lemmas 5 and 6 can be established without assuming completeness of finite-horizon binary relation, and thus one can verify that Sakai's (2009) proof can be established without assuming completeness of $\succsim$.

Atsumi, H. (1965) Neoclassical growth and the efficient program of capital accumulation. Rev Econ Stud 32: 127-136

Banerjee, K. (2006) On the extension of the utilitarian and Suppes-Sen social welfare relations to infinite utility streams. Soc Choice Welf 27: 327-339

Basu, K., Mitra, T. (2007) Utilitarianism for infinite utility streams: a new welfare criterion and its axiomatic characterization. J Econ Theory 133: 350-373

Blackorby, C., Bossert, W., Donaldson, D. (2002) Utilitarianism and the theory of justice. In: Arrow, K.J., Sen, A.K., Suzumura, K. (eds) Handbook of social choice and welfare vol. I. North-Holland, Amsterdam

Bossert, W., Sprumont, Y., Suzumura, K. (2007) Ordering infinite utility streams. J Econ Theory 135: 579-589

Bossert, W., Weymark, J.A. (2004) Utility in social choice. In: Barberà S, Hammond P, Seidl C (eds) Handbook of utility theory, Extensions, vol 2. Kluwer, Boston

Fleurbaey, M., Michel, P. (2003) Intertemporal equity and the extension of the Ramsey criterion. J Math Econ 39: 777-802

Hammond, P.J. (1976) Equity, Arrow's conditions, and Rawls' difference principle. Econometrica 44: 793-804

Hammond, P.J. (1979) Equity in two person situations – some consequences. Econometrica 47: 1127-1135

Kamaga, K. (2009) Complete intergenerational evaluations and coalitional veto of future generations. Mimeo, Waseda University

Kamaga, K., Kojima, T. (2009a) $\mathcal{Q}$-anonymous social welfare relations on infinite utility streams. Forthcoming in Soc Choice Welf (doi: 10.1007/s00355-009-0366-8)

Kamaga, K., Kojima, T. (2009b) On the leximin and utilitarian overtaking criteria with extended anonymity. Mimeo, Waseda University (http://globalcoe-glope2.jp/paper/GCOE_WP09.pdf)

Lauwers, L. (1997a) Rawlsian equity and generalised utilitarianism with and infinite population. Econ Theory 9: 143-150

Lauwers, L. (1997b) Infinite utility: insisting on strong monotonicity. Aust J Philos 75: 222-233

Lauwers, L. (1998) Intertemporal objective functions: strong Pareto versus anonymity. Math Soc Sci 35:37-55

Lauwers, L. (2006) Ordering infinite utility streams: completeness at the cost of a non-Ramsey set. Mimeo, Katholike Universiteit Leuven

Mitra, T., Basu, K. (2007) On the existence of Paretian social welfare quasi-orderings for infinite utility streams with extended anonymity. In: Roemer, J., Suzumura, K. (eds) Intergenerational equity and sustainability. Palgrave, London

Sakai, T. (2009) Intergenerational equity and an explicit construction of welfare criteria. Mimeo, Yokohama National University

Szpilrajn, E. (1930) Sur l'extension de l'ordre partiel. Fundam Math 16: 386-389

van Liedekerke, L. (1995) Should utilitarians be cautious about an infinite future. Aust J Philos 73: 405-407

von Weizsäcker, C.C. (1965) Existence of optimal programs of accumulation for an infinite time horizon. Rev Econ Stud 32: 85-104

Zame, W. (2007) Can intergenerational equity be operationalized? Theoret Econ 2: 187-202