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Welfare Cost of Inflation and Income Risks in an Incomplete Market Model: Application to the Japanese Economy

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Welfare Costs of Inflation and Income Risks in an Incomplete Market Model: Application to the Japanese Economy *

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Abstract

This paper quantitatively investigates the welfare costs of inflation and idiosyncratic and aggregate income risks in a Bewley-type incomplete market model. A calibrated model of the Japanese economy in the 1990s indicates that money growth generated a larger welfare cost equivalent to 0.334% of real GDP, than the estimated shoe-leather cost 0.2%, and that the cost of inflation was amplified through an increase in unemployment risk. The model also indicates that longer unemployment duration spells, and the presence of heterogeneous agents, augment the welfare costs of income risk. Finally, the aggregate risk of the business cycle is found to be of little importance to welfare.

Key Words: Aggregate Risk, Heterogeneity, Idiosyncratic Risk, Precautionary Saving, Welfare Cost of Inflation.

JEL Classification Codes: E21, E41, and E52.

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1 Introduction

This paper aims to quantitatively investigate, in an incomplete market situation, the welfare costs of idiosyncratic and aggregate income risks that motivate precautionary savings by households. The analysis pays attention to the welfare costs of inflation, because inflation deprives the purchasing power of monetary assets that are saved for consumption smoothing by households in the presence of uninsured income risk.

Bewley-type incomplete market models have been introduced in various studies (for example, Bewley, 1980; İmrohoroğlu, 1992; Kehoe, Levine and Woodford, 1992; Aiyagari, 1994). Although Krusell and Smith (1998) show that the precautionary saving motive against idiosyncratic risk is of little importance in determining aggregate saving, their findings do not necessarily reduce the importance of incomplete market models from a welfare perspective . The models can be applied to the sluggish Japanese economy[1] that is of particular interest in light of idiosyncratic income risk. The unemployment rate in Japan, which had never exceeded 3% from 1953 to the early 1990s, rose from 2.3% in 1990 to nearly 6% in 2003; the average spell of unemployment duration increased from three months to more than five months; and the ratio of those unemployed for more than one year in Japan in 2000 exceeded 25% of the total unemployed, as compared to 6% in the US economy (*OECD Employment Outlook*).

[1] There are a number of analyses concerned with the critical slump of the Japanese economy in the 1990s, including lowered productivity growth (Hayashi and Prescott, 2000), causes of the "lost-decade" (Bayoumi, 1999; Sato, 2001), macroeconomic and financial policies (Jinushi, Kuroki and Miyao, 2000; Krugman, 1999; Motonishi and Yoshikawa, 1999; Posen, 1998) or precautionary saving and habit formation (Carroll, 2000).

This paper introduces a model where households facing income risk hold fiat money for consumption-smoothing, as based on the model proposed by İmrohoroğlu (1992). We calibrate the model into the Japanese economy in the 1990s in order to evaluate the welfare costs of unemployment risk, with and without inflation. The welfare cost is then compared with the shoe-leather cost, which depends on the interest-rate elasticity of money demand for transaction motives (Bailey, 1956; Lucas, 1994).

We also examine effects of some assumptions implicit in the base model. First, a real rate of return is negative due to inflation in the base model. The effects of a positive real rate of return are examined. This is because İmrohoroğlu and Prescott (1991) and Diaz-Gimenez, Prescott, Fitzgerald and Alvarez (1992) show that after-tax *real* rates of return on liquid assets matter with welfare measures. Second, we incorporate into the model of heterogeneous rational agents "hand-to-mouth" agents who do not hold precautionary savings as in Campbell and Mankiw (1989), Lettau and Uhlig (1999) and Weil (1992). Such irrational agents introduce a negative externality through excessive consumption.

Using the model, we evaluate the welfare cost of increasing unemployment risk in Japan, especially from the viewpoint of longer spells of unemployment duration. We also evaluate the welfare cost of aggregate income risk from the business cycle, as well as idiosyncratic income risk. We postulate that aggregate risk augments idiosyncratic unemployment risk, based on the evidence presented by Storesletten, Telmer and Yaron (2001) on the relationship between individuals' earnings risk and aggregate risk.

The major findings from the simulation experiments are as follows: (1) the measured welfare cost of inflation is equivalent to 0.334% of real GDP, which is larger than the 0.2% of the estimated shoe-leather cost; (2) reducing the average unemployment spell from five months to three months reduces welfare cost by 0.486% of real GDP, even without changes in the unemployment rate; (3) the welfare cost of inflation is amplified through an increase in unemployment risk; (4) aggregate risk is a matter of little importance to welfare, even with fluctuations in unemployment risk; and (5) the presence of two hand-to-mouth agents of every three agents augments welfare costs equivalent to 1.141% of real GDP.

The structure of the paper is as follows. Section 2 describes an incomplete market model incorporating both idiosyncratic and aggregate risks with inflation, and selects the parameters necessary to calibrate the Japanese economy. Section 3 presents the simulation results. Section 4 provides discussion of the results. Section 5 concludes.

2 An Incomplete Market Model with Idiosyncratic Risk

2.1 Environment and Equilibrium

We consider an economy that consists of many infinitely-lived agents who differ at any point of time in their real cash balances and employment opportunities. An agent maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

where E_0 denotes an operator of expectation in the initial period, β is the discount rate ($0 < \beta < 1$), c_t is consumption in period t, and the utility function $u(\cdot)$ has the following CRRA (constant relative risk aversion) form:

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma}$$

with $\sigma > 0$. An agent is endowed with one indivisible unit of time in each period and faces an employment opportunity which is independent across agents. The employment state, w_t, is assumed to follow a first-order Markov process with two possible states, $w = e$ for employed and $w = u$ for unemployed. If employed, an agent receives y units of the consumption good ($y_t = y$). If unemployed, an agent receives θy ($0 < \theta < 1$) units of the consumption good ($y_t = \theta y$) through unemployment insurance.

Agents enter each period with individual nominal money balances m_t and a lump-sum transfer from the government equal to $g_t M_t$ where M_t is an average nominal money from time t-1 to t, and g_t is a growth rate of money supply. The money supply follows the law of motion

$$M_{t+1} = (1 + g_t) M_t.$$

Thus, the budget constraint of an agent becomes, in nominal terms,

$$p_t c_t + m_{t+1} \leq p_t y_t + m_t + g_t M_t \tag{2}$$

where p_t is the price of the consumption good at time t. Here, borrowing is not allowed; m_t is required to be nonnegative. Since state-contingent insurance is not permitted, an agent is insured only through money holdings [2]. An agent accumulates cash balances to cover unemployment risk during employed periods, dissaving during unemployed periods to smooth consumption.

An inflation rate π_t from time $t-1$ to t equals $\frac{p_t}{p_{t-1}} - 1$. An individual real asset saved at time t for the next period is defined by

$$a_{t+1} \equiv \frac{m_{t+1}}{p_t}$$

and an average real asset is

$$A_{t+1} \equiv \frac{M_{t+1}}{p_t}.$$

Therefore, the budget constraint of an agent is, in real terms,

$$c_t + a_{t+1} \leq y_t + \frac{1}{1+\pi_t} a_t + \frac{g_t}{1+\pi_t} A_t. \tag{3}$$

We introduce probabilistic processes for both an individual employment state and an aggregate state. First, transitional probabilities are assumed to be given for a first-order Markov process of employment status by

$$\chi_w = \Pr(w_{t+1} = w' | w_t = w)$$

for $w, w' \in W = \{e, u\}$. That is, the transitional probability matrix is denoted by

[2] Of the savings held by Japanese households in 1999, bank deposits and postal savings accounted for 56.9% and insurance equities 28.9%, while securities such as stocks and bonds represented only 11.7% (*The Family Saving Survey*, Statistics Bureau, Government of Japan).

$$\chi_w = \begin{bmatrix} \Pr(w' = e|w = e) & \Pr(w' = u|w = e) \\ \Pr(w' = e|w = u) & \Pr(w' = u|w = u) \end{bmatrix}.$$

In this process, the unemployment state is persistent rather than independent in each period. We follow İmrohoroğlu (1992) in the calculation of this probability matrix; $\chi_w(e, e)$ can be calculated from the constant unemployment rate of the economy using $\chi_w(u, u) = 1 - \frac{1}{D_u}$ where the average duration of unemployment is D_u.

Second, we also assume a first-order Markov process with two possible aggregate states, $z = h$ for good times, and $z = l$ for bad times. The transitional probabilities are given by

$$\chi_z = \Pr(z_{t+1} = z'|z_t = z)$$

In the matrix form,

$$\chi_z = \begin{bmatrix} \Pr(z' = h|z = h) & \Pr(z' = l|z = h) \\ \Pr(z' = h|z = l) & \Pr(z' = l|z = l) \end{bmatrix} = \begin{bmatrix} \rho_1 & 1 - \rho_1 \\ 1 - \rho_2 & \rho_2 \end{bmatrix}.$$

In this paper, we consider a symmetric process, i.e., $\rho_1 = \rho_2 = \rho$ for simplicity. The parameter ρ can be interpreted as a measure of persistence of the business cycle. The more detailed specification of aggregate risk will be covered in Sections 3.5 and 3.6.

In the equilibrium, the goods market clears such that

$$\sum_{i=1}^{N} c_t(i) = \sum_{i=1}^{N} y_t(i)$$

and the money market clears such that

$$\frac{1}{N}\sum_{i=1}^{N} a_{t+1}(i) = A_{t+1}$$

where N denotes the number of agents in the economy. Therefore, the inflation rate is $\pi_t = (1+g_t)A_t/A_{t+1} - 1$ from $A_{t+1} = A_t(1+g_t)/(1+\pi_t)$ by aggregating the individual budget constraint (3) over N agents. Thus, the inflation rate depends on the current and previous aggregate states: z_t and z_{t-1}. Without a change in z_t, the inflation rate equals the growth rate of money, i.e., $g_t = \pi_t$.

The optimality equation for this dynamic programming problem is now expressed as a Bellman's equation:

$$V(a, w, z, z_{-1}) = \max\{u(c) + \beta \cdot E[V(a', w', z', z)|(a, w, z, z_{-1})]\}$$

with a budget constraint (3), where z_{-1} indicates z one period before, maximization is over a', and $a' \geq 0$.

2.2 Computational Strategy

Given a set of parameters that characterize the economy, the individual policy a' to solve the problem (1) subject to (3) is obtained with numerical methods. However, since the average assets of A' are determined by aggregating individual behavior, the individual strategy is affected by the inflation rate. Therefore, the algorithm must solve the individual policy and average assets together. We apply the computational steps as follows:

(Step 1) Select an initial guess of A_h and A_l. Obtain the value function, as described later, numerically.

(Step 2) Using the value function in step 1, calculate an average asset of N agents with an initial distribution of endowments, after continuing the same aggregate state for a sufficiently long period. Obtain new A_h and A_l.

(Step 3) Repeat steps 1 and 2 until convergence is obtained with sufficiently small computational errors. Alternatively, and in practice, obtain optimal A_h and A_l that satisfy step 2 using any method to solve a set of nonlinear equations.

(Step 4) Using the obtained numerical solution of the value function and average assets, calculate the averages and standard deviations of asset, consumption and utility from the simulated time series of T for N agents.

In obtaining the value function numerically in step 1, we discretize states: $s = \{w, z, z_{-1}\}$ into eight categories: (e,h,h), (e,h,l), (e,l,h), (e,l,l), (u,h,h), (u,h,l), (u,l,h), and (u,l,l). We then treat the real asset (a) of a state variable, and also the real asset for the next period ($a\prime$) of the choice variable as a continuous state, while İmrohoroğlu (1992) discretizes this state. Rewriting the value function as

$$V(a,s) \;=\; \max_{a'}\{u(a,s,a') + \beta \cdot E[V(a',s')|(a,s)]\}.$$

the policy function

$$a'(a,s) = \arg\max_{a'}\{u(a,s,a') + \beta \cdot E[V(a',s')|(a,s)]\}$$

may be calculated by any constrained optimization methods. The optimal value function is approximated as a smoothed function

$$V_\alpha(a,s) = \sum_{k=1}^{K} \alpha(s)_k \cdot \phi_k(a)$$

where $\{\phi_1(a), \phi_2(a), ..., \phi_K(a)\}$ denotes a vector of the Chebyshev polynomials, and $\{\alpha(s)_1, \alpha(s)_2, ..., \alpha(s)_K\}$ denotes a vector of coefficients specific to each discretized state. The coefficient vectors are sought by projection methods following the algorithm developed in Judd (1992)[3] that makes the projected residual functions

$$\sum_{j=1}^{J}[V(a_j, s) - \Gamma(V(a_j, s))] \cdot \phi_k(a_j)$$

be zero for $k = 1, ..., K$, where the Bellman operator Γ is defined by

$$\Gamma(V)(a, s) = \max_{a'}\{u(a, s, a') + \beta \cdot E[V(a', s')|(a, s)]\}$$

and $\{a_1, a_2, ..., a_J\}$ are collocation grids. In our simulation, the number of collocation grid J equals 20 and the degree of polynomial K equals 10. For eight discrete states, 80 coefficients are calculated in total.

We use N equals 1000 and T equals 100 at step 2, following consideration of the improvements in accuracy and the computational costs. At step 4, the simulation length of time series is 1000 for fixed money growth rates; thus, the sample number is one million[4] .

2.3 Parameter Values for the Japanese Economy

The time period is selected to be one month. The selected discount factor β equals 0.995, and σ in the utility function equals 1.5 as in İmrohoroğlu (1992). Employed income y is given such that average GDP (social endowment including unemployed) or consumption is unity. The remaining

[3] The detailed algorithm and its explanation are also found in Judd (1998) and Rust (1996).

[4] Larger numbers for these variables were also attempted, although accuracy was not greatly improved.

parameters are selected with reference to Japanese data in the 1990s. The unemployment rate is assumed to be 4.9% in 2000, and the average duration of unemployment is five months. The transitional probability matrix of employment then becomes

$$\chi_w = \begin{bmatrix} 0.989 & 0.011 \\ 0.204 & 0.796 \end{bmatrix}.$$

The ratio of income when unemployed compared with employed income (θ) is assumed to be 0.36. This ratio is calculated based on three months unemployment insurance of 60% of monthly employed income, paid during the average unemployment duration of five months[5] .

Regarding the money growth rate g_t, we use an average rate 0.223% of 0.331% during the recession starting in 1997: 2Q to 1999: 1Q, and 0.114% during the recovery from 1999: 2Q to 2000: 3Q[6] . We also considered a countercyclical monetary policy, where the money growth rate fluctuated according to the aggregate state. However, this effect was negligible. Therefore, throughout the paper we present the results using average money growth rate. The parameters are effective throughout the simulation unless otherwise noted.

[5] As of 2001, the unemployment insurance system in Japan prescribes that the benefits paid for an unemployed person is equal to 60-80% of wages earned before his or her quitting employment, depending on the wage level. Although the spell of payment duration varies from 90-330 days, depending upon age, insurance period, reason for leaving job and so forth, we choose the general case of a minimum 90 days.

[6] In the 1990s, the average annual growth rate of nominal M2+CD was 9.5% during a period of contraction, and 2.6% during expansion.

3 Simulation Results

3.1 The Benchmark Case of Japan

Table 1 presents averages and standard deviations of the monetary asset (in real terms) and consumption, and associated welfare indices. The welfare level of the society is indicated as the average utility $\overline{u}$, and also expressed as an equivalent percentage change in real GDP, calculated as $(GDP - GDP_B)/GDP_B$ where $GDP = u^{-1}(\overline{u})$ and $GDP_B = u^{-1}(\overline{u}_B)$ with $\overline{u}_B$ being the utility level of a benchmark case.

In Table 1, Case (A) indicates the case without money growth, and Case (B) applies the average rate of money growth of 0.223%. The utility level shown is multiplied by 100 throughout the paper. The measured welfare cost of inflation is equivalent to 0.334% of real GDP. Inflation also reduces the value of financial assets.

In addition, inflation widens variation of consumption among agents. The idiosyncratic income risk generates both the consumption distribution and also the wealth distribution among agents with different histories of employment status; unemployed agents reduce monetary assets and also adjust consumption (because they cannot insure their idiosyncratic income risks by borrowing), while employed agents accumulate monetary assets.

3.2 Real Rates of Return on Liquid Asset

The benchmark case presumes that fiat money is the only means of precautionary saving, such that the real rates of return on the liquid asset are equal to the negative inflation rate[7] . It is possible for households to receive interests from the monetary asset and to smooth consumption easily, if a real rate of return is positive. On the other hand, inflation depreciates the purchasing power of the monetary asset. The former effect may dominate the latter.

In order to consider the effect of real rates on the welfare costs of inflation, we introduce a representative financial intermediary. The bank intermediates for households using exogenous production technology accessed only by the bank. The bank obtains real returns from its total output and issues interest-bearing deposits to households. The households' budget constraint in this case is

$$c_t + a_{t+1} \leq y_t + (1 + r_t)\ a_t + \frac{g_t}{(1 + g_t)}\ A_t,$$

where y_t is equal to $y - r_t A_t$ in employment, while in unemployment y_t is equal to $\theta(y - r_t A_t)$, with the average GDP equal to unity.

The simulation results presented in Table 2 suggest the following. First, the presence of real returns improves welfare, because a return on assets compensates for the reduction in income when unemployed. An increase in welfare is equivalent to 0.148% of real GDP without money growth and 0.182% with inflation. Second, with the same rate of real return, infla-

[7] There are differing views on whether real rates of interest have been negative or positive in Japan during the 1990s, when the nominal interest rates have been historically low, partly due to the zero interest rate policy of the Bank of Japan. Krugman (1998) suggests negativity in the presence of downturns in physical investments, in spite of the lower level of savings due to aging. On the other hand, Hayashi and Prescott (2001) show low but positive real rates of interest, where the shortened working hours are blamed for the decrease in total factor productivity, though *ex post* real rates of interest indicate that they were temporarily negative in both 1996 and 1998.

tion contributes to improvements in welfare equivalent to 0.034% of real GDP; without the real return, the effect of inflation is negligible at 0.004%. Third, with the same nominal interest rate, there are large welfare gains with greater real returns (and less inflation); with 0.223% of nominal interest rates, the same rate of monetary growth almost cancels the welfare gains equivalent to 0.413% of real GDP without inflation. These results support those of İmrohoroğlu and Prescott (1991) and Diaz-Gimenez, Prescott, Fitzgerald and Alvarez (1992) in the sense that real return matters with welfare measures.

3.3 Presence of "Hand-to-Mouth" Agents

An assumption of rational agents is relaxed; we introduce the *ex ante* heterogeneity of consumers (Campbell and Mankiw, 1989; Lettau and Uhlig, 1999; Weil, 1992). We consider the presence of two types of heterogeneous agents; one type is "rational" agents who prepare precautionary savings for idiosyncratic income risk, and the other type is "hand-to-mouth" agents who do not save to cover the risk of unemployment. Population estimates of irrational agents range between 0.30 and 0.66 (Campbell and Mankiw, 1989).

During the bubble period in the late 1980s in Japan, when the unemployment rate was 2.3% and unemployment duration was three months, our simulation results suggest that rational agents have little motivation to save money against unemployment risk. We assume that some ratio of agents do not save for the risk of unemployment with given rates of money growth, even after the collapse of the bubble. Thus, the budget constraint for agents without precautionary savings is, instead of constraint (3),

$$c_t \leq y_t + \frac{g_t}{1+\pi_t} A_t$$

With a positive money growth rate ($g_t > 0$), hand-to-mouth agents can afford more than the per capita GDP of this economy thanks to governmental allocation from money growth where rational agents consume less. In other words, there is a negative externality in the presence of hand-to-mouth agents with money growth.

Table 3 presents the simulation result; *GDP%* indicates a change from the benchmark case without money growth, and Δ*GDP%* indicates a change from the benchmark case with the money growth rate of 0.223% in Table 1. That is, "GDP%" includes both effects of inflation and heterogeneity, while Δ*GDP%* includes the effect of heterogeneity alone, give the rate of money growth. The selected ratio of agents without precautionary savings ("hand-to-mouth" ratio) is one-third or two-thirds.

In a comparison between rational agents and hand-to-mouth agents, rational agents consume less, but on average enjoy higher levels of welfare than hand-to-mouth agents; with a positive money growth rate, rational agents lose welfare, but hand-to-mouth agents gain, although money growth reduces welfare in total. With a two-thirds hand-to-mouth ratio, average welfare falls because of a lack of consumption smoothing for hand-to-mouth agents, and a reduction in average consumption for rational agents. The welfare cost of the presence of two hand-to-mouth agents out of three is equivalent to 1.141% (1.368%) of real GDP with (without) inflation.

3.4 Duration of Unemployment

It is obvious that a decrease in unemployment rate improves welfare even when total GDP remains the same; without unemployment risk, average utility level of –0.0321 can be calculated without numerical approximation, and the welfare gain is equivalent to 1.24% of real GDP compared to the benchmark case without inflation.

However, welfare effect of unemployment duration is not necessarily trivial. Here, we postulate a reduction in the average duration spell of unemployment, because an average unemployment period may be shortened by social systems to encourage mid-career placement. The selected average unemployment spell is then shortened to three months, which is equivalent to that found in Japan in 1990 or in the United States in İmrohoroğlu (1992), leaving the same unemployment rate as the benchmark case. The transitional probability matrix now becomes

$$\chi_w = \begin{bmatrix} 0.983 & 0.017 \\ 0.327 & 0.673 \end{bmatrix}.$$

Compared to the matrix with a spell of five months average duration, the probability of staying in the unemployment state declines by more than 10%, while the probability of moving from employed to unemployed increases from 1.1% to 1.7%.

Table 4 presents the simulation results with and without money growth. In both cases, two-month curtailments of the average spell of unemployment reduces the welfare cost equivalent to 0.486% (0.392%) with (without) the money growth, even after leaving the unemployment rate unchanged.

3.5 Aggregate Risk (1): Income Fluctuation

We now introduce two types of aggregate risk with fluctuations in aggregate output associated with the business cycle. It is assumed that the aggregate state in our model is twofold, either a good state ($z = h$) or a bad state ($z = l$). For the purpose of comparison, the average endowment is normalized to unity.

One type is an extreme case of a work-sharing system, as found with Japan's lifetime employment and as originally advocated by Weitzman (1984), where the unemployment rate is intact, but the individual income level fluctuates proportionally with aggregate endowment. We assume that average aggregate endowment is 1.02 in a good state, and 0.98 in a bad state[8] , where the individual income level for employed (unemployed) is $1.02y$ ($1.02\theta y$) in a good state and $0.98y$ ($0.98\theta y$) in a bad state. The transitional probability ρ from one aggregate state to another is assumed to be 10%, which means a boom or a recession continues, on average, for 10 months. Thus,

$$\chi_z = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}.$$

It should be noted that selection of this probability ρ (such as 50%, 5%, or 1%) affects little to average utility in this case.

Table 5 shows that measured welfare cost of this type of aggregate risk is small enough to be 0.039% (0.029%) of real GDP with (without) inflation.

[8] The 4% difference is based on the recent drop of income level from *the Family Income and Expenditure Surveys* (Statistics Bureau, Government of Japan).

3.6 Aggregate Risk (2): Unemployment-Rate Fluctuation

We also examine another type of aggregate risk that affects the idiosyncratic income risk rather than the individual income level; that is, in a good state, the unemployment rate falls and average unemployed duration is shorter than in a bad state.

We consider a case in which the average unemployment rate is 2.3% with three months duration in a good state, as in 1990, while the unemployment rate is 4.9% with five months duration in a bad state, like that in 2000. Since the average unemployment rate fluctuates across aggregate states in the model, we follow the transitional probability matrices of employment status χ_w^{ij} depending on transition of aggregate state from i to j one period ahead, where i and j mean either a good state (h) or a bad state (l). The transitional probability matrix of individual and aggregate states is:

$$\chi_w \equiv \begin{pmatrix} \chi_w^{hh} & \chi_w^{hl} \\ \chi_w^{lh} & \chi_w^{ll} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 0.992 & 0.008 \\ 0.348 & 0.652 \end{pmatrix} & \begin{pmatrix} 0.968 & 0.032 \\ 0.217 & 0.783 \end{pmatrix} \\ \begin{pmatrix} 0.992 & 0.008 \\ 0.694 & 0.306 \end{pmatrix} & \begin{pmatrix} 0.989 & 0.011 \\ 0.204 & 0.796 \end{pmatrix} \end{pmatrix}.$$

In the transition of aggregate states, the transitional probabilities are calculated such that the unemployment rate switches to that of the new aggregate state. For example, when the aggregate state transits from $z = h$ to $z' = l$, the unemployment rate must rise from 2.3% to 4.9%. In our calculation of the matrix, we first apply the transitional probability of employment at the new state z', then further move additional agents to adjust the unemployment rate. First, regarding the χ_w^{ll} matrix above, 1.1% of the employed at the previous state move to the unemployed. Also, 79.6% of the unemployed remain unemployed. This total of 2.9% is still lower than 4.9% with the new unemployment rate. Next, 2.0% (= 4.9% − 2.9%) of agents are removed from 96.6% (= 97.7% × 0.989) being (temporary) continuously employed, resulting 94.6% (= 96.6% − 2.0%) of agents continue to be employed. Thus, $Pr(w' = e|w = e)$ becomes 0.968 (= 94.6% ÷ 97.7%).

However, in this case, the welfare effect of the aggregate risk may not be comparable to the benchmark case. Therefore, for the purposes of comparison, the unemployment rate is assumed to be 2.3% and 7.5%, with the average spell of unemployment duration being, respectively, three months and seven months in a good state and a bad state[9] . Thus, the average unemployment rate is 4.9% and the average spell is five months, as in the benchmark case. The transitional probability matrix becomes:

$$\chi_w \equiv \begin{pmatrix} \chi_w^{hh} & \chi_w^{hl} \\ \chi_w^{lh} & \chi_w^{ll} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 0.992 & 0.008 \\ 0.348 & 0.652 \end{pmatrix} & \begin{pmatrix} 0.944 & 0.056 \\ 0.130 & 0.870 \end{pmatrix} \\ \begin{pmatrix} 0.991 & 0.009 \\ 0.800 & 0.200 \end{pmatrix} & \begin{pmatrix} 0.988 & 0.012 \\ 0.147 & 0.853 \end{pmatrix} \end{pmatrix}.$$

Table 6 shows the simulation results. In order to set income levels for the employed and the unemployed, the aggregate variation output is ±2.4% for the first assumption and ±3.2% for the second. With the first assumption, where unemployment risk in a good state improves, the welfare gain is equivalent to 0.413% (0.328%) of real GDP. This welfare improvement is still lower than the previous simulation with 5% unemployment rate with a spell of three months duration.

[9] The assumed unemployment risk in the bad state is still better than those found in many European countries in 2000, including Germany, France, or Italy (see, for example, Blanchard and Summers, 1987).

With the second assumption, the welfare gains are equivalent to 0.124% (0.082%) of real GDP with (without) inflation. This case employs the transitional probability between aggregate state ρ being 10%. However, with 5% of the transitional probability, welfare gain falls to 0.002% with inflation, and welfare loses are equivalent to 0.021% even without inflation. When high unemployment risk is not expected to last very long, the aggregate risk rather brings welfare gains by hedging the idiosyncratic income risk, while it might cause welfare loss when each aggregate state last long.

4 Discussion

4.1 The Welfare Cost of Inflation

In the transaction-motive models of money holdings (Bailey, 1956; Cooley and Hansen, 1995; Lucas, 1994), the shoe-leather cost of inflation depends on the forms of money demand and the magnitudes of either interest-rate (semi-) elasticity or income (for example, Chada, Haldane and Janssen, 1998). According to Shiratsuka (2001)[10] , the shoe-leather cost in Japan is estimated to be about 0.2% of real GDP [11] . Although Nakashima and Saito (2000) show a slight increase in the semi-interest-rate elasticity in their empirical study, the low interest sensitivity of money demand makes the estimated shoe-leather cost negligible for monetary policy in Japan.

[10] The estimates of the interest-rate elasticity of M1 money demand are nearly 0.10 in Japan for the entire postwar period. This is much lower than the interest-rate elasticity of 0.5 in the United States (Lucas, 1994). However, estimates of the semielasticity during the 1990s in Japan jump up to 0.41 for M1, 0.16 for M2+CD and 0.35 for currency (Nakashima and Saito, 2000).

[11] We assume that the annual money growth rate is 6%, a simple average of 9.5% (the average growth rate during the monetary contraction) and 2.6% (that during the expansion).

Moreover, in order to suggest how small the shoe-leather cost in the economy simulated above, our model can be used to construct a money demand curve. In the benchmark case, the real value of money decreases from 1.814 to 1.228 as the money growth rate or inflation rate changes from 0% to 0.223% (2.7% per annual). We assume an exponential form of money demand function with the income elasticity unity, as in Lucas (1994). Those figures yield a semi-interest-rate elasticity of 12.07 and the velocity of real cash balance as 6.62[12] . Under the estimated real money balance, the shoe-leather cost is calculated to be equivalent to be about 0.05% of real GDP[13] .

From either the actual estimated money demand curve in Japan or the constructed function for our simulated economy, money holding based on the transaction motive bears a small cost of inflation in the Japanese 1990s: 0.2% of real GDP at most.

As summarized in Table 7, the magnitude of the welfare cost is a little larger in our model than the estimate of the shoe-leather cost. The effect of money growth is relatively small with the presence of heterogeneous agents because hand-to-mouth agents are not affected by inflation. Also, the effect becomes smaller with smaller unemployment risks because inflation reduces the value of precautionary saving that is demanded more with higher unemployment risk.

[12] A 2.7% increase in the interest rate causes a decrease in the real money balance of 32.3%. Velocity is defined as the ratio of the annual income 12 of the economy to the cash balance 1.814 under zero inflation.

[13] The welfare cost of inflation is expressed as $\frac{1}{\eta \upsilon}[1-(1+\eta\pi)\exp^{-\eta\pi}]$, where η, υ and π denote the semi-interest elasticity, the velocity and inflation rate, respectively.

4.2 Comparing the Welfare Cost

Table 8 rank-orders the welfare costs in the 16 cases we addressed. One policy implication from the simulation may be found that a shorter spell of unemployment duration considerably raises welfare level, even without a decline in the unemployment rate. A two-month curtailment of the unemployment duration from five to three months turns out to bring considerable welfare gain equivalent to 0.392% of real GDP from rank (6) to (1) without inflation, and 0.486% of real GDP from rank (11) to (3) with the presence of 0.223% inflation. Also, the result indicates that the welfare gains (costs) are amplified with a decrease (increase) in the average spell of unemployment duration by the presence of inflation, because inflation reduces the value of precautionary saving that is more highly demanded with higher unemployment risk.

Another implication may be found in the welfare effects of the aggregate output fluctuation. The aggregate risk has a negligible effect on welfare costs when absorbed in income-level fluctuation without an increase in the unemployment rate. When the fluctuation in aggregate output is absorbed in the unemployment rate without changing employed income across the aggregate states, the effect appears to be twofold: When the aggregate state changes relatively swiftly, the aggregate fluctuation raise welfare by helping to hedge idiosyncratic income risk, because the unemployed face better chances to be employed after changing to a good state; when one aggregate state lasts for a relatively long period, then the aggregate fluctuation rather reduces welfare, particularly with the presence of money growth.

5 Conclusion

We have considered the sluggish Japanese economy from the 1990s facing increasing unemployment risk that motivates precautionary saving, and investigated the welfare cost of inflation in an economy with idiosyncratic and aggregate income risks. Money holding in our model is motivated by self-insurance, as different from the transaction motives in cash-in-advance constraint models. We have also considered the effect of the level of the real rate of interest, the presence of hand-to-mouth consumers lacking in preparation for unemployment risk, a change in the spell of unemployment duration, and aggregate risk in the business cycle with fluctuations in social endowment.

The aggregate income risk does not necessarily reduce the welfare level if the aggregate output fluctuation is absorbed by the fluctuation of employment payments as in the lifetime employment system, or when the business cycle is reasonably short. However, the sluggish economy of Japan without full-scale economic recovery in the business cycle for long periods could further reduce the level of welfare.

Associated with the worsening unemployment, Japanese society faces the collapse of the so-called lifetime employment system, without adequate social systems that support job placement to seek mid-career hiring or start-up ventures. One policy implication that may be drawn from our experiments is that shortened spells of unemployment duration considerably improve welfare, even without a reduction in the unemployment rate.

In addition, a positive real return on assets helps agents prepare for idiosyncratic risk, though the current nominal return on bank savings is negligible in Japan. Inflation without a positive real return is undesirable because the welfare costs of income risk are amplified by inflation.

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Table 1: The Benchmark Case of Japan

	Money Growth	Asset		Consumption		Utility	GDP%
(A)	0.0%	1.814	(0.497)	1.0	(0.095)	-1.184	(Base)
(B)	0.223%	1.228	(0.333)	1.0	(0.106)	-1.521	-0.334%

Note: Standard deviations are in parentheses.

Table 2: Real Rates of Return on Liquid Assets

Real Retrun	Money Growth	(Nominal Rate)	Asset		Consumption		Utility	GDP%
0.0%	0.0%	(0.0%)	1.814	(0.497)	1.000	(0.095)	-1.184	(Base)
0.0%	0.223%	(0.223%)	1.793	(0.492)	1.000	(0.095)	-1.180	0.004%
0.1%	0.0%	(0.1%)	2.219	(0.613)	1.000	(0.090)	-1.035	0.148%
0.1%	0.223%	(0.323%)	2.188	(0.606)	1.000	(0.089)	-1.001	0.182%
0.223%	0.0%	(0.223%)	2.938	(0.824)	1.000	(0.079)	-0.770	0.413%

Notes: GDP% is a comparison to the benchmark case without inflation.
Standard deviations are in parentheses.

Table 3: Hand-to-Mouth Consumers

H-to-M	Money	Rational			H-to-M		All		GDP%
Ratio	Growth	asset (s.d.)	cons. (s.d.)	utility	cons. (s.d.)	utility	cons. (s.d.)	utility	(ΔGDP%)
33.3%	0%	1.21 (0.3318)	1.0 (0.0954)	-1.184	1.0 (-)	-3.269	1.0 (0.0636)	-1.878	-0.687%
	0.223%	0.8225 (0.5313)	0.9991 (0.1061)	-1.620	1.0018 (0.1426)	-3.064	1.0 (0.1195)	-2.101	-0.905% (-0.571%)
66.7%	0%	0.6041 (0.1656)	1.0 (0.0954)	-1.184	1.0 (-)	-3.269	1.0 (0.0318)	-2.574	-1.368%
	0.223%	0.4101 (0.8887)	0.9982 (0.1060)	-1.717	1.0009 (0.1426)	-3.167	1.0 (0.1316)	-2.684	-1.475% (-1.141%)

Notes: "H-to-M" implies "Hand-to-Mouth"
GDP% (ΔGDP%) is a comparison to the benchmark case without (with) inflation.

Table 4: Shorter Unemployment Duration

Money Growth	Asset		Consumption		Utility	GDP%	(ΔGDP%)
0%	1.652	(0.459)	1.0	(0.080)	-0.791	0.392%	
0.223%	1.233	(0.342)	1.0	(0.090)	-1.031	0.152%	(0.486%)

Notes: GDP% (ΔGDP%) is a comparison to the benchmark case without (with) inflation. Standard deviations are in parentheses.

Table 5: Aggregate Risk (1): Income Fluctuation

Money Growth	Asset		Consumption		Utility	GDP%	(ΔGDP%)
0.0%	1.816	(0.500)	1.0	(0.097)	-1.213	-0.029%	
0.223%	1.218	(0.331)	1.0	(0.108)	-1.560	-0.373%	(-0.039%)

Notes: GDP% (ΔGDP%) is a comparison to the benchmark case without (with) inflation. Standard deviations are in parentheses.

Table 6: Aggregate Risk (2): Unemployment-Rate Fluctuation

Unemp. Risk	ρ	Money Growth	Asset	Consump -tion	Utility	GDP% (ΔGDP%)
2.3-4.9%	0.1	0.0%	1.347 (0.335)	1.0 (0.082)	-0.855	0.328%
(3-5 months)		0.223%	0.915 (0.222)	1.0 (0.090)	-1.105	0.079% (0.413%)
2.3-7.5%	0.1	0.0%	1.762 (0.488)	1.0 (0.092)	-1.102	0.082%
(3-7 months)		0.223%	1.230 (0.337)	1.0 (0.102)	-1.396	-0.210% (0.124%)
2.3-7.5%	0.05	0.0%	1.761 (0.486)	1.0 (0.096)	-1.205	-0.021%
(3-7 months)		0.223%	1.202 (0.327)	1.0 (0.106)	-1.519	-0.332% (0.002%)

Notes: GDP% (ΔGDP%) is a comparison to the benchmark case without (with) inflation. Standard deviations are in parentheses.

Table 7: Welfare Cost of Inflation

Ranking	Unemployment		Income	ρ	H-to-M	GDP%
	Rate	Duration	Risk		Ratio	
(1)	4.9%	5 months	-	-	66.7%	-0.107%
(2)	4.9%	5 months	-	-	33.3%	-0.218%
(3)	4.9%	3 months	-	-	-	-0.240%
(4)	2.3-4.9%	3-5 months	-	0.1	-	-0.249%
(5)	2.3-7.5%	3-7 months	-	0.1	-	-0.292%
(6)	2.3-7.5%	3-7 months	-	0.05	-	-0.311%
(7 Base)	4.9%	5 Months	-	-	-	-0.334%
(8)	4.9%	5 Months	0.98-1.02	0.1	-	-0.344%

Note: GDP% is the difference in cases with and without money growth of 0.223%.

Table 8: Welfare Ranking of Idiosyncratic and Aggregate Risks

Ranking	Money Growth	Unemployment Rate	Unemployment Duration	Income Risk	ρ	H-to-M Ratio	GDP%
(1)	0%	4.9%	3 Months	-	-	-	0.392%
(2)	0%	2.3-4.9%	3-5 Months	-	0.1	-	0.328%
(3)	0.223%	4.9%	3 Months	-	-	-	0.152%
(4)	0.%	2.3-7.5%	3-7 Months	-	0.1	-	0.082%
(5)	0.223%	2.3-4.9%	3-5 Months	-	0.1	-	0.079%
(6)	0%	4.9%	5 Months	-	-	-	[Base]
(7)	0%	2.3-7.5%	3-7 Months	-	0.05	-	-0.021%
(8)	0%	4.9%	5 Months	0.98-1.02	-	-	-0.029%
(9)	0.223%	2.3-7.5%	3-7 Months	-	0.1	-	-0.210%
(10)	0.223%	2.3-7.5%	3-7 Months	-	0.05	-	-0.332%
(11)	0.223%	4.9%	5 Months	-	-	-	-0.334%
(12)	0.223%	4.9%	5 Months	0.98-1.02	0.1	-	-0.373%
(13)	0%	4.9%	5 Months	-	-	33.3%	-0.687%
(14)	0.223%	4.9%	5 Months	-	-	33.3%	-0.905%
(15)	0%	4.9%	5 Months	-	-	66.7%	-1.368%
(16)	0.223%	4.9%	5 Months	-	-	66.7%	-1.475%

Note: GDP% is a comparison to the benchmark case without inflation.