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Incentive to Form FTA with Different Cost Conditions

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Incentive to Form FTA with Different Cost Conditions

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Abstract

This paper examines the formation of Free Trade Agreements between the countries with different cost conditions in the world where imcompletely substitutable goods are traded. We see that each country levies higher (lower) tariffs to a country with lower (higher) marginal cost when the substitutability between goods is sufficiently low. Further, we obtain that each country has an incentive to form FTA with a higher marginal cost-country and the countries which have similar marginal costs are likely to form FTA. It is also shown that gains from FTA decrease (increase) as FTA-partners' marginal costs become higher (lower).

Keywords: Free Trade Agreement, cost condition, social welfare

JEL Classification: F15

1 Introduction

Recently forming FTA have increased in the world. Representative examples are EU, NAFTA and AFTA. Japan has signed FTA with Singapore and Mexico and negotiates with other ASEAN countries now. Although many countries try to sign FTA with other countries, can they gain from the formation of FTA? Kemp and Wan (1976) showed that there exists tariff vectors, under which potential members gain from FTA and non-members don't lose from CU. Panagariya and Krishna (2002) showed the case of FTA. But the tariff vectors that they showed need not be optimal tariff vector. Further partner countries may be not able to adjust

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their tariff soon for some reason such as existence of a political process. FTA need not be beneficial for each country. So, what conditions makes FTA between countries beneficial? This issue is main topic in this paper.

The preceding research about this topic has been based mainly on the partial equilibrium analysis. The representative study is Viner (1950). But the analysis based on the partial equilibrium is restrictive in the way that the substitutablity between goods are ignored. Krugman (1991) and Bond and Syropoulos (1996) noticed that point. Nevertheless their model are not realistic. They ignored the cost conditions which countries have. They didn't study the relation between a cost condition and an incentive to form FTA. Which does the low technology country have an incentive to form FTA with low technology country or hightechnology country? This is the one of important issues. So in our paper we take into account the substilutablity between goods and investigate the incentive to form FTA in terms of differences in cost conditions countries have.

This paper is organized as follows. In Section 2, we present the model and derive the market equilibrium. We show a welfare effect decomposition arising from a change in tariffs, which is seen in tariff theory. We also analyze how optimal tariffs are changed by marginal cost changes. In Section 3 we analyze each country's incentive to form FTA. Section 4 is a conlusion of our paper.

2 The Model

We assume the world consists of n countries ($n \geq 2$), each of which has an identical representative consumer who consumes a numeraire good and a continuum of horizontally differentiated commodities that are indexed by $\omega \in [0, 1]$[1]. Each differentiated good is produced by one firm. The firm has a constant marginal cost technology. The firms in a same country have a same marginal cost whereas the firms in different countries do not necessarily have a same marginal cost. We assume that there is no entry of firms into each industry. Each firm is owned by a domestic consumer who receives all firm's profit. On the other hand, the numeraire good is produced competitively. A consumer in each country is endowed with L unit of labor,

[1]The following assumptions and model is mainly based on Furusawa and Konishi (2002). We introduced the cost condition into their model. In this paper, we focus on the effect of cost condition on incentives to form FTA. It is our originality.

which is used for production of differentiated goods and a numeraire good. One unit of labor produces one unit of the numeraire good, so that the wage rate is 1.

In country i $(i = 1, 2, \cdots, n)$, measure s_i of firms are located, in other words, measure s_i of differentiated goods are produced. We assume that all markets are segmented so that firms can perfectly price differentiated among different countries. Country i imposes a specific tariff t_j^i on the imports from country j. For simplicity, we assume that there is no commodity tax on domestic goods and same tariff rate t_j^i is imposed on all imports from country j. On the other hand, the numeraire good is traded freely in the world. Tariff revenue is redistributed to the domestic consumer.

2.1 Comsumer Demands

A utility function that a representative consumer in country i is given by the following quasi-linear utility function:

$$U^i = \alpha \int_0^1 q^i(\omega)d\omega - \frac{\beta}{2}\int_0^1 q^i(\omega)^2 d\omega - \frac{\delta}{2}\left[\int_0^1 q^i(\omega)d\omega\right]^2 + q_0^i, \tag{1}$$

where $q^i : [0,1] \to \mathbb{R}_+$ is an integrable consumption function, and q_0^i denotes a quantity of the numeraire good. The coefficient of the last second term δ means the substitutability among differentiated goods. Let y^i denote the consumer in country i. Then, the budget constraint can be written as

$$y^i = \int_0^1 \tilde{p}^i(\omega)q^i(\omega)d\omega + q_0^i, \tag{2}$$

where $\tilde{p}^i : [0,1] \to \mathbb{R}_+$ denotes the consumer price function. The first order condition for the consumer's utility maximization problem gives us the inverse demand function for each good ω:

$$\tilde{p}^i(\omega) = \alpha - \beta q^i(\omega) - \delta Q^i, \tag{3}$$

where $Q^i = \int_0^1 q^i(\omega)d\omega$. Q^i represents the average comsumer's demand in country i.

2.2 Equilibrium in Country i

Let $p^k(\omega)$ denote the production price of good ω for country k. We assume that all the firms in country j have technologies of a constant marginal cost c_j. The firm ω in country j chooses the set of output $\{q^k(\omega)\}_{k=1}^n$ in order to maximize its profits $\pi(\omega) = \sum_{k=1}^n \{p^k(\omega) - c_j\}q^k(\omega)$.

The first order condition for this maximization gives us

$$q^i(\omega) = \frac{\alpha}{2\beta} - \frac{\delta}{2\beta}Q^i - \frac{1}{2\beta}(t^i_j + c_j). \tag{4}$$

Notice that $q^i(\omega)$ does not vary with ω. Output of the good depends only on the exporting country's marginal cost c_j, the importing country's tariff t^i_j and total average consumer's demand in the importing country Q^i. For simplicity, we henceforth suppress the argument ω and let q^i_j denote country i's demand produced in the country j. We can rewrite equation (4) as the quasi-reaction function:

$$q^i_j = r^i_j(Q^i) = \frac{1}{2\beta}(\alpha - \delta Q^i - t^i_j - c_j). \tag{5}$$

In equilibrium, $Q^i = R^i(Q^i)$ holds, where $R^i(Q^i) = \sum_{k=1}^n s_k r^i_k(Q^i)$. Then, the equilibrium average output $(Q^i)^e$ is as follows.

$$(Q^i)^e = \frac{\alpha - \bar{t}^i - \bar{c}}{2\beta + \delta}. \tag{6}$$

$\bar{t}^i$ and $\bar{c}$ represent the average import tariff charged by country i ($\sum_{k=1}^n s_k t^i_k$) and the average marginal cost ($\sum_{k=1}^n s_k c_k$), respectively. We assume that $(Q^i)^e$ is positive. This is represented as the following Assumption 1.

Assumption 1: We assume that the following inequality is satisfied, i.e., $\bar{t}^i < \alpha - \bar{c}$.

This ensures existence of equilibrium average output $(Q^i)^e$. We also assume that this equilibrium is stable. As we know, the stability condition [2] is $\left|dR^i(Q^i)/dQ^i\right| < 1$. This condition can be rewritten as the following Assumption 2.

Assumption 2: The substitutability δ is sufficiently small, i.e., $\delta < 2\beta$.

Furthermore we assume that all countries produce or can produce their goods. This assumption is represented as the following Assumption 3.

Assumption 3: The demand size is sufficiently large, i.e., $\alpha > c_j$ (j=1, 2, …, n).

[2]In this paper, we use the following adjustment process: $Q^i_t = R^i(Q^i_{t-1})$, where Q^i_t means average output at the t th period.

Unless this assumption is satisfied, even when tariffs are not imposed on the goods, the goods are not produced since the demand curve intersects with the marginal cost curve. Under Assumption 1, 2, and 3, we drive the equilibrium output and producer price. Substituting (6) into (5) yields the equilibrium consumer's demand in country i for a commodity produced in country j, as the function of country i's tariff vector $\mathbf{t}^i = (t_1^i, \cdots, t_n^i)$, marginal cost vector $\mathbf{c} = (c_1, \cdots, c_n)$, and variety share vector of differentiated goods $\mathbf{s} = (s_1, \cdots, s_n)$:

$$q_j^i(\mathbf{t}^i, \mathbf{c}, \mathbf{s}) = \frac{\alpha}{2\beta + \delta} - \frac{1}{2\beta}(t_j^i + c_j) + \frac{\delta}{2\beta(2\beta + \delta)}(\bar{t}^i + \bar{c}).$$

Then it follows from (3) that the equilibrium producer price which each firm in country j charges for the market of country i is

$$p_j^i(\mathbf{t}^i, \mathbf{c}, \mathbf{s}) = \frac{\alpha\beta}{2\beta + \delta} - \frac{1}{2}(t_j^i - c_j) + \frac{\delta}{2(2\beta + \delta)}(\bar{t}^i + \bar{c}).$$

Note that $p_j^i(\mathbf{t}^i, \mathbf{c}, \mathbf{s}) = \beta q_j^i(\mathbf{t}^i, \mathbf{c}, \mathbf{s}) + c_j$ holds for any tariff vector $\mathbf{t}^i$, any marginal cost vector $\mathbf{c}$, and any variety share vector $\mathbf{s}$. We find the following results from these demands and prices,

Result:

$$\begin{aligned}
&\frac{\partial q_j^i}{\partial t_j^i} = -\frac{2\beta + \delta(1 - s_j)}{2\beta(2\beta + \delta)} < 0, \quad \frac{\partial q_j^i}{\partial t_k^i} = \frac{s_k\delta}{2\beta(2\beta + \delta)} > 0 \;\; (\forall k \neq j),\\
&\frac{\partial p_j^i}{\partial t_j^i} = -\frac{2\beta + \delta(1 - s_j)}{2(2\beta + \delta)} < 0, \quad \frac{\partial p_j^i}{\partial t_k^i} = \frac{s_k\delta}{2(2\beta + \delta)} > 0 \;\; (\forall k \neq j),\\
&\frac{\partial q_j^i}{\partial c_j} = -\frac{2\beta + \delta(1 - s_j)}{2\beta(2\beta + \delta)} < 0, \quad \frac{\partial q_j^i}{\partial c_k} = \frac{s_k\delta}{2\beta(2\beta + \delta)} > 0 \;\; (\forall k \neq j),\\
&\frac{\partial p_j^i}{\partial c_j} = \frac{2\beta + \delta(1 + s_j)}{2(2\beta + \delta)} > 0, \quad \frac{\partial p_j^i}{\partial c_k} = \frac{s_k\delta}{2\beta(2\beta + \delta)} > 0 \;\; (\forall k \neq j).
\end{aligned}$$

It should be also noticed that q_j^i and p_j^i $(j \neq i)$ don't depend on the tariffs imposed by the countries other than country i. It stems from the assumption that all markets are segmented.

2.3 Social Welfare

Under the world tariff vector $\mathbf{t} = (\mathbf{t}^1, \cdots, \mathbf{t}^n)$, each firm in country i earns the profits:

$$\pi_i(\mathbf{t}, \mathbf{c}, \mathbf{s}) = \sum_{k=1}^{n} \{p_i^k(\mathbf{t}^k, \mathbf{c}, \mathbf{s}) - c_i\} q_i^k(\mathbf{t}^k, \mathbf{c}, \mathbf{s}).$$

Country i's tariff revenue is

$$T^i(\mathbf{t}^i, \mathbf{c}, \mathbf{s}) = \sum_{k=1}^{n} s_k t_k^i q_k^i(\mathbf{t}^i, \mathbf{c}, \mathbf{s}).$$

A representative consumer's income is the sum of labor income, redistributed tariff revenue, and the profit of all the firms in country i:

$$y^i = L^i + T^i(\mathbf{t}^i, \mathbf{c}, \mathbf{s}) + s_i \pi_i(\mathbf{t}, \mathbf{c}, \mathbf{s}).$$

Then it follows from (2) that

$$\begin{aligned} q_0^i(\mathbf{t}, \mathbf{c}, \mathbf{s}) &= y^i - \sum_{k=1}^{n} s_k \{p_k^i + t_k^i\} q_k^i, \\ &= - \sum_{k \neq i} s_k p_k^i q_k^i + s_i \sum_{k \neq i} p_i^k q_i^k - s_i c_i \sum_{k=1}^{n} q_i^k + L^i. \end{aligned}$$

Here we omit the independent variables of functions to keep notation to a minimum. Substituting this equilibrium demand into (1), we obtain

$$\begin{aligned} W^i = {} & \alpha \sum_{k=1}^{n} s_k q_k^i - \frac{\beta}{2} \sum_{k=1}^{n} s_k (q_k^i)^2 - \frac{\delta}{2} \left[\sum_{k=1}^{n} s_k q_k^i \right]^2 \\ & - \sum_{k \neq i} s_k p_k^i q_k^i + s_i \sum_{k \neq i} p_i^k q_i^k - s_i c_i \sum_{k=1}^{n} q_i^k + L^i, \\ = {} & W^i(\mathbf{p}, \mathbf{q}, \mathbf{c}, \mathbf{s}). \end{aligned}$$

$\mathbf{q}$ and $\mathbf{p}$ represent the equilibrium consumers' demand vector in the world $\mathbf{q} = (q_1^1, \cdots, q_n^1, \cdots, q_1^n, \cdots, q_n^n)$ and the equilibrium producers' price vector $\mathbf{p} = (p_1^1, \cdots, p_n^1, \cdots, p_1^n, \cdots, p_n^n)$, respectively. In equilibrium $\mathbf{q}$ and $\mathbf{p}$ depend on the world tariff vector $\mathbf{t}$, the marginal cost vector $\mathbf{c}$, and the variety vector $\mathbf{s}$. Then we gives the following definition of welfare function.

$$\widehat{W}^i(\mathbf{t}, \mathbf{c}, \mathbf{s}) = W^i(\mathbf{p}(\mathbf{t}, \mathbf{c}, \mathbf{s}), \mathbf{q}(\mathbf{t}, \mathbf{c}, \mathbf{s}), \mathbf{c}, \mathbf{s})$$

Note that the welfare function W^i can be decomposed as below[3],

$$W^i = V^i - M^i + E^i - TC^i + L^i, \tag{7}$$

where

$$\begin{aligned} V^i &= \alpha \sum_{k=1}^{n} q_k^i - \frac{\beta}{2} \sum_{k=1}^{n} s_k (q_k^i)^2 - \frac{\delta}{2} \left[\sum_{k=1}^{n} s_k q_k^i \right]^2, \\ M^i &= \sum_{k \neq i} s_k p_k^i q_k^i, \quad E^i = \sum_{k \neq i} s_i p_i^k q_i^k, \quad TC^i = \sum_{k=1}^{n} s_i c_i q_i^k. \end{aligned}$$

[3]See Furusawa and Konishi (2004) about this decomposition.

V^i represents consumer's gross utility. E^i and M^i represent export payments of country i and import payment of country i, respectively. TC^i stands for the total cost of production in country i. Thus, country i's welfare increases if E^i increases or M^i and TC^i decrease. At first glance, country i's seems to have an incentive to impose higher tariffs on foreign countries so as to decrease country i's imports. However, this is not necessarily true. Consider the case where country i levies a higher tariff on country j. Then country i's imports from country j decrease. In spite of that, total imports of country i may increase since prices of goods from the other foreign countries in country i become relatively lower than country j and country i's imports from these countries increase. Moreover, TC^i becomes large because prices of goods produced in country i also becomes relatively lower than goods produced in country j and demands for goods produced in country i (or supplies) become large.

For our convenience we derive the differential coefficient of country i's welfare function over t_j^i.

$$\begin{aligned}
\frac{\partial \widehat{W}^i}{\partial t_j^i} &= \sum_{k=1}^{n} \frac{\partial V^i}{\partial q_k^i}\frac{\partial q_k^i}{\partial t_j^i} - \sum_{k\neq i} s_k q_k^i \frac{\partial p_k^i}{\partial t_j^i} - \sum_{k\neq i} s_k p_k^i \frac{\partial q_k^i}{\partial t_j^i} - s_i c_i \frac{\partial q_i^i}{\partial t_j^i}, \\
&= \sum_{k=1}^{n} s_k (p_k^i + t_k^i)\frac{\partial q_k^i}{\partial t_j^i} - \sum_{k\neq i} s_k q_k^i \frac{\partial p_k^i}{\partial t_j^i} - \sum_{k\neq i} s_k p_k^i \frac{\partial q_k^i}{\partial t_j^i} - s_i c_i \frac{\partial q_i^i}{\partial t_j^i}, \\
&= \sum_{k\neq i} s_k t_k^i \frac{\partial q_k^i}{\partial t_j^i} + s_i (p_i^i - c_i)\frac{\partial q_i^i}{\partial t_j^i} - \sum_{k\neq i} s_k q_k^i \frac{\partial p_k^i}{\partial t_j^i}.
\end{aligned}$$

The first term in the second equation results from the fact that the marginal utility $\partial V^i/\partial q_k^i$ is equal to the consumer price $\tilde{p}_k^i = p_k^i + t_k^i$. Then

$$\frac{\partial \widehat{W}^i}{\partial t_j^i} = -\sum_{k\neq i} s_k q_k^i \frac{\partial p_k^i}{\partial t_j^i} + s_i (p_i^i - c_i)\frac{\partial q_i^i}{\partial t_j^i} + \sum_{k\neq i} s_k t_k^i \frac{\partial q_k^i}{\partial t_j^i} \tag{8}$$

Therefore we can find that the effect of country i's tariff t_j^i on its welfare $\widehat{W}^i$ is decomposed into three effects. The first, second, and third term are called *terms of trade effect*, *allocation effect*, and *tariff revenue effect* respectively[4]. Terms of trade effect means how improvement of terms of trade through raising the tariff influences the welfare. Resource allocation effect means how increasing production by raising the tariff influences the welfare when allocation is not efficient. Tariff revenue effect is that of increasing the tariff revenue by raising the tariff.

[4] This decomposition follows from Bond (1990) and Kowalczyk (2000)

2.4 Optimal Tariffs

In this subsection we derive the optimal tariffs. Before doing this, we see the concavity of the welfare function. We gave the assumption about the substitutability δ above(Assumption 2). We put the stronger assumption here. This is the following Assumption 4.

Assumption 4: The substitutability δ satisfies $\delta < \beta/2$.

We obtain Lemma 1 when Assumption 4 is satisfied.

Lemma 1: Let Assumption 4 be satisfied. Then $\widehat{W}^i$ is concave in $\mathbf{t}^i$.
Proof: see Appendix.

Now we can derive the optimal tariffs which country i chooses to maximize its welfare by Lemma 1. The first order condition is

$$\frac{\partial}{\partial t^i_j}\widehat{W}^i(\mathbf{t}, \mathbf{c}, \mathrm{s}) = 0 \quad j = 1, 2, \ldots, n \quad\quad j \neq i. \tag{9}$$

Solving this simultaneous equations, we obtain the optimal tariffs and an important result about this optimal tariffs. This is shown by Lemma 2.

Lemma 2: The optimal tariff is positive.
Proof: see Appendix.

Now we define the optimal tariff imposed by country i on the imports from country j as $(t^i_j)^*$:

$$(t^i_j)^* = t^i_j(\mathbf{c}, \mathbf{s}). \tag{10}$$

At first glance (9) seems to give us the optimal tariff function of $\mathbf{c}$, $\mathbf{s}$, and the tariff vector of other countries $\mathbf{t}^{-i}$. As we show in Appendix, the first order differential coefficients of country i's welfare function over its tariffs for the other countries do not depend on the other countries' tariffs, i.e.,

$$\frac{\partial^2 \widehat{W}^i}{\partial t^k_h \partial t^i_j} = 0 \quad \forall k \neq i \quad\quad \forall h \neq j.$$

Thus the optimal tariffs rely only on $\mathbf{c}$ and $\mathbf{s}$.

Heceforth, for simplicity, we continue to assume that all the countries have the same variety share ($s_i = s = 1/n$, $i = 1, 2, \cdots n$). Then we can calculate and derive optimal tariffs by solving multiple equations (9). This result is given by Lemma 3.

Lemma 3: Country i's optimal tariff levied on country j is

$$(t_j^i)^* = \frac{A_j^i}{3\{12\beta^2 + 4(1+2s)\beta\delta + s(1+2s)\delta^2\}} > 0, \tag{11}$$

where $A_j^i = 12(\alpha - c_j)\beta^2 + 4(\bar{c} - c_j - 4sc_i - 2s\bar{c} + 3s\alpha)\beta\delta + (s\bar{c} - 9sc_i - sc_j + 2s^2 c_i - 2s^2 c_j)\delta^2$.

2.5 Comparative Statics

In above subsection, we derived the optimal tariffs in the case where countries' variety shares are same in the world. For tractability, we continue to assume that this holds. In this subsection, using Lemma 3, we analyze the effect of the change in a domestic country i's marginal cost and foreign countries' marginal costs on the optimal domestic country i's tariffs. The foreign countries' marginal costs are divided into two groups. If we focus on the tariff $(t_j^i)^*$, the country j's marginal cost and the marginal costs of the countries other than country j are so. In view of these divisions we conduct the comparative statistics. Then we obtain the following proposition.

Proposition 1: When Assumption 1, 3, and 4 are satisfied and all the countries have the same variety share (i.e., $s = 1/n$), the optimal tariffs have the following relations to marginal costs [5].

$$
\begin{aligned}
&(i)\ \frac{\partial (t_j^i)^*}{\partial c_i} = -\frac{4(3+2s)\beta\delta + 3s(3-s)}{3\{12\beta^2 + 4(1+2s)\beta\delta + s(1+2s)\delta^2\}} < 0,\\
&(ii)\ \frac{\partial (t_j^i)^*}{\partial c_j} = -\frac{12\beta^2 + 4(1-s+2s^2)\beta\delta + s(1+s)\delta^2}{3\{12\beta^2 + 4(1+2s)\beta\delta + s(1+2s)\delta^2\}} < 0,\\
&(iii)\ \frac{\partial (t_j^i)^*}{\partial c_k} = \frac{4s(1-2s)\beta\delta + s^2\delta^2}{3\{12\beta^2 + 4(1+2s)\beta\delta + s(1+2s)\delta^2\}} > 0 \quad (k \neq i, j).
\end{aligned}
$$

In order to understand an implication of Proposition 1, consider the following functions,

$$t_j^i = r_j^i(\mathbf{t}_{-j}^i, \mathbf{c}, \mathbf{s}) = \arg\max_{t_j^i} \widehat{W}^i(\mathbf{t}, \mathbf{c}, \mathbf{s})$$

where $\mathbf{t}_{-j}^i = (t_1^i, \cdots, t_{j-1}^i, t_{j+1}^i, \cdots, t_n^i)$. This function satisfies

[5] We can show that when strengthening Assumption 2' a little, even though an importing country has a variety share s and each exporting country has a variety share $(1-s)/(n-1)$, we obtain the same result as the one of Proposition 1. The result of Proposition 1 is relatively robust in this sense.

$$(a)\ \frac{\partial r_j^i}{\partial c_i} = -\frac{2(1-s)\beta + (1-2s)\delta}{12\beta^2 + 2(6-3s-s^2)\beta\delta + (3-3s-2s^2)\delta^2} < 0$$
$$(b)\ \frac{\partial r_j^i}{\partial c_j} = -\frac{4\beta^2 + 2(2-s-s^2)\beta\delta + (1-s-2s^2)\delta^2}{12\beta^2 + 2(6-3s-s^2)\beta\delta + (3-3s-2s^2)\delta^2} < 0$$
$$(c)\ \frac{\partial r_j^i}{\partial c_k} = \frac{s\delta\{2\beta + \delta + 2(\beta+\delta)s\}}{12\beta^2 + 2(6-3s-s^2)\beta\delta + (3-3s-2s^2)\delta^2} > 0$$

In words, increase in an importing country i's marginal cost or an exporting country j's marginal cost shifts the curve $r_j^i r_j^i$ inward while increase in other exporting country k's marginal cost shifts the curve $r_j^i r_j^i$ outward. On the other hand, effects of changes in the tariffs imposed on other countries on the function r_j^i are

$$\frac{\partial r_j^i}{\partial t_k^i} = \frac{\delta\{3(2\beta+\delta) + 2(\beta+\delta)s\}}{12\beta^2 + 2(6-3s-s^2)\beta\delta + (3-3s-2s^2)\delta^2} \quad (k \neq i, j) > 0. \tag{12}$$

For simplicity, consider the case where country 1 determines the optimal tariffs in 3-country model. This is drawn in Figure 1 and 2 as $r_j^1 r_j^1$ $(j = 2, 3)$. The curve $r_j^1 r_j^1$ is of upward-sloping from (12). We explain the reason for this. The sign of $\partial^2 \widehat{W}^1/\partial t_k^1 \partial t_j^1$ determines the slope of the curve $r_j^1 r_j^1$.

$$\frac{\partial^2 \widehat{W}^1}{\partial t_k^1 \partial t_j^1} = \sum_{h\neq 1} s_h \frac{\partial q_h^1}{\partial t_k^1}\frac{\partial p_h^1}{\partial t_j^1} + s_1 \frac{\partial q_1^1}{\partial t_j^1}\frac{\partial p_1^1}{\partial t_k^1} + s_k \frac{\partial q_k^1}{\partial t_j^1} \quad (k \neq j).$$

The first, second, and third terms on the right-hand side are the changes in terms of trade effect, allocation effect, and trade revenue effect, respectively. The signs of the changes in allocation effect and trade revenue effect are necessarily positive but the one in terms of trade effect is ambiguous. However, from (12), we know that $\partial^2 \widehat{W}^1/\partial t_k^1 \partial t_j^1$ is positive. The changes in tariff revenue effect and allocation effect dominate the one in terms of trade effect. This makes marginal welfare of the tariff on country j large and country 1 has an incentive to raise the tariff on country j. Thus, the curve $r_j^1 r_j^1$ is upward-sloping as in Figure 1 and 2. The equilibrium is represented by E in Figure 1 and 2.

[INSERT Figure 1]

[INSERT Figure 2]

Next we try to analyze how optimal tariffs change as marginal costs change. We can do it by checking the $\partial^2 \widehat{W}^1/\partial c_1 \partial t_j^1$, $\partial^2 \widehat{W}^1/\partial c_j \partial t_j^1$, and $\partial^2 \widehat{W}^1/\partial c_k \partial t_j^1$ in the same way as the above.

By doing a little arithmetic, we find that allocation effect determines the sign of (a) and the terms of trade effect determines the sign of (b) and (c). These facts tell us two important things. First, the curve $r_j^1 r_j^1$ shifts outward if country 1's marginal cost becomes lower. This is drawn in Figure 1. Then, the equilibrium changes into E' and country 1 raises the tariffs on country 2 and 3. This is because if country 1 lowers the tariffs, then goods produced in efficient country 1 are substituted for goods produced in relatively inefficient country 2 and 3 and this fact makes large distortion. Second, we also see that if country 2's marginal cost decreases, then the curve $r_2^1 r_2^1$ and $r_3^1 r_3^1$ shift outward and inward, respectively. This is drawn in Figure 2. Then, the equilibrium changes into E' and country 1 raises the tariff on country 2 and lowers the tariff on country 3. When country 2's marginal cost lowers, if country 1 decreases the tariff on country 2, then terms of trade for country 2 worsens largely and country 1's welfare decreases. Similarly, if country 1 increases the tariff on country 3, then terms of trade for country 2 and country 1's welfare decreases.

3 Incentive to Form FTA

In this section we discuss the incentives for countries to form FTA. Each country has an incentive to form FTA if a partner and it improve their welfare when they eliminate their tariffs on the partner. To analyze this incentive, we define the country i's benefit from FTA between country i and j as follows.

Definition: The country i's benefit from FTA between country i and j, $\Delta\widehat{W}^i$ is defined as

$$\Delta\widehat{W}^i = \widehat{W}^i(\mathbf{t}_F^{ij}, \mathbf{s}, \mathbf{c}) - \widehat{W}^i(\mathbf{t}^*, \mathbf{s}, \mathbf{c}). \tag{13}$$

where $\mathbf{t}^* = (\mathbf{t}^{1*}, \ldots, \mathbf{t}^{k*}, \ldots, \mathbf{t}^{n*})$ is the world optimal tariff vector, $\mathbf{t}^{i*} = ((t_1^i)^*, \cdots, (t_n^i)^*)$ is country i's optimal tariff vector, and $\mathbf{t}_F^{ij}$ is the tariff vector , the elements of which are $t_h^k = (t_h^k)^*$ $(h, k \neq i, j)$, $t_h^i = (t_h^i)^*$ $(h \neq j)$, $t_h^j = (t_h^i)^*$ and $t_j^i = t_i^j = 0$.

FTA between two countries may be beneficial for one country but be not for the other. Under no transfer, the country which loses from FTA does not have an incentive to form FTA. If the country which obtain the benefit from FTA transfers to the other country and both countries obtain higher welfare than before forming FTA, both countries may have incentives to form FTA. So, we utilize the joint welfare function to examine what condition should be satisfied for two countries to form FTA. The joint welfare function between country i and j is

$$\widehat{W}^{i,j}(\mathbf{t},\mathbf{s},\mathbf{c}) = \widehat{W}^{i}(\mathbf{t},\mathbf{s},\mathbf{c}) + \widehat{W}^{j}(\mathbf{t},\mathbf{s},\mathbf{c}). \tag{14}$$

We assume that this joint welfare function has the following property:

Assumption 5: The joint welfare function $\widehat{W}^{i,j}(\mathbf{t},\mathbf{s},\mathbf{c})$ is concave in $(\mathbf{t}^i,\mathbf{t}^j)$.

By using equation (13) and (14), both countries' benefit from FTA is given by

$$\Delta\widehat{W}^{i,j} = \widehat{W}^{i,j}(\mathbf{t}_F^{ij},\mathbf{s},\mathbf{c}) - \widehat{W}^{i,j}(\mathbf{t}^*,\mathbf{s},\mathbf{c}). \tag{15}$$

Since the joint welfare function $\widehat{W}^{i,j}(\mathbf{t},\mathbf{s},\mathbf{c})$ is concave in $(\mathbf{t}^i,\mathbf{t}^j)$ by Assumption 5,

$$\Delta\widehat{W}^{i,j} \geq -(t_j^i)^*\frac{\partial}{\partial t_j^i}\widehat{W}^{i,j}((\mathbf{t}^{-i})^*,(\mathbf{t}_{-j}^i)^*,0,\mathbf{s},\mathbf{c}) - (t_i^j)^*\frac{\partial}{\partial t_i^j}\widehat{W}^{i,j}((\mathbf{t}^{-j})^*,(\mathbf{t}_{-i}^j)^*,0,\mathbf{s},\mathbf{c}), \tag{16}$$

where $(\mathbf{t}^{-k})^*$ represents optimal tariff vector that all the country other than country k impose and $(\mathbf{t}_{-h}^k)^*$ represents the optimal tariff vector that country i imposes on all the countries other than country j. LHS in this equation is positive if RHS is positive. We know that the optimal tariffs are positive. Then we see $\partial\hat{W}^{i,j}((\mathbf{t}^{-i})^*,(\mathbf{t}_{-j}^i)^*,0,\mathbf{s},\mathbf{c})/\partial t_j^i$. This is calculated as follows,

$$\begin{aligned}\frac{\partial}{\partial t_j^i}\widehat{W}^{i,j}((\mathbf{t}^{-i})^*,(\mathbf{t}_{-j}^i)^*,0,\mathbf{s},\mathbf{c}) =& \frac{s\delta}{4\beta(2\beta+\delta)^2}\{4\beta+(3+2s)\delta\}\sum_{k\neq i,j}s(t_k^i)^* - \frac{s}{2\beta(2\beta+\delta)^2}\{2(1-s)\beta\delta+2\beta^2\}\alpha \\ &- \frac{s\delta}{4\beta(2\beta+\delta)^2}\{(3-s)\beta\delta+\delta^2\}\bar{c} + \frac{s\beta\delta}{4\beta^2(2\beta+\delta)}\sum_{k\neq i}sc_k + \frac{s}{4\beta}c_j - \frac{s^2\delta}{4\beta(2\beta+\delta)}c_i.\end{aligned}$$

The second line in RHS is negative by doing a little arithmetic. The first line can be rewritten as

$$\frac{s}{4\beta(2\beta+\delta)^2}\left[4\beta\left\{\delta\sum_{k\neq i,j}s(t_k^i)^* - \alpha\beta\right\} + \delta\left\{(3+2s)\delta\sum_{k\neq i,j}s(t_k^i)^* - 4(1-s)\alpha\beta\right\}\right].$$

The first term in the brackets is negative because of Assumption 1 and 4. Under Assumption 1 and 4 the second term in brackets is

$$\delta\left\{(3+2s)\delta\sum_{k\neq i,j}s(t^i_k)^*-4(1-s)\alpha\beta\right\}<\alpha\delta\{(3+2s)\delta-4(1-s)\beta\}\ \text{(by Assumption 1)},$$

$$<\{2(3+2s)-4(1-s)\}\alpha\beta\delta\ \text{(by Assumption 4)},$$

$$<0.$$

Hence we obtain that the sign of $\partial\widehat{W}^{i,j}((\mathbf{t}^{-i})^*,(\mathbf{t}^i_{-j})^*,0,\mathbf{s},\mathbf{c})/\partial t^i_j$ relies on the sign of $sc_j/4\beta - s^2\delta c_i/4\beta(2\beta+\delta)$. Namely the sign of RHS in inequality (14) depends on the sign of

$$-(t^i_j)^*\left\{\frac{s}{4\beta}c_j-\frac{s^2\delta}{4\beta(2\beta+\delta)}c_i\right\}-(t^j_i)^*\left\{\frac{s}{4\beta}c_i-\frac{s^2\delta}{4\beta(2\beta+\delta)}c_j\right\},$$

because we get the similar result from $\partial\widehat{W}^{i,j}((\mathbf{t}^{-j})^*,(\mathbf{t}^j_{-i})^*,0,\mathbf{s},\mathbf{c})/\partial t^j_i$ too. If this is positive, RHS in (16) is positive. This condition can be written as

$$\left\{-(t^i_j)^*+\frac{s\delta}{2\beta+\delta}(t^j_i)^*\right\}c_i+\left\{-(t^j_i)^*+\frac{s\delta}{2\beta+\delta}(t^i_j)^*\right\}c_j>0. \tag{17}$$

(17) is satisfied when both two terms are positive in (17). In other words,

$$\frac{s\delta}{2\beta+\delta}<\frac{(t^i_j)^*}{(t^j_i)^*}<\frac{2\beta+\delta}{s\delta} \tag{18}$$

Proposition 2: When all the countries have the same variety share and Assumption 1, 4, and 5 are satisfied, if two countries impose the tariffs that satisfy the condition (16), then the two countries have incentives to form FTA.

Proposition 2 tells us that if two countries levies similar optimal tariff on each other, then they have incentives to form FTA. Intuitively, there are two points to explain this fact. First, FTA means that imports from the partner increase while exports to the partner increase. We find from Result 1 that the effects of eliminateing the tariff on exports to and imports from the partner is similar between FTA partners since all countries have same variety shares and FTA partners levies similar tariffs on each other. Changes in trade flows between partners are roughly offset. Thus from (7), this does not influence both countries' welfare. Second, eliminating the tariff on the partner means that prices of goods from the rest of the world become relatively lower than the one from the partner. Thus imports from the rest of the world decrease. From (7), this leads to an increase in both FTA partners' welfare.

Corollary: If two countries have the same marginal costs, then they have incentives to form FTA with each other.

Note that an FTA deteriorates welfare of the rest of the world. In fact, consider the change in welfare of country k in the rest of the world when country i and j form FTA. From (7),

$$\frac{\partial \widehat{W}^k}{\partial t^i_j} = \frac{\partial}{\partial t^i_j}\left[s_k \sum_{h\neq k}(p^h_k - c_k)q^h_k\right] = \frac{s_k s_j \delta}{2\beta+\delta} > 0.$$

Hence, FTA between country i and j results in a decrease in welfare of the rest of world. Neverthless, the countries in the rest of the world may accept the FTA if adjustment of tariffs after forming FTA between country i and j is considered. Countries in the rest of the world don't have incentives to change their tariffs since determining tariffs on country i and j don't depend on tariffs imposed by country i and j. On the other hand, country i and j have incentives to change their tariffs imposed on the rest of the world. We show that. Evaluated at $t^i_j = 0$, $t^i_k = (t^i_k)^*$ $(k \neq i, j)$, in words, if country i imposes the same tariffs (i.e., optimal tariffs levied before FTA) on the rest of the world even after FTA,

$$\begin{aligned}
\frac{\partial \widehat{W}}{\partial t^i_k} &= -\sum_{h\neq i} s_h q^i_h \frac{\partial p^i_h}{\partial t^i_k} + s_i(p^i_i - c_i)\frac{\partial q^i_i}{\partial t^i_k} + \sum_{h\neq i,j} s_h t^i_h \frac{\partial q^i_h}{\partial t^i_k} \\
&= \frac{s_k s_j \delta (t^i_j)^*}{4\beta(2\beta+\delta)^2}\left[\delta \sum_{h\neq i,j,k} s_h - 8\beta - \delta(4 - s_k - s_j) - s_i\delta\right] \\
&< 0,
\end{aligned}$$

where the second equality follows from envelope theorem and Result 1.

Thus, country i has an incentive to lower the tariffs imposed on the foreign countries other than FTA partner, country j, since country i's welfare function $\widehat{W}^i$ satisfies the second order condition. The rest of the world can enjoy an increase in exports to FTA partners through such adjustment of tariffs by country i and j. Even if country i and j sign to form FTA and the other countries suffer from a decrease in exports to FTA partners by forming FTA between country i and j itself, then the countries' welfare in the rest of the world may improve as long as FTA partners adjust their tariffs levied on the others. This is because an increase in exports to FTA partners through country i's and j's adjustment of tariffs may offset by decrease in export to them by forming FTA itself (or, by setting t^i_j and t^j_i to zero).

When we consider adjustment of tariffs after FTA just like the above, we can obtain the following important proposition about the relationship between gains from FTA and each country's cost condition.

Proposition 3: When a country has the same marginal cost as a potential partner, if they have lower marginal costs, then they gain higher benefits from FTA.
Proof: see Appendix.

That is, the higher marginal costs of FTA partners become, the lower their welfare becomes. the higher marginal costs mean that marginal costs of the countries in the rest of the world are relatively lower. Thus FTA leads to largely worsening the terms of trade of FTA partners for the other countries. According to Viner's terminology, FTA generates a big trade diversion. This diversion lowers gains from FTA. Neverthless, from Proposition 2, FTA between country i and j guarantees increase in their welfare for them as long as their marginal costs are same.

4 Concluding Remarks

To understand the relations between each country's incentive to form FTA with another country and cost condition, we utilized the model in which incompletely substituables are traded. In particular we could get a grasp of the fact that the subtitutability δ plays an important role. We have found that if δ is sufficiently low, then each country imposes a higher (lower) tariff on a lower (higher) marginal cost-country and raises tariffs on one country when another country's marginal cost lower (Proposition 1). The latter result is different from that of Kiyono (1993), which led to the adverse result by using the model where a complete substitutable is traded. We also have found that each country has an incentive to form FTA with a higher marginal cost-country (Proposition 2) and two countries with similar marginal costs are likely to form FTA with each other (Proposition 3). Propostion 2 may show that FTA with Singapore and Mexico who have relatively high cost condition gives Japan the benefit. On the other hand, Proposition 3 may show that AFTA or MERCOSUR are beneficial. Hoever we must not forget that these results depend on the assumption that partners and non-partners don't change their tariffs after FTA formation. How do our propositions change without this assumption? This issue is important. We leave this extension for future research.

Appendix

Proof of Lemma 1

Let the Hessian of country i's welfare function $\widehat{W}^i$ denote $|H|$. Then $|H|$ is

$$|H| = \begin{vmatrix} \dfrac{\partial^2 \widehat{W}^i}{\partial (t_1^i)^2} & \dfrac{\partial^2 \widehat{W}^i}{\partial t_2^i \partial t_1^i} & \cdots & \dfrac{\partial^2 \widehat{W}^i}{\partial t_n^i \partial t_1^i} \\ \dfrac{\partial^2 \widehat{W}^i}{\partial t_1^i \partial t_2^i} & \dfrac{\partial^2 \widehat{W}^i}{\partial (t_2^i)^2} & \cdots & \dfrac{\partial^2 \widehat{W}^i}{\partial t_n^i \partial t_2^i} \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{\partial^2 \widehat{W}^i}{\partial t_1^i \partial t_n^i} & \dfrac{\partial^2 \widehat{W}^i}{\partial t_2^i \partial t_n^i} & \cdots & \dfrac{\partial^2 \widehat{W}^i}{\partial (t_n^i)^2} \end{vmatrix}$$

$$= \frac{\prod_{h \neq i} s_h}{\{4\beta(2\beta+\delta)^2\}^{n-1}} \begin{vmatrix} -D_1 & s_2\delta E & s_3\delta E & s_4\delta E & \ldots & s_n\delta E \\ s_1\delta E & -D_2 & s_3\delta E & s_4\delta E & \ldots & s_n\delta E \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ s_1\delta E & s_2\delta E & s_3\delta E & s_4\delta E & \ldots & -D_n \end{vmatrix}$$

where

$$\frac{\partial^2 \widehat{W}^i}{\partial (t_j^i)^2} = -\frac{s_j}{4\beta(2\beta+\delta)^2} D_j, \quad \frac{\partial^2 \widehat{W}^i}{\partial t_k^i \partial t_j^i} = \frac{s_j s_k \delta}{2\beta(2\beta+\delta)^2} E$$

$$D_k = \{2\beta + \delta(1-s_k)\}\{2\beta(1+2s_k) + \delta(1+s_k)\} + s_k\delta^2(1-s_k) > 0$$

$$E = 8\beta + \delta(4 + s_k - \sum_{h \neq i} s_h) > 0.$$

Let the k th principal minor of the Hessian denote $|H_k|$. By using the Hessian and the principal minor $|H_k|$, we prove the concavity of country i's welfare function. We do by using the mathematical induction.

(I) $m = 1, 2$

$$|H_1| = -D_1 < 0, \quad |H_2| = D_1 D_2 - \delta^2 s_1 s_2 E^2.$$

$|H_2|$ is positive under Assumption 4. We can rewrite $|H_2|$ the function of s_j.

$$f(s_j) \equiv -2\delta(2\beta+\delta)s_j^2 + \{8\beta^2 - 4\beta\delta - 2(1+s_i)\delta^2\}s_j + (2\beta+\delta)^2 = |H_2|.$$

Obviously $f(s_j)$ is positive for any $s \in [0,1]$ if $f(1)$ is positive. Then we see the sign of $f(1)$.

$$
\begin{aligned}
f(1) &= 12\beta^2 - 4\beta\delta - (3+2s_i)\delta^2 \\
&> (12 \cdot 2 - 4)\,\beta\delta - (3+2s_i)\delta^2 \quad \text{(by Assumption 4)} \\
&= 20\beta\delta - (3+2_i)\delta^2 \\
&> \{10 \cdot 2 - (3+2s_i)\}\,\delta^2 \\
&> 0.
\end{aligned}
$$

Hence we obtain $|H_2| > 0$.

(Ⅱ) Let $|H_{k-1}|$ satisfy the concavity condition, in other words,

$$
sgn|H_{k-1}| = \begin{cases} +1 & k-1 = 2t \\ -1 & k-1 = 2t-1 \end{cases} \tag{19}
$$

We can write out $|H_k|$ as follows.

$$
\begin{aligned}
|H_k| &= \frac{\Pi_{h\neq i} s_h}{\{4\beta(2\beta+\delta)^2\}^k}
\begin{vmatrix}
-D_1 & s_1\delta E & s_3\delta E & \dots & s_k\delta E \\
s_1\delta E + D_1 & -(s_2\delta E + D_2) & 0 & \dots & 0 \\
\dots & \dots & \dots & \dots & \dots \\
s_1\delta E + D_1 & 0 & 0 & \dots & -(s_k\delta E + D_k)
\end{vmatrix} \\
&= -(s_k\delta E + D_k)|H_{k-1}| + (-1)^{k+1}\{-(s_1\delta E + D_1)\}|D_0|.
\end{aligned}
$$

$|D_0|$ represents the determinant of the cofactor corresponding to $(k,1)$ element. We focus on $|D_0|$.

$$
\begin{aligned}
|D_0| &= \begin{vmatrix}
s_2\delta E & s_3\delta E & s_4\delta E & \dots & s_{k-1}\delta E & s_k\delta E \\
-(s_2\delta E + D_2) & 0 & 0 & \dots & 0 & 0 \\
\dots & \dots & \dots & \dots & \dots & \dots \\
0 & 0 & 0 & \dots & -(s_{k-1}\delta E + D_{k-1}) & 0
\end{vmatrix} \\
&= \prod_{h=2}^{k-1}\{(s_h\delta E + D_h)\}(\delta E)^{k-1}(-1)^{k-2}
\begin{vmatrix}
s_2 & s_3 & s_4 & \dots & s_{k-1} & s_k \\
1 & 0 & 0 & \dots & 0 & 0 \\
\dots & \dots & \dots & \dots & \dots & \dots \\
0 & 0 & 0 & \dots & 1 & 0
\end{vmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&= \prod_{h=2}^{k-1} s_h\{(s_h\delta E + D_h)\}(\delta E)^{k-1}(-1)^{k-2}s_k \begin{vmatrix} 0 & 0 & \dots & 0 & 1 \\ 1 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 1 \end{vmatrix} \\
&= \prod_{h=2}^{k-1} s_k s_h\{(s_h\delta E + D_h)\}(\delta E)^{k-1}(-1)^{2(k-1)} \\
&> 0.
\end{aligned}
$$

Hence we obtain $|D_0| > 0$. Then we see the sign of $|H_k|$ by considering both $k = 2t$ and $k = 2t - 1$.

(i) $k = 2t$

In this case $|H_{k-1}|$ is negative. Hence

$$
\begin{aligned}
|H_k| &= -(s_k\delta E + D_k)|H_{k-1}| + (-1)^{k+1}\{-(s_1\delta E + D_1)\}|D_0| \\
&= -(s_k\delta E + D_k)|H_{k-1}| + (-1)^{2(t+1)}(s_1\delta E + D_1)|D_0| \\
&> 0.
\end{aligned}
$$

(ii)$k = 2t - 1$

In this case $|H_{k-1}|$ is positive. Hence

$$
\begin{aligned}
|H_k| &= -(s_k\delta E + D_k)|H_{k-1}| + (-1)^{k+1}\{-(s_1\delta E + D_1)\}|D_0| \\
&= -(s_k\delta E + D_k)|H_{k-1}| + (-1)^{2t+1}(s_1\delta E + D_1)|D_0| \\
&< 0.
\end{aligned}
$$

From (i) , (ii), $sgn|H_{k-1}| = -sgn|H_k|$. Thus we found that $\widehat{W}^i$ is concave in $\mathbf{t}^i$. □

Proof of Lemma 2

First of all we redefine the welfare function as follows from the fact that $p^i_j = \beta q^i_j + c_j$.

$$
\widehat{W}^i(\mathbf{t}, \mathbf{c}, \mathbf{s}) = \widetilde{W}^i(\mathbf{q}(\mathbf{t}, \mathbf{c}, \mathbf{s}), \mathbf{c}, \mathbf{s}) = W^i(\beta\mathbf{q}(\mathbf{t}, \mathbf{c}, \mathbf{s}), \mathbf{q}(\mathbf{t}, \mathbf{c}, \mathbf{s}), \mathbf{c}, \mathbf{s}).
$$

Then the equations for deriving the optimal tariffs are

$$\frac{\partial \widehat{W}^i}{\partial t^i_j} = \sum_{k=1}^{n} \frac{\partial \widetilde{W}^i}{\partial q^i_k}\frac{\partial q^i_k}{\partial t^i_j} = 0 \qquad \forall j = 1, \ldots, n, \;\; j \neq i.$$

We can rewrite these equations in the form of matrix:

$$\begin{pmatrix} \frac{\partial \widetilde{W}^i}{\partial q^i_i}\frac{\partial q^i_i}{\partial t^i_1} \\ \vdots \\ \frac{\partial \widetilde{W}^i}{\partial q^i_i}\frac{\partial q^i_i}{\partial t^i_n} \end{pmatrix} = \begin{pmatrix} -\frac{\partial q^i_1}{\partial t^i_1} & -\frac{\partial q^i_2}{\partial t^i_1} & \cdots & -\frac{\partial q^i_n}{\partial t^i_1} \\ \cdots & \cdots & \cdots & \cdots \\ -\frac{\partial q^i_1}{\partial t^i_n} & -\frac{\partial q^i_2}{\partial t^i_n} & \cdots & -\frac{\partial q^i_n}{\partial t^i_n} \end{pmatrix} \begin{pmatrix} \frac{\partial \widetilde{W}^i}{\partial q^i_1} \\ \vdots \\ \frac{\partial \widetilde{W}^i}{\partial q^i_n} \end{pmatrix}. \tag{20}$$

To prove the optimal tariff is positive, we show that the coefficient matrix of the equations satisfies Hawkins-Simon condition with the mathematical induction. We define the determinant of the coefficient matrix as Δ and the determinant of the m th principal minor as Δ_m.

(I) $m = 1$

$$\Delta_m = -\frac{\partial q^i_1}{\partial t^i_1} = \frac{2\beta + \delta(1 - s_1)}{2\beta(2\beta + \delta)} > 0.$$

(II) We assume that Δ_m is positive when $m = k$. Then,

$$\begin{aligned}
\Delta_{m+1} &= \begin{vmatrix} -\frac{\partial q^i_1}{\partial t^i_1} & \cdots & -\frac{\partial q^i_{m+2}}{\partial t^i_1} \\ \cdots & \cdots & \cdots \\ -\frac{\partial q^i_{m+2}}{\partial t^i_{m+2}} & \cdots & -\frac{\partial q^i_{m+2}}{\partial t^i_{m+2}} \end{vmatrix} \\
&= \begin{vmatrix} \frac{2\beta + \delta(1 - s_1)}{2\beta(2\beta + \delta)} & -\frac{\delta s_1}{2\beta(2\beta + \delta)} & \cdots & -\frac{\delta s_1}{2\beta(2\beta + \delta)} \\ \cdots & \cdots & \cdots & \cdots \\ -\frac{\delta s_{m+2}}{2\beta(2\beta + \delta)} & -\frac{\delta s_{m+2}}{2\beta(2\beta + \delta)} & \cdots & \frac{2\beta + \delta(1 - s_{m+2})}{2\beta(2\beta + \delta)} \end{vmatrix} \\
&= (-1)^{m+1}\left[\frac{\delta}{2\beta(2\beta + \delta)}\right]^{m+1} \prod_{h \neq i} s_h \begin{vmatrix} -\frac{2\beta + \delta(1 - s_1)}{\delta s_1} & 1 & \cdots & 1 \\ \cdots & \cdots & \cdots & \cdots \\ 1 & 1 & \cdots & \frac{2\beta + \delta(1 - s_{m+2})}{\delta s_{m+2}} \end{vmatrix} \\
&= (-1)^{m+1}\left[\frac{\delta}{2\beta(2\beta + \delta)}\right]^{m+1} \frac{2\beta + \delta}{\delta s_1} \prod_{h \neq i} s_h \begin{vmatrix} -\frac{2\beta + \delta(1 - s_1)}{2\beta + \delta} & 1 & 1 & \cdots & 1 & 1 \\ 1 & -\frac{2\beta + \delta}{\delta s_2} & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & \cdots & 0 & -\frac{2\beta + \delta}{\delta s_{m+2}} \end{vmatrix}.
\end{aligned}$$

Let the determinant of RHS in the last equation be $\tilde{\Delta}_{m+1}$. Then $\tilde{\Delta}_{m+1}$ is

$$\tilde{\Delta}_{m+1} = -\frac{2\beta+\delta}{\delta s_{m+2}}\bar{\Delta}_m + (-1)^{m+2}\bar{\bar{\Delta}}_m$$

where

$$\bar{\Delta}_m = \left[(-1)^m \left\{\frac{\delta}{2\beta(2\beta+\delta)}\right\}^m \frac{2\beta+\delta}{\delta s_1}\prod_{h\neq i} s_h\right]^{-1} \Delta_m,\ \bar{\bar{\Delta}}_m = \begin{vmatrix} 1 & 1 & \dots & 1 & 1 \\ -\dfrac{2\beta+\delta}{\delta s_2} & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & -\dfrac{2\beta+\delta}{\delta s_{m+1}} & 0 \end{vmatrix}.$$

We focus on $\bar{\bar{\Delta}}_m$. We can rewrite this as follows.

$$\bar{\bar{\Delta}}_m = (-1)^{m-1}\left[\frac{2\beta+\delta}{\delta}\right]^{m-1}\prod_{h\neq 1,i} s_h^{-1} \begin{vmatrix} 1 & 1 & \dots & 1 & 1 \\ 1 & 0 & \dots & 1 & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 0 \end{vmatrix} = (-1)^{2m-3}\left[\frac{2\beta+\delta}{\delta}\right]^{m-1}\prod_{h\neq 1,i} s_h^{-1}.$$

By taking account on $\bar{\Delta}_m$ and $\bar{\bar{\Delta}}_m$, we see how the sign of Δ_{m+1} and of $\tilde{\Delta}_{m+1}$ are related. In fact the relation between the two is $sgn(\Delta_{m+1}) = sgn\{(-1)^{m+1}\tilde{\Delta}_{m+1}\}$. So, we show that Δ_{m+1} is positive as Δ_m is so by showing $(-1)^{m+1}\tilde{\Delta}_{m+1}$ is positive. $(-1)^{m+1}\tilde{\Delta}_{m+1}$ satisfies

$$\begin{aligned}
(-1)^{m+1}\tilde{\Delta}_{m+1} &= (-1)^{m+1}\left[-\frac{2\beta+\delta}{\delta s_{m+2}}\bar{\Delta}_m + (-1)^{3m-1}\left\{\frac{2\beta+\delta}{\delta}\right\}^{m-1}\prod_{h\neq 1,i} s_h^{-1}\right] \\
&= -(-1)^{m+1}\frac{2\beta+\delta}{\delta s_{m+2}}\left\{(-1)^m\left(\frac{\delta}{2\beta(2\beta+\delta)}\right)^m \frac{2\beta+\delta}{\delta s_1}\prod_{h\neq 1,i} s_h\right\}^{-1}\Delta_m \\
&\quad + (-1)^{m+1}(-1)^{3m-1}\left\{\frac{2\beta+\delta}{\delta}\right\}^{m-1}\prod_{h\neq 1,i} s_h^{-1} \\
&= (-1)^{m+1+1-m}\frac{2\beta+\delta}{\delta s_{m+2}}\left\{\left(\frac{\delta}{2\beta(2\beta+\delta)}\right)^m \frac{2\beta+\delta}{\delta s_1}\prod_{h\neq 1,i} s_h\right\}^{-1}\Delta_m \\
&\quad + (-1)^{m+1+3m-1}\left\{\frac{2\beta+\delta}{\delta}\right\}^{m-1}\prod_{h\neq 1,i} s_h^{-1} \\
&= (-1)^2\frac{2\beta+\delta}{\delta s_{m+2}}\left\{\left(\frac{\delta}{2\beta(2\beta+\delta)}\right)^m \frac{2\beta+\delta}{\delta s_1}\prod_{h\neq 1,i} s_h\right\}^{-1}\Delta_m + (-1)^{4m}\left\{\frac{2\beta+\delta}{\delta}\right\}^{m-1}\prod_{h\neq 1,i} s_h^{-1} \\
&> 0.
\end{aligned}$$

Hence we obtain that any principal minor of the coefficient is positive from (I), (II), so that the simultaneous equations (8) satisfy Hawkins-Simon condition. Since the simultaneous equations have non-negative solutions when the equations satisfy Hawkins-Simon condition, we see the simultaneous equations (8) have non-negative solution $\partial\widetilde{W}^i/\partial q_j^i$. Notice that

$$\frac{\partial\widetilde{W}^i}{\partial q_j^i} = s_j(t_j^i - \beta q_j^i) > 0.$$

Therefore we obtain the optimal tariffs $(t_j^i)^*$ is positive. $\square$

Proof of Proposition 3

Consider FTA between country i and j. Further we assume that they have same marginal costs (i.e., $c_i = c_j$). For convenience, we define the following welfare function.

$$\overline{W}^i(\mathbf{c}, \mathbf{s}) = \widehat{W}^i(\mathbf{t}^{**}(\mathbf{c}, \mathbf{s}), \mathbf{c}, \mathbf{s}),$$

where $\mathbf{t}^{**}$ is an optimal tariff vector after FTA and depends on a marginal cost vector $\mathbf{c}$ and a variety share vector $\mathbf{s}$. Then $\partial\overline{W}^i/\partial c_i$ is

$$\begin{aligned}
\frac{\partial\overline{W}^i}{\partial c_i} &= \sum_{k\neq i,j} \frac{\partial\widehat{W}^i}{\partial t_k^i}\frac{\partial(t_k^i)^{**}}{\partial c_i} + \sum_{k\neq i,j} \frac{\partial\widehat{W}^i}{\partial t_i^k}\frac{\partial(t_i^k)^{**}}{\partial c_i} + \frac{\partial\widehat{W}^i}{\partial c_i} \\
&= \sum_{k\neq i,j} \frac{\partial\widehat{W}^i}{\partial t_i^k}\frac{\partial(t_i^k)^{**}}{\partial c_i} + \frac{\partial\widehat{W}^i}{\partial c_i}.
\end{aligned}$$

The second line follows from envelope theorem. Similarly,

$$\frac{\partial\overline{W}^i}{\partial c_j} = \sum_{k\neq i,j} \frac{\partial\widehat{W}^i}{\partial t_i^k}\frac{\partial(t_i^k)^{**}}{\partial c_j} + \frac{\partial\widehat{W}^i}{\partial c_j}.$$

Thus we show $\partial\overline{W}^i/\partial c_i + \partial\overline{W}^i/\partial c_j < 0$ in order to prove Proposition 3. From Result, we know that

$$\frac{\partial\widehat{W}^i}{\partial t_i^k} = s_i\left[(p_i^k - c_i)\frac{\partial q_i^k}{\partial t_i^k} + q_i^k\frac{\partial p_i^k}{\partial t_i^k}\right] = 2s_i\beta q_i^k\frac{\partial q_i^k}{\partial t_i^k} < 0.$$

By Proposition 1,

$$\sum_{k\neq i,j} \frac{\partial\widehat{W}^i}{\partial t_i^k}\frac{\partial(t_i^k)^{**}}{\partial c_j} < 0.$$

Hence, in order to prove $\partial \overline{W}^i/\partial c_i + \partial \overline{W}^i/\partial c_j < 0$, it's sufficient to show that

$$B = \sum_{k \neq i,j} \frac{\partial \widehat{W}^i}{\partial t_i^k} \frac{\partial (t_i^k)^{**}}{\partial c_i} + \frac{\partial \widehat{W}^i}{\partial c_i} + \frac{\partial \widehat{W}^i}{\partial c_j} < 0.$$

Simple calculations give us

$$\frac{\partial \widehat{W}^i}{\partial c_i} = s_i \sum_{k=1}^{n} (p_i^k - c_i) \frac{\partial q_i^k}{\partial c_i} - \sum_{k \neq i} s_k q_k^i \frac{\partial p_k^i}{\partial c_i} + \sum_{k \neq i,j} s_k (t_k^i)^{**} \frac{\partial q_k^i}{\partial c_i} - \sum_{k=1}^{n} s_k q_i^k + s_i \sum_{k \neq i} q_i^k \frac{\partial p_i^k}{\partial c_i}$$

and

$$\begin{aligned}
\frac{\partial \widehat{W}^i}{\partial c_j} &= \left[s_i (p_i^i - c_i) \frac{\partial q_i^i}{\partial c_j} - \sum_{k \neq i} s_k q_k^i \frac{\partial p_k^i}{\partial c_j} + \sum_{k \neq i,j} s_k (t_k^i)^{**} \frac{\partial q_k^i}{\partial c_j} \right] + s_i \sum_{k \neq i} (p_i^k - c_i) \frac{\partial q_i^k}{\partial c_j} + s_i \sum_{k \neq i} q_i^k \frac{\partial p_i^k}{\partial c_j} \\
&= s_i \sum_{k \neq i} (p_i^k - c_i) \frac{\partial q_i^k}{\partial c_j} + s_i \sum_{k \neq i} q_i^k \frac{\partial p_i^k}{\partial c_j} - s_j q_j^i.
\end{aligned}$$

The second equality of $\partial \widehat{W}^i/\partial c_j$ follows from Result ($\partial q_k^i/\partial c_j = \partial q_k^i/\partial t_j^i$, $\partial p_k^i/\partial c_j = \partial p_k^i/\partial t_j^i$, and $\partial p_j^i/\partial c_j = \partial p_j^i/\partial t_j^i + 1$) and envelope theorem. Then $\partial \widehat{W}^i/\partial c_i + \partial \widehat{W}^i/\partial c_j$ is

$$\begin{aligned}
\frac{\partial \widehat{W}^i}{\partial c_i} + \frac{\partial \widehat{W}^i}{\partial c_j} = &\left[s_i (p_i^i - c_i) \frac{\partial q_i^i}{\partial c_i} - \sum_{k \neq i}^{n} s_k q_k^i \frac{\partial p_k^i}{\partial c_i} + \sum_{k \neq i,j} s_k (t_k^i)^{**} \frac{\partial q_k^i}{\partial c_i} \right] \\
&+ s_i \sum_{k \neq i} (p_i^k - c_i) \left(\frac{\partial q_i^k}{\partial c_j} + \frac{\partial q_i^k}{\partial c_i} \right) + s_i \sum_{k \neq i} q_i^k \left(\frac{\partial p_i^k}{\partial c_j} + \frac{\partial p_i^k}{\partial c_j} \right) - s_j q_j^i \\
&< \sum_{k \neq i,j} s_k (t_k^i)^{**} \frac{\partial q_k^i}{\partial c_i} - s_i \sum_{k=1}^{n} q_i^k.
\end{aligned}$$

Then, we show that B is negative.

$$\begin{aligned}
B &< -\sum_{k \neq i} \frac{\partial \widehat{W}^i}{\partial t_i^k} + \frac{\partial \widehat{W}^i}{\partial c_i} + \frac{\partial \widehat{W}^i}{\partial c_j} \\
&= -2 s_i \beta \sum_{k \neq i} q_i^k \frac{\partial q_i^k}{\partial t_i^k} + \frac{\partial \widehat{W}^i}{\partial c_i} + \frac{\partial \widehat{W}^i}{\partial c_j} \\
&< -2 s_i \beta \sum_{k \neq i} q_i^k \frac{\partial q_i^k}{\partial t_i^k} + \sum_{k \neq i,j} s_k (t_k^i)^{**} \frac{\partial q_k^i}{\partial c_i} - s_i \sum_{k=1}^{n} q_i^k.
\end{aligned}$$

The first inequality follows from the fact that $|\partial (t_i^k)^*/\partial c_i| < 1$ as we know from Proposition 1. When all the countries in the world have same variety shares, we can rewrite the last inequality.

$$
\begin{aligned}
&-2s_i\beta\sum_{k\neq i} q_i^k \frac{\partial q_i^k}{\partial t_i^k} + \sum_{k\neq i,j} s_k(t_k^i)^{**}\frac{\partial q_k^i}{\partial c_i} - s_i\sum_{k=1}^{n} q_i^k \\
&\quad = \frac{\{2\beta+\delta(1-s)\}s}{2\beta+\delta}\sum_{k\neq i} q_i^k + \frac{\delta s}{2\beta(2\beta+\delta)}\bar{t}^i - s\sum_{k=1}^{n}\left[\frac{\alpha}{2\beta+\delta} - \frac{t_i^k+c_i}{2\beta} + \frac{\delta(\bar{t}^k+\bar{c})}{2\beta(2\beta+\delta)}\right] \\
&\quad = s\left[\frac{2\beta+\delta(1-s)}{2\beta+\delta}\sum_{k\neq i} q_i^k - \sum_{k\neq i} q_i^k\right] - s\left[\frac{\alpha}{2\beta+\delta} - \frac{c_i}{2\beta} + \frac{\delta\bar{c}}{2\beta(2\beta+\delta)}\right] \\
&\quad = -\frac{\delta s^2}{2\beta+\delta}\sum_{k\neq i} q_i^k - s q_i^i(\mathbf{0},\mathbf{c},\mathbf{s}) \\
&\quad < 0,
\end{aligned}
$$

where $\mathbf{0} = (0, 0, \cdots, 0)$. Hence, B is negative. The proposition was proved. $\square$

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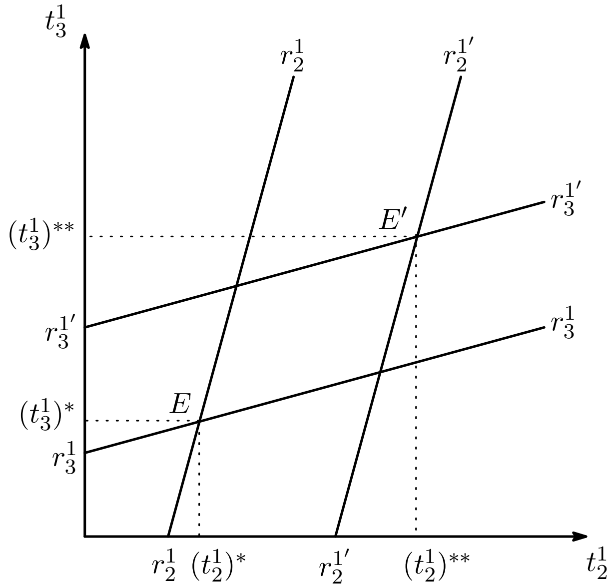

Figure 1: the case of decrease in c_1

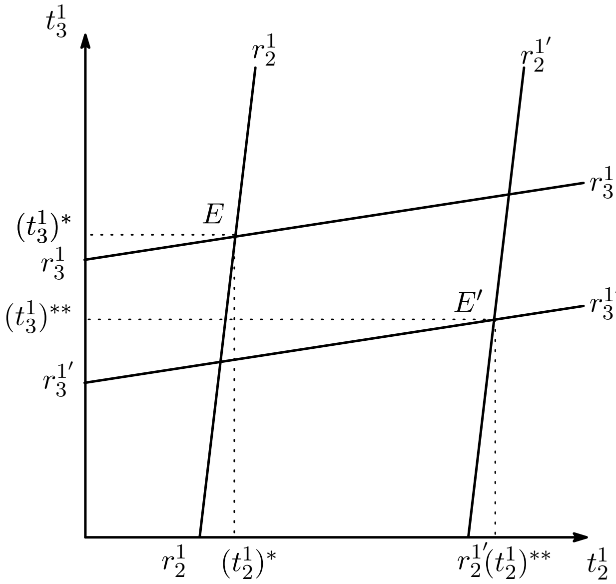

Figure 2: the case of decrease in c_2