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A Theory of Civil Conflict and Democracy in Unequal Societies

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Abstract

This paper examines the endogenous choice between democracy and conflict in a scenario with different social classes in terms of income inequality and with parties representing each of the two social classes. We consider how the change in economic inequality between the poor and rich people affects the sustainability of democracy against conflict and how it impacts the equilibrium levels of tax rate and public expenditure under democracy. We show that the increase in economic inequality destabilizes of democracy since the poor hardly has the incentive to sustain the democracy; Further the increase is positively associated with the equilibrium levels of both the tax rate and public expenditure. Therefore, we successfully provide theoretic justification for the fact that sufficiently large economic inequality decreases the possibility of a self-enforcing democracy.

JEL Classification Numbers: H11; D72; D74
Keywords: self-enforcing democracy; civil conflict; economic inequality

1 Introduction

This paper presents a theoretical analysis of the endogenous choice between democracy and civil conflict by considering political parties as the representatives of different social groups, which are classified by income inequality. Several empirical studies, for example, Bulte and Damania (2008) and Ross (2004) show that resource-abundant countries are less democratic than resource-scarce countries. Many papers challenge the phenomenon of so-called *political* "resource-curse." In particular, until recently works using game-theoretic modeling could be divided

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into two main groups. One group adopts the model in which resource rents affect the intensity and duration of civil conflict. Mehlum and Moene (2002) and Skaperdas (2002) showed that social welfare reduces as natural resources are wasted on unproductive arming and fighting. Torvik (2002) developed a simple mechanism to explain why natural resource abundance may lower income and welfare using the rent-seeking game-theoretic model. The other group, which has emerged relatively recently, suggests the model where voters are explicitly considered and studies the impact of resource abundance on the political equilibrium. Robinson and Torvik (2005) explicitly modeled politicians to explain the mis-allocation of investment. Acemoglu and Robinson (2006) constructed a simple model in which political elites may block technological and institutional development. Moreover, Robinson et al. (2006) studied the political incentives generated by resource rents and resource booms. The first group comprises of the works that try to explain the relation between civil conflicts and resource rents, whereas the second group investigates the cause of the resource-curse in a more political context. Recent literature on resource-curse provides an integrated analysis in the above two groups. The pioneering work of Aslaksen and Torvik (2006) analyzed the model in which the form of political competition–more precisely, electoral competition or civil conflict–is endogenous under the super-game theoretic framework. In this model, they obtained the result that in the (self-enforcing) equilibrium, the likelihood of democracy is inversely proportional to the size of the resource rents relative to national income.

The purpose of our work is to explore the endogenous choice between democracy and civil conflict taking into consideration the existence of different social classes in society. We add the following two considerable points to the model presented by Aslaksen and Torvik (2006): First; the constituent members in the society are different with respect to economic level; they are classified into three classes, the *poor*, *middle*, and *wealthy*. Second, both the political parties are regarded as representatives of the poor and wealthy, respectively. When we take into account of the difference between each individual's economic standard, the attitude toward the optimal size of the government under democracy should differ across classes. Furthermore, it is obvious that under civil conflict, the opportunity cost of the effort to civil conflict is higher for the *wealthy* than for the *poor*. Thus, we can state that political preferences differ with respect to the economic level, Later, we examine how the change in economic inequality between the *poor* and *wealthy* affects the sustainability of democracy in the context of civil conflict and how it influences the equilibrium levels of tax rate and public expenditure under a self-enforcing democracy. In this paper, we obtain the following three results. First, the increase in economic inequality results in the instability of the democratic state since the party, as the representative of the *poor*, hardly has the incentive to sustain the democracy. Second, in a society where economic inequality is relatively large, the equilibrium tax rate might increase since the party as the representative of the *wealthy* may choose a political platform favorable to the *poor*. This result is closely related to that of Przeworski (2005) who also showed that the equilibrium platform becomes hopeful for the *poor* in the context of resource-curse. Akin to Przeworski, we as well as he present the asymmetric equilibrium different from the standard median voting equilibrium. In addition to the above two results, we get our third result, which is similar to the second result, that the party as the representative of the *wealthy* chooses a platform that is relatively favorable to the *poor* with the increase in natural resources, resulting in an increase in the public expenditure becomes larger. At the end of our analysis, we confirm that all the results in this paper hold against an extension to an infinitely repeated game model. Our results theoretically support the observations in the real world economy and the conclusion of several empirical works. We believe that our formulation and findings have sufficient importance to investigate the resource-curse.

The remainder of the paper is organized as follows. In Section 2, we construct the basic setting. In Section 3, we examine the endogenous choice between democracy and civil conflict and derive the conditions for the different political outcomes. Furthermore, we extend the model to the one which is formulated under an indefinitely repeated-game framework. Section 4 concludes with some remarks. The proofs of all the propositions are presented in the Appendix.

2 The Model

We formulate the model based on the works of Przeworski (2005) and Aslaksen and Torvik (2006). Assume that a society consists of three types of income earners: the poor, middle, and wealthy. These are indexed by $i \in \{p,m,w\}$. Further, their proportions in the society are such that $\pi_i \in (0,0.5)$ for all i. These types may be interpreted on the basis of their ethnic, regional, or religious affiliations. A multiple of the average income is α_i for all $i \in \{p,m,w\}$; thus, $\alpha_p < \alpha_m < \alpha_w$. The average income is normalized to 1. Two political parties vie for political power in the society; the left party represents group p and the right party represents group w. We also index the parties by the groups that they represent. The objectives of the parties are to maximize the expected value of $v_i \equiv u_i - \beta_i$ for all i, where (i) β_i is the gross cost of conflict when a conflict occurs at t and (ii) u_i is the benefit for each person of group i evaluated by party i. We assume that u_i is determined by the amount of private consumption c_i, government spending for public goods g, and rent of the group r_i. The benefit for each person in group $i \in \{w,m,p\}$, u_i, is given by $u_i = h(g) + c_i + \gamma_i r_i$, where h denotes the benefit from public goods and γ_i is the multiplier of the rents relative to private consumption. We specify function h as follows:

$$h(g) \equiv k \log\left(\frac{g+\delta}{\delta}\right). \tag{1}$$

Thus, function h satisfies the following normal conditions: $h(0) = 0$, $h' > 0$ and $h'' < 0$. We assume that (i) $h'(0) > 1$ and (ii) $\gamma_i > h'(0)$ for each $i \in \{p,w\}$. Assumption (i) implies $k > \delta$, i.e. the public goods is at minimum efficient relative to private consumption for the society while assumption (ii) implies that the rents are more attractive than public goods. Moreover, we assume that the income inequality between groups p and w is sufficiently large with respect to the ratio of the multipliers of the rents: $\alpha_w/\alpha_p > (\gamma_w/\gamma_p)^2$. The timing of events in the game is given as follows:

1. Each of the two parties announce a political platform (a tax rate).
2. An election is held. Each party decides whether to accept the electoral result of to initiate conflict.
3. If (at least) one of the parties chooses to initiate a conflict, a conflict is initiated. The winner of the conflict then decides the new policy and executes it. If no party initiates a conflict, the political platform announced by the winner of the election is implemented.

The tax rate on income in each political platform is given by $\tau \in [0, \overline{\tau}]$, where $\overline{\tau}$ is the socially limited maximum tax rate. When a conflict is not initiated and the elected party i proposed τ_i as its platform, the party i must execute the tax rate and use all its finance for public goods: $g = \tau_i + R$, where R is the amount of natural (or non-tax) resources relative to the gross income of the society. At this point, for the members of each group $i = \{p, m, w\}$, private consumption is $(1-\tau)\alpha_i$ and rents are zero. Thus, when the implemented tax rate is τ, we describe the benefit for group i, $u_i^d(\tau)$ as follows:

$$u_i^d(\tau) = h(\tau + R) + (1-\tau)\alpha_i = k \log\left(\frac{\tau + R + \delta}{\delta}\right) + (1-\tau)\alpha_i. \qquad (2)$$

We assume that the voters are sincere. Thus, when the platforms of both the parties are represented by τ_p and τ_w, respectively, a voter in group $i \in \{p, m, w\}$ supports party p only if $u_i^d(\tau_p) \geq u_i^d(\tau_w)$. Since $\pi_i < 0.5$ for all i, party p wins in the election with probability 1 if $u_m^d(\tau_p) > u_m^d(\tau_w)$, probability $\rho_p \in (0,1)$ if $u_m^d(\tau_p) = u_m^d(\tau_w)$, and probability 0 if $u_m^d(\tau_p) < u_m^d(\tau_w)$. The probability at which party w wins is given in the same way.

Let τ^i be the optimal tax rate for group i under democracy: $\tau^i \equiv \arg\max u_i(\tau)$. We assume that the optimal tax rates of group p and m have an interior solution; this implies that

$$\tau^i = \frac{k}{\alpha_i} - \delta - R > 0 \quad \forall i \in \{p, m\} \qquad (3)$$

and $\overline{\tau} > \tau^p$. Thus, the assumption regarding efficiency of public goods ensures that $\tau^p > \tau^m > \tau^w$.

In the case of conflict, the probability P_p at which party p wins the conflict depends on the fighting efforts of the two parties, e_p and e_w. The military contest success function follows the standard specification of Tullock (1975). That is, we specify P_p as follows:

$$P_p(e_p, e_w) \equiv \frac{e_p}{e_p + e_w}. \qquad (4)$$

The probability at which party w wins, P_w, is given as $P_w = 1 - P_p$. For each party $i \in \{p, w\}$, the marginal cost per unit of fighting efforts is equal to the income of the group, α_i. Thus, the gross cost β_i of conflict for party i with effort e_i is denoted by $\alpha_i e_i$.

When party i is the winner of the conflict, party i provides and executes a new policy. Since $\gamma_i > g'(0)$, party i chooses the maximum tax rate ($\tau = \overline{\tau}$) and spends all finance on the rents of group i after the conflict. Thus, $g = 0$ and the private consumption of group i is $(1 - \overline{\tau})\alpha_i$. The rents of group i are $R + \overline{\tau}$. For each group $j \neq i$, the consumption is $(1 - \overline{\tau})\alpha_j$ and the rents are 0. Now, we describe u^c_{ii} as the benefit of group i when party i is the winner of the conflict:

$$u^c_{ii} = \gamma_i\,(\overline{\tau} + R) + \alpha_i(1 - \overline{\tau}). \tag{5}$$

Similarly, the benefit of group j when party i wins the conflict, u^c_{ij}, is denoted as follows:

$$u^c_{ij} = \alpha_j(1 - \overline{\tau}). \tag{6}$$

3 Analysis

First, we consider the strategy of each party when a conflict is initiated. Since the winning party chooses a policy that maximizes its benefit, before the conflict is initiated, for each $i \in \{p, w\}$, the expected benefit for group i with effort e_p and e_w in the period is given by

$$\frac{e_i}{e_i + e_j} u^c_{ii} + \frac{e_j}{e_i + e_j} u^c_{ji} - \alpha_i e_i, \tag{7}$$

where $j \in \{p, w\}$ and $j \neq i$. Since each group $i \in \{p, w\}$ simultaneously decides its fighting effort e^*_i to maximize the expected benefit given by the opponent group's effort, the effort in the equilibrium is given by

$$e^*_i = \frac{\frac{\alpha_j}{\gamma_j}}{(\frac{\alpha_i}{\gamma_i} + \frac{\alpha_j}{\gamma_j})^2} \frac{1}{(\overline{\tau} + R)}, \tag{8}$$

where $i, j \in \{p, w\}$ and $j \neq i$. By substituting this result into equation (7), the expected benefit $\overline{u^c_i}$ for each group $i \in \{p, w\}$ per a period in a conflict is given by

$$\overline{u^c_i} = \overline{P_i^2}\,(\overline{\tau} + R) + \alpha_i(1 - \overline{\tau}), \tag{9}$$

where $\overline{P_i} \equiv \frac{\alpha_j}{\gamma_j} / \left(\frac{\alpha_i}{\gamma_i} + \frac{\alpha_j}{\gamma_j} \right)$. If the expected benefit of continuing democracy is less than $\overline{u^c_i}$, then party i chooses to initiate a conflict.

Now, we consider the situation wherein a conflict is not initiated. When a conflict is not initiated, each group announces a tax rate as its policy platform. After the election, the tax rate directed by the winner is implemented.

The following fact is satisfied.

Fact 1: For each $\tau \in [0, \overline{\tau}]$, $(u_r^d(\tau) - \overline{u_r^c}) > (u_p^d(\tau) - \overline{u_p^c})$.

Proof. From easy calculations,

$$(u_r^d(\tau) - \overline{u_r^c}) - (u_p^d(\tau) - \overline{u_p^c}) = (\alpha_w - \alpha_p)\left[(\overline{\tau} - \tau) + R\right] + (R + \overline{\tau})\left(\gamma_p \overline{P_p^2} - \gamma_w \overline{P_w^2}\right) \tag{10}$$

is positive since we assume that $\frac{\alpha_w}{\gamma_w^2} > \frac{\alpha_p}{\gamma_p^2}$. □

Now, we obtain the following proposition.

Proposition 1. *The property of the self-enforcing democracy and the equilibrium policy in democracy is as follows:*

1. *If $\overline{u_p^c} > u_p^d(\tau^p)$, then democracy is not self-enforcing.*
2. *If $u_p^d(\tau^p) \geq \overline{u_p^c} \geq u_p^d(\tau^m)$, then there exists a (unique) tax rate $\tau^* \in [\tau^m, \tau^p]$ such that $u_p^d(\tau^*) - \overline{u_p^c} = 0$ and $u_p^d(\tau) - \overline{u_p^c} < 0$ for each $\tau < \tau^*$, and each group announces tax rate τ^* as its platform. Thus, the tax rate τ^* is implemented for each period.*
3. *If $u_p^d(\tau^m) > \overline{u_p^c}$, then each group announces tax rate τ^m as its platform. Thus, tax rate τ^m is implemented for each period.*

The proof is provided in the Appendix. $u_p^d(\tau^p) < \overline{u_p^c}$ implies that party p has no incentive to support the election even if the result of the election is the most preferable policy for its group. Thus, democracy is not self-enforcing. On the other hand, $u_p^d(\tau^m) > \overline{u_p^c}$ implies that party p prefers to accept the most hopeful policy for the median voter (in group m) than to initiate conflict. Thus, the result of the election competition follows in accordance with the median voter theorem.

The reason that τ^* becomes an equilibrium tax rate when $u_p^d(\tau^p) \geq \overline{u_p^c} > u_p^d(\tau^m)$ is as follows. By Fact 1 and the definition of τ^*, group w strictly prefers tax rate τ^* over conflict. However, for group p, democracy and conflict are indifferent at tax rate τ^*. Thus, party p has no incentive to support the result of the election if party w proposes a tax rate less than τ^*. Therefore, the threat of initiating conflict by party p is credible and party w makes a concession to party p in the election.

The second point in Proposition 1 is interesting in the manner that the equilibrium policy is sensitive to the incentive of group p for sustaining democracy in that case; in the median voter rule, when the preferences of voters is different from that of the median voter, the result of the election are not influenced. This has a significant impact on the following propositions.

Next, we consider the effect of natural resources on democracy and equilibrium policy. By Proposition 1, self-enforcing democracy is possible if and only if

$u_p^d(\tau^p) \geq \bar{u}_p^c$; alternatively,

$$u_p^d(\tau^p) - \bar{u}_p^c = \left(k\log(\frac{k}{\alpha_p\delta}) + \alpha_p - k + \alpha_p\delta + \alpha_p R\right) - \left(\gamma_p \bar{P}_p^2(\bar{\tau}+R) + \alpha_p(1-\bar{\tau})\right)$$
$$= \left(k\log(\frac{k}{\alpha_p\delta}) + (-k+\alpha_p\delta) + (\alpha_p - \gamma\bar{P}_p^2)\bar{\tau}\right) + \left(\alpha_p - \gamma\bar{P}_p^2\right)R, \tag{11}$$

is nonnegative. Note that this is a linear expression of R. There exist two cases with respect to the coefficient of R, $\alpha_p - \gamma\bar{P}_p^2$: negative or nonnegative.

Proposition 2. *The relation between natural resources and democracy is as follows:*

1. *If $\alpha_p - \gamma\bar{P}_p^2 \geq 0$, then democracy is possible regardless of the amount of natural resource.*
2. *If $\alpha_p - \gamma\bar{P}_p^2 < 0$, then democracy is possible when*

$$R \leq \frac{k\log(\frac{k}{\alpha_p\delta}) + (-k+\alpha_p\delta) + (\alpha_p - \gamma\bar{P}_p^2)\bar{\tau}}{\gamma\bar{P}_p^2 - \alpha_p}. \tag{12}$$

Proposition 2 implies a kind of resource curse; the increase of natural resources relative to the average income has a positive effect on civil conflict.

Next, we consider the equilibrium policy when the increase in natural resources reduces the likelihood of a democracy.

Proposition 3. *When $\alpha_p - \gamma\bar{P}_p^2 < 0$, government expenditure increases with the increase in natural resources.*

The increase in natural resources in itself has no effect on the optimal supply of public goods for each group. However, since the increase of natural resources weakens the incentive of group p to support democracy, group w is required to make additional concessions to group p. Thus, there is an increase in government spending.

Finally, we consider the relation between the economic inequality of two groups, democracy, and the equilibrium policy. For any $\tau \in [0,\bar{\tau}]$, differentiating $u_p(\tau) - \bar{u}_p$ with respect to α_p and α_w, we obtain the following:

$$\frac{\partial}{\partial\alpha_w}\left(u_p^d(\tau) - \bar{u}_p^c\right) = -\gamma(\bar{\tau}+R)\frac{\partial\bar{P}_p^2}{\partial\alpha_w} < 0, \tag{13}$$

$$\frac{\partial}{\partial\alpha_p}\left(u_p^d(\tau) - \bar{u}_p^c\right) = (\bar{\tau}-\tau)\alpha_p - \gamma(\bar{\tau}+R)\frac{\partial\bar{P}_p^2}{\partial\alpha_p} > 0. \tag{14}$$

Note that $\frac{\partial\bar{P}_p}{\partial\alpha_p} < 0$, $\frac{\partial\bar{P}_p}{\partial\alpha_w} > 0$.

Proposition 4. *An increase in the economic inequality between groups p and w makes democracy less likely.*

An increase in the income of group *w* depresses its ability to fight in conflict since it raises the cost of hiring soldiers. It makes group *p* favorable for conflict. Thus, it decreases the incentive for group *p* to support democracy instead of conflict. The increase of group *p*'s income also depresses its ability to fight. The effect of an increase in disposal income under democracy must exceed that of an increase in the remaining income after group *p* loses the conflict. Thus, group *p* has more incentive to support democracy. Therefore, an increase in the economic inequality reduces hte likelihood of a democracy.

Finally, we obtain the result that an increase in the economic inequality results in increased concessions from party *w* in the election since a large inequality decreases the incentive for group *p* to support democracy.

Proposition 5. *An increase in the economic inequality between groups p and w raises the equilibrium tax rate.*

Przeworski (2005) and Aslaksen and Torvik (2006) analyzed the endogenous choice between democracy and civil conflict by infinite-period models. In their model, the parties choose whether or not to initiate a conflict before the election. The self-enforcing democracy is achieved by a trigger-strategy equilibrium; when the conflict is initiated at a period, the party chooses to pursue the conflict after the period.

Our propositions can be also considered as the results of an infinite period model if we define self-enforcing democracy as the dynamic equilibrium satisfying the following condition:

- whenever democracy prevails, each party does not initiate conflict, announces a (period-independent) policy platform maximizing its expected utility of the period, and accept the election result, and further,
- when it is initiated, the parties choose to pursue the conflict. The chosen fighting efforts maximize the expected utilities of the period.

This is similar to the definitions of self-enforcing democracy in Przeworski (2005) and Aslaksen and Torvik (2006).

The condition for the parties to choose initiating a conflict after an election in an infinite period model is equivalent with the condition of the single period model since, in our model, the equilibrium tax rate is unique independently of the result of an election and both the benefits to continue democracy and initiate a conflict are equivalent between before and after an election.

4 Concluding Remarks

In this paper, we examined the endogenous choice between democracy and civil conflict in line of with the literature on the resource-curse. In particular, we extended the model of Aslaksen and Torvik (2006) to include the scenario wherein there exists three social classes and two parties supported by the two classes of the three. We obtained the result that an increase in economic inequality between poor and rich people tends to weaken democracy in equilibrium. Thus, from this result, not only in the existing literature in this field but also in our new model assuming the existence of different social classes and the two parties as the representative of the two classes, the phenomenon of the resource-curse can be explained. Moreover, we found that economic inequality is positively associated with both the tax rate levels and public expenditure. We theoretically succeed to show that sufficiently large economic inequality incurs decreasing of the possibility to self-enforce the democracy.

There are two interesting extensions of our model. We assumed that the middle class does not have a political party that represents their own political idea and philosophy. The next obvious step is to consider the issue of the instability of democracy on the condition that there exist a political party as the representative of the middle class. Furthermore, we restricted our scope to the analysis of a simple single-period model, and thus in our model, the relation between the quantity of natural resources and durability of civil conflict cannot explicitly be considered in our model. These issues are left for future research.

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Appendix

Proof of Proposition 1.

The case of $\bar{u}_p^c > u_p^d(\tau^p)$: The proposition obviously follows.

The case of $u_p^d(\tau^p) \geq \bar{u}_p^c \geq u_p^d(\tau^m)$: Since $u_p''(\tau) < 0$ follows from $h'' < 0$, there must be a (unique) tax rate $\tau^* \in [\tau^m, \tau^p]$ such that $u_p(\tau^*) - \bar{u}_p = 0$ and $u_p(\tau) - \bar{u}_p < 0$ for each $\tau < \tau^*$. Based on Fact 1, $u_w^d(\tau) \geq \bar{u}_w^c$ for each $\tau \in [\tau^w, \tau^*]$.

Claim 1: If an equilibrium of democracy exists, τ^* must be implemented.

Assume that there exists an equilibrium of democracy. Let τ_i be the equilibrium platform for party $i \in \{p, w\}$.

Step 1: There exists no party $i \in \{p, w\}$ such that group m strictly prefers τ_i to τ^*.

If not, the result of the election must not be acceptable for party p.

Step 2: Both parties cannot win the election at the same time in spite of $\tau_p \neq \tau_w$.

Otherwise, since party p does not initiate conflict before the election, it follows that

$$\rho_p u_p^d(\tau_p) + (1 - \rho_p) u_p^d(\tau_w) \geq \bar{u}_p^c. \tag{15}$$

By this equation, it follows that $\tau_i > \tau^* > \tau_j$ with $i, j \in \{p, w\}$ and party p can make a profit by announcing a platform strictly lower than τ_i but sufficiently close to $\min\{\tau_i, \tau^p\}$.

Step 3: There exists party i with $\tau_i = \tau^*$

If not, party w can make a profit by announcing a platform strictly higher than τ^* and sufficiently close to τ^*.

By Step 1, 2, and 3, there exists a party that announces τ^* and no party announces a platform that is strictly preferred by group m.

Next, we show that there exists an equilibrium. In fact, if each party announces τ^*, it becomes an equilibrium. In this case, party p clearly chooses a best response strategy. Party w also cannot make a profit; if it announces a platform that is strictly preferred by group m to τ^*, party p must initiate a conflict after the election; otherwise, party w must lose the election. Thus, the proposition follows.

The case of $u_p^d(\tau^m) > \overline{u_p^c}$: By Fact 1, $u_w^d(\tau) > \overline{u_w^c}$ for each $\tau \in [\tau^w, \tau^p]$. Thus, It is clearly an equilibrium that each party announces τ^m as its platform.

We show that the equilibrium is unique. Assume not; thus, there exists another equilibrium. Let τ_i be the equilibrium platform for party $i \in \{p, w\}$. Now, both parties cannot win the election at the same time in spite of $\tau_p \neq \tau_w$; otherwise, τ_p and τ_w are indifferent for group m and $\tau_i > \tau_m > \tau_j$ with $i, j \in \{p, w\}$, and party p can make a profit by announcing a platform lower than τ_i but sufficiently close to $\min\{\tau_i, \tau^p\}$.

Thus, if the chosen tax rate is higher than τ^m, party w can make a profit by announcing a platform lower than the tax rate but sufficiently close to the tax rate. If the chosen tax rate is lower than τ^m, party p can make a profit by announcing a platform higher than the tax rate but sufficiently close to the tax rate. This contradicts the assumption. □

Proof of Proposition 2.

The case of $\alpha_p - \gamma\overline{P}_p^2 \geq 0$: Since $1 > \alpha_p$ and g's efficiency ensure that $k > \delta$, it follows that $k\log(\frac{k}{\alpha_p\delta}) + (-k + \alpha_p\delta) > 0$. Thus, $\alpha_p - \gamma\overline{P}_p^2 \geq 0$ implies that equation (11) is positive. By Proposition 1, the proposition follows.

The case of $\alpha_p - \gamma\overline{P}_p^2 < 0$: The proposition follows from equation (11) and proposition 1.

□

Proof of Proposition 3.

Since the increase in natural resources makes democracy less likely, $\alpha_p - \gamma\overline{P}_p^2 < 0$. There exist three types of relation between $u_p^d(\tau^p), u_p^d(\tau^m)$, and $\overline{u_p^c}$.

The case of $\overline{u_p^c} \geq u_p^d(\tau^p)$: Proposition 1 and 2 implies that no democracy is self-enforcing when the natural resources R increase.

The case of $u_p^d(\tau^m) \geq \overline{u_p^c} > u_p^d(\tau^p)$: Proposition 1 implies that the equilibrium tax rate τ^* is in $[\tau^m, \tau^p)$. By differentiating $u_p(\tau^*(R)) - \overline{u}_p = 0$ with respect to R, we obtain:

$$\frac{d\tau^*}{dR} = -\frac{h_g - \gamma_p\overline{P}_p^2}{(h_g - \alpha_p)}, \tag{16}$$

where $h_g \equiv \frac{\partial}{\partial g} h(\tau^* + R)$. (Note $h_g < \frac{\partial}{\partial g} h(\tau^p + R) = \alpha_p$.) Thus,

$$\frac{d}{dR}(\tau^* + R) = \frac{\gamma \overline{P}_p^2 - \alpha_p}{h_g - \alpha_p} > 0, \tag{17}$$

and proposition follows.

The case of $u_p^d(\tau^m) > \overline{u}_p^c$: Proposition 1 and Equation (3) imply that government expenditure is $\tau^m + R = \frac{k}{\alpha_m} - \delta$ and constant to R.

□

Proof of proposition 4.

The condition for a self-enforcing democracy is that $u_p^d(\tau^p) - \overline{u}_p^c \geq 0$. Since τ^p maximizes $u_p^d(\tau^p) - \overline{u}_p^c$, Equations (13), (14), and the envelope theorem imply that

$$\frac{d}{d\alpha_w}\left(u_p^d(\tau^p) - \overline{u}_p^c\right) < 0, \tag{18}$$

$$\frac{d}{d\alpha_p}\left(u_p^d(\tau^p) - \overline{u}_p^c\right) > 0, \tag{19}$$

and the proposition follows. □

Proof of Proposition 5.

There exist three types of relation between $u_p^d(\tau^p), u_p^d(\tau^m)$, and $\overline{u}_p^c$.

The case of $\overline{u}_p^c \geq u_p^d(\tau^p)$: Proposition 1 and 4 implies that no democracy is self-enforcing when α_w increases or α_p decreases.

The case of $u_p^d(\tau^p) > \overline{u}_p^c \geq u_p^d(\tau^m)$: Since the equilibrium tax rate τ^* is in $[\tau^m, \tau^p)$, we have $\frac{d}{d\tau} u_p^d(\tau^*) > 0$. By differentiating $u_p(\tau^*) - \overline{u}_p^c$ with respect to α_p and α_w, we obtain

$$\frac{d\tau^*}{d\alpha_w} = -\frac{\partial}{\partial \alpha_w}\left(u_p^d(\tau^*) - \overline{u}_p^c\right) / \frac{\partial}{\partial \tau}\left(u_p^d(\tau^*) - \overline{u}_p^c\right) > 0, \tag{20}$$

$$\frac{d\tau^*}{d\alpha_p} = -\frac{\partial}{\partial \alpha_p}\left(u_p^d(\tau^*) - \overline{u}_p^c\right) / \frac{\partial}{\partial \tau}\left(u_p^d(\tau^*) - \overline{u}_p^c\right) < 0, \tag{21}$$

and the proposition follows.

The case of $u_p^d(\tau^m) > \overline{u}_p^c$: Proposition 1 implies that the equilibrium tax rate is $\tau^m = \frac{k}{\alpha_m} - \delta - R$ and constant to α_p and α_w. □