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# The Core and Productivity-Improving Mergers in Stackelberg Mixed Oligopoly

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**Abstract**

We analyze productivity-improving mergers in Stackelberg mixed triopoly especially focusing on the stability of coalition formations of owners of firms. We adopt the core as the solution concept of coalition formations and examine the core of market structures for each of the two alternative forms of the Stackelberg competitions: one is the case where a public firm or a public-private merged firm acts as a Stackelberg leader, and the other is the case where each of these firms acts as a Stackelberg follower. In contrast to the case of the Cournot competition in which it is known that the core is non-empty, a striking impossibility result is obtained that, in each of the two alternative forms of the Stackelberg competition, the core must be empty, i.e. none of the market structures is stable.

*Keywords*: Mergers; Mixed oligopoly; The core of market structures; Stackelberg

*JEL classification*: D21; L13; L33

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## 1 Introduction

This paper provides a theoretical analysis on merger activities in the oligopoly composed not only of private firms which maximize their profits but also of a public firm which is a welfare maximizer. Such an industry is usually referred to as *mixed oligopoly*, and the studies of the mixed oligopoly go back to De Fraja and Delbono (1989). In contrast to the extensive literature on a merger activity in private oligopoly,[1] not so many efforts have been carried out in studying a merger activity in mixed oligopoly. Exceptions are Bárcena-Ruiz and Gárzon (2003), Coloma (2006), Kamijo and Nakamura (2007), and Kamaga and Nakamura (2007). Among them, the papers of Kamijo and Nakamura and of Kamaga and Nakamura provided the analyses on a merger activity especially focusing on the stability of coalition formations of firms' owners in the similar manner to Barros (1998), Horn and Persson (2001), and Straume (2006).

In the current paper, we work with the framework set up by Kamaga and Nakamura (2007). In their paper, Kamaga and Nakamura considered the mixed triopoly of a single homogeneous product, i.e. the industry composed of a public firm and two private firms producing a homogeneous product, and they examined the case of the Cournot competition, i.e. the case of simultaneous moves of the three firms. They adopted the *core* as the solution concept of coalition formations. Then, they obtained the result that if a merger entails the improvement on productivity in a merged firm,[2] the core is non-empty and it consists solely of the market structures of the merger between the public firm and one of the two private firms with the shareholding ratio by the owner of the pre-merged public firm in the merged firm near around 0.57. This result shows that, as in the case of the private oligopoly,[3] a merger activity in the mixed oligopoly increases social welfare if it entails the improvement on productivity in a merged firm. Moreover, it suggests a possible resolution to the surprising and somewhat counterintuitive result by Bárcena-Ruiz and Gárzon (2003). Bárcena-Ruiz and Gárzon, in their model of mixed duopoly of two heterogeneous products, assumed that a merger does not entail the improvement on productivity in the merged firm and showed that a public firm and a private firm can reach an agreement on their merger only in the case of *sufficiently low degree of substitution* between the two products and will *never* merge in the case of *perfect substitution*. The result of non-empty core by Kamaga and Nakamura (2007) shows that a merger will also take place in the case of perfect substitution if a merger yields the improvement on productivity.

[1] For example, Salant et al. (1983), Deneckere and Davidson (1985), and Farrell and Shapiro (1990).

[2] They adopted the similar type of quadratic cost functions to those considered in McAfee and Williams (1992).

[3] On the productivity-improving mergers in the private oligopoly, see Farrell and Shapiro (1990).

The purpose of the paper is to examine whether or not the result of Kamaga and Nakamura (2007) obtained for the Cournot equilibrium will change in accordance with an adopted equilibrium concept. The robustness of a result concerning an adopted equilibrium concept has been extensively analyzed in the literature on the mixed oligopoly.[4] As an alternative to the Cournot equilibrium, we consider two forms of the Stackelberg competition. One is the Stackelberg equilibrium where the public firm or a merged firm in which the public firm participates acts as a Stackelberg leader, and the other is the one where each of these firms acts as a Stackelberg follower. In contrast to the result by Kamaga and Nakamura (2007), we obtained a striking result that the core is empty for each of the two forms of the Stackelberg competition. In other words, none of the market structures is stable in either case.

The paper is organized as follows. In Section 2.1 we introduce the model of mixed triopoly set up by Kamaga and Nakamura (2007). In the model, there are four regimes in accordance with the forms of a merger: (a) mixed triopoly; (b) merger between private firms; (c) merger between a public firm and a private firm; and (d) merger among all the three firms. For each of the regimes, we provide the equilibrium outcomes for each of the Cournot equilibrium and the two forms of the Stackelberg equilibrium. In Section 2.2 we elaborate the motivation to analyze the stability problem of owners' coalition formations and also provide a formal definition of the core of market structures. After reviewing the result obtained in Kamaga and Nakamura (2007) in Section 3, we provide the main results of the current paper in Section 4. Section 5 concludes with some remarks.

# 2 Model

## 2.1 Four regimes in mixed triopoly model

We analyze a merger activity in the mixed triopoly, i.e. in the industry composed of a public firm and two private firms. We work with the model set up by Kamaga and Nakamura (2007). A public firm is denoted by 0 and two private firms by 1 and 2, respectively. Each firm produces a single homogeneous good and is assumed to be entrepreneurial one, i.e. the owners themselves make every managerial decision making of their firms. To make the analysis simple, the public firm is assumed to be owned by the government and each of the private firms by a single private shareholder, respectively. In the mixed triopoly model,

[4] For example, in the context of privatization and subsidization, White (1996) and the subsequent papers: Poyago-Theotoky (2001); Myles (2002); Fjell and Heywood (2004); and Hashimzade et al. (2007).

we should consider the following four different market regimes (a) to (d) in accordance with which type of mergers is actually realized among the firms: (a) *mixed triopoly*, i.e. the case where none of the firms reach an agreement on a merger; (b) *merger between private firms*; (c) *merger between a public firm and a private firm*; and (d) *merger among all the three firms*.

The four regimes are slightly different in their details of the formal descriptions. We begin with introducing the regime of the mixed triopoly (a). The inverse demand function is linear in the total output $Q$,

$$P(Q) = a - Q, \tag{1}$$

where $a$ is a sufficiently large positive number. As assumed in Bárcena-Ruiz and Garzón (2003), each firm $i$ $(= 0, 1, 2)$ has an identical technology represented by the quadratic cost function:

$$C(q_i) = q_i^2, \tag{2}$$

where $q_i$ is the quantity of the output by the firm $i$. The profit function of the firm $i$ $(= 0, 1, 2)$ is given as:

$$\Pi_i = (a - Q)q_i - q_i^2. \tag{3}$$

As usual, social welfare, denoted by $W$, is measured by the sum of consumer surplus $CS = Q^2/2$, and the firms' profits. The public firm is assumed to be a welfare maximizer, and then, its objective function $U_0$ is given as:

$$U_0(q_0; q_1, q_2) = W = CS + \Pi_0 + \sum_{i=1}^{2} \Pi_i. \tag{4}$$

On the other hand, the private firms are assumed to maximize their own profits, and their objective functions $U_i$ are

$$U_i(q_i; q_0, q_j) = \Pi_i \quad (i, j = 1, 2,\ i \neq j). \tag{5}$$

Next, we introduce the other three regimes, (b), (c), and (d). Each of these three regimes is described as an extension of the mixed triopoly. In our model, what makes these regimes different from the mixed triopoly is not only the number of the firms but also cost functions of the merged firms which reflect a synergy effect yielded by a merger. Although Bárcena-Ruiz and Garzón (2003) have not discussed the case where a merger entails a synergy effect on productivity in a merged firm, we assume that a merger yields a positive effect on productivity of a merged firm. In the case of homogeneous goods market, such an assumption can be easily justified: e.g. operating the plants owned by the pre-merged firms in the most efficient way, or sharing the patents of the pre-merged firms. In the literature on a merger activity, McAfee and Williams (1992) have analyzed a productivity-improving merger. We basically follow McAfee and Williams, and assume that if $n$ $(= 2, 3)$ firms merge into one firm, the total cost of the merged firm $C_m$ is represented as:

$$C_m(q_m) = \frac{q_m^2}{n}, \tag{6}$$

where $q_m$ is the output of the merged firm $m$.[5] As we have suggested above, such a cost function is supported by the assumption that the merged firm adopts the most efficient operation plan of the plants previously owned by the pre-merged firms. The profit of the merged firm, $\Pi_m$, is given by replacing $q_i^2$ with $q_m^2/n$ in (3). Throughout the paper, a merged firm organized by pre-merged firms $i$ and $j$ is denoted by simply combining the notation just as $ij$. We summarize the firms' profit functions in each of the four regimes in Table 1, where profit functions with superscript $r$ $(= a, b, c, d)$ denote those considered in the regime $r$.

We now move to the objectives of the merged firms in the regimes (b), (c), and (d). In the regime (b), the merged firm 12 is still completely private-owned, and thus its objective is to maximize the profit $\Pi_{12}$. On the other hand, in the regimes (c) and (d), the merged firm 01 (or 02) or 012 is jointly owned by the owners of the pre-merged private firm 1 and/or 2 and the pre-merged public firm 0. Therefore, the objective of the merged firm in these two regimes should reflect both of the objectives of the pre-merged public firm and the pre-merged private firm(s). In this paper, we define the objectives of the merged firms 01, 02, and 012 as the weighted sum of social welfare and the profit of the merged firm.[6] Let $\beta \in [0, 1]$ be

[5] The cost function defined here is a special case of those considered by McAfee and Williams. Heywood and McGinty (2007a; 2007b) and Nakamura and Inoue (2007) have also assumed this type of cost functions.

[6] The weighted sum of social welfare and the profit has first been suggested by Matsumura (1998) for the objective of a firm jointly owned by a public shareholder and private shareholders and have also been adopted in Bárcena-Ruiz and Garzón (2003).

Table 1: Firms' profit functions in each regime (r)

| (r) | profit $\Pi_i^r$ |
|---|---|
| (a) | $\Pi_i^a = \left[a-(q_0+\sum_{j=1}^2 q_j)\right]q_i-(q_i)^2 \quad (i=0,1,2)$ |
| (b) | $\Pi_0^b = \left[a-(q_0+q_{12})\right]q_0-(q_0)^2$,<br>$\Pi_{12}^b = \left[a-(q_0+q_{12})\right]q_{12}-(q_{12})^2/2$ |
| (c) | $\Pi_{0i}^c = \left[a-(q_{0i}+q_j)\right]q_{0i}-(q_{0i})^2/2$,<br>$\Pi_j^c = \left[a-(q_{0i}+q_j)\right]q_j-(q_j)^2 \quad (i,j=1,2,\ i\neq j)$ |
| (d) | $\Pi_{012}^d = (a-q_{012})q_{012}-(q_{012})^2/3$ |

the shareholding ratio by the owner of the public firm 0 in the merged firm $0i$ ($i=1,2$). We assume that the shareholding ratio $\beta$ by the owner of the public firm 0 directly measures the weight on social welfare in the objective of the merged firm $0i$. Similarly, let $\gamma \in [0,1]$ denote the shareholding ratio by the owner of the firm 0 in the merged firm 012 and measure the weight on social welfare in the firm's objective. Table 2 provides the firms' objective functions in each of the four regimes, where the functions with superscript $r$ $(=a,b,c,d)$ denote those considered in the regime $r$.

Table 2: Firms' objectives in each regime (r)

| (r) | objective function $U_i^r$ |
|---|---|
| (a) | $U_0^a = W^a = (q_0+\sum_{j=1}^2 q_j)^2/2+\Pi_0^a+\sum_{j=1}^2 \Pi_j^a$,<br>$U_i^a = \Pi_i^a \quad (i=1,2)$ |
| (b) | $U_0^b = W^b = (q_0+q_{12})^2/2+\Pi_0^b+\Pi_{12}^b$,<br>$U_{12}^b = \Pi_{12}^b$ |
| (c) | $U_{0i}^c = \beta W^c+(1-\beta)\Pi_{0i}^c = \beta\left[(q_{0i}+q_j)^2/2+\Pi_{0i}^c+\Pi_j^c\right]+(1-\beta)\Pi_{0i}^c$,<br>$U_j^c = \Pi_j^c \quad (i,j=1,2,\ j\neq i)$ |
| (d) | $U_{012}^d = \gamma W^d+(1-\gamma)\Pi_{012}^d = \gamma\left[(q_{012})^2/2+\Pi_{012}^d\right]+(1-\gamma)\Pi_{012}^d$ |

In this paper, we consider the following three equilibrium concepts mainly focusing on the last two cases: (i) *the Cournot equilibrium*; (ii) *the Stackelberg equilibrium* where the public firm or the merged firm in which the public firm participates acts as a Stackelberg leader; and (iii) *the Stackelberg equilibrium* where the public firm or the merged firm in which the public firm participates acts as a Stackelberg follower. In the rest of the paper, we refer to the last two equilibrium concepts as *L-Stackelberg equilibrium* and *F-Stackelberg equilibrium*, respectively.

In each of the regimes, every firm chooses the output to maximize its objective given in the Table 2. From the routine calculation, the equilibrium outputs for each of the three equilibrium concepts are given in Table 3.

Table 3: Equilibrium outputs $q^{r*}$ in each regime (r)

| (r) | | Cournot | L-Stackelberg | F-Stackelberg |
|---|---|---|---|---|
| (a) | $q^{a*} = \left(q_0^{a*}, q_1^{a*}, q_2^{a*}\right)$ | $\left(\frac{3a}{13}, \frac{2a}{13}, \frac{2a}{13}\right)$ | $\left(\frac{13a}{63}, \frac{10a}{63}, \frac{10a}{63}\right)$ | $\left(\frac{2a}{9}, \frac{a}{6}, \frac{a}{6}\right)$ |
| (b) | $q^{b*} = \left(q_0^{b*}, q_{12}^{b*}\right)$ | $\left(\frac{a}{4}, \frac{a}{4}\right)$ | $\left(\frac{5a}{23}, \frac{6a}{23}\right)$ | $\left(\frac{5a}{21}, \frac{2a}{7}\right)$ |
| (c) | $q^{c*} = \left(q_{0i}^{c*}, q_j^{c*}\right)$ | $\left(\frac{3a}{11-4\beta}, \frac{(2-\beta)a}{11-4\beta}\right)$ | $\left(\frac{(12-\beta)a}{40-13\beta}, \frac{(7-3\beta)a}{40-13\beta}\right)$ | $\left(\frac{(8-3\beta)a}{2(5-2\beta)(3-\beta)}, \frac{(2-\beta)a}{2(5-2\beta)}\right)$ |
| (d) | $q^{d*} = q_{012}^{d*}$ | $\frac{3a}{8-3\gamma}$ | same as left | same as left |

## 2.2 merger as a coalition formation and stable market structures

Assuming the firms are entrepreneurial ones, a managerial decision making about a merger is the one conducted by the owner of a firm such as "agree or not agree to merge" and "break off a merger." Consequently, merger activities of the three firms, 0, 1, and 2, can be analyzed in terms of coalition formations among the owners of the firms. In the analysis of coalition formations, the most fundamental and most important problem is the stability of coalition formations. As in the papers of Barros (1998), Horn and Persson (2001), Straume (2006), Kamijo and Nakamura (2007), and Kamaga and Nakamura (2007), we adopt the core as the solution concept of coalition formations and explore the stable coalition formations, or the stable market structures. In the rest of the paper, the notation 0, 1, and 2 is also used to denote the owners of these firms 0, 1, and 2, respectively. A market structure is characterized in terms of coalition formations among the three owners 0, 1, and 2, and also of the shareholding ratios by the participating owners in the merged firm. In the preceding subsection, we introduced $\beta$ and $\gamma$ as the shareholding ratio by the owner 0 in the merged firm in the regimes (c) and (d), respectively. To complete the formal description of each market structure, we need to introduce the additional notation. Let $\alpha \in [0,1]$ be the shareholding ratio by the owner 1 in the merged firm of the regime (b), i.e. in the private merged firm 12, and also $\delta \in [0,1]$ be the distribution ratio of the profit of the merged firm 012 between the owners 1 and 2. We interpret that $(1-\gamma)\delta$ (resp. $(1-\gamma)(1-\delta)$) is the shareholding ratio by the owner 1 (resp. the owner 2) in the merged firm 012. We denote by $M_t^{\{C\}}$ a market structure composed of a coalition formation $C$ with a shareholding ratio $t$ in the merged firm (if exists), where $\{C\}$ is a partition of the set of the owners $\{0,1,2\}$ and $t$ will be $\alpha$ in the regime (b); $\beta$ in (c); and a pair of $\gamma$ and $\delta$ in (d).[7] For example, the market structure of the merger between the public firm 0 and the private firm 1 with the owner 0's shareholding ratio $\beta = 0.5$ is denoted by $M_{\beta=0.5}^{\{\{0,1\},\{2\}\}}$ (see also Figure 1).

We next define the payoffs to the owners. Each of the owners determines the managerial decision on a merger to maximize her/his own payoff. The owner of the public firm 0 is assumed to be a welfare maximizer and the owners of the two private firms 1 and 2 to be profit maximizers. Then, the payoff to the owner 0 in a regime r $(= a,b,c,d)$, denoted by $V_0^r$, is the equilibrium social welfare $W^{r*}$ in the regime r, and those to the owners 1 and 2 are (i) the equilibrium profits of their own firms in the regime (a) and (ii) the distributed equilibrium profits determined according to their shareholding ratio(s) in the merged firm in the regimes (b) to (d). Table 4 describes the payoffs to the owners in each regime (r), where the variables with asterisk denote those evaluated in an adopted equilibrium.

The purpose of the paper is to examine which of the market structures is/are stable in terms of the owners' coalition formations. In the presence of more than two owners (one for each firm), it is not sufficient to simply analyze incentives of the owners in each particular case of the merger. An example will help understanding the importance of analyzing the stability problem. Consider the following four market structures (one for each regime (r)): $M^{\{\{0\},\{1\},\{2\}\}}$ in (a); $M_{\alpha=0.5}^{\{\{0\},\{1,2\}\}}$ in (b); $M_{\beta=0.5}^{\{\{0,1\},\{2\}\}}$ in (c); and $M_{\gamma=0.33,\delta=0.5}^{\{\{0,1,2\}\}}$ in (d). Suppose that the owners' preference orderings over these four market structures are derived as follows according to the payoffs to them:

[7] A partition of a set $A$ is a set of subsets of $A$ such that (i) the union of its elements is equal to $A$, and (ii) the intersection of any two of its elements is empty. We allow the case where a partition of $\{0,1,2\}$ contains an empty set as an element.

Table 4: Owners' payoffs $V_i^r$ in each regime (r)

| (r) | payoffs $(V_0^a, V_1^a, V_2^a)$ |
|---|---|
| (a) | $\left(V_0^a, V_1^a, V_2^a\right) = \left(W^{a*},\ \Pi_1^{a*},\ \Pi_2^{a*}\right)$ |
| (b) | $\left(V_0^b, V_1^b, V_2^b\right) = \left(W^{b*},\ \alpha\Pi_{12}^{b*},\ (1-\alpha)\Pi_{12}^{b*}\right)$ |
| (c) | $\left(V_0^c, V_i^c, V_j^c\right) = \left(W^{c*},\ (1-\beta)\Pi_{0i}^{c*},\ \Pi_j^{c*}\right), \quad (i,j=1,2,\ i\neq j)$ |
| (d) | $\left(V_0^d, V_1^d, V_2^d\right) = \left(W^{d*},\ (1-\gamma)\delta\Pi_{012}^{d*},\ (1-\gamma)(1-\delta)\Pi_{012}^{d*}\right)$ |

$$M^{\{\{0\},\{1\},\{2\}\}} \succ_0 M_{\gamma=0.33,\delta=0.5}^{\{\{0,1,2\}\}} \succ_0 M_{\alpha=0.5}^{\{\{0\},\{1,2\}\}} \succ_0 M_{\beta=0.5}^{\{\{0,1\},\{2\}\}};$$

$$M_{\beta=0.5}^{\{\{0,1\},\{2\}\}} \succ_1 M_{\gamma=0.33,\delta=0.5}^{\{\{0,1,2\}\}} \succ_1 M_{\alpha=0.5}^{\{\{0\},\{1,2\}\}} \succ_1 M^{\{\{0\},\{1\},\{2\}\}};$$

$$M_{\beta=0.5}^{\{\{0,1\},\{2\}\}} \succ_2 M_{\gamma=0.33,\delta=0.5}^{\{\{0,1,2\}\}} \succ_2 M_{\alpha=0.5}^{\{\{0\},\{1,2\}\}} \succ_2 M^{\{\{0\},\{1\},\{2\}\}};$$

where $a \succ_i b$ means the owner $i$ prefers $a$ to $b$. Now, suppose that $M^{\{\{0\},\{1\},\{2\}\}}$ is temporarily realized. Then, the owners 1 and 2 will jointly deviate and $M^{\{\{0\},\{1\},\{2\}\}}$ will shift into $M_{\alpha=0.5}^{\{\{0\},\{1,2\}\}}$ because both of them can gain higher payoffs. By the same reason, every owner now wants to shift into $M_{\gamma=0.33,\delta=0.5}^{\{\{0,1,2\}\}}$ from $M_{\alpha=0.5}^{\{\{0\},\{1,2\}\}}$. $M_{\gamma=0.33,\delta=0.5}^{\{\{0,1,2\}\}}$ is still unstable because the owner 2 can gain much higher payoff if s/he deviates from the merger and operate her/his own firm in $M_{\beta=0.5}^{\{\{0,1\},\{2\}\}}$. Thus, $M_{\gamma=0.33,\delta=0.5}^{\{\{0,1,2\}\}}$ will shift into $M_{\beta=0.5}^{\{\{0,1\},\{2\}\}}$ through the deviation by the owner 2. Now, in the market structure $M_{\beta=0.5}^{\{\{0,1\},\{2\}\}}$, the owner of the firm 0 in turn has an incentive to deviate and to shift into $M^{\{\{0\},\{1\},\{2\}\}}$. Then, we inevitably come back to the market structure we started this observation. The cycle we just observed is depicted in Figure 1.

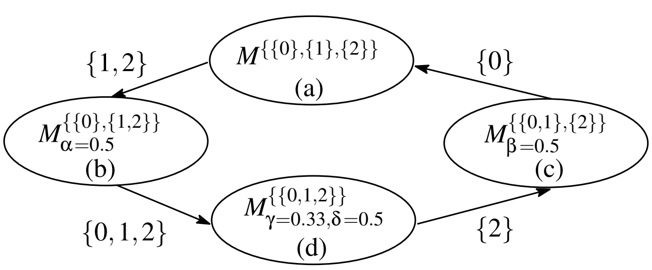


Figure 1: A typical market structure in regime (r) and a cycle in deviations

In this paper, we invoke the core, the well-established solution concept in cooperative game theory to examine the stable market structures. In order to define the core of the market structures, we should start with the definitions of inducement relations among the market structures and of a blocking market structure.

A market structure $M$ is said to be **inducible from $M'$ via** a coalition $S \subseteq \{0,1,2\}$ if, given that the coalition structure in $M'$ is represented as the partition $\{S'_1,\ldots,S'_n\}$, the coalition structure in $M$ is described as the partition $\{S,S_1,\ldots,S_n\}$ such that

$$S_i = \begin{cases} S'_i & \text{if } S\cap S'_i = \varnothing, \\ S'_i \backslash S & \text{if } S\cap S'_i \neq \varnothing, \end{cases} \tag{7}$$

for all $i = 1,\ldots,n$. In other words, a market structure $M$ is inducible from a structure $M'$ via a coalition $S$ if the deviant coalition $S$ can generate the new structure $M$ only through their deviation without any cooperation of the owners outside of the coalition. We should give two remarks about the inducibility we just defined in general form. The first is that the above definition of inducibility among market structures allows the case where the deviation by the stand-alone coalition $\{0\}$ (resp. $\{i\}$ $(i=1,2)$) from $M^{\{\{0,1,2\}\}}_{\gamma\in[0,1],\delta\in[0,1]}$ can generate the market structure $M^{\{\{0\},\{1,2\}\}}_{\alpha}$ with any of her/his desired ratios $\alpha \in [0,1]$ (resp. $M^{\{0,j\},\{i\}}_{\beta}$ with any of her/his desired ratios $\beta \in [0,1]$). Since it seems unreasonable to assume that the deviant owner has a decisive influence on the shareholding ratios in the merged firm organized by the rest of the owners, we restrict admissible inducement relations as follows: in the case of the deviation by $\{0\}$ from $M^{\{\{0,1,2\}\}}_{\gamma\in[0,1],\delta\in[0,1]}$,

$$M^{\{\{0\},\{1,2\}\}}_{\alpha} \text{ with } \alpha = \delta \tag{8}$$

is solely inducible; and in the case of the deviation by $\{1\}$ (resp. $\{2\}$) from $M^{\{\{0,1,2\}\}}_{\gamma\in[0,1],\delta\in[0,1]}$,

$$M^{\{\{0,1\},\{2\}\}}_{\beta} \text{ with } \beta = \frac{\gamma}{\gamma+(1-\gamma)\delta} \quad \left(\text{resp. } M^{\{\{0,2\},\{1\}\}}_{\beta} \text{ with } \beta = \frac{\gamma}{\gamma+(1-\gamma)(1-\delta)}\right) \tag{9}$$

is solely inducible, i.e. the shareholding ratios are determined according to those of the two owners in the merged firm 012. In the rest of the paper, we restrict our analysis to the admissible inducement relations and use the terms “inducement relations” and “inducible” to mean those restricted to the admissible cases. The second remark is that our definition does not allow the case where market structure of the mixed triopoly is directly induced from a market structure of the merger among the three firms, i.e. that of the regime (d). We eliminate such a case to make our analysis simple. However, as will be shown later, this simplification never detracts the generality and relevance of our core analysis.

A market structure $M$ is said to **block** a market structure $M'$ **via** a coalition $S \subseteq \{0,1,2\}$ if (i) $M$ is inducible from $M'$ via $S$, and (ii) $V_i > V_i'$ for all $i \in S$, where $V_i$ (resp. $V_i'$) denotes the payoff to $i$ in $M$ (resp. in $M'$). By definition, if a market structure $M$ blocks $M'$ via a coalition $S$, each of the owners in the coalition $S$ has an incentive to deviate from $M'$ and induce $M$. Consequently, a blocked market structure will never be realized. We write $M \succ_S M'$ to mean $M$ blocks $M'$ via $S$. We are now ready to define the core of the market structures.

The **core** is the set of market structures each of which is never blocked by any other market structure. In other words, a market structure $M$ belongs to the core if there is no market structure $M'$ such that $M' \succ_S M$ for some $S \subseteq \{0,1,2\}$. We denote the core of the market structures by $\mathfrak{Co}$. By definition, if a market structure is in the core, all of the three owners have no incentive to deviate and to induce any other market structure. In this sense, a market structure in the core is regarded as a stable one.

## 3 Cournot competition

We briefly review the case of the Cournot competition which was analyzed in Kamaga and Nakamura (2007). In the Cournot equilibrium of each regime (r), we have the following equilibrium payoffs to the three owners $(V_0^r, V_1^r, V_2^r)$:

$$(V_0^a, V_1^a, V_2^a) = \left(\frac{99a^2}{338}, \frac{8a^2}{169}, \frac{8a^2}{169}\right); \tag{10a}$$

$$(V_0^b, V_1^b, V_2^b) = \left(\frac{9a^2}{32}, \frac{3\alpha a^2}{32}, \frac{3(1-\alpha)a^2}{32}\right); \tag{10b}$$

$$(V_0^c, V_i^c, V_j^c) = \left(\frac{(68-44\beta+5\beta^2)a^2}{2(11-4\beta)^2}, \frac{9(3-2\beta)(1-\beta)a^2}{2(11-4\beta)^2}, \frac{2(2-\beta)^2a^2}{(11-4\beta)^2}\right); \tag{10c}$$

$$(V_0^d, V_1^d, V_2^d) = \left(\frac{3(11-6\gamma)a^2}{2(8-3\gamma)^2}, \frac{3(4-3\gamma)(1-\gamma)\delta a^2}{(8-3\gamma)^2}, \frac{3(4-3\gamma)(1-\gamma)(1-\delta)a^2}{(8-3\gamma)^2}\right) \tag{10d}$$

In the paper of Kamaga and Nakamura, it has been shown that the core is non-empty in the case of the Cournot competition and also that the core consists solely of the market structures of the merger between the public firm 0 and one of the two private firm $i$ with the share ratio $\beta$ in $[\underline{\beta}, \bar{\beta}]$, where

$$\underline{\beta} = \frac{638 - 39\sqrt{31}}{739} \approx 0.56950 \quad \text{and} \quad \bar{\beta} = \frac{6197 - 39\sqrt{6001}}{5572} \approx 0.56996. \tag{11}$$

Their result (Kamaga and Nakamura (2007), Proposition 1) is formally stated as follows.

**Proposition 1.** *In the case of the Cournot competition, the market structure of the public-private merged firm 0i and the private firm* $j \neq i$, $M_{\beta}^{\{\{0,i\},\{j\}\}}$, *is in the core whenever the ratio of shareholding by the owner 0 in the merged firm 0i,* $\beta$, *is in the closed interval* $[\underline{\beta}, \bar{\beta}]$, *and moreover, the core consists solely of these market structures, i.e.* $\mathfrak{Co} = \left\{M_{\beta}^{\{\{0,i\},\{j\}\}} : \beta \in [\underline{\beta}, \bar{\beta}],\ i,j = 1,2,\ i \neq j\right\}$.

In the next section, we examine to what extent an adopted equilibrium concept affects the core of market structures for each of the L-Stackelberg equilibrium and the F-Stackelberg equilibrium.

# 4 L-Stackelberg competition and F-Stackelberg competition

In the case of the merger among all the three firms, i.e. the regime (d), the L-Stackelberg equilibrium coincides with the Cournot equilibrium. Thus, the equilibrium payoffs in the regime (d), $(V_0^d, V_1^d, V_2^d)$, are given by (10d). In the other three regimes (a)-(c), the equilibrium payoffs to the three owners are given as follows:

$$(V_0^a, V_1^a, V_2^a) = \left(\frac{37a^2}{126}, \frac{200a^2}{3969}, \frac{200a^2}{3969}\right); \tag{12a}$$

$$(V_0^b, V_1^b, V_2^b) = \left(\frac{13a^2}{46}, \frac{54\alpha a^2}{529}, \frac{54(1-\alpha)a^2}{529}\right); \tag{12b}$$

$$(V_0^c, V_i^c, V_j^c) = \left(\frac{(917 - 554\beta + 69\beta^2)a^2}{2(40 - 13\beta)^2}, \frac{(1-\beta)(12-\beta)(30-17\beta)a^2}{2(40-13\beta)^2}, \frac{2(7-3\beta)^2 a^2}{(40-13\beta)^2}\right) \tag{12c}$$

In the case of the L-Stackelberg competition, we obtain the following striking but serious impossibility result.

**Proposition 2.** *In the case of the L-Stackelberg competition, none of the market structures belongs to the core, i.e.* $\mathfrak{Co} = \varnothing$.

*Proof.* The proof proceeds through a series of claims (a) to (d). In each claim (r) (= a, b, c, d), it will be shown that, for any market structure $M$ in the regime (r), we can find some other market structure which blocks $M$ via some coalition $S \subseteq \{0,1,2\}$, i.e. we explicitly provide $M'$ and $S$ such that $M' \succ_S M$.

**Claim (a)**: $M^{\{\{0\},\{1,2\}\}}_{\alpha\in(\underline{\alpha},\bar{\alpha})} \succ_{\{1,2\}} M^{\{\{0\},\{1\},\{2\}\}}$, where $\underline{\alpha} = \frac{52900}{107163}$ and $\bar{\alpha} = \frac{54263}{107163}$.

This claim is easily checked as follows:

$$V_1^b(\alpha) - V_1^a = \frac{-2a^2(52900 - 107163\alpha)}{2099601} > 0 \Leftrightarrow \alpha > \frac{52900}{107163}, \tag{13}$$

$$V_2^b(\alpha) - V_2^a = \frac{2a^2(54263 - 107163\alpha)}{2099601} > 0 \Leftrightarrow \alpha < \frac{54263}{107163}. \tag{14}$$

Thus, the joint deviation by $\{0,1\}$ will take place if $\alpha \in \left(\frac{52900}{107163}, \frac{54263}{107163}\right)$.

**Claim (b)**: (i) $M^{\{\{0\},\{1\},\{2\}\}} \succ_{\{1\}} M^{\{\{0\},\{1,2\}\}}_{\alpha}$ in any case of $\alpha \in \left[0, \frac{52900}{107163}\right)$;
(ii) $M^{\{\{0,i\},\{j\}\}}_{\beta=1/2} \succ_{\{0,1\}} M^{\{\{0\},\{1,2\}\}}_{\alpha}$ in any case of $\alpha \in \left[\frac{52900}{107163}, \frac{54263}{107163}\right]$;[8]
(iii) $M^{\{\{0\},\{1\},\{2\}\}} \succ_{\{2\}} M^{\{\{0\},\{1,2\}\}}_{\alpha}$ in any case of $\alpha \in \left(\frac{54263}{107163}, 1\right]$.

Since the equivalence assertions in (13) and (14) still hold when we reverse the inequality signs, (i) and (iii) are straightforward. We prove (ii). Assume, without loss of generality, $i = 1$. For the owner 0, we have

$$V_0^c(\beta) - V_0^b = \frac{a^2(291 + 778\beta - 610\beta^2)}{46(40 - 13\beta)^2} > 0, \quad \forall\beta \in [0,1], \tag{15}$$

because $\frac{d(V_0^c(\beta) - V_0^b)}{d\beta} = \frac{841a^2(1-\beta)}{(40-13\beta)^3} \geq 0$ for all $\beta \in [0,1]$ and $V_0^c(\beta)|_{\beta=0} - V_0^b = \frac{291a^2}{73600} > 0$. On the other hand, for the owner 1, we have

$$V_1^c(\beta)|_{\beta=\frac{1}{2}} - V_1^b(\alpha)|_{\alpha=\frac{54263}{107163}} = \frac{127812533a^2}{37700435556} > 0. \tag{16}$$

[8] This result can be generalized for any $\beta \in [0, 0.529309)$.

Note that $\frac{54263}{107163} = \arg\max_{\alpha \in [\frac{52900}{107163}, \frac{54263}{107163}]} V_1^b(\alpha)$. Thus, by (15) and (16), $M_{\beta=1/2}^{\{\{0,1\},\{2\}\}} \succ_{\{0,1\}} M_{\alpha}^{\{\{0\},\{1,2\}\}}$ in any case of $\alpha \in [\frac{52900}{107163}, \frac{54263}{107163}]$.

**Claim (c)**: (i) $M^{\{\{0\},\{1\},\{2\}\}} \succ_{\{0\}} M_{\beta}^{\{\{0,i\},\{j\}\}}$ in any case of $\beta \in \big[0, \frac{1789-261\sqrt{7}}{1906}\big)$;
(ii) $M^{\{\{0\},\{1\},\{2\}\}} \succ_{\{i\}} M_{\beta}^{\{\{0,i\},\{j\}\}}$ in any case of $\beta \in \big[\frac{1789-261\sqrt{7}}{1906}, 1\big]$.

Assume, without loss of generality, $i=1$. For the owner 0, we have

$$V_0^a - V_0^c(\beta) = \frac{a^2(1429 - 3578\beta + 1906\beta^2)}{126(40-13\beta)^2} > 0 \Leftrightarrow 0 \leq \beta < \frac{1789-261\sqrt{7}}{1906} \approx 0.576316. \tag{17}$$

On the other hand, taking into account that $\frac{d(V_1^a - V_1^c(\beta))}{d\beta} = \frac{a^2(14400-12358\beta+2040\beta^2-221\beta^3)}{2(40-13\beta)^3} > 0$ for all $\beta \in [0,1]$, we have

$$V_1^a - V_1^c(\beta) = \frac{-a^2(788840 - 1941586\beta + 928619\beta^2 - 67473\beta^3)}{7938(40-13\beta)^2} > 0, \quad \forall \beta \in \Big[\frac{1789-261\sqrt{7}}{1906}, 1\Big], \tag{18}$$

since $V_1^a - V_1^c(\beta)|_{\beta=\frac{1789-261\sqrt{7}}{1906}} = \frac{(592531-200361\sqrt{7})a^2}{15129828} > 0$. Thus, the statement (i) follows from (17), and (ii) does form (18), respectively.

**Claim (d)**: (i) $M_{\beta=\gamma}^{\{\{0,1\},\{2\}\}} \succ_{\{0,1\}} M_{\gamma,\delta}^{\{\{0,1,2\}\}}$ in any case of $\gamma \in \big[0, \frac{3}{4}\big)$ and $\delta \in [0,1]$;
(ii) $M_{\beta=\frac{\gamma}{\gamma+(1-\gamma)\delta}}^{\{\{0,1\},\{2\}\}} \succ_{\{2\}} M_{\gamma,\delta}^{\{\{0,1,2\}\}}$ in any case of $\gamma \in \big[\frac{3}{4}, 1\big]$ and $\delta \in [0,1]$.

We begin with the proof of (i). For the payoff to the owner of the public firm 0, we have

$$V_0^c(\beta)|_{\beta=\gamma} - V_0^d(\gamma) > 0 \quad \forall \gamma < \frac{3}{4} \tag{19}$$

from the fact that $\frac{d(V_0^c(\beta)|_{\beta=\gamma} - V_0^d(\gamma))}{d\gamma} = \frac{-4a^2(1-\gamma)(324352-300096\gamma+91476\gamma^2-9153\gamma^3)}{(8-3\gamma)^3(40-13\gamma)^3} \leq 0$ for all $\gamma \in [0,1]$ and $V_0^c(\beta)|_{\beta=\frac{3}{4}} - V_0^d(\gamma)|_{\gamma=\frac{3}{4}} = \frac{5213a^2}{15490178} > 0$. For the payoff to the owner of the private firm 1, we also obtain,

$$V_1^c(\beta)|_{\beta=\gamma} - V_1^d(\gamma,\delta)|_{\delta=\frac{1}{2}} > 0, \quad \forall\gamma < \frac{3}{4}, \tag{20}$$

because we have $\frac{d\left(V_1^c(\beta)|_{\beta=\gamma}-V_1^d(\gamma,\delta)|_{\delta=\frac{1}{2}}\right)}{d\gamma} = \frac{-a^2(1228800-3447296\gamma+4271808\gamma^2-2492728\gamma^3+723645\gamma^4-102816\gamma^5+5967\gamma^6)}{2(8-3\gamma)^3(40-13\gamma)^3} < 0$ for all $\gamma \in [0,1]$ and $V_1^c(\beta)|_{\beta=\frac{3}{4}} - V_1^d(\gamma,\delta)|_{\gamma=\frac{3}{4},\delta=\frac{1}{2}} = \frac{412701a^2}{61960712} > 0$. Thus, by (19) and (20), we obtain $M_{\beta=\gamma}^{\{\{0,1\},\{2\}\}} \succ_{\{0,1\}} M_{\gamma<\frac{3}{4},\delta=\frac{1}{2}}^{\{\{0,1,2\}\}}$. Since in the cases of $\delta \neq \frac{1}{2}$ either of the two owners 1 and 2 will receive smaller payoff than in the case of $\delta = \frac{1}{2}$, we can apply the above argument to the owner with the smaller payoff and complete the proof of (i).

Next, we prove (ii). Because $\delta = 0$ is the most favorable case of $\delta$ for the owner 2, it is sufficient to show that $M_{\gamma\geq\frac{3}{4},\delta=0}^{\{\{0,1,2\}\}}$ is blocked by $M_{\beta=\frac{\gamma}{\gamma+(1-\gamma)\delta}}^{\{\{0,1\},\{2\}\}}$, i.e. by $M_{\beta=1}^{\{\{0,1\},\{2\}\}}$, via $\{2\}$. In $M_{\gamma\geq\frac{3}{4},\delta=0}^{\{\{0,1,2\}\}}$, the payoff to the owner of the private firm 2 is given as:

$$V_2^d(\gamma,\delta)|_{\delta=0} = \frac{3a^2(1-\gamma)(4-3\gamma)}{(8-3\gamma)^2}. \tag{21}$$

From the fact that $\frac{dV_2^d(\gamma,\delta)|_{\delta=0}}{d\gamma} = \frac{-3a^2(32-27\gamma)}{(8-3\gamma)^3} < 0$ for all $\gamma \in \left[\frac{3}{4},1\right]$,

$$\max_{\gamma\in[\frac{3}{4},1]} V_2^d(\gamma,\delta)|_{\delta=0} = V^d\left(\frac{3}{4},0\right) = \frac{21a^2}{529}. \tag{22}$$

On the other hand, in the market structure $M_{\beta=1}^{\{\{0,1\},\{2\}\}}$, the payoff to the owner of the private firm 2 is

$$V_2^c(\beta)|_{\beta=1} = \frac{2a^2(7-3\beta)^2}{(40-13\beta)^2}\Big|_{\beta=1} = \frac{32a^2}{729} > \frac{21a^2}{529} = \max_{\gamma\in[\frac{3}{4},1]} V_2^d(\gamma,\delta)|_{\delta=0}. \tag{23}$$

Thus, $M_{\beta=1}^{\{\{0,1\},\{2\}\}} \succ_{\{2\}} M_{\gamma,\delta=0}^{\{\{0,1,2\}\}}$ for any case of $\gamma \in \left[\frac{3}{4},1\right]$. ■

By the definition of the core, in the case of the L-Stackelberg competition, none of the market structures is stable in the sense that there always exists at least one owner who wants to deviate and induce a new market structure: in the regime (a), the private owners 1 and 2; in (b), the coalition of the owners 0 and $i$ $(=1,2)$ or a single private owner $j$ $(=1,2)$; in (c), either of the owners 0 and $i$ $(=1,2)$; and in (d), the two owners 0 and $i$ $(=1,2)$, or a single private owner $j$ $(=1,2)$.

As we have shown in Claim (d), the market structures in the regime (d) are blocked by those in the regime (c). This result is due to that the positive effect of the improvement on productivity entailed through the shift of a market structure from the regime (c) (or also (b)) into (d) is relatively smaller than in the cases of the shift from (a) into (c) (or also (b)). We obtain basically the same result for each of the other equilibrium concepts: the Cournot equilibrium and the F-Stackelberg equilibrium (on this, see Lemma 1 in Kamaga and Nakamura (2007) and our Claim (d)' we will present later). As a consequence, although our definition of the inducibility among the market structures does not allow the case where the market structure of the mixed triopoly is directly induced from those in the regime (d), it never detracts the generality of our core analysis.

Comparing the result in the Cournot competition and that of the L-Stackelberg competition, it can be said that in our model of the productivity-improving mergers the order of the firms' moves affects the stability of market structures. The difference between the two results is due to the two facts. One is that every owner gains higher payoff in the mixed triopoly in the L-Stackelberg equilibrium than in the Cournot equilibrium. The other is that the profit of the private merged firm 12 in the L-Stackelberg equilibrium is higher than in the Cournot equilibrium and, moreover, it increases at a higher rate than the profit of a private firm in the mixed triopoly: the former is $\left(\frac{54a^2}{529}-\frac{3a^2}{32}\right)/\frac{3a^2}{32}\approx 0.089$, and the latter is $\left(\frac{200a^2}{3969}-\frac{8a^2}{169}\right)/\frac{8a^2}{169}\approx 0.064$, i.e. the improvement on productivity by the merger between the private firms has relatively large impact in the L-Stackelberg equilibrium rather than in the Cournot equilibrium. This latter fact is ascribed to the coordinating role of the public firm 0 which, anticipating the subsequent decision making by the private merged firm 12 that has relatively efficient production technology, decreases its output level $q_0^{b*}$ from the level in the Cournot equilibrium, $\frac{a}{4}$, to $\frac{5a}{23}$ so as to maximize social welfare through a possible subsequent increase of the output by the merged firm 12. Kamaga and Nakamura (2007), in their lemma 2, showed that in the Cournot competition the market structure of the mixed triopoly $M^{\{\{0\},\{1\},\{2\}\}}$ blocks any market structure of the merger between the private firms $M_{\alpha}^{\{\{0\},\{1,2\}\}}$, and, in their lemma 3, that the market structure of the merger between the public firm and a private firm $M_{\beta\in(\underline{\beta},\bar{\beta})}^{\{\{0,i\},\{j\}\}}$ blocks the mixed triopoly. In the L-Stackelberg equilibrium, these two forms of blocking are no longer possible. Because of the expansion of its output level in the regime (b) from $\frac{a}{4}$ to $\frac{6a}{23}$, the profit of the merged firm 12 becomes high enough to induce the deviation of the private owners $\{1,2\}$ from the mixed triopoly (Claim (a)). However, since the regime (b) is less competitive than the mixed triopoly, the total output in the regime (b), $\frac{11a}{23}$, is now smaller than in the mixed triopoly, $\frac{11a}{21}$, and the social welfare decreases from the level in the regime (a), $\frac{37a^2}{126}$, to $\frac{13a^2}{46}$. Consequently, while any market structure of the regime (c) is blocked by the mixed triopoly (Claim (c)), some of the market structures of the regime (c) blocks those in the regime (b) with $\alpha \in \left[\frac{52900}{107163}, \frac{54263}{107163}\right]$ (Claim (b)).

We next examine another form of the Stackelberg competition where the public firm 0 and the public-private firm $0i$ now act as a Stackelberg follower, i.e. the F-Stackelberg competition. As in the case of the L-Stackelberg competition, the equilibrium payoffs in the regime (d), $(V_0^d, V_1^d, V_2^d)$, in the F-Stackelberg equilibrium are given by (10d). In the other three regimes, the equilibrium payoffs in the F-Stackelberg equilibrium are obtained as:

$$(V_0^a, V_1^a, V_2^a) = \left(\frac{8a^2}{27}, \frac{5a^2}{108}, \frac{5a^2}{108}\right); \tag{24a}$$

$$(V_0^b, V_1^b, V_2^b) = \left(\frac{85a^2}{294}, \frac{2\alpha a^2}{21}, \frac{2(1-\alpha)a^2}{21}\right); \tag{24b}$$

$$(V_0^c, V_i^c, V_j^c) = \left(\frac{(508-704\beta+349\beta^2-72\beta^3+5\beta^4)a^2}{8(15-11\beta+2\beta^2)^2}, \frac{(8-3\beta)^2(1-\beta)(3-2\beta)a^2}{8(15-11\beta+2\beta^2)^2}, \frac{(2-\beta)^2a^2}{60-44\beta+8\beta^2}\right) \tag{24c}$$

Unfortunately, we still obtain the following impossibility result in the F-Stackelberg competition, either.

**Proposition 3.** *In the case of the L-Stackelberg competition, none of the market structures belongs to the core, i.e.* $\mathfrak{Co} = \emptyset$.

*Proof.* The proof is basically same as that of Proposition 2. We limit ourselves to providing the following claims (a)' to (d)', and omit the detailed proofs similar to those in the proof of Proposition 2.

**Claim (a)'**: $M^{\{\{0\},\{1,2\}\}}_{\alpha\in(\underline{\alpha},\bar{\alpha})} \succ_{\{1,2\}} M^{\{\{0\},\{1\},\{2\}\}}$, where $\underline{\alpha} = \frac{35}{72}$ and $\bar{\alpha} = \frac{37}{72}$.

**Claim (b)'**: (i) $M^{\{\{0\},\{1\},\{2\}\}} \succ_{\{1\}} M^{\{\{0\},\{1,2\}\}}_{\alpha}$ in any case of $\alpha \in \left[0, \frac{35}{72}\right)$;
(ii) $M^{\{\{0,1\},\{2\}\}}_{\beta=1/2} \succ_{\{0,1\}} M^{\{\{0\},\{1,2\}\}}_{\alpha}$ in any case of $\alpha \in \left[\frac{35}{72}, \frac{37}{72}\right]$;

(iii) $M^{\{\{0\},\{1\},\{2\}\}} \succ_{\{2\}} M_{\alpha}^{\{\{0\},\{1,2\}\}}$ in any case of $\alpha \in \left(\frac{37}{72}, 1\right]$.

**Claim (c)'**: (i) $\mathscr{M}^{\{\{0\},\{1\},\{2\}\}} \succ_{\{0\}} M_{\beta}^{\{\{0,i\},\{j\}\}}$ in any case of $\beta \in \left[0, \frac{3}{5}\right)$;
(ii) $\mathscr{M}^{\{\{0\},\{1\},\{2\}\}} \succ_{\{i\}} M_{\beta}^{\{\{0,i\},\{j\}\}}$ in any case of $\beta \in \left[\frac{3}{5}, 1\right]$.

**Claim (d)'**: (i) $M_{\beta=\gamma}^{\{\{0,1\},\{2\}\}} \succ_{\{0,1\}} M_{\gamma,\delta}^{\{\{0,1,2\}\}}$ in any case of $\gamma \in \left[0, \frac{4}{5}\right)$ and $\delta \in [0,1]$;
(ii) $M_{\beta=\frac{\gamma}{\gamma+(1-\gamma)\delta}}^{\{\{0,1\},\{2\}\}} \succ_{\{2\}} M_{\gamma,\delta}^{\{\{0,1,2\}\}}$ in any case of $\gamma \in \left[\frac{4}{5}, 1\right]$ and $\delta \in [0,1]$. ■

In the case of the F-Stackelberg competition, we obtain the same pattern of blockings between the market structures. As in the case of the L-Stackelberg equilibrium, the key is that, by the coordinating role of the public firm 0, the merged firm 12 in the regime (b) can achieve relatively higher profit than in the case of the Cournot equilibrium.

Finally, it should be noted that the two impossibility results obtained in Propositions 2 and 3 are due to the positive effect of the improvement on productivity entailed by a merger which is formulated in (6). From a series of deviations demonstrated in the proofs of the propositions, it is easily verified that if this positive effect is weakened the market structure of the mixed triopoly will solely belong to the core.

# 5 Concluding remarks

This paper examined a merger activity in mixed oligopoly especially focusing on the stability of owners' coalition formations. As in the paper of Kamaga and Nakamura (2007), we adopted the core as the solution concept to analyze the stability of coalition formations, and we mainly considered the two alternative forms of the Stackelberg competition: one is the L-Stackelberg equilibrium where the public firm or the public-private merged firm acts as a Stackelberg leader, and the other is the F-Stackelberg equilibrium where each of these firms acts as a Stackelberg follower. In contrast to the case of the Cournot equilibrium examined in Kamaga and Nakamura (2007), we obtained a striking but serious impossibility result that in each case of the two alternative forms of the Stackelberg competition the core must be empty. In other words, none of the market structures is stable in either case. As noted in the last section, the mixed triopoly will uniquely belong to the core if the positive effect on the production cost entailed by a merger is weakened.

Two interesting extensions of our model remain. The first is to assume the separation between ownership and management and to introduce managerial delegation in a firm. In the current paper, we assumed that all of the three firms are entrepreneurial ones to make our analysis as simple as possible. In the case of the separation of ownership and management, we should reformulate the model to consider the problem of strategic delegation in the contract à la Fershtman, Judd, and Sklivas (Fershtman and Judd (1987); Sklivas (1987)). The other possible extension is to consider the model in which the foreign shareholders are taken into account. In this case, social welfare that the government aims to maximize should become independent of the profits of the foreign-owned firms, and thus, the public firm's decision making and resulting equilibrium outcomes will change. These issues are left for future research.

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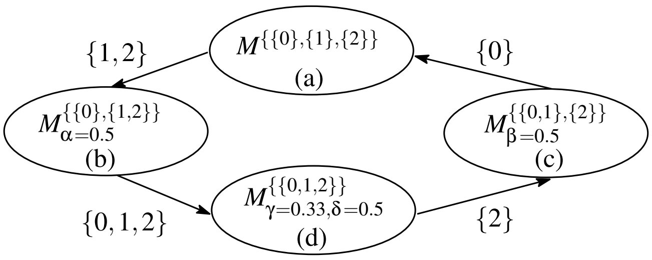


Figure 1: A typical market structure in each regime (r) and a cycle in deviations

Table 1: Firms' profit functions in each regime (r)

| (r) | profit $\Pi_i^r$ |
|---|---|
| (a) | $\Pi_i^a = \left[a - (q_0 + \sum_{j=1}^2 q_j)\right] q_i - (q_i)^2 \quad (i = 0,1,2)$ |
| (b) | $\Pi_0^b = \left[a - (q_0 + q_{12})\right] q_0 - (q_0)^2$, <br> $\Pi_{12}^b = \left[a - (q_0 + q_{12})\right] q_{12} - (q_{12})^2/2$ |
| (c) | $\Pi_{0i}^c = \left[a - (q_{0i} + q_j)\right] q_{0i} - (q_{0i})^2/2$, <br> $\Pi_j^c = \left[a - (q_{0i} + q_j)\right] q_j - (q_j)^2 \quad (i,j = 1,2,\ i \neq j)$ |
| (d) | $\Pi_{012}^d = (a - q_{012}) q_{012} - (q_{012})^2/3$ |

Table 2: Firms' objectives in each regime (r)

| (r) | objective function $U_i^r$ |
|---|---|
| (a) | $U_0^a = W^a = (q_0 + \sum_{j=1}^2 q_j)^2/2 + \Pi_0^a + \sum_{j=1}^2 \Pi_j^a$, <br> $U_i^a = \Pi_i^a \quad (i = 1,2)$ |
| (b) | $U_0^b = W^b = (q_0 + q_{12})^2/2 + \Pi_0^b + \Pi_{12}^b$, <br> $U_{12}^b = \Pi_{12}^b$ |
| (c) | $U_{0i}^c = \beta W^c + (1-\beta)\Pi_{0i}^c = \beta\left[(q_{0i} + q_j)^2/2 + \Pi_{0i}^c + \Pi_j^c\right] + (1-\beta)\Pi_{0i}^c$, <br> $U_j^c = \Pi_j^c \quad (i,j = 1,2,\ j \neq i)$ |
| (d) | $U_{012}^d = \gamma W^d + (1-\gamma)\Pi_{012}^d = \gamma\left[(q_{012})^2/2 + \Pi_{012}^d\right] + (1-\gamma)\Pi_{012}^d$ |

Table 3: Equilibrium outputs $q^{r*}$ in each regime (r)

| (r) | | Cournot | L-Stackelberg | F-Stackelberg |
|---|---|---|---|---|
| (a) | $q^{a*} = \left(q_0^{a*}, q_1^{a*}, q_2^{a*}\right)$ | $\left(\frac{3a}{13}, \frac{2a}{13}, \frac{2a}{13}\right)$ | $\left(\frac{13a}{63}, \frac{10a}{63}, \frac{10a}{63}\right)$ | $\left(\frac{2a}{9}, \frac{a}{6}, \frac{a}{6}\right)$ |
| (b) | $q^{b*} = \left(q_0^{b*}, q_{12}^{b*}\right)$ | $\left(\frac{a}{4}, \frac{a}{4}\right)$ | $\left(\frac{5a}{23}, \frac{6a}{23}\right)$ | $\left(\frac{5a}{21}, \frac{2a}{7}\right)$ |
| (c) | $q^{c*} = \left(q_{0i}^{c*}, q_j^{c*}\right)$ | $\left(\frac{3a}{11-4\beta}, \frac{(2-\beta)a}{11-4\beta}\right)$ | $\left(\frac{(12-\beta)a}{40-13\beta}, \frac{(7-3\beta)a}{40-13\beta}\right)$ | $\left(\frac{(8-3\beta)a}{(5-2\beta)(6-2\beta)}, \frac{(2-\beta)a}{2(5-2\beta)}\right)$ |
| (d) | $q^{d*} = q_{012}^{d*}$ | $\frac{3a}{8-3\gamma}$ | same as left | same as left |

Table 4: Owners' payoffs $V_i^r$ in each regime (r)

| (r) | payoffs $(V_0^a, V_1^a, V_2^a)$ |
|---|---|
| (a) | $\left(V_0^a, V_1^a, V_2^a\right) = \left(W^{a*},\ \Pi_1^{a*},\ \Pi_2^{a*}\right)$ |
| (b) | $\left(V_0^b, V_1^b, V_2^b\right) = \left(W^{b*},\ \alpha\Pi_{12}^{b*},\ (1-\alpha)\Pi_{12}^{b*}\right)$ |
| (c) | $\left(V_0^c, V_i^c, V_j^c\right) = \left(W^{c*},\ (1-\beta)\Pi_{0i}^{c*},\ \Pi_j^{c*}\right), \quad (i, j = 1, 2,\ i \neq j)$ |
| (d) | $\left(V_0^d, V_1^d, V_2^d\right) = \left(W^{d*},\ (1-\gamma)\delta\Pi_{012}^{d*},\ (1-\gamma)(1-\delta)\Pi_{012}^{d*}\right)$ |