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## The Core of Productivity-Improving Mergers in Mixed Oligopoly: A Case of Managerial Delegation

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# The Core of Productivity-Improving Mergers in Mixed Oligopoly: A Case of Managerial Delegation

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**Abstract**

We analyze the stability problem of productivity-improving mergers in mixed oligopoly. While it is known that the core of market structures is non-empty in the case of entrepreneurial firms, we show that the core is empty in the case where managerial delegation is introduced.

*Keywords*: Mergers; Managerial delegation; Mixed oligopoly; The core of market structures

*JEL classification*: D21; L13; L33

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# 1 Introduction

This paper provides a theoretical analysis on merger activities in the oligopoly composed not only of private firms which maximize their profits but also of a public firm which is a welfare maximizer. This type of market is usually referred to as *mixed oligopoly*. The studies of the mixed oligopoly go back to De Fraja and Delbono (1989). Merger activities in mixed oligopoly have been analyzed by Bárcena-Ruiz and Gárzon (2003), Coloma (2006), Kamijo and Nakamura (2007), and Kamaga and Nakamura (2007). Among them, Kamijo and Nakamura (2007) and Kamaga and Nakamura (2007) focus on the stability of coalition formations of firms' owners. In the literature on private oligopoly, Barros (1998), Horn and Persson (2001), and Straume (2006) also discuss the stability problem of merger activities. Along the lines of these works, Kamijo and Nakamura (2007) and Kamaga and Nakamura (2007) adopt the *core* as the solution concept of owners' coalition formations. In these two papers, each of a public firm and private firms is assumed to be an entrepreneurial one, i.e. every managerial decision making is carried out by the owner of the firm in question. Kamijo and Nakamura consider the case of linear cost functions and show that the core of owners' coalition formations is non-empty. On the other hand, Kamaga and Nakamura work with the model of quadratic cost functions. They assume that, as in the paper of McAfee and Williams (1992), a merger among firms entail a synergy effect on the productivity of the merged firm, and show that the core of such productivity-improving mergers is also non-empty.

The purpose of this paper is to examine whether the core is still non-empty when we introduce managerial delegation in the model of Kamaga and Nakamura (2007). In the literature on mixed oligopoly, Barros (1995) and White (2001) provide the analyses of the managerial delegation à la Fershtman and Judd (1987) and Sklivas (1987), or so-called FJS contract. In this paper, we extend the model of Kamaga and Nakamura (2007) by introducing the FJS contract in the same manner as in Straume (2006) that works with the model of private oligopoly and examines the core of mergers. In contrast to the case of entrepreneurial firms, each firm's owner(s) delegates the output decision to a manager. Each manager sets the output to maximize her/his payoff defined by an incentive contract provided by the firm's owner(s). Our interest lies on which type of ownership structure, or which type of merger, chosen by the owners will be stable. The result obtained in this paper is striking but seriously negative. In contrast to the result in Kamaga and Nakamura (2007), we obtain that the core of the owners' coalition formations is empty in our delegation model. In other words, if we additionally introduce managerial delegation in their model, none of the market structures is stable in the sense that in any market structure there always exists at least one firm's owner who decides to deviate from this current market structure.

The rest of the paper is organized as follows. In Section 2, we extend the model of Kamaga and Nakamura (2007) by introducing managerial delegation. In Section 3, we provide the formal definition of the core of market structures and show that none of the market structures belongs to the core in our delegation model. Section 4 concludes.

# 2 Model

## 2.1 Incentive contracts

We analyze a merger activity in the mixed triopoly, i.e. in the industry composed of a public firm and two private firms. We extend the model set up by Kamaga and Nakamura (2007) by introducing managerial delegation. A public firm is denoted by 0 and two private firms by 1 and 2, respectively. A merged firm organized by pre-merged firms $i$ and $j$ is denoted by simply combining the notation just as $ij$. Each firm competes in the same market, i.e. producing a homogeneous good. In this paper, each of the three firms is assumed to be managerial one, i.e. each owner delegates the output decision to a manager. To make the analysis simple, we assume that the public firm is owned by the government and each of the private firms by a single private shareholder, i.e. each firm is owned by a single agent.

To formalize managerial delegation, we mainly follow Straume (2006). In each firm, an owner delegates the output decision to a manager. Each manager sets the output to maximize his payoff defined by an incentive contract provided by the owner of the firm. Let $q_i$ and $\Pi_i$ denote the output and profit of a firm $i$. In each firm $i$, an owner provides the following type of incentive contract $\phi_i$ to a manager:

$$\phi_i(\Pi_i(q_i), q_i; \theta_i) = \theta_i \Pi_i(q_i) + (1-\theta_i)q_i. \tag{1}$$

where $\theta_i$ is a contract parameter chosen by the owner (or owners in a merged firm). A manager of a firm $i$ can maximize her/his payoff by choosing the output $q_i$ which maximizes $\phi_i$. This can be supported by the assumption that the payoff to a manager of a firm $i$ is represented as $\lambda_i + \mu_i\phi_i$ for some real number $\lambda_i$ and some positive real number $\mu_i$.

## 2.2 Productivity-improving mergers and four market regimes

In the model of mixed triopoly, we should distinguish the following four different market regimes (a) to (d) in accordance with which type of merger is actually realized among the owners: (a) *mixed triopoly*, i.e. the case where a merger does not take place; (b) *merger between private firms*; (c) *merger between a public firm and a private firm*; and (d) *merger among all the three firms*. We start with introducing the regime of the mixed triopoly, i.e. (a). The inverse demand function is linear in the total output $Q$,

$$P(Q) = a - Q, \tag{2}$$

where $a$ is sufficiently large positive number. Each firm $i$ $(= 0, 1, 2)$ has an identical technology represented by the quadratic cost function

$$C(q_i) = q_i^2, \tag{3}$$

Consequently, the profit function of the firm $i$ $(= 0, 1, 2)$ is given as:

$$\Pi_i = (a - Q)q_i - q_i^2. \tag{4}$$

Next, we introduce the other three regimes (b), (c), and (d). We assume that a merger entails the improvement on productivity in a merged firm. Consequently, the three regimes (b), (c), and (d) are different from the mixed triopoly especially in the definition of the cost function of merged firms which reflect the synergy effect entailed by mergers. To formalize such a positive effect of mergers, we follow McAfee and Williams (1992). If $n$ $(= 2,3)$ firms merge into one firm, the total cost of the merged firm $C_m$ is represented as:

$$C_m(q_m) = \frac{q_m^2}{n}, \tag{5}$$

where $q_m$ is the output of the merged firm $m$. This type of cost function is also considered in Heywood and McGinty (2007a; 2007b), Nakamura and Inoue (2007), and Kamaga and Nakamura (2007) and is supported by the assumption that the merged firm operates the plants previously owned by the pre-merged firms in the most efficient way. The profit of a merged firm is given by replacing $q_i^2$ with $q_m^2/n$ in (4). We summarize firms' profit functions in each of the four regimes in Table 1, where profit functions with superscript $r$ $(= a,b,c,d)$ denote those considered in the regime $r$. In the rest of the paper, functions and variables with superscript $r$ $(= a,b,c,d)$ denote those considered in the regime $r$.

[Insert Table 1 around here]

In our delegation model, owners and managers play the following two-stage game: in the first stage, owners simultaneously choose incentive contracts for managers, then in the second stage, the firms' managers simultaneously set the outputs. The equilibrium outcomes of each regime are those derived by the subgame perfect Nash equilibrium.

From the routine backward calculation, for a given list of contract parameters, managers' equilibrium outputs, each denoted $q_i^{mr}$, are determined as follows. In regime (a), the equilibrium outputs set by the managers of the public firm 0 and private firms $i$ ($i = 1,2$) are

$$q_0^{ma} = \frac{1}{18}\left(3(a-1) + \frac{5}{\theta_0^a} - \frac{1}{\theta_1^a} - \frac{1}{\theta_2^a}\right) \text{ and } q_i^{ma} = \frac{1}{18}\left(3(a-1) - \frac{1}{\theta_0^a} + \frac{5}{\theta_i^a} - \frac{1}{\theta_j^a}\right), \quad (6)$$

where $j = 1,2$ and $j \neq i$. In regime (b), the equilibrium outputs of the managers of the public firm 0 and the private merged firm 12 are given as

$$q_0^{mb} = \frac{1}{11}\left(2(a-1) + \frac{3}{\theta_0^b} - \frac{1}{\theta_{12}^b}\right) \text{ and } q_{12}^{mb} = \frac{1}{11}\left(3(a-1) - \frac{1}{\theta_0^b} + \frac{4}{\theta_{12}^b}\right). \quad (7)$$

In regime (c), the equilibrium outputs set by the managers of the public-private merged firm $0i$ and private firm $j$ ($i, j = 1,2$ with $i \neq j$) are

$$q_{0i}^{mc} = \frac{1}{11}\left(3(a-1) + \frac{4}{\theta_{0i}^c} - \frac{1}{\theta_j^c}\right) \text{ and } q_j^{mc} = \frac{1}{11}\left(2(a-1) - \frac{1}{\theta_{0i}^c} + \frac{3}{\theta_j^c}\right). \quad (8)$$

Finally, in regime (d), the manager of the merged firm 012 sets the following output

$$q_{012}^{md} = \frac{1}{8}\left(3(a-1) + \frac{3}{\theta_{012}^d}\right). \quad (9)$$

To derive equilibrium outcomes, we next define the owners' objectives which they will maximize through their choices of contract parameters. The owner of the public firm 0 is assumed to be a welfare maximizer. On the other hand, the owners of the private firms are profit maximizers. As usual, social welfare $W$ is measured by the sum of consumer surplus $CS = Q^2/2$, and firms' profits. Let $U_i^r$ denote an objective function that the owner(s) of firm $i$ maximizes in regime $r$ ($= a,b,c,d$). In the regime (a), the owners' objectives are given as:

$$U_0^a(\theta_0^a;\theta_1^a,\theta_2^a) = W^a = \frac{1}{2}\left(q_0^{ma} + \textstyle\sum_{i=1}^2 q_i^{ma}\right)^2 + \Pi_0(\mathbf{q}^{ma}) + \textstyle\sum_{i=1}^2 \Pi_i(\mathbf{q}^{ma}), \quad (10)$$

$$U_i^a(\theta_i^a;\theta_0^a,\theta_j^a) = \Pi_i(\mathbf{q}^{ma}), \quad (i, j = 1,2 \text{ and } i \neq j). \quad (11)$$

The other three regimes (b), (c), and (d) contain a joint-ownership in the merged firms. The owners of a merged firm need to jointly constitute the objective which they maximize through the choice of their most favorable incentive contract to a manager. In the regime (b), the merged firm 12 is still completely private-owned, and thus the common objective of the owners in the firm 12 is defined as the maximization of the profit $\Pi_{12}$. On the other hand, in the regimes (c) and (d), the merged firm 01 (or 02) and 012 are jointly owned by the owners of the pre-merged public firm 0 and the pre-merged private firm 1 and/or 2. Therefore, the objective constituted by the owners in each of these merged firms should reflect both two different kinds of objective: welfare maximization and profit maximization. On the objective constituted by the owners of a public-private merged firm, we follow Matsumura (1998) and consider the weighted sum of social welfare and the profit of the merged firm. Let $\beta \in [0,1]$ be the shareholding ratio by the owner of the public firm 0 in the merged firm $0i$ ($i = 1,2$), and also $\gamma \in [0,1]$ denote the shareholding ratio by the owner of the firm 0 in the merged firm 012. We assume that the shareholding ratio $\beta$ (resp. $\gamma$) by the owner of the public firm 0 directly measures the weight on social welfare in the constituted objective in the merged firm $0i$ (resp. 012). In Table 2, we summarize the (constituted) objectives of owners for each regime r.

[Insert Table 2 around here]

### 2.3 Equilibrium outcomes

We are ready to present the equilibrium outcomes for each of the four market regimes. Taking into account of the outputs subsequently realized in the second stage: (6), (7), (8), and (9), the owners choose their optimal incentive contracts to maximize their (constituted) objectives. Tables 3 and 4 summarize the equilibrium incentive contracts $\theta_i^{r*}$ and the equilibrium outputs $q^{r*}$, profits $\Pi_i^{r*}$, and social welfare $W^{r*}$ in each regime r.

[Insert Table 3 around here]

[Insert Table 4 around here]

# 3 Merger as a coalition formation and the core

It is the owners' decision-making on mergers that determines which of the market regimes is actually realized. Merger activities can be analyzed in terms of coalition formations among firms' owners. In the analysis of coalition formations, the most fundamental and most important problem is the stability of coalition formations. In the literature on mergers in private oligopoly, Barros (1998), Horn and Persson (2001), Straume (2006) adopted the core as the solution concept of coalition formations and analyzed the stability of mergers. Along the lines of these works, Kamijo and Nakamura (2007) and Kamaga and Nakamura (2007) examined the stability of mergers in mixed oligopoly. In the rest of the paper, we explore which types of coalition formation is stable in our delegation model.

To analyze the owners' decision-making on merger activities, we need to define the payoffs to the owners. In what follows, we use the notation 0, 1, and 2 to denote the owner of the firm 0, 1, and 2, respectively, as well as to denote each of these firms. Let $\alpha \in [0,1]$ be the shareholding ratio by the owner 1 in the merged firm of the regime (b), i.e. in the private merged firm 12, and also $\delta \in [0,1]$ be the distribution ratio of the profit of the merged firm 012 between the owners 1 and 2. We interpret that $(1-\gamma)\delta$ (resp. $(1-\gamma)(1-\delta)$) is the shareholding ratio by the owner 1 (resp. the owner 2) in the merged firm 012. The payoff to the owner 0 in a regime r $(= a,b,c,d)$, denoted by $V_0^r$, is defined as the equilibrium social welfare $W^{r*}$ in the regime r, and those to the owners 1 and 2 are (i) the equilibrium profits of their own firms in the regime (a) and (ii) the distributed equilibrium profits determined according to their shareholding ratio(s) in the merged firm in the regimes (b), (c), and (d). Since the owner of the public firm 0 is assumed to be a welfare maximizer and the owners of the two private firms 1 and 2 to be profit maximizers, the payoffs defined in this way will seem quite reasonable. Table 4 summarizes the payoffs to the owners for each regime (r).

[Insert Table 5 around here]

From the equilibrium outcomes presented in Table 4, the payoffs to the owners, $(V_0^r, V_1^r, V_2^r)$, are explicitly given as follows:

$$(V_0^a, V_1^a, V_2^a) = \left(\frac{49659}{167042}a^2, \frac{4000}{83521}a^2, \frac{4000}{83521}a^2\right); \tag{12a}$$

$$(V_0^b, V_1^b, V_2^b) = \left(\frac{113}{392}a^2, \frac{5\alpha}{49}a^2, \frac{5(1-\alpha)}{49}a^2\right); \tag{12b}$$

$$(V_0^c, V_i^c, V_j^c) = \left(\frac{(6839-4058\beta+483\beta^2)}{2(109-35\beta)^2}a^2, \frac{2(1-\beta)(640-408\beta+23\beta^2)}{(109-35\beta)^2}a^2, \frac{15(7-3\beta)^2}{(109-35\beta)^2}a^2\right); \tag{12c}$$

$$(V_0^d, V_1^d, V_2^d) = \left(\frac{3(11-6\gamma)}{2(8-3\gamma)^2}a^2, \frac{3(1-\gamma)(4-3\gamma)\delta}{(8-3\gamma)^2}a^2, \frac{3(1-\gamma)(4-3\gamma)(1-\delta)}{(8-3\gamma)^2}a^2\right). \tag{12d}$$

We now move to examine the core of market structures. To introduce the formal definition of the core of market structures, we start with providing the formal description of a market structure. A market structure is characterized in terms of coalition formations among the three owners 0, 1, and 2, and also of the shareholding ratios by the participating owners in the merged firm. We denote by $M_t^{\{C\}}$ a market structure composed of a coalition formation $C$ with a shareholding ratio $t$ in the merged firm (if exists), where $\{C\}$ is a partition of the set of the owners $\{0,1,2\}$ and $t$ will be $\alpha$ in the regime (b); $\beta$ in (c); and a pair of $\gamma$ and $\delta$ in (d). A partition of a set $A$ is a set of subsets of $A$ such that (i) the union of its elements is equal to $A$, and (ii) the intersection of any two of its elements is empty. We allow the case where a partition of $\{0,1,2\}$ contains an empty set as an element. For example, the market structure of the merger between the public firm 0 and the private firm 1 with the owner 0's shareholding ratio $\beta = 0.5$ is denoted by $M_{\beta=0.5}^{\{\{0,1\},\{2\}\}}$.

Next, we define the inducement relations among the market structures. A market structure $M$ is said to be **inducible from $M'$ via** a coalition $S \subseteq \{0,1,2\}$ if, given that the coalition structure in $M'$ is represented as the partition $\{S'_1, \ldots, S'_n\}$, the coalition structure in $M$ is described as the partition $\{S, S_1, \ldots, S_n\}$ such that

$$S_i = \begin{cases} S'_i & \text{if } S \cap S'_i = \varnothing, \\ S'_i \backslash S & \text{if } S \cap S'_i \neq \varnothing, \end{cases} \tag{13}$$

for all $i = 1, \ldots, n$. In other words, a market structure $M$ is inducible from a structure $M'$ via a coalition $S$ if the deviant coalition $S$ can generate the new structure $M$ only through their deviation without any cooperation of the owners outside of the coalition. We should give two remarks about the inducibility defined above. The first is that the above definition of inducibility among market structures allows the case where the deviation by the stand-alone coalition $\{0\}$ (resp. $\{i\}$ $(i = 1, 2)$) from $M^{\{\{0,1,2\}\}}_{\gamma\in[0,1],\delta\in[0,1]}$ can generate the market structure $M^{\{\{0\},\{1,2\}\}}_{\alpha}$ with any of her/his desired ratios $\alpha \in [0, 1]$ (resp. $M^{\{0,j\},\{i\}}_{\beta}$ with any of her/his desired ratios $\beta \in [0, 1]$). Since it seems unreasonable to assume that the deviant owner has a decisive influence on the shareholding ratios in the merged firm organized by the rest of the owners, we restrict admissible inducement relations as follows: in the case of the deviation by $\{0\}$ from $M^{\{\{0,1,2\}\}}_{\gamma\in[0,1],\delta\in[0,1]}$,

$$M^{\{\{0\},\{1,2\}\}}_{\alpha} \text{ with } \alpha = \delta \tag{14}$$

is solely inducible; and in the case of the deviation by $\{1\}$ (resp. $\{2\}$) from $M^{\{\{0,1,2\}\}}_{\gamma\in[0,1],\delta\in[0,1]}$,

$$M^{\{\{0,1\},\{2\}\}}_{\beta} \text{ with } \beta = \frac{\gamma}{\gamma + (1-\gamma)\delta} \quad \left(\text{resp. } M^{\{\{0,2\},\{1\}\}}_{\beta} \text{ with } \beta = \frac{\gamma}{\gamma + (1-\gamma)(1-\delta)}\right) \tag{15}$$

is solely inducible, i.e. the shareholding ratios are determined according to those of the two owners in the merged firm 012. In the rest of the paper, we restrict our analysis to the admissible inducement relations and use the terms "inducement relations" and "inducible" to mean those restricted to the admissible ones. The second remark is that our definition does not allow the case where market structure of the mixed triopoly is directly induced from the market structure of the merger among all the three firms, i.e. that of the regime (d). We eliminate such a case to make the analysis simple. However, as will be shown later, this simplification never detracts the generality and relevance of our core analysis.

A market structure $M$ is said to **block** a market structure $M'$ **via** a coalition $S \subseteq \{0,1,2\}$ if (i) $M$ is inducible from $M'$ via $S$, and (ii) $V_i > V_i'$ for all $i \in S$, where $V_i$ (resp. $V_i'$) denotes the payoff to $i$ in $M$ (resp. in $M'$). By definition, if a market structure $M$ blocks $M'$ via a coalition $S$, each of the owners in the coalition $S$ has an incentive to deviate from $M'$ and induce $M$. Consequently, a blocked market structure will never be realized. We write $M \succ_S M'$ to mean $M$ blocks $M'$ via $S$. We are now ready to define the core of the market structures.

The **core** is the set of market structures each of which is never blocked by any other market structure. In other words, a market structure $M$ belongs to the core if there is no market structure $M'$ such that $M' \succ_S M$ for some $S \subseteq \{0,1,2\}$. We denote the core of the market structures by $\mathfrak{Co}$. By definition, if a market structure is in the core, all of the three owners have no incentive to deviate and to induce any other market structure. In this sense, a market structure in the core is regarded as a stable one.

Kamaga and Nakamura (2007) has shown that the core is non-empty in the case of entrepreneurial firms, and that the core consists solely of the market structures of the merger between the public firm 0 and one of the two private firms with the share ratio near around 0.57. In contrast to their result, we obtain the following striking but serious impossibility result in our delegation model.

**Proposition 1.** *None of the market structures belongs to the core, i.e.* $\mathfrak{Co} = \varnothing$.

*Proof.* The proof proceeds through a series of claims (a) to (d). In each claim (r) (= a, b, c, d), it will be shown that, for any market structure $M$ in the regime (r), we can find some other market structure which blocks $M$ via some coalition $S \subseteq \{0,1,2\}$, i.e. we prove by explicitly presenting $M'$ and $S$ such that $M' \succ_S M$.

**Claim (a)**: $M^{\{\{0\},\{1,2\}\}}_{\alpha\in(\underline{\alpha},\bar{\alpha})} \succ_{\{1,2\}} M^{\{\{0\},\{1\},\{2\}\}}$, where $\underline{\alpha}=\frac{39200}{83521}$ and $\bar{\alpha}=\frac{44321}{83521}$.

This claim is easily checked as follows:

$$V_1^b(\alpha)-V_1^a=\frac{-5a^2(39200-83521\alpha)}{4092529}>0 \;\Leftrightarrow\; \alpha>\frac{39200}{83521}, \tag{16}$$

$$V_2^b(\alpha)-V_2^a=\frac{5a^2(44321-83521\alpha)}{4092529}>0 \;\Leftrightarrow\; \alpha<\frac{44321}{83521}. \tag{17}$$

Thus, the joint deviation by $\{0,1\}$ will take place if $\alpha\in\left(\frac{39200}{83521},\frac{44321}{83521}\right)$.

**Claim (b)**: (i) $M^{\{\{0\},\{1\},\{2\}\}} \succ_{\{1\}} M^{\{\{0\},\{1,2\}\}}_{\alpha}$ in any case of $\alpha\in\left[0,\frac{39200}{83521}\right)$;
(ii) $M^{\{\{0,i\},\{j\}\}}_{\beta=1/3} \succ_{\{0,1\}} M^{\{\{0\},\{1,2\}\}}_{\alpha}$ in any case of $\alpha\in\left[\frac{39200}{83521},\frac{44321}{83521}\right]$;
(iii) $M^{\{\{0\},\{1\},\{2\}\}} \succ_{\{2\}} M^{\{\{0\},\{1,2\}\}}_{\alpha}$ in any case of $\alpha\in\left(\frac{44321}{83521},1\right]$.

The statements (i) and (iii) are straightforward from the fact that the equivalence assertions in (16) and (17) still hold when we reverse the inequality signs. We provide the proof of (ii). Let, without loss of generality, $i=1$. For the owner 0, we have

$$V_0^c(\beta)-V_0^b=\frac{-a^2(2109-66822\beta+43757\beta^2)}{392(109-35\beta)^2}>0\Leftrightarrow\beta>\frac{4773-164\sqrt{777}}{6251}\approx 0.0322422. \tag{18}$$

On the other hand, for the owner of the firm 1, we have

$$V_1^c(\beta)|_{\beta=\frac{1}{3}}-V_1^b(\alpha)|_{\alpha=\frac{44321}{83521}}=\frac{4486643171a^2}{261709044492}>0. \tag{19}$$

Note that $\frac{44321}{83521}=\arg\max_{\alpha\in[\frac{39200}{83521},\frac{44321}{83521}]}V_1^b(\alpha)$. Thus, by (18) and (19), $M^{\{\{0,1\},\{2\}\}}_{\beta=1/3} \succ_{\{0,1\}} M^{\{\{0\},\{1,2\}\}}_{\alpha}$ in any case of $\alpha\in[\frac{39200}{83521},\frac{44321}{83521}]$.

**Claim (c)**: (i) $M^{\{\{0\},\{1\},\{2\}\}} \succ_{\{i\}} M^{\{\{0,i\},\{j\}\}}_{\beta}$ in any case of $\beta\in\left(\frac{3}{5},1\right]$;

(ii) $M^{\{\{0\},\{1\},\{2\}\}} \succ_{\{0\}} M_{\beta}^{\{\{0,i\},\{j\}\}}$ in any case of $\beta \in \left[0, \frac{3}{5}\right]$.

Let, without loss of generality, $i = 1$. For the owner 1, we have

$$V_1^a - V_1^c(\beta) = \frac{-2a^2(29691440 - 72270008\beta + 33547551\beta^2 - 1920983\beta^3)}{83521(109 - 35\beta)^2} > 0,$$
$$\forall \beta \in \left(0.543873,\ 1\right] \supseteq \left(\frac{3}{5},\ 1\right]. \tag{20}$$

On the other hand, taking into account that $\frac{d(V_0^a - V_0^c(\beta))}{d\beta} = \frac{-169a^2(111-112\beta)}{(109-35\beta)^3} < 0$ for all $\beta \in [0, \frac{3}{5}]$, we have

$$V_0^a - V_0^c(\beta) = \frac{2a^2(4699615 - 9992488\beta + 5122908\beta^2)}{83521(109 - 35\beta)^2} > 0 \Leftrightarrow \forall \beta \in \left[0,\ \frac{3}{5}\right], \tag{21}$$

since $V_0^a - V_0^c(\beta)|_{\beta=\frac{3}{5}} = \frac{13709227}{8084832800}a^2 > 0$. Thus, the statement (i) follows from (20), and (ii) does from (21), respectively.

**Claim (d)**: (i) $M_{\beta=\gamma}^{\{\{0,1\},\{2\}\}} \succ_{\{0,1\}} M_{\gamma,\delta}^{\{\{0,1,2\}\}}$ in any case of $\gamma \in \left[0, \frac{4}{5}\right)$ and $\delta \in [0,1]$;
(ii) $M_{\beta=\frac{\gamma}{\gamma+(1-\gamma)\delta}}^{\{\{0,1\},\{2\}\}} \succ_{\{2\}} M_{\gamma,\delta}^{\{\{0,1,2\}\}}$ in any case of $\gamma \in \left[\frac{4}{5}, 1\right]$ and $\delta \in [0,1]$.

We begin with the proof of (i). For the payoff to the owner of the public firm 0, we have

$$V_0^c(\beta)|_{\beta=\gamma} - V_0^d(\gamma) > 0, \quad \forall \gamma < \frac{4}{5}, \tag{22}$$

from the fact that $\frac{d(V_0^c(\beta)|_{\beta=\gamma} - V_0^d(\gamma))}{d\gamma} = \frac{-a^2(25645335-48758498\gamma+29986128\gamma^2-7514154\gamma^3+661689\gamma^4)}{(8-3\gamma)^3(109-35\gamma)^3} < 0$ for all $\gamma \in [0,1]$ and $V_0^c(\beta)|_{\beta=\frac{4}{5}} - V_0^d(\gamma)|_{\gamma=\frac{4}{5}} = \frac{202087a^2}{257191200} > 0$. For the payoff to the owner of the private firm 1, we also obtain,

$$V_1^c(\beta)|_{\beta=\gamma} - V_1^d(\gamma, \delta)|_{\delta=\frac{1}{2}} > 0, \quad \forall \gamma < \frac{4}{5}, \tag{23}$$

because we have

$$\frac{d\left(V_1^c(\beta)|_{\beta=\gamma} - V_1^d(\gamma,\delta)|_{\delta=\frac{1}{2}}\right)}{d\gamma}$$
$$= \frac{-a^2\left[\begin{array}{c}17873952 - 52618843\gamma + 67857663\gamma^2 - 39401297\gamma^3 \\ +11066013\gamma^4 - 1507788\gamma^5 + 86940\gamma^6\end{array}\right]}{2(8-3\gamma)^3(109-35\gamma)^3} < 0, \quad \forall\gamma \in [0,1], \tag{24}$$

and $V_1^c(\beta)|_{\beta=\frac{4}{5}} - V_1^d(\gamma,\delta)|_{\gamma=\frac{4}{5},\delta=\frac{1}{2}} = \frac{28043a^2}{5953500} > 0$. Thus, by (22) and (23), we obtain $M_{\beta=\gamma}^{\{\{0,1\},\{2\}\}} \succ_{\{0,1\}} M_{\gamma<\frac{4}{5},\delta=\frac{1}{2}}^{\{\{0,1,2\}\}}$. Since in the cases of $\delta \neq \frac{1}{2}$ either of the two owners 1 and 2 will receive smaller payoff than in the case of $\delta = \frac{1}{2}$, we can apply the above argument to the owner with the smaller payoff and complete the proof of (i).

Next, we prove (ii). Because $\delta = 0$ is the most favorable case of $\delta$ for the owner 2 and $\frac{dV_2^c(\beta)}{d\beta} = \frac{-2460a^2(7-3\beta)}{(109-35\beta)^3} < 0$ for all $\beta \in [0,1]$, it is sufficient to show that $M_{\gamma\geq\frac{4}{5},\delta=0}^{\{\{0,1,2\}\}}$ is blocked by $M_{\beta=\frac{\gamma}{\gamma+(1-\gamma)\delta}}^{\{\{0,1\},\{2\}\}}$, i.e. by $M_{\beta=1}^{\{\{0,1\},\{2\}\}}$, via $\{2\}$. In $M_{\gamma\geq\frac{4}{5},\delta=0}^{\{\{0,1,2\}\}}$, the payoff to the owner of the private firm 2 is given as:

$$V_2^d(\gamma,\delta)|_{\delta=0} = \frac{3a^2(1-\gamma)(4-3\gamma)}{(8-3\gamma)^2}. \tag{25}$$

From the fact that $\frac{dV_2^d(\gamma,\delta)|_{\delta=0}}{d\gamma} = \frac{-3a^2(32-27\gamma)}{(8-3\gamma)^3} < 0$ for all $\gamma \in \left[\frac{4}{5},1\right]$,

$$\max_{\gamma\in[\frac{4}{5},1]} V_2^d(\gamma,\delta)|_{\delta=0} = V^d\left(\frac{4}{5},0\right) = \frac{3a^2}{98}. \tag{26}$$

On the other hand, in the market structure $M_{\beta=1}^{\{\{0,1\},\{2\}\}}$, the payoff to the owner of the private firm 2 is

$$V_2^c(\beta)|_{\beta=1} = \frac{15a^2(7-3\beta)^2}{(109-35\beta)^2}\bigg|_{\beta=1} = \frac{60a^2}{1369} > \frac{3a^2}{98} = \max_{\gamma\in[\frac{4}{5},1]} V_2^d(\gamma,\delta)|_{\delta=0}, \tag{27}$$

Thus, $M_{\beta=1}^{\{\{0,1\},\{2\}\}} \succ_{\{2\}} M_{\gamma,\delta=0}^{\{\{0,1,2\}\}}$ for any case of $\gamma \in \left[\frac{4}{5}, 1\right]$. ■

By the definition of the core, none of the market structures is stable in the sense that there always exists at least one owner who wants to deviate and induce a new market structure: in the regime (a), the private owners 1 and 2; in (b), the coalition of the owners 0 and $i$ $(= 1,2)$ or a single private owner $j$ $(= 1,2)$; in (c), either of the owners 0 and $i$ $(= 1,2)$; and in (d), the two owners 0 and $i$ $(= 1,2)$, or a single private owner $j$ $(= 1,2)$. In what follows, we explain the reasoning behind the results we stated as claims in the proof of our proposition.

Our claim (a) tells that the market structure of the mixed triopoly is blocked by those structures of the regime (b) with $\alpha \in (\underline{\alpha}, \bar{\alpha})$. In the regime (b), the private merged firm 12 enjoys improved production technology represented by the cost function $C(q_{12}) = \frac{1}{2}q_{12}^2$ and competes with the public firm 0 whose production technology is less efficient than the firm 12, while every firm in the regime (a) has a symmetric production technology. Faced with this asymmetry of technologies in the regime (b), the owner of the public firm, a welfare maximizer, chooses the contract parameter $\theta_0^b$ larger than the one in the regime (a) (note that $\theta_0^b - \theta_0^a = \frac{40a}{(7+a)(289+47a)} > 0$) to induce more of the production by the merged firm 12 which now operating more efficient production technology. Consequently, both of the two owners of the merged firm 12 can achieve higher payoffs in the regime (b) than in the regime (a).

In their model of mixed triopoly of entrepreneurial firms, Kamaga and Nakamura (2007) show that the market structures of the regime (c) with $\beta \in [0.56950, 0.56996]$ belong to the core, where 0.56996 (resp. 0.56950) is the highest (resp. lowest) value of $\beta$ that the owner of the private firm $i$ (resp. the public firm 0) agrees on the merger. Our claim (c), however, tells that, in the case of managerial firms, any market structure of the regime (c) is blocked by the market structure of the mixed triopoly. To explain the reasoning behind this result, we compare the equilibrium outcomes obtained in their entrepreneurial model and those in our managerial model. We use a superscript $e$ to denote equilibrium outcomes in the entrepreneurial model. In the entrepreneurial model of Kamaga and Nakamura (2007), the equilibrium outputs of the public firm 0 and private firm $i$ and the equilibrium profit of the private firm $i$ in the regime (a) are obtained as follows:

$$q_0^{ae} = \frac{3}{13}a, \quad q_i^{ae} = \frac{2}{13}a, \quad \Pi_i^{ae} = \frac{8}{169}a^2;$$

and the equilibrium outputs of the merged firm $0i$ and private firm $j$ and the equilibrium profit of the merged firm $0i$ in the regime (c) are given as:

$$q_{0i}^{ce} = \frac{3}{11-4\beta}a, \quad q_j^{ce} = \frac{2-\beta}{11-4\beta}a, \quad \Pi_{0i}^{ae} = \frac{9(3-2\beta)}{2(11-4\beta)^2}a^2.$$

Comparing these outcomes with those obtained in our model, we will observe the following changes brought by managerial delegation: (i) in the regime (a), the output of the private firm $i$ increases $\left(q_i^{a*} - q_i^{ae} = \frac{72}{3757}a\right)$ while the output of the public firm, now operated by a manager whose objective is not welfare maximization, decreases $(q_0^{a*} - q_0^{ae} = -\frac{100}{3757}a)$; (ii) consequently, the profit of the private firm $i$ in the regime (a) becomes higher in the managerial case than in the entrepreneurial case $(\Pi_i^{a*} - \Pi_i^{ae} = \frac{7832}{14115049}a^2)$; (iii) but in the regime (c) where the merged firm $0i$ increases the output for the case of relatively small $\beta$ $(q_{0i}^{c*} - q_{0i}^{ce} = \frac{(5-\beta)(5-8\beta)}{(11-4\beta)(109-35\beta)}a > 0$ for all $\beta \in [0, \frac{5}{8}))$, the competitor of the merged firm $0i$ is not a public firm but solely the private firm $j$ which also increases the output $(q_j^{c*} - q_j^{ce} = \frac{13-4\beta+\beta^2}{(11-4\beta)(109-35\beta)}a > 0$ for all $\beta \in [0,1])$, and such an expansion of the output by the private firm $j$ will decrease the profit of the merged firm $0i$ for almost all cases of $\beta$ $(\Pi_{0i}^{c*} - \Pi_{0i}^{ae} = -[a^2(11027 + 2884\beta - 25293\beta^2 + 12158\beta^3 - 1472\beta^4)/2(11-4\beta)^2(109-35\beta)^2] < 0$ for all $\beta \in [0, 0.959536))$. As a consequence, the shareholding ratio by the owner $i$, i.e. $(1-\beta)$, need to become higher in the managerial case than in the entrepreneurial case to induce an agreement of the owner $i$ on the merger between the public firm 0 and private firm $i$. In fact, as show in (20), the value of $1-\beta$ must be larger than 0.456127, or equivalently $\beta$ smaller than 0.543873 in our managerial model, whereas $\beta$ must be smaller than 0.56996 in the entrepreneurial case. However, as shown in (21), the owner of the public firm 0 never agrees on the merger as long as $\beta$ is lower than 0.543873.

Although the market structures of the regime (b) with $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ blocks the mixed triopoly, these market structures, as stated in (ii) of Claim (b), are blocked by those of the regime (c) with $\beta = \frac{1}{3}$. By (18), (19), and the derivative of $V_1^c(\beta)$, it is easily checked that this result also holds for any $\beta \in \left(\frac{4773-164\sqrt{777}}{6251}, \frac{1}{3}\right]$. The reasoning behind this result is explained as follows. Because of the regime (b) is less competitive than the regime (a), social welfare which the owner of the public firm 0 wants to maximize becomes lower than the level in regime (a). Consequently, while the owners of the public firm 0 and private firm $i$ in the regime (a) never reach an agreement on the merger between these two firms, the owner of the public firm 0 in the regime (b) now has an incentive to make a conciliatory offer to the owner of the private firm $i$ on the merger of these two firms, and they can reach an agreement on the merger.

Finally, Claim (d) shows that the market structures in the regime (d) are blocked by those in the regime (c). This result is due to the fact that the positive effect of the improvement on productivity in a merged firm is relatively smaller in the case of the shift from the regime (c) (or (b)) into (d) than in the cases of the shift from (a) into (c) (or (b)). As a consequence, although our definition of the inducibility among the market structures does not allow the case where the market structure of the mixed triopoly is directly induced from those in the regime (d), it never detracts the generality of our core analysis.

# 4 Concluding remarks

This paper examined a merger activity in the mixed oligopoly, especially focusing on the stability of owners' coalition formations. We introduced managerial delegation into the model of Kamaga and Nakamura (2007) and examined the core of market structures, i.e. stable market structures. In contrast to the result in Kamaga and Nakamura (2007), we obtained a striking but seriously negative result that the core must be empty. In other words, none of the market structures is stable in the sense that there always exists at least one owner who wants to deviate and induce a new market structure.

In this paper, we assumed that firms' managers set the outputs simultaneously. The interesting extension of our model is to examine the case of Stackelberg competition in the second stage, i.e. non-simultaneous moves of the managers. While such an extension is considered by Lambertini (2000) in the model of private oligopoly, none of the existing works of mixed oligopoly analyzes this type of extension in the context of managerial delegation. This is left for future research.

# References

Bárcena-Ruiz, J.C. and M.B. Garzón, (2003), "Mixed Duopoly, Merger and Multiproduct Firms," *Journal of Economics*, 80, 27-42.

Barros, F., (1995), "Incentive Schemes as Strategic Variables: An Application to a Mixed Duopoly," *International Journal of Industrial Organization*, 13, 373-386.

Barros, P.P., (1998), "Endogenous Merger and Size Asymmetry of Merger Participants," *Economics Letters*, 60, 113-119.

Coloma, G., (2006), "Mergers and Acquisitions in Mixed-Oligopoly Markets," *International Journal of Business and Economics*, 5, 147-159.

De Fraja, G. and F. Delbono, (1989), "Alternative Strategies of a Public Enterprise in Oligopoly," *Oxford Economic Papers*, 41, 302-311.

Fershtman, C. and K.L. Judd, (1987), "Equilibrium Incentives in Oligopoly," *American Economic Review*, 77, 927-940.

Heywood, J.S. and M. McGinty, (2007a), "Convex Costs and the Merger Paradox Revisited," *Economic Inquiry*, 45, 342-349.

Heywood, J.S. and M. McGinty, (2007b), "Merger among Leaders and Merger among Followers," *Economics Bulletin*, 12 (12), 1-7.

Horn, H. and L. Persson, (2001), "Endogenous Merger in Concentrated Markets," *International Journal of Industrial Organization*, 19, 1213-1244.

Kamaga, K. and Y. Nakamura, (2007), "The Core and the Productivity-Improving Mergers in Mixed Oligopoly," 21COE-GLOPE Working Paper No. 23, Waseda University.

Kamijo, Y. and Y. Nakamura, (2007), “Stable Market Structures from Merger Activities in Mixed Oligopoly with Asymmetric Costs,” 21COE-GLOPE Working Paper No. 21, Waseda University.

Lambertini, L., (2000), “Extended Games Played by Managerial Firms,” *Japanese Economic Review*, 51, 274-283.

McAfee, R.P. and M.A. Williams, (1992), “Horizontal Mergers and Antitrust Policy,” *Journal of Industrial Economics*, 40, 181-187.

Matsumura, T., (1998), “Partial Privatization in Mixed Duopoly,” *Journal of Public Economics*, 70, 473-483.

Nakamura, Y. and T. Inoue, (2007), “Mixed Oligopoly and Productive-Improving Mergers,” *Economics Bulletin*, 12(20), 1-9.

Sklivas, S.D., (1987), “The Strategic Choice of Managerial Incentives,” *RAND Journal of Economics*, 18, 452-458.

Straume, O.R., (2006), “Managerial Delegation and Merger Incentives with Asymmetric Costs,” *Journal of Institutional and Theoretical Economics*, 162, 450-469.

White, M.D., (2001)., “Managerial Incentives and the Decision to Hire Managers in Markets with Public and Private Firms,” *European Journal of Political Economy*, 17, 877-896.

Table 1: Firms' profit functions in each regime (r)

| (r) | profit $\Pi_i^r$ |
|---|---|
| (a) | $\Pi_i^a = \left[a-(q_0+\sum_{j=1}^{2} q_j)\right]q_i-(q_i)^2 \quad (i=0,1,2)$ |
| (b) | $\Pi_0^b = \left[a-(q_0+q_{12})\right]q_0-(q_0)^2$,<br>$\Pi_{12}^b = \left[a-(q_0+q_{12})\right]q_{12}-(q_{12})^2/2$ |
| (c) | $\Pi_{0i}^c = \left[a-(q_{0i}+q_j)\right]q_{0i}-(q_{0i})^2/2$,<br>$\Pi_j^c = \left[a-(q_{0i}+q_j)\right]q_j-(q_j)^2 \quad (i,j=1,2,\ i\neq j)$ |
| (d) | $\Pi_{012}^d = (a-q_{012})q_{012}-(q_{012})^2/3$ |

Table 2: Owners' (constituted) objectives in each regime (r)

| (r) | objective function $U_i^r$ |
|---|---|
| (a) | $U_0^a(\theta_0^a;\theta_1^a,\theta_2^a) = W^a$<br>$= (q_0^{ma} + \sum_{j=1}^2 q_j^{ma})^2/2 + \Pi_0^a(\mathbf{q}^{ma}) + \sum_{j=1}^2 \Pi_j^a(\mathbf{q}^{ma}),$<br>$U_i^a(\theta_i^a;\theta_0^a,\theta_j^a) = \Pi_i^a(\mathbf{q}^{ma}) \quad (i=1,2,\ j\neq i)$ |
| (b) | $U_0^b(\theta_0^b;\theta_{12}^b) = W^b$<br>$= (q_0^{mb} + q_{12}^{mb})^2/2 + \Pi_0^b(\mathbf{q}^{mb}) + \Pi_{12}^b(\mathbf{q}^{mb}),$<br>$U_{12}^b(\theta_0^b;\theta_{12}^b) = \Pi_{12}^b(\mathbf{q}^{mb})$ |
| (c) | $U_{0i}^c(\theta_{0i}^b;\theta_j^b) = \beta W^c + (1-\beta)\Pi_{0i}^c$<br>$= \beta\left[(q_{0i}^{mc} + q_j^{mc})^2/2 + \Pi_{0i}^c(\mathbf{q}^{mc}) + \Pi_j^c(\mathbf{q}^{mc})\right] + (1-\beta)\Pi_{0i}^c(\mathbf{q}^{mc}),$<br>$U_j^c(\theta_{0i}^b;\theta_j^b) = \Pi_j^c(\mathbf{q}^{mc}) \quad (i,j=1,2,\ j\neq i)$ |
| (d) | $U_{012}^d(\theta_{012}^d) = \gamma W^d + (1-\gamma)\Pi_{012}^d$<br>$= \gamma\left[(q_{012}^{md})^2/2 + \Pi_{012}^d(q_{012}^{md})\right] + (1-\gamma)\Pi_{012}^d(q_{012}^{md})$ |

Table 3: Equilibrium incentive contracts in each regime (r)

| (r) | incentive contract $\theta_i^{r*}$ | |
|---|---|---|
| (a) | $\theta_0^{a*} = \frac{289}{289+47a}$ | $\theta_i^{a*} = \frac{289}{289+20a} \quad (i=1,2)$ |
| (b) | $\theta_0^{b*} = \frac{7}{7+a}$ | $\theta_{12}^{b*} = \frac{14}{14+a}$ |
| (c) | $\theta_{0i}^{c*} = \frac{109-35\beta}{109-35\beta+a(8+20\beta)}$ | $\theta_j^{c*} = \frac{109-35\beta}{109-35\beta+a(7-3\beta)} \quad (i,j=1,2; i\neq j)$ |
| (d) | $\theta_{012}^{d*} = \frac{8-3\gamma}{8-3\gamma+3a\gamma}$ | |

Table 4: Equilibrium outputs, profits, and social welfare in each regime (r)

| (r) | equilibrium outcomes |
|---|---|
| (a) | $q_0^{a*} = \frac{59}{289}a, \quad q_i^{a*} = \frac{50}{289}a \ \ (i=1,2)$; <br> $\Pi_0^{a*} = \frac{4189}{83521}a^2, \quad \Pi_i^{a*} = \frac{4000}{83521}a^2 \ \ (i=1,2)$; <br> $W^{a*} = \frac{49659}{167042}a^2$ |
| (b) | $q_0^{b*} = \frac{3}{14}a, \quad q_{12}^{b*} = \frac{2}{7}a$; <br> $\Pi_0^{b*} = \frac{3}{49}a^2, \quad \Pi_{12}^{b*} = \frac{5}{49}a^2$; <br> $W^{b*} = \frac{113}{392}a^2$ |
| (c) | $q_{0i}^{c*} = \frac{2a(16-\beta)}{109-35\beta}, \quad q_j^{c*} = \frac{3a(7-3\beta)}{109-35\beta}, \ (i,j=1,2; i \neq j)$; <br> $\Pi_{0i}^{c*} = \frac{2a^2\left(640-408\beta+23\beta^2\right)}{(109-35\beta)^2}, \quad \Pi_j^{c*} = \frac{15a^2(7-3\beta)^2}{(109-35\beta)^2} \ \ (i,j=1,2; i \neq j)$; <br> $W^{c*} = \frac{a^2\left(6839-4058\beta+483\beta^2\right)}{2(109-35\beta)^2}$ |
| (d) | $q_{012}^{d*} = \frac{3a}{8-3\gamma}$; <br> $\Pi_{012}^{d*} = \frac{3a^2(4-3\gamma)}{(8-3\gamma)^2}$; <br> $W^{d*} = \frac{3a^2(11-6\gamma)}{2(8-3\gamma)^2}$ |

Table 5: Owners' payoffs $V_i^r$ in each regime (r)

| (r) | payoffs $(V_0^r, V_1^r, V_2^r)$ |
|---|---|
| (a) | $\left(V_0^a, V_1^a, V_2^a\right) = \left(W^{a*}, \ \Pi_1^{a*}, \ \Pi_2^{a*}\right)$ |
| (b) | $\left(V_0^b, V_1^b, V_2^b\right) = \left(W^{b*}, \ \alpha\Pi_{12}^{b*}, \ (1-\alpha)\Pi_{12}^{b*}\right)$ |
| (c) | $\left(V_0^c, V_i^c, V_j^c\right) = \left(W^{c*}, \ (1-\beta)\Pi_{0i}^{c*}, \ \Pi_j^{c*}\right), \quad (i,j=1,2, \ i \neq j)$ |
| (d) | $\left(V_0^d, V_1^d, V_2^d\right) = \left(W^{d*}, \ (1-\gamma)\delta\Pi_{012}^{d*}, \ (1-\gamma)(1-\delta)\Pi_{012}^{d*}\right)$ |