21COE-GLOPE Working Paper Series

Bargaining over Managerial Contracts in Delegation Games: The Differentiated Goods Case

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Working Paper No. 40

1 Introduction

This paper presents a theoretical analysis of bargaining on managerial delegation between owner-shareholders and managers in a differentiated-products market. Modern corporate governance codes includes clauses retiring the disclosure of managerial compensation in order to protect owner-shareholders against the opportunistic behavior of managers. This paper aims to explain the effects of managerial power and the disclosure of managerial compensation on market outcomes in the case wherein each firm produces a differentiated good.

A theoretical analysis of the separation of ownership and management challenges the validity of the traditional assumption of a firm as the sole profit maximizing agent. With the development of a principal-agent model, studies have been conducted to clarify the interaction between owner-manager contracts and market behavior in the context of an oligopolistic industry with managerial delegation. In their well-known articles, Fershtman and Judd (1987) and Sklivas (1987) revealed that profit-maximizing owners sometimes choose to divert the preference of their managers away from profit maximization. They investigated a two-stage duopolistic game where in the first stage, each profit-maximizing owner chooses a compensation scheme for his or her manager that is a linear combination of profit and sales. Based on the two seminal works, the literature on delegation games extended its scope to alternative compensation schemes. Salas Fumas (1992) and Miller and Pazgal (2002) considered the compensation scheme as a linear combination of the profit of a firm and its rival. Jansen et al. (2007) investigated the case in which each manager's remuneration is a weighted sum of the firm's profit and market share.

Another major branch of the literature on managerial delegation considers bargaining issues between an owner and a manager in various environments. Since the pioneering work by Fershtman (1985), this type of model has particularly focused on wage bargaining. Szymanski (1994) and Bughin (1995) analyzed the influence of the delegation of wage bargaining with unions to managers on product market competition, depending on the type of the managerial compensation scheme. Along the lines of these works, Witteloostuijn *et al.* (2007) considered the bargaining between owners and managers over their compensation contracts, in order to explain the disclosure obligation that is central to many modern corporate governance codes. More precisely, they analyzed the two-stage duopolistic game where in the first stage owners negotiate with their managers over managerial delegation contracts. Subsequently, in the second stage, *Cournot* competition with a homogeneous good is observed.

The main purpose of this paper is to expand the scope of the results of Witteloostuijn *et al.* (2007) with respect to the sales delegation case to those of a differentiated-products duopolistic game. In this paper, the game operates in the same manner as that in Witteloostuijn *et al.* (2007). However, we investigate both *Cournot* and *Bertrand* competitions. In short, we extend the scope of the strategic managerial delegation model with bargaining between owners and managers by considering both product differentiation and the change in firms' strategic variables. Through these analyses, we intend to determine whether or not the results of Witteloostuijn *et al.* (2007) of the sales delegation is robust against a change(s) in the degree of product differentiation and/or the firms' strategic variables. We obtain the result that competition is promoted in both *Cournot* and *Bertrand* fashions irrespective of the degree of product differentiation, as the relative bargaining power of the manager increases. Thus, in the context of a symmetric duopolistic competition with bargaining between owners and managers, we find that the results of *Cournot* competition with homogeneous goods are robust against the changes in the degree of product differentiation and firms' (managers') strategic variables.

The remainder of this paper is organized as follows. In Section 2, we formulate the basic setting of the delegation game of the two types of models considered in this paper. In Section 3, we consider the case of quantity competition. In Section 4, we investigate the case of price competition. Section 5 presents the concluding remarks.

2 Model

We consider a symmetric duopolistic model using a standard product differentiation model as in Singh and Vives (1984). A representative consumer's utility is denoted as

$$U(q_1, q_2) = a\,(q_1 + q_2) - \frac{1}{2}\left[(q_1)^2 + 2bq_1q_2 + (q_2)^2\right] + q,$$

where q_i represents the quantity of good i $(= 1, 2)$, and $b \in (0, 1)$ represents the degree of product differentiation. Note that q denotes a numeraire good. The utility function generates the system of linear demand functions:

$$q_i = \frac{a\,(1-b) - p_i + bp_j}{1 - b^2}, \qquad b \in (0, 1)\,, \qquad i, j = 1, 2,\ i \neq j.$$

Then, the inverse demand functions can be inverted to obtain

$$p_i = a - q_i - bq_j, \qquad b \in (0, 1)\,, \qquad i, j = 1, 2,\ i \neq j,$$

where p_i represents the price of good i $(= 1, 2)$. We assume that both the firms have constant marginal costs of production, $c\,(< a)$. The profit of firm i is given by

$$\begin{aligned}\Pi_i &= (a - q_i - bq_j - c)q_i \\ &= \frac{a\,(1-b) - p_i + bp_j}{1-b^2}\,(p_i - c)\,, \qquad i, j = 1, 2,\ i \neq j.\end{aligned}$$

Usually, social welfare, denoted by W, is measured as the sum of consumer surplus (CS) and producer surplus (PS):

$$W = CS + PS,$$

where $PS = \Pi_1 + \Pi_2$, and CS is given by

$$\begin{aligned}CS &= \frac{1}{2}\left[(q_1)^2 + 2bq_1q_2 + (q_2)^2\right], \\ &= \frac{2a^2\,(1-b) + (p_1)^2 - 2bp_1p_2 + (p_2)^2 - 2a\,(1-b)\,(p_1 + p_2)}{2\,(1-b^2)}.\end{aligned}$$

We consider the situation in which a firm's owner decides to delegate the control of his or her assets to a manager, similar to the case in Fershtman and Judd (1987) and Sklivas (1987). Both the firms' owners can assess the performance of their managers according to two readily observable indicators–the output and profit of the firm. In this case, the objective of each manager is given by

$$U_i = \Pi_i + \theta_i q_i, \qquad \theta_i \in \mathbb{R}, \quad i = 1, 2,$$

where parameter θ_i measures the relevance of sales. The manager of firm i can maximize his or her payoff by choosing the output q_i or the price p_i that maximizes U_i $(i = 1, 2)$. This can be supported by the assumption that the payoff to the manager of firm i is represented as $\lambda_i + \mu_i V_i$ for some real number λ_i and some positive number μ_i $(i = 1, 2)$. Similar to many existing works, we assume that the payoffs to the managers are negligible as compared to profits, because we emphasize the impact of managerial delegation on the equilibrium outcomes.

We propose the following two-stage delegation game. In the first stage, an owner and his or her manager bargain over the incentive parameter θ_i in each firm $(i = 1, 2)$. Subsequently, in the second stage, each manager simultaneously decides his or her output or price while being aware of the level of his or her own incentive parameter. Thus, owners and managers bargain over their incentive parameters in the sales delegation contracts such that both their profits and payoffs are enhanced through their collusive market behaviors.

The bargaining in the first stage is generally formulated in the same manner as in Binmore *et al.* (1986). For each firm, the equilibrium outcome of the bargaining process coincides with the Nash bargaining solution of the following equation in terms of the incentive parameter θ_i:

$$B_i = U_i^{\beta} \cdot \Pi_i^{1-\beta}, \qquad i = 1, 2,$$

where $\beta \in [0, 1)$ denotes the measure for the relative bargaining power of the manager. Note that in this paper, we restrict our attention to the symmetric case in which all managers have the same bargaining powers β. As a result, we assume that owners select sufficiently homogeneous managers both in terms of their bargaining powers and the preferences to their payoffs. The disagreement point of both owner and manager is zero: If the bargaining process breaks down, the managers are unable to procure the other jobs, and the owners are unable to operate run their firms since they do not have adequate time or managerial skills.

3 Quantity Competition

In this section, we consider how profits, consumer surplus, and social welfare depend on the relative bargaining power of the manager, β, for the sales delegation in the case of quantity competition in a differentiated-products duopolistic setting.

As usual, we begin by solving the second stage. Given a pair of incentive parameters θ_1 and θ_2, each manager maximizes his or her objective U_i with respect to q_i $(i = 1, 2)$. The condition for each manager is given by

$$\frac{\partial U_i}{\partial q_i} = a - c - 2q_i - bq_j + \theta_i = 0, \qquad i, j = 1, 2, \ i \neq j.$$

Solving the system of the above equations for q_0 and q_1, we obtain the output of each firm as follows:

$$q_i = \frac{a\,(2-b) - c\,(2-b) + 2\theta_i - b\theta_j}{4 - b^2}, \qquad i, j = 1, 2, \ i \neq j.$$

According to the first-order condition of the above system, we have

$$U_i = (q_i)^2, \qquad \Pi_i = (q_i)^2 - \theta_i q_i, \qquad i = 1, 2.$$

Thus, we obtain the following results:

$$\frac{\partial U_i}{\partial \theta_i} = \frac{4q_i}{4 - b^2} > 0, \qquad \frac{\partial^2 \Pi_i}{\partial \theta_i^2} = -\frac{4\left(2 - b^2\right)}{\left(4 - b^2\right)^2} < 0, \qquad \forall b \in (0, 1), \qquad i = 1, 2.$$

In short, similar to the results of the sales delegation case of Witteloostuijn *et al.* (2007), we obtain the result that the objective function of the manager U_i is an increasing function of θ_i, and the profit of each firm is an concave function of θ_i $(i = 1, 2)$. The interests of the owner and manager of each firm are different. Therefore, it is appropriate for them to bargain over the incentive parameter, θ_i $(i = 1, 2)$.

We now proceed to the analysis of the first stage of the game. Considering the outcomes in the first stage, we rewrite the Nash product for the bargaining problem of each firm as follows:

$$B_i = U_i^{\beta} \cdot \Pi_i^{1-\beta} = (q_i)^{2\beta} \cdot \left[(q_i)^2 - \theta_i q_i\right]^{1-\beta}, \qquad i = 1, 2.$$

Thus, analogous to Witteloostuijn *et al.* (2007), we obtain the following first-order condition for the bargaining of each firm:

$$\frac{\partial B_i}{\partial \theta_i} = 0 \Leftrightarrow \left[2q_i - \theta_i (1 + \beta)\right] \frac{\partial q_i}{\partial \theta_i} - q_i (1 - \beta) = 0, \qquad i = 1, 2.$$

Taking into account that $\partial q_i / \partial \theta_i = 2/\left(4 - b^2\right)$ and that the problem's symmetry $\theta = \theta_1 = \theta_2$, we obtain

$$\frac{\partial B_i}{\partial \theta_i} = 0 \Leftrightarrow \frac{a\left[b^2 (1 - \beta) + 4\beta\right] - b^2 (1 - \beta)(c - \theta) - 2b\theta (1 + \beta) - 4 (c\beta + \theta)}{(2 - b)(2 + b)^2} = 0,$$

yielding

$$\theta^* = \theta_i^* = \frac{b^2 (1 - \beta) + 4\beta}{4 + 2b (1 + \beta) - b^2 (1 - \beta)} (a - c), \qquad i = 1, 2.$$

The equilibrium outcomes (denoted by superscript "$*$") are obtained as follows:

$$\begin{aligned}
q_i^* &= \frac{2(1 + \beta)}{4 + 2b (1 + \beta) - b^2 (1 - \beta)} (a - c); \quad p_i^* = \frac{a\left(2 - b^2\right)(1 - \beta) + 2c (1 + b)(1 + \beta)}{4 + 2b (1 + \beta) - b^2 (1 - \beta)}; \\
\Pi_i^* &= \frac{2\left(2 - b^2\right)(1 - \beta)(1 + \beta)}{\left[4 + 2b (1 + \beta) - b^2 (1 - \beta)\right]^2} (a - c)^2; \qquad i = 1, 2; \\
CS^* &= \frac{4 (1 + b)(1 + \beta)^2}{\left[4 + 2b (1 + \beta) - b^2 (1 - \beta)\right]^2} (a - c)^2; \\
W^* &= \frac{4 (1 + \beta)\left[3 - \beta + b (1 + \beta) - b^2 (1 - \beta)\right]}{\left[4 + 2b (1 + \beta) - b^2 (1 - \beta)\right]^2} (a - c)^2.
\end{aligned}$$

Furthermore, we obtain the following result:

$$\frac{\partial \Pi_i^*}{\partial \beta} = -\frac{4\left(2 - b^2\right)\left[4\beta + 2b (1 + \beta) + b^2 (1 - \beta)\right]}{\left[4 + 2b (1 + \beta) - b^2 (1 - \beta)\right]^3} (a - c)^2 < 0,$$

$$\forall b \in (0,1)\,,\ \forall \beta \in [0,1)\,, \quad i = 1,2.$$

In short, the partial derivative of the equilibrium profit of each firm with β is negative for all the values of $b \in (0,1)$ and $\beta \in [0,1)$. On the other hand, we have

$$\frac{\partial CS^*}{\partial \beta} = \frac{16\,(1+b)\left(2-b^2\right)(1+\beta)}{\left[4+2b\,(1+\beta)-b^2\,(1-\beta)\right]^3}\,(a-c)^2 > 0, \qquad \forall b \in (0,1)\,,\ \forall \beta \in [0,1)\,,$$

and

$$\frac{\partial W^*}{\partial \beta} = \frac{8\left(2-b^2\right)^2(1-\beta)}{\left[4+2b\,(1+\beta)-b^2\,(1-\beta)\right]^3}\,(a-c)^2 > 0, \qquad \forall b \in (0,1)\,,\ \forall \beta \in [0,1)\,.$$

With respect to the equilibrium consumer surplus and social welfare, the partial derivatives with β are positive for all the values of $b \in (0,1)$ and $\beta \in [0,1)$. The preceding results are summarized in Proposition 1.

Proposition 1. *If each owner in a symmetric Cournot duopoly hires a manager who receives a payoff through the managerial incentive contract based on profits and sales, and if the incentive parameter on sales is the outcome of a bargaining process, then the corresponding delegation game has a unique subgame perfect Nash equilibrium, which depends on the bargaining power β of the managers. Moreover, if the bargaining power of the managers increases, the equilibrium profit of each firm decreases, whereas the consumer surplus and social welfare increases.*

Note that similar to Witteloostuijn *et al.* (2007), as the measure for the relative bargaining power of the manager β approaches 1, the equilibrium profit of each firm, Π_i^* approaches 0, whereas the consumer surplus, CS^* and social welfare, W^* gradually increase.

4 Price Competition

In this section, we analyze the price competition case in which each firm produces a differentiated good. We again use backward induction and consider a two-stage process in which the owners and managers bargain over the incentive parameter in the first stage, and both the firms compete with regard to prices in the second stage.

Given a pair of incentive parameters θ_1 and θ_2, each manager maximizes the objective function U_i with respect to p_i $(i = 1,2)$. The first-order condition for each manager is given by

$$\frac{\partial U_i}{\partial p_i} = \frac{a\,(1-b)+c-2p_i+bp_j-\theta_i}{1-b^2} = 0, \qquad i,j = 1,2,\ i \neq j.$$

Solving the system of the above equations for p_0 and p_1, we obtain the price of each firm as follows:

$$p_i = \frac{a\left(2-b-b^2\right)+c\,(2+b)-2\theta_i-b\theta_j}{4-b^2}, \qquad i,j = 1,2,\ i \neq j.$$

Considering the first-order condition of each manager, we obtain

$$U_i = \frac{\left[a\,(1-b)-p_i+bp_j\right]^2}{1-b^2},$$

$$\Pi_i = \frac{[a(1-b) - p_i + bp_j][a(1-b) - p_i + bp_j - \theta_i]}{1-b^2}, \qquad i = 1, 2.$$

Thus, we obtain the following results:

$$\frac{\partial U_i}{\partial \theta_i} = \frac{2(2-b^2)}{(4-b^2)} q_i > 0, \quad \frac{\partial^2 \Pi_i}{\partial \theta_i^2} = -\frac{4(2-b^2)}{(4-b^2)^2(1-b^2)} < 0, \quad \forall b \in (0,1), \ i = 1, 2.$$

Therefore, similar to quantity competition, we find that in price competition the objective function of each manager U_i is an increasing function of θ_i, and the profit of each firm is a concave function of θ_i $(i = 1, 2)$.

Next, we analyze the bargaining between owners and managers in the first stage. In the case of price competition, the bargaining problem is as follows:

$$\begin{aligned} B_i &= U_i^{\beta} \cdot \Pi_i^{1-\beta} \\ &= \left\{ \frac{[a(1-b) - p_i + bp_j]^2}{1-b^2} \right\}^{\beta} \cdot \left\{ \frac{[a(1-b) - p_i + bp_j][a(1-b) - p_i + bp_j - \theta_i]}{1-b^2} \right\}^{1-\beta}, \\ &\quad i = 1, 2. \end{aligned}$$

Considering the problem's symmetry, $\theta = \theta_1 = \theta_2$, we obtain

$$\begin{aligned} &\frac{\partial B_i}{\partial \theta_i} = 0 \\ &\Leftrightarrow a(1-b)\left[4\beta - b^2(1+\beta)\right] - b^3 c(1+\beta) + b^2(1+\beta)(c+\theta) \\ &\quad - 4(c\beta + \theta) + 2b[2c\beta + \theta(1-\beta)] = 0, \qquad i = 1, 2, \end{aligned}$$

yielding

$$\theta^* = \theta_i^* = \frac{(1-b)\left[4\beta - b^2(1+\beta)\right]}{4 - 2b(1-\beta) - b^2(1+\beta)}(a-c), \qquad i = 1, 2.$$

We obtain the equilibrium outcomes as follows:

$$\begin{aligned} p_i^* &= \frac{2a(1 - b - \beta + b\beta) + c(2-b^2)(1+\beta)}{4 - 2b(1-\beta) - b^2(1+\beta)}, \quad q_i^* = \frac{(2-b^2)(1+\beta)}{(1+b)[4 - 2b(1-\beta) - b^2(1+\beta)]}(a-c); \\ \Pi_i^* &= \frac{2(2 - 2b - b^2 + b^3)(1-\beta^2)}{(1+b)[4 - 2b(1-\beta) - b^2(1+\beta)]^2}(a-c)^2; \qquad i = 1, 2; \\ CS^* &= \frac{(2-b^2)^2(1+\beta)^2}{(1+b)[4 - 2b(1-\beta) - b^2(1+\beta)]^2}(a-c)^2; \\ W^* &= \frac{(2-b^2)(1+\beta)\left[2(3-\beta) - 4b(1-\beta) - b^2(1+\beta)\right]}{(1+b)[4 - 2b(1-\beta) - b^2(1+\beta)]^2}(a-c)^2. \end{aligned}$$

Furthermore, we obtain the following result:

$$\begin{aligned} \frac{\partial \Pi_i^*}{\partial \beta} &= -\frac{4(2 - 2b - b^2 + b^3)\left[2b(1-\beta) + 4\beta - b^2(1+\beta)\right]}{(1+b)[4 - 2b(1-\beta) - b^2(1+\beta)]^3}(a-c)^2 < 0, \\ &\forall b \in (0,1), \ \forall \beta \in [0,1). \end{aligned}$$

Thus, we find that the equilibrium profit of each firm decreases for all the values of $b \in (0,1)$ and $\beta \in [0,1)$ as the relative bargaining power of the manager, β, increases. On the other hand, we obtain

$$\frac{\partial CS^*}{\partial \beta} = \frac{8(1-b)\left(2-b^2\right)^2(1+\beta)}{(1+b)\left[4-2b(1-\beta)-b^2(1+\beta)\right]^3}(a-c)^2 > 0, \qquad \forall b \in (0,1)\,,\ \forall \beta \in [0,1)\,,$$

and

$$\frac{\partial W^*}{\partial \beta} = \frac{16(1-b)^2\left(2-b^2\right)(1-\beta)}{(1+b)\left[4-2b(1-\beta)-b^2(1+\beta)\right]^3}(a-c)^2 > 0, \qquad \forall b \in (0,1)\,,\ \forall \beta \in [0,1)\,.$$

Thus, we find that both the equilibrium consumer surplus and social welfare improves as the relative bargaining power of the managers, β, increases in contrast to the equilibrium profit of each firm. The preceding results are summarized in Proposition 2.

Proposition 2. *If each owner in a symmetric Bertrand duopoly hires a manager who receives a payoff through the managerial delegation contract based on profits and sales, and if the incentive parameter on sales is the outcome of a bargaining process, then the corresponding delegation game has a unique subgame perfect Nash equilibrium, which depends on the bargaining power β of the managers. Moreover, if the bargaining power of the managers increases, the equilibrium profit of each firm decreases, whereas the consumer surplus and social welfare increase.*

Note that analogous to quantity competition, as β approaches 1, the equilibrium profit of each firm, Π_i^* approaches 0, whereas the consumer surplus, CS^* and social welfare, W^* gradually increase.

5 Conclusion

This paper examined the bargaining between owners and managers over managerial incentive contracts for the sales delegation case in two types of differentiated-products markets with *Cournot* and *Bertrand* competitions. We analyzed whether or not the results of Witteloostuijn *et al.* (2007) are robust against the changes in the degree of product differentiation and/or firms' (managers') strategic variables. We obtained the result that the managerial power of the managers, β, is positively associated with the equilibrium consumer surplus and social welfare, similar ot that in Witteloostuijn *et al.* (2007). Therefore, the result of the managerial delegation game with bargaining between owners and managers is robust against such extensions.

Similar to most existing works, we assume the simultaneous-movement of both the firms in the market stage. The interesting extension of our model is to examine whether or not the results of this paper are robust in the case of the sequential-movement of both the firms. Futuere research should focus on other compensation schemes, e.g., the relative profit delegation and market share delegation cases, and should confirm the robustness of the results in this paper.

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