21COE-GLOPE Working Paper Series

Interregional Mixed Duopoly, Location and Welfare

Tomohiro Inoue, Yoshio Kamijo and Yoshihiro Tomaru

This paper is the revised version of 21COE-GLOPE Working Paper No. 16. The revised points are as follows: (i) the intuitions of equilibria, (ii) eliminations of Equilibrium selection (Section 4.2 of the previous version) and Decentralization of government and privatization (Section 5 of the previous version), and (iii) introduction of Extension (Section 4).

Working Paper No. 37

Interregional Mixed Duopoly, Location and Welfare

Tomohiro Inoue[a,*] Yoshio Kamijo[b]

Yoshihiro Tomaru[c]

[a] Graduate School of Economics, Waseda University

1-6-1, Nishi-waseda, Shinjuku-ku, Tokyo, 169-8050, Japan

[b,c] Faculty of Political Science and Economics, Waseda University

1-6-1, Nishi-waseda, Shinjuku-ku, Tokyo, 169-8050, Japan

Abstract

We investigate an interregional mixed duopoly where a local pubic firm competes against a private firm. We employ a spatial model with price competition. The public firm is owned by the local government of the left half of the linear city, called Region 1, and maximizes the welfare of Region 1. We demonstrate that our two-stage game composed of location choice and price competition has two types of equilibria. In one equilibrium, the public firm locates in Region 1 and the private firm locates in the outside of the region. In the other equilibrium, both the firms are located in Region 1.

JEL classification: L13; L32; R32; R59

Keywords: Interregional mixed duopoly; Local public firm; Spatial model

1 Introduction

In the recent waves of privatization, not only state-owned firms but also local public firms have been privatized. Nevertheless, local public firms still exist in many developing countries as well as in developed countries. One reason for this is that they usually supply, necessities such as natural gas, electricity, water services, medical services, and educational services. In most cases, such

*Corresponding author. Email-address: tomo-ino@suou.waseda.jp Telephone number: (+81) 3-3208-8560

goods and services are also provided by private firms. The purpose of this paper is to investigate mixed markets where private and local public firms compete.

Competition among public and private firms has been studied in the literature on mixed oligopoly (for example, see De Fraja and Delbono, 1989). It usually assumes one country or one market in which one public firm and several private firms compete, and examines the effect of the privatization of the public firm on social welfare. Thus the literature has not established an appropriate model of the behaviors of local public firms in a country consisting of a number of regions or provinces. Certainly, a few previous studies such as Fjell and Pal (1996), Pal and White (1998), and Matsushima and Matsumura (2006) investigate the effect of imports from foreign firms on the domestic mixed market. If we regard the domestic and foreign countries as provinces or counties, it seems that some works analyze the mixed market, which includes a local public firm. However, in the real world, the local public firms in one region often supply goods and services to consumers in other regions. For example, state hospitals and state universities supply services to the residents of other states. Therefore, in this paper, we establish a model where a local public firm in a region competes against a private firm and supplies goods and/or services to consumers who live outside the region.

For this purpose, we employ a Hotelling-type spatial model (Hotelling, 1929) in which population is dispersed and each consumer has a specific personal address on the line with a length of unity (hence, so-called the linear city). In this model, firms locate at a point on the line and the purchase of the goods from one of them involves transportation costs that vary according to the consumer's location. Because consumers have to incur the transportation costs of goods, they choose which firm they purchase from, taking into account the transportation costs in addition to prices. Some works on mixed oligopoly use a spatial model (see, *e.g.*, Cremer et al., 1991; Matsumura and Matsushima, 2003, 2004; Matsushima and Matsumura, 2003). Cremer et al. (1991) carried out a pioneering work on the spatial mixed oligopoly, in which they assumed that state-owned and private firms exist in a linear city and decide their locations and prices.[1] We extend their model by dividing the city into two symmetric districts, Regions 1 and 2, each of which is run by a local government, and thus, a firm owned by the government is regarded as a local public firm.

[1]In their models, the state-owned firm indicates the firm that maximizes the welfare of the entire city (social welfare). For instance, Cremer et al. (1991) analyze how social welfare is affected by the number of state-owned and private firms.

This paper is different from Cremer et al. (1991) in terms of the ownership of the public enterprise. In Cremer et al. (1991) the owner is a central government that has authority over the entire linear city. In contrast, in this paper, it is the local government that runs a half of the city. We suppose that the local government of Region 1 owns the public firm and the owners of the private firm are in Region 2. In addition, we also assume that the local public firm aims at maximizing the local welfare in Region 1 and that the local welfare does not include the profit of the private firm.

In the above setting, we find that our model of location choice and price competition has multiple equilibria. In one equilibrium, the local public firm locates in the region run by the government and the private firm locates outside the region. In this equilibrium, all the goods to the residents in Region 1 are supplied by the local public firm. Moreover, this firm also provides goods to some of the residents in Region 2. In the other equilibrium, both the firms locate in Region 1. In contrast to the former equilibrium, a large portion of the consumers in Region 1 is supplied by the private firm and the local public firm monopolizes the demand of the residents in Region 2. An important welfare implication is that the latter equilibrium is more preferable than the former in terms of the payoffs of the local public firm and the private firm. In other words, both the local welfare of Region 1 and the profit of the private firm are higher in the equilibrium where both the firms locate in Region 1 than in the equilibrium where they locate in different regions.

Our model can be interpreted as one of foreign direct investment. Let us consider Region 1 as a domestic country and, thus, a local public firm as a domestic public firm. Region 2 is considered as a foreign country that is adjacent to the domestic country and where the market for goods is unified among the two countries. Consider a situation that the entry of foreign firms is substantively prohibited by an exorbitant corporate tax on them. In this case, the first equilibrium can arise because the foreign private firm prefers a location at Region 2. On the other hand, in the case where the domestic government attracts the foreign firm into the domestic country, the second equilibrium occurs because the profit of the foreign firm in the second is greater than that in the first equilibrium. Thus, our result indicates that the domestic country can improve its welfare through opening the domestic market to the foreign firms.

There exist some studies that are closely related to our model. Tharakan and Thisse (2002) analyze the model in which two regions are divided by a boundary point on the linear city and each region has a private firm. However they assume that each private firm locates at the center of its region, although in our model, firms' locations are determined endogenously and they can locate in either country. Bárcena-Ruiz and Garzón (2005) analyze the model where two regions (or countries) trade with one another and their governments strategically decide whether they privatize their own public firms. They show that the decision of privatization and the trade patterns are determined by the difference of the marginal costs between the local public and private firms. In their model, there is no trade if there is no difference in the marginal costs. We present a model where a local public firm exports to the other region even though both the private and public firm's marginal costs are identical.

The remainder of this paper proceeds as follows. In Section 2, we explain our basic framework of the spatial model. In Section 3, we first explore the subgame perfect equilibrium for the two-stage game: in the first stage, a local public firm and a private firm choose their location, and in the second stage, they compete in price. We then discuss the properties of two types of equilibria. In Section 4, we extend the basic model. In Section 5, we offer some concluding remarks and discuss the possibility for future research.

2 Model

We consider a spatial model in which there is a linear city of length one represented by the interval $[0, 1]$ and consumers that are uniformly with density one. This city comprises two districts - $[0, 1/2)$ and $[1/2, 1]$ - referred to as Region 1 and Region 2, respectively.

There are two firms that produce a homogeneous good at zero cost. One firm, indexed by A, is owned by the local government of Region 1 and the other, indexed by B, is owned by private shareholders in Region 2.[2] Let $a \in [0, 1]$ and $b \in [0, 1]$ denote the locations of Firms A and B, respectively. We assume that each consumer purchases one unit of the good irrespective of the price. To purchase the good, the consumer must incur the transportation cost that is quadratic in distance between the firm's and his/her locations, in addition to the mill price. For instance, a consumer living at point $y \in [0, 1]$ incurs a transportation cost of $t(y - a)^2$ and mill price P_A when he/she purchases good from Firm A.[3] Since both firms produce a homogeneous good, each consumer patronizes the firm with the lower full price, *i.e.*, the sum of the transportation cost and

[2]This assumption is modified in Section 4.

[3]The value of t (> 0) does not matter to the results obtained from our analysis.

the mill price.

For any $a \in [0,1]$ and $b \in [0,1]$, let x denote the location of the consumer who is indifferent between buying from Firms A and B; thus we obtain the solution to the following equation:

$$P_A + t(x-a)^2 = P_B + t(x-b)^2,$$

where P_i $(i = A, B)$ is the price charged by Firm i. Thus, when $a \neq b$, x is given by

$$x = \frac{a+b}{2} + \frac{P_A - P_B}{2(a-b)t}. \tag{1}$$

If Firm A is located to the left of the location of Firm B, *i.e.*, $a < b$, the consumers who live on the left side of x purchase from Firm A whereas those living on the right side of x purchase from Firm B, and vice versa. Accordingly, Firm A faces the demand given by

$$D_A(P_A, P_B, a, b) = \begin{cases} x & \text{if} \quad a < b, \\ 1-x & \text{if} \quad a > b, \\ 0 & \text{if} \quad a = b \text{ and } P_A > P_B, \\ \frac{1}{2} & \text{if} \quad a = b \text{ and } P_A = P_B, \\ 1 & \text{if} \quad a = b \text{ and } P_A < P_B, \end{cases}$$

where in the case in which both firms locate at the same point, all the consumers purchase from the firm with the lower price because the transportation cost is the same whether they purchase from Firm A or Firm B. When both the firms set the same price, it is assumed that the total demand is equally divided among the two firms.

The demand for Firm B is defined by the total demand minus the demand for Firm A, that is, $D_B = 1 - D_A$.

Since there is no cost, the profit of each firm is given by

$$\Pi_i = P_i D_i \quad i = A, B. \tag{2}$$

The social welfare W is defined by

$$W = \begin{cases} -\int_0^x t(a-z)^2 dz - \int_x^1 t(b-z)^2 dz & \text{if} \quad a \leq b, \\ -\int_0^x t(b-z)^2 dz - \int_x^1 t(a-z)^2 dz & \text{otherwise.} \end{cases} \tag{3}$$

Since individual demands are perfectly inelastic, positive prices (*i.e.*, the prices above marginal costs) do not create any distortions in the allocation of resources. Thus, the maximization of social welfare is equivalent to the minimization of the transportation cost. The sum of the transportation costs of all the consumers depends on the locations of the two firms, and so does social welfare.

Next, we consider the local welfare of Region 1. In contrast to social welfare, local welfare is affected by the firm's profit and the prices of the products, which are borne by the local residents. Accordingly, the local welfare of Region 1 is given by

$$W_1 = \Pi_A - T_1 - C_1,$$

where T_1 and C_1 denote the total transportation cost and the total burden of the price, respectively, which are borne by the residents of Region 1. Note that Π_A, T_1, and C_1 vary with the locations of the two firms and the corresponding boundary x. These relations in the case of $a \neq b$ are summarized in Table 1. Let F_i denote W_1 in Case i $(i = 1, 2, 3, 4)$. On the other hand, when both the firms locate at the same point $(a = b)$, the local welfare of Region 1 is given by

$$W_1 = \begin{cases} -\int_0^{1/2} t(a-z)^2 dz + \frac{1}{2}P_A & \text{if} \quad P_A < P_B, \\ -\int_0^{1/2} t(a-z)^2 dz & \text{if} \quad P_A = P_B, \\ -\int_0^{1/2} t(a-z)^2 dz - \frac{1}{2}P_B & \text{if} \quad P_A > P_B. \end{cases}$$

[Insert Table 1 Here]

Since social welfare is the sum of the local welfare of the two regions, the local welfare of Region 2 is given by

$$W_2 = W - W_1. \tag{4}$$

While Firm B maximizes its profit, Π_B, Firm A, which is owned by the government of Region 1, aims at maximizing the local welfare of Region 1, W_1.

We consider the following standard two-stage game: In the first stage, each firm chooses its location simultaneously, and in the second stage, the firms choose their prices simultaneously, having observed their locations chosen at the first stage. We assume that each firm can locate at any point in the interval $[0, 1]$ without any restriction. Thus, we allow the local public firm to locate outside its home region.

3 Results

We use a subgame perfect equilibrium as our solution concept. Thus, we first consider the subgame after location choice, that is, the price-setting game. Then, we consider the location choice stage.

3.1 *Price competition*

Given the firms' locations a, b chosen at the first stage, the two firms compete with respect to price in the second stage of our two-stage game. Since the objectives of the two firms vary with their locations, we should separate three cases to analyze the equilibrium: (I) $a > b$, (II) $b < a$, and (III) $a = b$.

We first consider case (III) because the equilibrium of the price-setting game in this case is easily attained as compared to the other cases. Firm B, a profit-maximizing private firm, always prefers to choose a price that is slightly lower than P_A whenever P_A is positive. Moreover, for any price of Firm B that satisfies $0 < P_B < P_A$, there is another price P'_B such that $P_B < P'_B < P_A$ holds. Thus, given such a price P_B, Firm B has an incentive to change the price from P_B to P'_B because the profit of Firm B is better off by doing so. Thus, Firm B does not have an optimal action in the price-setting game provided $P_A > 0$. Consequently, $P_A = P_B = 0$ is a unique equilibrium when the two firms choose the same location. In fact, both the firms do not have incentives to change the prices from this situation. In addition, $W_1 = T_1$ since $P_A = P_B = 0$ in the equilibrium.

Next, we consider the other two cases. Let us denote the objectives of the two firms in (I) and (II) by $U_i^j(P_A, P_B),\ i = A, B,\ j = I, II$. Then, the first order conditions for the maximizations are as follows:

$$\frac{\partial U_A^j}{\partial P_A} = 0, \quad \frac{\partial U_B^j}{\partial P_B} = 0.$$

The reaction functions of Firms A and B, denoted by $r_A^j(P_B)$ and $r_B^j(P_A)$, respectively, satisfy

$$\frac{\partial U_A^j}{\partial P_A}(r_A^j(P_B), P_B) = 0, \quad \frac{\partial U_B^j}{\partial P_B}(P_A, r_B^j(P_A)) = 0.$$

The equilibrium price of the second stage is defined by the fixed point of the composite function of the two reaction functions. In other words, $P_A^* = r_A^j(r_B^j(P_A^*))$ and $P_B^* = r_B^j(P_A^*)$ hold.

(I) $\mathbf{a < b}$. This case corresponds to Cases 1 and 2 in Table 1. Since $a < b$, we obtain

$$F_1 - F_2 = -\frac{(P_A - P_B + \alpha)^2}{4(b-a)t} = -\frac{(P_A - \widetilde{P}_A)^2}{4(b-a)t} \leq 0,$$

where F_i is the local welfare of Region 1 in Case i, $\alpha := (b-a)(1-a-b)t$, and $\widetilde{P}_A := P_B - \alpha$. Thus, $F_1 < F_2$ holds unless $P_A = \widetilde{P}_A$. In addition, it is easily verified that F_1 and F_2 are the concave functions in P_A. Moreover,

$$\left.\frac{\partial F_1}{\partial P_A}\right|_{P_A=\widetilde{P}_A} = \left.\frac{\partial F_2}{\partial P_A}\right|_{P_A=\widetilde{P}_A} = \frac{P_B - \alpha}{2(a-b)t} = -\frac{\widetilde{P}_A}{2(b-a)t}.$$

Thus, the signs of the slopes of F_1 and F_2 at $\widetilde{P}_A$ are changed according to the sign of $\widetilde{P}_A$. As a result, we have the relationship of F_1 and F_2, as depicted in Figure 1. The thin and thick curves denote F_1 and F_2, respectively.

(insert Figure 1 here)

Equation (1) implies

$$P_A = (a-b)(2x-1)t + P_B + (a-b)(1-a-b)t = (a-b)(2x-1)t + \widetilde{P}_A.$$

Thus, we have under $a < b$,

$$\begin{cases} P_A \leq \widetilde{P}_A \iff x \geq \frac{1}{2} & \Longrightarrow \quad \text{Case 1 and } W_1 = F_1, \\ P_A > \widetilde{P}_A \iff x < \frac{1}{2} & \Longrightarrow \quad \text{Case 2 and } W_1 = F_2, \end{cases}$$

and $P_A = \widetilde{P}_A \iff x = 1/2$. This indicates that the curves with the shaded portion in Figure 1 represent the local welfare W_1. Thus, the maximum of W_1 is attained by the maximization of F_2 when $\widetilde{P}_A < 0$ and by the maximization of F_1 when $\widetilde{P}_A \geq 0$. By the first order conditions for the maximization of F_1 and F_2, we obtain

$$r_A^I(P_B) = \begin{cases} \frac{P_B - \alpha}{2} & \text{if} \quad \widetilde{P}_A \geq 0 \quad (P_B \geq \alpha), \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

In contrast with Firm A, the objective of Firm B is $U_B^I = \Pi_B = P_B(1-x)$ irrespective of Case 1 or 2. Thus, we have, by the first order condition of maximizing U_B^I,[4]

$$r_B^I(P_A) = \frac{P_A - (a-b)(2-a-b)t}{2}. \tag{6}$$

Equations (5) and (6) yield the following equilibrium prices:

$$P_A^1(a,b) = -\frac{(a-b)(a+b)t}{3}, \quad P_B^1(a,b) = -\frac{(a-b)(3-a-b)t}{3}.$$

[4]The second order condition is satisfied.

The superscript 1 indicates that the equilibrium holds for the range of Case 1 ($x \geq 1/2$).

(II) $\mathbf{a > b}$. This corresponds to Cases 3 and 4 in Table 1. We obtain, by $a > b$,

$$F_3 - F_4 = -\frac{(P_A - P_B + \alpha)^2}{4(a-b)t} \leq 0.$$

Thus, $F_3 < F_4$ holds except for the case that $P_A = \widetilde{P}_A$. Further, both F_3 and F_4 are concave in P_A and

$$\left.\frac{\partial F_3}{\partial P_A}\right|_{P_A=\widetilde{P}_A} = \left.\frac{\partial F_4}{\partial P_A}\right|_{P_A=\widetilde{P}_A} = -\frac{P_B - \beta}{2(a-b)t},$$

where $\beta := (a-b)(a+b)t > 0$. We have the relationship of F_3 and F_4, as illustrated in Figure 2. In this figure, the thin and thick curves represent F_3 and F_4, respectively. Note that the signs of the slopes of F_3 and F_4 at $P_A = \widetilde{P}_A$ vary according to the sign of $P_B - \beta$. Therefore, we obtain the two different graphs as depicted in Figure 2.

(insert Figure 2 here)

Equation (1) and Table 1 imply that under $a > b$,

$$\begin{cases} P_A \geq \widetilde{P}_A \iff x \geq \frac{1}{2} & \Longrightarrow \quad \text{Case 3 and } W_1 = F_3, \\ P_A < \widetilde{P}_A \iff x < \frac{1}{2} & \Longrightarrow \quad \text{Case 4 and } W_1 = F_4, \end{cases}$$

and $P_A = \widetilde{P}_A \iff x = 1/2$. The curves with the shaded portion represent the local welfare W_1. Thus, we have, by the maximization of F_3 and F_4,

$$r_A^{II}(P_B) = \begin{cases} \frac{P_B+(a-b)(2-a-b)t}{2} & \text{if} \quad P_B < \beta, \\ (a-b)t & \text{otherwise.} \end{cases} \tag{7}$$

Finally, the objective of firm B is $U_B^{II} = \Pi_B = P_B x$. By the maximization of Π_B,[5] we have

$$r_B^{II}(P_A) = \frac{P_A + (a-b)(a+b)t}{2}. \tag{8}$$

By Equations (7) and (8), we obtain the relationship of the two reaction curves when $a > b$, as presented in Figure 3. Since the reaction curve of Firm A is kinked at $P_B = \beta$, there are two possible types of intersections. As is shown in Figure 3, when $\beta > (a-b)t \iff a+b > 1$, the intersection occurs in the area of $P_A < \beta$ and $P_B < \beta$. On the other hand, when $\beta \leq (a-b)t \iff a+b \leq 1$,

[5]The second order condition holds.

the intersection lies in the area of $P_A \geq \beta$ and $P_B \geq \beta$. Thus, we obtain the equilibrium prices in the second stage as follows:

$$\begin{cases} P_A^3(a,b) = \frac{(a-b)(4-a-b)t}{3}, & P_B^3(a,b) = \frac{(a-b)(2+a+b)t}{3} \quad \text{if} \quad a+b>1, \\ P_A^4(a,b) = (a-b)t, & P_B^4(a,b) = \frac{(a-b)(1+a+b)t}{2} \quad \text{otherwise.} \end{cases}$$

The superscripts 3 and 4 indicate that the equilibria hold for the ranges of Case 3 ($x \geq 1/2$) and Case 4 ($x < 1/2$), respectively.

(insert Figure 3 here)

These results are summarized in the following lemma.

Lemma 1. *The relation between locations a, b and the equilibrium prices in the second stage is as follows:*

(I) $a < b$

$$P_A^1(a,b) = -\frac{(a-b)(a+b)t}{3}, \quad P_B^1(a,b) = -\frac{(a-b)(3-a-b)t}{3}.$$

(II) $a > b$

$$\begin{cases} P_A^3(a,b) = \frac{(a-b)(4-a-b)t}{3}, & P_B^3(a,b) = \frac{(a-b)(2+a+b)t}{3} \quad \textit{if} \quad a+b>1, \\ P_A^4(a,b) = (a-b)t, & P_B^4(a,b) = \frac{(a-b)(1+a+b)t}{2} \quad \textit{otherwise.} \end{cases}$$

(III) $a = b$.

$$P_A(a,b) = 0, \quad P_B(a,b) = 0.$$

The superscript j of the equilibrium price $P_i^j(a,b)$ corresponds to Case j in Table 1.

Lemma 1 shows that the equilibrium prices are increasing functions of the distance between the two firms if $a \neq b$. Thus, for example, $P_A^1(a,b)$ is a decreasing function of a, whereas $P_B^1(a,b)$ is an increasing function of b.

Let $\mathbf{P}^j(a,b) = (P_A^j(a,b), P_B^j(a,b))$ denote the pair of equilibrium prices for Case j ($j = 1,3,4$). Based on Lemma 1, we present Figure 4 that shows the range in which $\mathbf{P}^j(a,b)$ holds. The boundary, in which the consumer is indifferent to either of the two firms from which he/she purchases the good, in each case, is obtained as follows:

$$\begin{cases} x^1(a,b) = \frac{3+a+b}{6} & \text{if} \quad \mathbf{P} = \mathbf{P}^1(a,b), \\ x^3(a,b) = \frac{2+a+b}{6} & \text{if} \quad \mathbf{P} = \mathbf{P}^3(a,b), \\ x^4(a,b) = \frac{1+a+b}{4} & \text{if} \quad \mathbf{P} = \mathbf{P}^4(a,b). \end{cases} \tag{9}$$

In the next subsection, we consider the location problem in the first stage.

(insert Figure 4 here)

3.2 *Location choice*

We now consider the location choices of the two firms in the first stage. We observe from Figure 4 that if Firm B is located in $[0, 1/2)$, as Firm A moves to right (that is, a increases), the equilibrium price of Firm A changes from $P_A^1(a, b)$ to $P_A^4(a, b)$ at $a = b$ and from $P_A^4(a, b)$ to $P_A^3(a, b)$ at $a = 1 - b$. Similarly, if Firm B is located in $[1/2, 1]$, it changes from $P_A^1(a, b)$ to $P_A^3(a, b)$ at $a = b$ as a increases. Accordingly, in order to obtain the reaction function of Firm A in the first stage, we need to distinguish between the two cases.

When $b \in [0, 1/2)$, the objective of Firm A is given by

$$W_1(a,b) = \begin{cases} F_1 = P_A^1(x^1 - \frac{1}{2}) - \int_0^{1/2} t(a-z)^2 dz & \text{if} \quad a \le b, \\ F_4 = \frac{P_A^4}{2} - P_B^4 x^4 - \int_0^{x^4} t(b-z)^2 dz - \int_{x^4}^{1/2} t(a-z)^2 dz & \text{if} \quad b < a \le 1-b, \\ F_3 = P_A^3(1 - x^3) - \frac{P_B^3}{2} - \int_0^{1/2} t(b-z)^2 dz & \text{otherwise}, \end{cases} \tag{10}$$

and when $b \in [1/2, 1]$, it is given by

$$W_1(a,b) = \begin{cases} F_1 = P_A^1(x^1 - \frac{1}{2}) - \int_0^{1/2} t(a-z)^2 dz & \text{if} \quad a \le b, \\ F_3 = P_A^3(1 - x^3) - \frac{P_B^3}{2} - \int_0^{1/2} t(b-z)^2 dz & \text{otherwise}. \end{cases} \tag{11}$$

Thus, we can obtain the reaction function of Firm A as follows:

$$R_A(b) = \begin{cases} \frac{10 - b - \sqrt{73 - 20b + 4b^2}}{3} & \text{if} \quad b < \bar{b}, \\ \frac{-18 - 2b + \sqrt{378 + 72b + 16b^2}}{6} & \text{otherwise}, \end{cases} \tag{12}$$

where $\bar{b} \approx 0.3656$. See the appendix for the derivation of Equation (12).

If Firm A is located in $[0, 1/2)$, the equilibrium price of Firm B changes from $P_B^4(a, b)$ to $P_B^1(a, b)$ at $b = a$ as b increases. Similarly, if Firm A is located in $[1/2, 1]$, it changes from $P_B^4(a, b)$ to $P_B^3(a, b)$ at $b = 1 - a$ and from $P_B^3(a, b)$ to $P_B^1(a, b)$ at $b = a$ as b increases. Thus, we need to classify the objective of Firm B into two cases. When $a \in [0, 1/2)$, the objective is given by

$$\Pi_B(a,b) = \begin{cases} G_4 = P_B^4 x^4 & \text{if} \quad b < a, \\ G_1 = P_B^1(1 - x^1) & \text{otherwise}, \end{cases} \tag{13}$$

and when $a \in [1/2, 1]$,

$$\Pi_B(a,b) = \begin{cases} G_4 = P_B^4 x^4 & \text{if} \quad b \leq 1-a, \\ G_3 = P_B^3 x^3 & \text{if} \quad 1-a < b < a, \\ G_1 = P_B^1(1-x^1) & \text{otherwise,} \end{cases} \tag{14}$$

where G_i $(i = 1, 3, 4)$ is the profit of Firm B corresponding to each equilibrium price. We can derive the reaction function of Firm B as follows:

$$R_B(a) = \begin{cases} 1 & \text{if} \quad a < \bar{a}, \\ 0 & \text{otherwise,} \end{cases} \tag{15}$$

where $\bar{a} \approx 0.3799$. See the appendix for the derivation of Equation (15).

Figure 5 describes the reaction functions $R_A(b)$ and $R_B(a)$. $R_A(b)$ is jumped at $b = \bar{b}$ and $R_B(a)$ is jumped at $a = \bar{a}$. As is shown in Figure 5, this model has two subgame perfect equilibria, E_1 and E_2. Let (a_i^*, b_i^*) denote the pair of equilibrium location points in E_i $(i = 1, 2)$. Thus, we have the following proposition.

(insert Figure 5 here)

Proposition 1. *In the first stage, there are two equilibria, E_1 and E_2. Each location point is as follows:*

$$E_1 : \begin{cases} a_1^* = \frac{-20+\sqrt{466}}{6} \approx 0.2645 \\ b_1^* = 1 \end{cases} \qquad E_2 : \begin{cases} a_2^* = \frac{10-\sqrt{73}}{3} \approx 0.4853 \\ b_2^* = 0 \end{cases}$$

Proposition 1 shows that our two-stage game has two types of equilibria. Each firm is located in its home district in equilibrium E_1, whereas both the firms are located in Region 1 whose government owns Firm A in equilibrium E_2. Figure 6 describes the locations of the two firms in the interval $[0, 1]$ at the two equilibria. Here, the boundary x in equilibrium E_k is denoted by x_k $(k = 1, 2)$.

(insert Figure 6 here)

As is shown by d'Aspremont et al. (1979), if both the firms are private, they locate at both the edges of the linear city. However, Proposition 1 shows that this is not the case when one of the firms is owned by a local government. Our local public firm, Firm A, has W_1 as its objective function. Let Π_{ij} $(i = A, B,\ j = 1, 2)$ denote a profit of Firm i obtained from the residents in Region j. Then, this objective function can clearly be rewritten as

$$W_1 = \Pi_{A2} - \Pi_{B1} - T_1 = P_A \cdot x_{A2} - P_B \cdot x_{B1} - T_1,$$

where x_{A2} and x_{B1} represent Firm A's supply to the residents in Region 2 (or Firm A's market share in Region 2) and that of Firm B to the residents in Region 1. Totally differentiating this function, we obtain

$$dW_1 = x_{A2} dP_A + P_A dx_{A2} - d(P_B \cdot x_{B1}) - dT_1. \tag{16}$$

This equation states that local welfare in Region 1 improves if the price P_A and market share x_{A2} increase or if the transportation cost T_1 or the payment to Firm B by residents in Region 1, $P_B \cdot x_{B1}$, decreases. In particular, the price P_A increases when Firm A departs from the rival firm, because this makes the demand inelastic. Henceforth, we refer to the terms $x_{A2}dP_A$, $P_A dx_{A2}$, and dT_1 as the *price-raising effect*, *market share effect*, and *transportation cost effect*, respectively. We also term $d(P_B \cdot x_{B1})$ as the *payment effect*.

We now explain why the locations in Proposition 1 can be equilibria. First, we consider equilibrium E_1. To examine this equilibrium, we analyze what happens if Firms A and B locate at points 0 and 1, respectively. In this case, by Lemma 1 and Equation (9), the demand that Firm A faces, x, is 2/3. Hence, the residents in Region 1 purchase the goods from only Firm A, and we can reduce Equation (16) as follows:

$$dW_1 = x_{A2} dP_A + P_A dx_{A2} - dT_1.$$

First, the transportation cost effect gives Firm A an incentive to move to point 1/4 where the transportation costs in Region 1 is minimized. Further, at point 1/4, Firm A moves its location to the right because this move leads to an expansion of its market share in Region 2 (through the market share effect). It is certain that Firm A has an incentive to move left for fear that competition between firms are tough (through the price-raising effect). However, this price-raising effect is small since the local welfare W_1 does not include Firm A's profits from Region 1, which weakens the positive aspect of the price-raising effect. In fact, when evaluated at $a = 1/4$ and $b = 1$ (the transportation cost effect vanishes), from Lemma 1 and (9),

$$\frac{\partial W_1}{\partial a} = x_{A2}\frac{\partial P_A}{\partial a} + P_A \frac{\partial x_{A2}}{\partial a} - \frac{\partial T_1}{\partial a} = -\frac{t}{18}(a+b)(3a-b) = \frac{5t}{288} > 0.$$

Next, we explain equilibrium E_2. Let us start with the case where the transportation cost T_1 is minimized, *i.e.*, $dT_1 = 0$ when $b = 0$. This is achieved when $a = 1/3$ as easily seen by Lemma 1 and (9). Then, the marginal consumer x is also 1/3, and thus, all the residents in Region 2 purchase goods from Firm A. This means that the market share effect evaporates. Based on this fact, we can rewrite (16) as

$$dW_1 = x_{A2} dP_A - d(P_B \cdot x_{B1}),$$

at $a = 1/3$ and $b = 0$. As shown in d'Aspremont et al. (1979), the firms' profits become higher as the firms depart from each other; in other words, a increases. Hence, the payment effect is negative while the price-raising effect is positive. To analyze the relative strength of these two partially offsetting effects, we evaluate, at $a = 1/3$ and $b = 0$,

$$\frac{\partial W_1}{\partial a} = x_{A2}\frac{\partial P_A}{\partial a} - \frac{\partial}{\partial a}(P_B \cdot x_{B1}) = \frac{t}{2} - (1+a+b)(1+3a-b)\frac{t}{8} = \frac{t}{3} > 0.$$

This equation means that the local welfare gain from an increase in profits from the residents in Region 2 is larger than the loss from an increase in the payment to Firm B. This results in the increase in a. Note that although Firm A moves its location a from 1/3 to the right, a is nevertheless in Region 1.

3.3 *Equilibrium comparison*

In this subsection, we compare two equilibria, E_1 and E_2. The equilibrium payoffs are as follows:

$$E_1 : \begin{cases} \text{Firm } A: & W_1(a_1^*, b_1^*) = \frac{(-9953+466\sqrt{466})t}{1944} \approx 0.0548t, \\ \text{Firm } B: & \Pi_B(a_1^*, b_1^*) = \frac{(26-\sqrt{466})(32-\sqrt{466})^2 t}{3888} \approx 0.1231t, \end{cases}$$

$$E_2 : \begin{cases} \text{Firm } A: & W_1(a_2^*, b_2^*) = \frac{(-604+73\sqrt{73})t}{216} \approx 0.0913t, \\ \text{Firm } B: & \Pi_B(a_2^*, b_2^*) = \frac{(10-\sqrt{73})(13-\sqrt{73})^2 t}{216} \approx 0.1338t. \end{cases}$$

The above equations show that equilibrium E_2 is preferable for the two firms to the equilibrium E_1 in terms of their payoffs since $W_1(a_2^*, b_2^*) > W_1(a_1^*, b_1^*)$ and $\Pi_B(a_2^*, b_2^*) > \Pi_B(a_1^*, b_1^*)$ hold.

The reason for this is as follows. In equilibrium E_2, both the firms obtain the benefits at the sacrifice of the consumer surplus of Region 2. We explain this using the each firm's price in equilibrium E_2:

$$(P_A(a_2^*, b_2^*), P_B(a_2^*, b_2^*)) = \left(\frac{(10-\sqrt{73})t}{3}, \frac{(13-\sqrt{73})(10-\sqrt{73})t}{18}\right) \approx (0.4853t, 0.3604t).$$

On the one hand, Firm A receives the benefit from Region 2 by selling the products at a relatively high price. On the other hand, Firm B receives the benefit because it can sell a considerable amount to consumers in Region 1 due to the high price that Firm A sets. Consequently, the residents in Region 2 face dual hardships. One is that they face the high price set by Firm A and the other is that the transportation costs they incur are very high since both the firms are located in Region 1 as pointed out in the previous subsection. Thus, the local welfare of Region 2 in E_2 is lower than in E_1. In fact, these hardships for Region 2 are so serious that in equilibrium E_2, not only the local welfare of Region 2 but also social welfare decreases as compared to equilibrium E_1, as follows:

$$E_1 : \begin{cases} W_2(a_1^*, b_1^*) = -\frac{(-20056+935\sqrt{466})t}{1296} \approx -0.0987t, \\ W(a_1^*, b_1^*) = -\frac{(-40262+1873\sqrt{466})t}{3888} \approx -0.0439t, \end{cases}$$

$$E_2 : \begin{cases} W_2(a_2^*, b_2^*) = -\frac{7(-337+40\sqrt{73})t}{216} \approx -0.1543t, \\ W(a_2^*, b_2^*) = -\frac{(-195+23\sqrt{73})t}{24} \approx -0.0630t. \end{cases}$$

Thus, we obtain the following proposition.

Proposition 2. *Equilibrium E_2 are preferable for the two firms to equilibrium E_1 in terms of their payoffs. However, social welfare in E_2 is lower than that in E_1 because the efficiency of E_2 is at the expense of Region 2.*

We include some remarks on social welfare and the firms' locations in a market as a whole. Cremer et al. (1991), which analyze the mixed duopoly with a state-owned firm and a private firm, show that in equilibrium, the state-owned firm is located at point 1/4 that minimizes the total transportation cost in the market when the private firm is at point 3/4, which maximizes social welfare in the entire market. d'Aspremont et al. (1979), which examine the private duopoly, show that the two firms are located at points 0 and 1, respectively, where social welfare is low. In our equilibria E_1 and E_2, Firm A does not set a to points 0, 1, 1/4, and 3/4, and thus, social welfare in our equilibria is intermediate between those in Cremer et al. (1991) and d'Aspremont et al. (1979).

4 Extensions

In the preceding sections, we discussed a simultaneous-move interregional mixed duopoly between a local pubic firm and a private firm owned by the residents of Region 2. In this section, we briefly discuss some related topics: leadership of the local public firm and ownership of the two firms.

4.1 *Leadership by the local public firm*

We investigate whether the timing of entry into the market matters. An importance of the timing problem arises from an interpretation of our model in terms of international trade or foreign direct investment because in such situations, a domestic public firm has an advantage in entering the domestic market to foreign firms. We consider the timing as follows. First, the local public firm chooses its location. Second, having observed the location of the local public firm, the private firm chooses its location. After the sequential location choice, the two firms simultaneously choose their prices. As the following proposition will show, we find that the timing consideration of our model fortunately serves as a criterion for selection from two equilibria.

In the sequential location choice game mentioned above, the public firm chooses its location in considering the best response behavior of the private firm against its location. As Figure 5 shows, the best response location curve of the private firm consists of two horizontal lines. Thus, if the local public firm chooses location $a < \bar{a}$ in the first stage, the private firm is located at $b = 1$ in the next stage. On the other hand, the private firm chooses location $b = 0$ when the local public firm chooses $a > \bar{a}$.

The fact that a pair of locations $[(a_1^*, b_1^*) = (a_1^*, 1)]$ is the equilibrium point of the simultaneous location choice game implies that

$$W_1(a_1^*, 1) > W_1(a, 1) \text{ for any } a \in [0, 1],\ a \neq a_1^*. \tag{17}$$

Applying the similar reasoning to $[(a_2^*, b_2^*) = (a_2^*, 0)]$, we obtain

$$W_1(a_2^*, 0) > W_1(a, 0) \text{ for any } a \in [0, 1],\ a \neq a_2^*. \tag{18}$$

Since $W_1(a_2^*, 0) > W_1(a_1^*, 1)$ by Proposition 2, with Equation (17) we obtain

$$W_1(a_2^*, 0) > W_1(a, 1) \text{ for any } a \in [0, 1]. \tag{19}$$

By Equations (18) and (19), in the equilibrium of the sequential-move game, the local public firm chooses location $a = a_2^*$ in the first stage and then the private firm locates at $b = 0$.

Proposition 3. *In the equilibrium of the sequential-move game, first the local public firm locates at $a = a_2^*$ and then the private firm locates at $b = b_2^* = 0$.*

Thus, the location points of E_2 are realized through the sequential-move game.

4.2 *Ownership of the firms*

In this subsection, we provide an inclusive discussion about the ownership of firms in a linear city model. As mentioned, our model considered in previous sections (henceforth, called a basic model) is different from those of the other existing studies on the ownership of firms: given an opponent (Firm B) as private firm, Firm A is a state-owned firm in Cremer et al. (1991); a local public firm in the basic model; and a private firm in d'Aspremont et al. (1979). Because an opponent is fixed to a private firm, these studies do not consider competition among public firms. Thus, we treat such situations in this subsection. In addition, who owns the private firm or where the private shareholders live also matters because local and social welfare include the profits of private firms, which are owned by the residents of its own region, but not those of private firms, which are owned by the residents of other regions. This motivates us to study the case where a public firm competes against a foreign private firm.

Table 2 compare the existing studies with our basic model in terms of categorizing each case according to the ownership of the two firms. (i), (ii), and (iii) denote competition among public firms: they represent state-owned vs. state-owned, local public vs. state-owned, and local public vs. local public competition, respectively. (iv) is the case where a local public firm in Region 1 competes against a local private firm of the region; that is, the shareholders of the private firm are residents in Region 1. Thus, the local public firm should take into account the profit of the private firm in this case, in contrast to the basic model (see the definition of the local welfare of Region 1). This is also viewed as a special model of competition among a state-owned firm and a domestic private firm in a global market including a domestic market. Competition between a state-owned firm and a foreign private firm is denoted by (v).

In the following, we discuss the equilibrium location pattern from (i) to (v).

[Insert Table 2 Here]

(i), (ii), and (iii): competition among public firms. In the case of (i), namely, competition between two state-owned firms, the two firms have the same objective function, that is, the social welfare of the entire city. Thus, there is no strategic confrontation among them. They easily reach a unique Pareto-optimal outcome. Therefore, in the equilibrium of our two-stage game, the two firms locate at points $1/4$ and $3/4$, respectively, and the marginal consumer is at $1/2$.

A rather strong result is obtained in the case of (ii) and (iii). In these cases, the Pareto-optimal outcome of the entire city is attained as an equilibrium of our two-stage game, despite the different objective functions of the two firms (for a detailed discussion, see Inoue et al., 2007). However, this result in (ii) and (iii) strongly depends on the symmetric division of the city. When there is size-asymmetry in the two regions, the efficiency result in (ii) and (iii) is no longer assured.

(iv): local public firm vs. local private firm. We consider the model where not only the local public firm but also the private firm belong to Region 1. In other words, the private firms are owned by the residents in Region 1. Here, the local welfare of Region 1 is the sum of the consumer surplus of the region and the profits of both the firms. Thus, W_1 in this model is written as

$$W_1 = \Pi_A + \Pi_B - T_1 - C_1 = \Pi_{A2} + \Pi_{B2} - T_1.$$

Thus, the local public firm in this case does not compete against the private firm so severely as in the basic model (note that in the basic model, the objective of the local public firm is $\Pi_{A2} - \Pi_{B1} - T_1$). We thus obtain the following proposition.

Proposition 4. *In the two-stage game of location choice and price competition between a local public firm and a private firm owned by the residents in Region* 1*, there are two equilibria.*

1. *In one equilibrium, the relevant equilibrium variables are*

$$a = 0,\ b = \frac{2}{3},\ P_A = \frac{8t}{9},\ P_B = \frac{8t}{9},\ x = \frac{1}{3}.$$

2. *In the other equilibrium, the relevant equilibrium variables are*

$$a = 1,\ b = 0,\ P_A = 2t,\ P_B = \frac{3t}{2},\ x = \frac{3}{4}.$$

Proof. See the appendix.

Thus, as the above proposition shows, in the equilibrium with $a < b$, the local public firm does not move right to capture the demand from Region 2 because this depletes the profit of Firm B, which is obtained from Region 2. In fact, Firm A locates at the left edge of the linear city even though moving right would reduce the transportation cost of the residents of Region 1. On the other hand, in the equilibrium with $a > b$, the public firm does not move left so as to protect the residents in Region 1 against exploitation by Firm B; this is because it does not see the profit of

Firm B from Region 1 as a negative factor in its payoff, in contrast with the basic model. Rather, in this equilibrium, the local public firm is mainly concerned with the large profit obtained from Region 2.

(v): state-owned firm vs. foreign private firm. We consider the model in which a state-owned firm (Firm A), which is owned by the central government of the city as a whole, competes against a foreign private firm (Firm B). The aim of the state-owned firm is to maximize the social welfare of the entire city. That is, it takes into account the burden of the residents in *both* the regions. In this model, the two firms have no relation with Regions 1 and 2. Thus, without loss of generality, we assume that Firm A locates left to the location of Firm B, that is, $a < b$.

We obtain the following proposition.

Proposition 5. *In the two-stage game of location choice and price competition between a state-owned firm and a foreign private firm, the equilibrium locations are*

$$a = \frac{3\sqrt{7} - 7}{2} \approx 0.4686, \quad b = \frac{\sqrt{7} - 1}{2} \approx 0.8229.$$

The equilibrium prices are

$$P_A = 0, \quad P_B = (3 - \sqrt{7})^2 t \approx 0.1255t.$$

Besides, the location of the marginal consumer is $x = (\sqrt{7} - 1)/2 = b$.

Proof. See the appendix.

The difference from Cremer et al. (1991) is an ownership of the private firm. In their paper, the private firm is a domestic one, and thus, the state-owned firm takes the profit of the private into consideration. In contrast, in this model, the state-owned firm does not consider the profit of the private firm because the private firm is a foreign one. Thus, in this model, the state-owned firm engages in severe competition against the private firm as compared to Cremer et al. (1991), resulting in the aggressive location choice of the state-owned firm. That is, the location of the state-owned firm in this model, $a \approx 0.4686$, is closer to the center than in Cremer et al. (1991), $a = 0.25$.

In comparison with the basic model, the difference is the range of residents that the public firm considers. Thus, while the public firm in these two models are identical in not considering the profit of private firms, the public firm in the basic model only considers the residents in Region 1. In other words, while the objective of the public firm in this model is

$$W = -T - \Pi_B,$$

where T is the total transportation cost of the entire city, the local public firm in the basic model considers the transportation cost of only Region 1. Thus, the public firm in the basic model does not want to locate near the center of the linear city because as it goes away from the center of Region 1, the negative effect of the transportation cost tends to dominate the other positive effects including the market share effect and the price-raising effect. This is the reason why the location of the local public firm in equilibrium E_1 is quite left to the location of the public firm in the model here.

Finally, Table 3 shows the equilibrium location point in every case classified by the ownership of the firms.

[Insert Table 3 Here]

5 Conclusion

In this paper, we investigate a mixed duopoly involving a private firm and a local public firm, which is owned by a local government. We construct a two-stage game and show that the game has two subgame perfect equilibria (E_1 and E_2). One of them (E_1) is that each firm is located in its home district, and the other (E_2) is that both the firms are located in the same region, Region 1.

We introduce a local public firm that supplies to other regions, whereas most of the literature on mixed oligopoly treat public firms, that supply only to their own regions. In addition to this, we analyze the strategic decisions of each government by considering multiple regions as done in Bárcena-Ruiz and Garzón (2005). Our setting can be applied in the context of international relationships such as the location choice of multinational enterprises. In this context, equilibrium E_2 indicates a foreign firm's direct investment. As pointed out by Bárcena-Ruiz and Garzón (2005), particularly in the EU, although the Single Market was introduced, the decision of whether or not to privatize firms is a national issue. Thus, there are strategic interactions among member countries in the market.

This model is restrictive since we employ a simple duopoly model. However, this paper has a significant impact on the argument of mixed oligopoly due to the above reason. Moreover, this research could be extended the cases of asymmetric regions and quantity-setting oligopoly, so as to consider a variety of conditions. It is necessary to analyze the mixed oligopoly involving local public firms in order to clarify the desirable ownership patterns of firms.

Appendix

Derivation of Equation (12). In Figure 7, we present the local welfare of Region A as described in Equations (10) and (11). If $b < 1/2$ (left figure), W_1 is maximized when $a \in [0, b]$ ($W_1 = F_1$) or $a \in (b, 1-b]$ ($W_1 = F_4$). If $b \geq 1/2$ (right figure), W_1 is maximized when $a \in [0, b]$ ($W_1 = F_1$). In $b < 1/2$, whether the maximum value of W_1 exists in $a \in [0, b]$ or $a \in (b, 1-b]$ depends on the value of b. To derive this condition, we calculate the following equation:

$$
\begin{aligned}
& W_1(R_A^1(b), b) - W_1(R_A^4(b), b) \\
& \quad = -\frac{[9(215 + 73\gamma - 42\delta) + 18b(213 - 10\gamma - 4\delta) + 4b^2(27 - 34b + 9\gamma - 4\delta)]t}{1944}, \\
& \qquad\qquad\qquad\qquad \gamma \equiv \sqrt{73 - 4b(5-b)}, \quad \delta \equiv \sqrt{378 + 8b(9+2b)},
\end{aligned}
$$

where $R_A^j(b) := \operatorname{argmax}_a F_j$ $(j = 1, 4)$. This equation is a monotonically increasing function of b. In addition, when $b = \bar{b} \approx 0.3656$, the equation equals to zero. Thus, when $b \leq \bar{b}$, $R_A^4(b)$ maximizes W_1, otherwise $R_A^1(b)$ maximizes W_1. Hence, in the first stage, the reaction function of Firm A is expressed by

$$
R_A(b) = \begin{cases} \frac{10 - b - \sqrt{73 - 20b + 4b^2}}{3} & \text{if} \quad b < \bar{b}, \\ \frac{-18 - 2b + \sqrt{378 + 72b + 16b^2}}{6} & \text{otherwise.} \end{cases}
$$

(insert Figure 7 here)

Derivation of Equation (15). In Figure 8, we show the profit of Firm B described in Equations (13) and (14). If $a < 1/2$ (left figure), Π_B is maximized when $b = 0$ ($\Pi_B = G_4$) or $b = 1$ ($\Pi_B = G_1$). If $a \geq 1/2$ (right figure), Π_B is maximized when $b = 0$ ($\Pi_B = G_4$). In $a < 1/2$, whether the maximum value of Π_B exists at $b = 0$ or $b = 1$ depends on the value of a. To derive this condition, we calculate following equation:

$$
\Pi_B(a, R_B^1(a)) - \Pi_B(a, R_B^4(a)) = \frac{(16 - 41a + 2a^2 - 13a^3)t}{72},
$$

where $R_B^j(b) := \text{argmax}_b\ G_j\ (j = 1, 4)$. This equation is a monotonically decreasing function of a. In addition, when $a = \bar{a} \approx 0.3799$, the equation equals zero. Thus, when $a < \bar{a}$, $R_B^1(a)$ maximizes Π_B, otherwise $R_B^4(a)$ maximizes Π_B. Hence, in the first stage, the reaction function of Firm B is expressed by

$$R_B(a) = \begin{cases} 1 & \text{if} \quad a < \bar{a}, \\ 0 & \text{otherwise.} \end{cases}$$

(insert Figure 8 here)

Proof of Proposition 4. The market is divided into two regions, and thus, the objective functions of both the local public firm (Firm A) and the private firm (Firm B) are different between $a < b$ and $a > b$. In the case of $a < b$, the objective functions of both the firms are expressed by

$$W_1 = \begin{cases} \Pi_A + \Pi_B - \int_0^{1/2} t(a-z)^2 dz - \frac{P_A}{2} & \text{if} \quad x \geq \frac{1}{2}, \\ \Pi_A + \Pi_B - \int_0^x t(a-z)^2 dz - \int_x^{1/2} t(b-z)^2 dz - P_A x - P_B \left(\frac{1}{2} - x\right) & \text{if} \quad x < \frac{1}{2}, \end{cases}$$

$$\Pi_B = P_B(1-x),$$

where $\Pi_A = P_A x$. The reaction functions in the second stage are given by

$$r_A(P_B) = \begin{cases} \frac{2P_B + (a-b)(1-a-b)t}{2} & \text{if} \quad a+b > 1, \\ P_B & \text{otherwise,} \end{cases}$$

$$r_B(P_A) = \frac{P_A - (a-b)(2-a-b)t}{2},$$

and the pair of equilibrium prices in the second stage is as follows:

$$\begin{cases} P_A = -(a-b)t, & P_B = -\frac{(a-b)(3-a-b)t}{3} & \text{if} \quad a+b > 1, \\ P_A = -(a-b)(2-a-b)t, & P_B = -(a-b)(2-a-b)t & \text{otherwise.} \end{cases}$$

Next, in the case of $a > b$, the objective functions of both firms are expressed by

$$W_1 = \begin{cases} \Pi_A + \Pi_B - \int_0^{1/2} t(b-z)^2 dz - \frac{P_B}{2} & \text{if} \quad x \geq \frac{1}{2}, \\ \Pi_A + \Pi_B - \int_0^x t(b-z)^2 dz - \int_x^{1/2} t(a-z)^2 dz - P_B x - P_A \left(\frac{1}{2} - x\right) & \text{if} \quad x < \frac{1}{2}, \end{cases}$$

$$\Pi_B = P_B x.$$

where $\Pi_A = P_A(1-x)$. The reaction functions in the second stage are given by

$$r_A(P_B) = \frac{2P_B + (a-b)(2-a-b)t}{2}$$

$$r_B(P_A) = \frac{P_A + (a-b)(a+b)t}{2},$$

and the pair of the equilibrium prices in the second stage is as follows:

$$P_A = 2(a-b)t, \quad P_B = \frac{(a-b)(2+a+b)t}{2}.$$

Using the above results, the reaction functions in the first stage are given by

$$R_A(b) = \begin{cases} 0 & \text{if} \quad b > \tilde{b} \\ 1 & \text{otherwise} \end{cases} \qquad \text{where} \quad \tilde{b} \approx 0.5306,$$

$$R_B(a) = \begin{cases} \frac{2+a}{3} & \text{if} \quad a < \tilde{a}_1 \\ 1 & \text{if} \quad \tilde{a}_1 \leq a \leq \tilde{a}_2 \\ 0 & \text{otherwise} \end{cases} \qquad \text{where} \quad \tilde{a}_1 \approx 0.1506, \quad \tilde{a}_2 \approx 0.3364.$$

Therefore, the equilibrium variables are

$$\begin{cases} a = 0, \quad b = \frac{2}{3}, \quad P_A = \frac{8t}{9}, \quad P_B = \frac{8t}{9}, \quad x = \frac{1}{3}, \\ a = 1, \quad b = 0, \quad P_A = 2t, \quad P_B = \frac{3t}{2}, \quad x = \frac{3}{4}. \end{cases}$$

$\square$

Proof of Proposition 5. The state-owned firm (Firm A) is the only domestic firm. Thus, the social welfare of the entire city is as follows:

$$W = -\int_0^x t(a-z)^2 dz - \int_x^1 t(b-z)^2 - \Pi_B,$$

where $a < b$. Since the state-owned firm is the social welfare maximizer, its reaction function in the second stage is expressed by

$$r_A(P_B) = 0.$$

The firm chooses $P_A = 0$ regardless of the foreign firm's price. On the other hand, since the private firm, whether domestic or foreign, maximizes its own profit, the reaction function of the foreign private firm (Firm B) is given by Equation (6). Therefore, the pair of equilibrium prices in the second stage is as follows:

$$P_A = 0, \quad P_B = -\frac{(a-b)(2-a-b)t}{2}.$$

Next, the reaction functions of both the firms in the first stage are as follows:

$$R_A(b) = \frac{-12 - b + 2\sqrt{45 + 6b + b^2}}{3}, \quad R_B(a) = \frac{2+a}{3}.$$

Accordingly, the equilibrium locations are

$$a = \frac{3\sqrt{7} - 7}{2}, \quad b = \frac{\sqrt{7} - 1}{2},$$

and the location of the marginal consumer is $x = (\sqrt{7} - 1)/2 = b$. $\square$

References

d'Aspremont C, Gabszewicz JJ, Thisse J-F. On Hotelling's 'stability in competition'. Econometrica 1979;47; 1145-1150.

Bárcena-Ruiz JC, Garzón MB. International trade and strategic privatization. Review of Development Economics 2005;9; 502-513.

Cremer H, Marchand M, Thisse J-F. Mixed oligopoly with differentiated products. International Journal of Industrial Organization 1991;9; 43-53.

De Fraja G, Delbono F. Alternative strategies of a public enterprise in oligopoly. Oxford Economic Papers 1989;41; 302-311.

Fjell K, Pal D. A mixed oligopoly in the presence of foreign private firms. Canadian Journal of Economics 1996;29; 737-743.

Hotelling H. Stability in competition. Economic Journal 1929;39; 41-57.

Inoue T, Kamijo Y, Tomaru Y. Interregional mixed duopoly, location and welfare. 21COE-GLOPE Working Paper No.16. Waseda University; 2007 (http://21coe-glope.com/en/paper/index.html).

Matsumura T, Matsushima N. Mixed duopoly with product differentiation: Sequential choice of location. Australian Economic Papers 2003;42; 18-34.

Matsumura T, Matsushima N. Endogenous cost differentials between public and private enterprises: A mixed duopoly approach. Economica 2004;71; 671-688.

Matsushima N, Matsumura T. Mixed oligopoly and spatial agglomeration. Canadian Journal of Economics 2003;36; 62-87.

Matsushima N, Matsumura T. Mixed oligopoly, foreign firms, and location choice. Regional Science and Urban Economics 2006;36; 753-772.

Pal D, White M. Mixed oligopoly, privatization and strategic trade policy. Southern Economic Journal 1998;65; 264-281.

Tharakan J, Thisse J-F. The importance of being small. Or when countries are areas and not points. Regional Science and Urban Economics 2002;32; 381-408.

Tables and Figures

Table 1: The classification of four cases ($a \neq b$)

	location	boundary	W_1	Π_A	T_1	C_1
Case 1	$a < b$	$x \geq 1/2$	F_1	$P_A x$	$\int_0^{1/2} t(a-z)^2 dz$	$P_A/2$
Case 2		$x < 1/2$	F_2		$\int_0^x t(a-z)^2 dz + \int_x^{1/2} t(b-z)^2 dz$	$P_A x + P_B(1/2 - x)$
Case 3	$a > b$	$x \geq 1/2$	F_3	$P_A(1-x)$	$\int_0^{1/2} t(b-z)^2 dz$	$P_B/2$
Case 4		$x < 1/2$	F_4		$\int_0^x t(b-z)^2 dz + \int_x^{1/2} t(a-z)^2 dz$	$P_B x + P_A(1/2 - x)$

Table 2: Classification of ownership

<table>
<tr><td></td><td colspan="6">Firm B</td></tr>
<tr><td rowspan="3"></td><td rowspan="3"></td><td rowspan="3">center</td><td rowspan="3">local
(Region 2)</td><td colspan="3">private</td></tr>
<tr><td colspan="2">domestic</td><td rowspan="2">foreign</td></tr>
<tr><td>Region 1</td><td>Region 2</td></tr>
<tr><td rowspan="3">Firm A</td><td>center</td><td>(i)</td><td>—</td><td colspan="2">Cremer et al. (1991)</td><td>(v)</td></tr>
<tr><td>local
(Region 1)</td><td>(ii)</td><td>(iii)</td><td>(iv)</td><td colspan="2">model in previous sections
(basic model)</td></tr>
<tr><td>private</td><td>—</td><td>—</td><td colspan="3">d'Aspremont et al. (1979)</td></tr>
</table>

Table 3: Equilibrium location pairs (a, b)

		Firm B				
		center	local (Region 2)	private		
				domestic		foreign
				Region 1	Region 2	
Firm A	center	$(\frac{1}{4}, \frac{3}{4})^*$	—	$(\frac{1}{4}, \frac{3}{4})^*$		$(0.47, 0.82)^*$
	local (Region 1)	$(\frac{1}{4}, \frac{3}{4})$	$(\frac{1}{4}, \frac{3}{4})$	$(0, \frac{2}{3})$, $(1, 0)$	$(0.26, 1)$, $(0.49, 0)$	
	private	—	—	$(0, 1)^*$		

* (b, a) is also a pair of equilibrium locations.

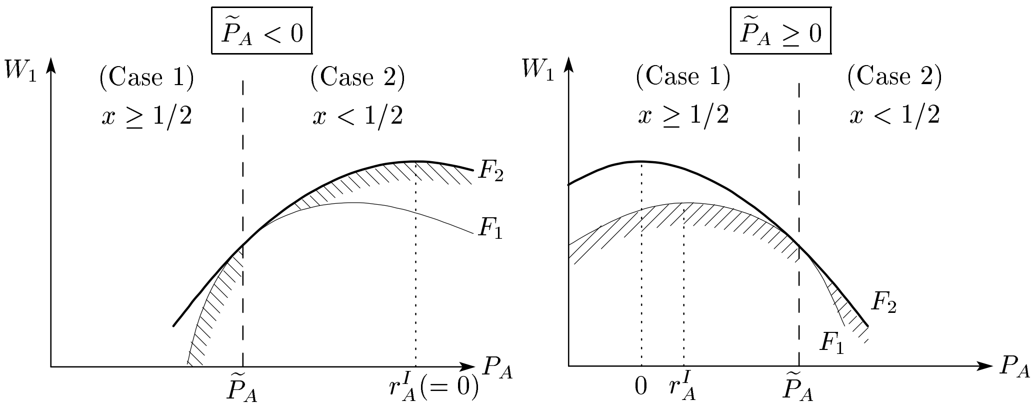

Figure 1: The local welfare of Region 1 in (I) $a < b$

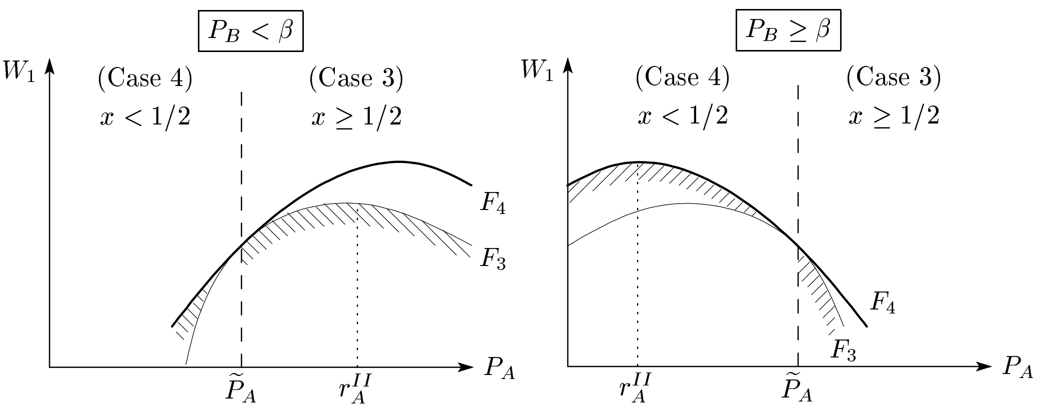

Figure 2: The local welfare of Region 1 in (II) $a > b$

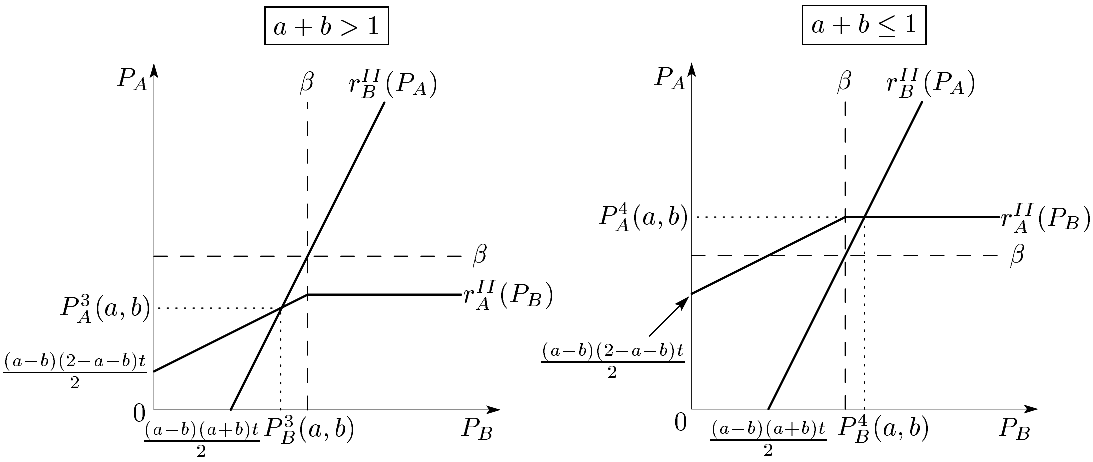

Figure 3: The reaction curves in price competition (in (II) $a > b$)

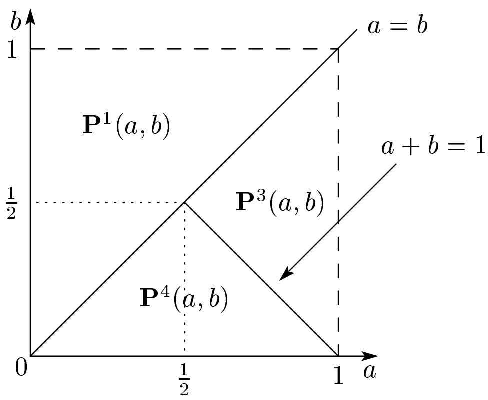

Figure 4: The ranges of equilibrium prices

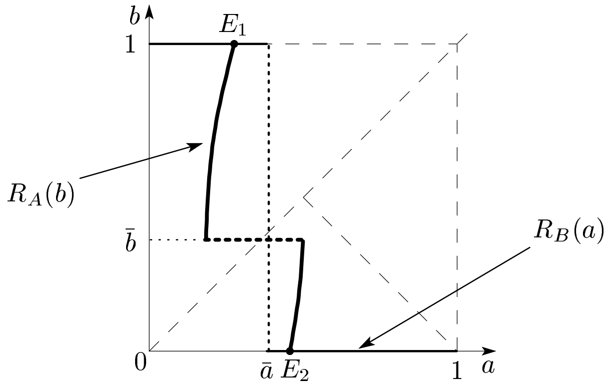

Figure 5: The reaction curves in location choice

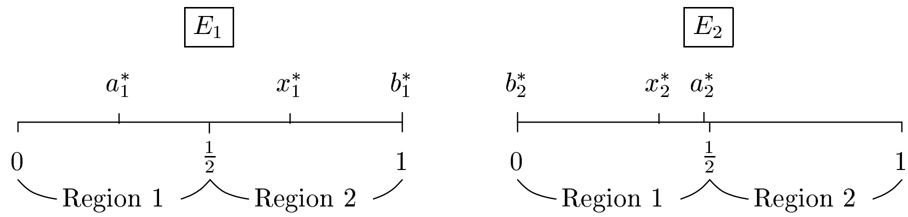

Figure 6: The locations of firms in two types of equilibria

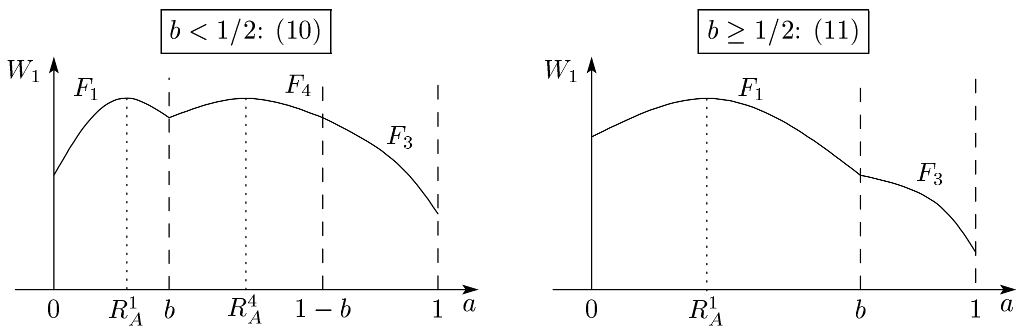

Figure 7: Derivation of Equation (12)

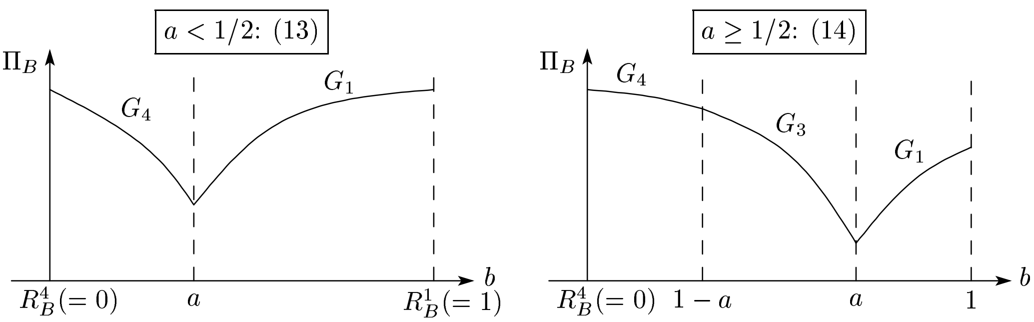

Figure 8: Derivation of Equation (15)