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An endogenous objective function of a partially privatized firm: A Nash bargaining approach

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Abstract

We consider a mixed duopoly comprising a private firm and a partially privatized firm jointly owned by the government and a private capitalist. The objective function of the private firm is its profit while that of the partially privatized firm is endogenously determined through bargaining between both owners. Usually, it is considered that the more shares the shareholders have, the more strongly they can reflect their objectives in the firm's objective. However, we find that when the government has more shares, it may attempt to reflect its objective in the partially privatized firm's objective.

JEL classification: L13; L33; C78

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1 Introduction

In this paper, we demonstrate how a firm's objective function is determined when each owner has a different interest. For this purpose, we use a "mixed duopoly" model where a profit maximizing private firm competes against a partially privatized firm. The privatized firm is owned by two types of owners: one is a private capitalist and the other is the government. The private capitalist usually expects the firm to maximize its own profits Π_0 whereas the government, the social welfare W. This implies that the owners have contradictory interests, and thus, it is not easy for them to set the privatized firm's objective function. Against this backdrop, this paper aims to explain the process of setting the function as a bargaining process.

Since the 1980s, many public firms have been privatized, and the private sector has owned such firms fully or partially.[1] DeFraja and Delbono (1989) examine the effect of privatization of a public firm on social welfare and show that in some situations, privatizing a public firm enhances social welfare despite it not involving an improvement in production efficiency but only a change in the firm's objective and behavior. This result is extended to partial privatization by Matsumura (1998). A partially privatized firm is a mixed joint stock company owned by a profit maximizing private capitalist and the welfare maximizing public sector (or the government). In his model, a partially privatized firm is assumed to maximize $\alpha W + (1-\alpha)\Pi_0$, $\alpha \in [0,1]$, the weighted average of owners' interests. It is also assumed that this weight increases with the corresponding owner's shareholding ratio (i.e., α is an increasing function of the public sector's shareholding ratio). In other words, if an owner increases shares in the firm, then the firm gives extra consideration to the owner's concern. Matsumura shows that partial privatization is always a more effective means for achieving high social welfare than both full nationalization and full privatization.

These works can also be analyzed from the viewpoint of what objective a player should pursue in strategic environments. The possibility that a player who complies with some behavioral principle distinct from his objective receive better returns than when he acts so as to maximize

[1] We can see such privatized firms in a wide range of industries such as the airlines, gas, electricity, telecommunications, banking, and education indsutries. The Japanese government established four corporations in Japan — Japan Post Network Corporation, Japan Post Service Corporation, Japan Bank Corporation and Japan Post Insurance Corporation — and made Japan Post Holdings Corporation (JP) have these corporations as subsidiaries, in October 2007. By 2017, the Japanese government intends to sell two-thirds of its shares in JP. Thus, Japan Post will be a typical partially privatized firm in Japan.

the real objective is already known in several contexts.[2] A problem that arises for a player who recognizes that changing his objective is beneficial for him pertains to how he credibly reports the change in the objective or the utility function to his rivals. As Schelling (1980) indicates, the useful way to credibly change the objective is to lose or restrict the power of the player in a legal manner. Thus, privatization and partial privatization constitute credible means to change the objective of a public firm because the rivals believe that the firm now concerns the interests of both the owners and behaves so as to harmonize their contradicting interests.

The problem discussed here is related to how two parties in a partially privatized firm agree on an objective of the firm. In the growing literature on mixed oligopoly, Matsumura's model and its variations are intensively used to analyze the market outcome in various conditions, without considering how a partially privatized firm makes decisions.[3] Moreover, in Matsumura (1998), it is assumed that the owner who has a larger part of shares of the firm strongly reflects his objective in the partially privatized firm's behavior. However, it can so happen that the majority may dare to reflect its objective in the partially privatized firm's objective because as we explained in the previous paragraph, the pursuit of a different objective by a player can prove to be beneficial to his true objective. Thus, in this paper, we provide a model where the objective of a partially privatized firm is endogenously determined through bargaining between the two sectors. Further, we examine the validity of the assumption adopted by Matsumura (1998). We also consider the welfare implications of the endogenously determined objective model.

To explore how a partially privatized firm makes decisions or how two parties determine the objective of the firm, we consider a two-stage game described as follows. In the first stage, the public and private sectors discuss the management policy of the firm, which is well represented

[2] For instance, Crawford and Varian (1979) and Sobel (1981) show that in the Nash bargaining problem, distorting the player's utility function might benefit the player. In the context of strategic delegation, it is known that hiring agents who participate in the game on behalf of its real player gives the player (called the principal) a first mover or other advantage over the opponents (e.g., Vickers 1985, Fershtman 1985, Fershtman and Judd 1987, Sklivas 1987, Fershtman, Judd, and Kalai 1991). However, when a contract between the principal and the agent can not be observed by the opponents, using such delegation does not change the equilibrium outcome from the one when the principal himself plays the game (Katz 1991).

[3] For example, Matsumura and Kanda (2005) show that when firms are allowed to enter in the market freely, full nationalization is desirable in terms of social welfare, unlike Matsumura (1998). Further, some research studies the relationship between the partial privatization policy and other policies. Chao and Yu (2006) show that the partial privatization policy is substitutable for import tariff as a trade policy.

by the parameter $\alpha \in [0,1]$. This parameter indicates the weight attached to the management policy by the two sectors. In the process of reaching an agreement though bargaining, this information becomes public, and in the next stage, the privatized firm competes against the other private firms in Cournot fashion. On the other hand, when they fail to reach an agreement through negotiation, they play the *defund game* to decide to either continue operating the business of the partially privatized firm or defund and liquidate it. When both sectors choose in favor of the continuation of the firm, the majority party asserts the total control over the firm by resorting to a shareholder meeting. Thereafter, the firm acts so as to maximize the majority's objective. In contrast, when one of them chooses to defund the firm, each party is returned funds in proportion its shareholding ratio, which it then uses to invest in their other opportunities.

We first conduct a comparative statics of the agreed value of α with respect to the share $s \in (0,1)$ of the public sector. We find that this crucially depends on the outcome of the defund game. Specifically, when the continuation of the firm is chosen in the defund game, an increment of s does not affect the agreed value of the weight of the public sector, α^*. On the other hand, when the defunding the firm is decided on, the effect of an infinitesimal increment in s on α^* relies on the difference between the return rates of public and private investments. If the former rate is higher than the latter, then the weight on social welfare in the privatized firm's objective function becomes larger as the government's share increases; if the return rate of public investment is lower than that of private investment, the result is reversed. Thus, our endogenous determined objective model indicates that it might be difficult to support Matsumura's assumption. Moreover, we obtain different implications pertaining to the effectiveness of privatization or partial privatization from DeFraja and Delbono (1989) and Matsumura (1998). We find that when the marginal cost of the public firm is higher than that of the private firm, with the difference not being substantial, and the outcome of the defund game is liquidation of the firm, not privatizing the firm is the optimal choice for the government that is concerned with social welfare.

To conclude the introduction, we note a few characteristics of our approach that are derived from existing literature. First, we analyze a bargaining situation of the first stage using a cooperative game framework similar to that employed by Aoki (1980, 1982) to analyze modern corporations as coalitions of several stakeholders. We use the *Nash solution* as our solution concept for the first stage game. Second, we do not characterize the partially privatized firm as one that chooses its output so as to maximize the Nash product of the two parties, given the output of the other private firm. Instead, we adopt the two stage-game where in the first stage, the two parties determine the objective of the partially privatized firm because it is difficult to imagine that the owners of the firm make decisions on the daily output determination. Third, even though the majority party can always resort to the general shareholders' meeting to control the firm, we assume that it cooperatively bargains with the minority to determine the firm's objective for as long as there is scope for mutual benefit through bargaining. Therfore, resorting to the general shareholders' meeting is one of possible threats posed by the majority party in order to obtain a better outcome from negotiations. Finally, we do not consider the problem of delegation because it totally changes the context and makes it difficult to set the comparison in our research against that in existing works such as DeFraja and Delbono (1989) and Matsumura (1998) and other studies in this field (for research considering delegation in the mixed oligopoly, see White 2001).

This paper proceeds as follows. In Section 2, we explain the standard mixed duopoly where a private firm competes against a partially privatized firm jointly owned by a profit maximizing private capitalist and the welfare maximizing government. We find that the discrepancy between their interests gives rise to some room for bargaining over what the partially privatized firm should maximize. In Section 3, we provide the model of bargaining between the two sectors and conduct a comparative statics of α^* on s. In Section 4, we consider the welfare implications obtained from our endogenously determined objective model. Section 5 presents the conclusion.

2 Model

We consider an industry where a partially privatized firm (firm 0) and a private firm (firm 1) are engaged in Cournot competition. These firms produce a homogeneous commodity, and demand for this commodity is given by the inverse demand function $P = P(Q) = 1 - Q$. Here, P represents the price, $Q = q_0 + q_1$, the total quantity produced by the two firms; and q_i represents the output of the firm i $(i = 0, 1)$. Let the cost functions of these firms be given by $C_i(q_i) = F + c_i q_i$. Since issues of entry are not considered in this paper, we assume that $F = 0$.

Furhter, we assume that the partially privatized firm's marginal cost c_0 is higher than the private firm's marginal cost c_1. For simplicity, we suppose that $c_0 = c > 0 = c_1$. This assumption of the partially privatized firm's inefficiency is standard in a mixed oligopoly with linear costs. Technically speaking, the assumption guarantees that the private firm is active in the market. This is evident later. Furthermore, many empirical studies indicate that the productive efficiency of public firms is low relative to that of private firms.[4] Some theoretical papers show that public firms strategically adopt a lower level of cost-reducing R&D investment.[5] Therefore, our assumption of inefficiency of the partially privatized firm appears reasonable.

Private firm 1 maximizes its profit:

$$\Pi_1(q_1, q_0) = (P(Q) - C_1(q_1))\, q_1 = (1 - q_0 - q_1)q_1.$$

Firm 0 is a partially privatized firm which is jointly owned by a profit maximizing private capitalist and the welfare maximizing government. Since the privatized firm with mixed ownership must respect both owners, it cannot be either a pure welfare maximizer or a pure profit maximizer. Therefore it should take into consideration its own profit, given by

$$\Pi_0(q_0, q_1) = (P(Q) - C_0(q_0))\, q_0 = (1 - q_0 - q_1 - c)q_0,$$

as well as social welfare, given by

$$W(q_0, q_1) = \int_0^Q P(z)dz - C_0(q_0) - C_1(q_1) = (q_0 + q_1) - \frac{1}{2}(q_0 + q_1)^2 - cq_0.$$

Following Matsumura (1998), we assume that firm 0 maximizes the weighted average of social welfare and its own profit that is given by

$$V_0(q_0, q_1, \alpha) = \alpha W(q_0, q_1) + (1 - \alpha)\Pi_0(q_0, q_1),$$

where $\alpha \in [0, 1]$ denotes the weight of the payoff of the government in firm 0's objective. An interpretation of this parameter is that it represents the power of the government to reflect its objective in the partially privatized firm's objective function. In fact, if this power is very strong such that the government can set α to 1, then the partially privatized firm becomes a welfare maximizer. On the other hand, if the power is very weak such that the other owner, the private capitalist, can set α to 0, the firm becomes a profit maximizer.

The first-order conditions for maximizing V_0 and Π_1 with respect to q_0 and q_1 are

$$\frac{\partial V_0}{\partial q_0} = 1 - (2 - \alpha)q_0 - q_1 - c = 0 \quad \text{and} \quad \frac{\partial \Pi_1}{\partial q_1} = 1 - q_0 - 2q_1 = 0,$$

[4] For example, see Mizutani (2004) and Megginson and Netter (2001).

[5] For the discussions pertaining to public firms' inefficiency and R&D investments, see Matsumura and Matsushima (2004), Nishimori and Ogawa (2002), and Tomaru (2007).

respectively. By solving these first-order conditions for both firms, we derive their reaction functions:

$$q_0 = R_0(q_1, \alpha) = \frac{1 - q_1 - c}{2 - \alpha} \quad \text{and} \quad q_1 = R_1(q_0) = \frac{1 - q_0}{2}.$$

Although the private firm's reaction function is derived from the ordinary Cournot model, the same is not true for the partially privatized firm's reaction function. An increase in the weight α on social welfare shifts it outward. This increase implies that the partially privatized firm places more emphasis on social welfare, particularly on consumer surplus, which in turn implies an increase in the total output. Thus, the partially privatized firm has an incentive to expand its output so as to increase the total output.

From the above reaction functions, we can obtain the following equilibrium outputs:

$$q_0^*(\alpha) = \frac{1 - 2c}{3 - 2\alpha}, \quad q_1^*(\alpha) = \frac{1 - \alpha + c}{3 - 2\alpha}, \quad \text{and} \quad Q^*(\alpha) = \frac{2 - \alpha - c}{3 - 2\alpha}. \tag{1}$$

It can be easily shown that the partially privatized firm's output $q_0^*(\alpha)$ and the total output $Q^*(\alpha)$ increase and that of the private firm, $q_1^*(\alpha)$, decreases as the weight α increases. The outward shift of the partially privatized firm's reaction curve raises its equilibrium output q_0^*, which results in a decrease in the output of the private firm, q_1^*, as a strategic substitute for that of the partially privatized firm. The total output Q^* rises with α because the slope of the private firm's reaction curve is, in absolute value, less than unity. An increment in q_0^* is greater than a decrement in q_1^*; therefore, Q^* increases.

Using (1), we find that as given α, profits and social welfare at the equilibrium are

$$\Pi_0^*(\alpha) = \frac{(1 - \alpha)(1 - 2c)^2}{(3 - 2\alpha)^2}, \quad \Pi_1^*(\alpha) = \frac{(1 - \alpha + c)^2}{(3 - 2\alpha)^2}, \tag{2}$$

$$W^*(\alpha) = \frac{(11 - 8\alpha)c^2 - 2(4 - 3\alpha)c + 8 - 10\alpha + 3\alpha^2}{2(3 - 2\alpha)^2}. \tag{3}$$

By differentiating both firms' equilibrium profits Π_0^* and Π_1^* and social welfare W^* with respect to α, we obtain

$$\Pi_1^{*\prime}(\alpha) = -\frac{2(1 - 2c)(1 + c - \alpha)}{(3 - 2\alpha)^3} < 0,$$

$$\Pi_0^{*\prime}(\alpha) = \frac{(1 - 2c)^2(1 - 2\alpha)}{(3 - 2\alpha)^3}, \tag{4}$$

$$W^{*\prime}(\alpha) = \frac{(1 - 2c)\left[1 - 5c - (1 - 4c)\alpha\right]}{(3 - 2\alpha)^3}. \tag{5}$$

As seen above, since an increase in α increases the total output and thereby lowers price, it reduces the private firm's profit. However, this is not the case when the partially privatized firm's profit and social welfare are considered. Differentiating (4) and (5) with respect to α, we find that

$$\Pi_0^{*\prime\prime}(\alpha) = -\frac{8(1-2c)^2\alpha}{(3-2\alpha)^4} < 0, \quad \text{and} \quad W^{*\prime\prime}(\alpha) = \frac{(1-2c)\,[3(1-6c)-4(1-4c)\alpha]}{(3-2\alpha)^4}. \tag{6}$$

Since $\Pi_0^*(\alpha)$ is concave in α, $\Pi_0^*(\alpha)$ is maximized at $\alpha = 1/2$ from (4). Thus, this level of α is the most desirable for the private capitalist. Note that at $\alpha = 1/2$, the partially privatized firm 0 gains the same profit as when it is a profit maximizer and Stackelberg leader. The reason for this is as follows. At any $\alpha \in [0, 1]$, equilibrium is realized along the private firm's reaction curve. A profit maximizing firm as a Stackelberg leader selects its output on the curve which gives it the highest profit. This process is tantamount to choosing α to maximize the partially privatized firm's profits.

On the other hand, whether $W^*(\alpha)$ is concave or convex relies on the marginal cost of the partially privatized firm c. Thus, we must apply another approach in order to determine the welfare maximizing α. Since $W^*(\alpha)$ is continuous in $[0, 1]$, by comparing $W^*(0)$, $W^*(1)$, and $W^*((1-5c)/(1-4c))$, we can derive α which maximizes $W^*(\alpha)$. It is straightforward to show that

$$W^*(0) = \frac{8-8c+11c^2}{18}, \quad W^*(1) = \frac{1-2c+3c^2}{2}, \quad W^*\left(\frac{1-5c}{1-4c}\right) = \frac{1-2c+4c^2}{2},$$

and simple calculation yields $W^*((1-5c)/(1-4c)) > \max\{W^*(0), W^*(1)\}$ as long as $(1-5c)/(1-4c) \in (0, 1)$, i.e., $c < 1/5$. Thus, it is at $\alpha = (1-5c)/(1-4c)$ that $W^*(\cdot)$ is maximized. By using the more general model, Matsumura (1998) shows that there exists such welfare maximizing α. This level of α leads to the same social welfare as when the partially privatized firm 0 is a welfare maximizer and Stackelberg leader.[6]

The above discussion implies that the levels of α which maximize $\Pi_0^*(\alpha)$ and $W^*(\alpha)$ are not necessarily the same. They are the same only if $c = 1/6$. The preceding results are summarized in Proposition 1.

Proposition 1. *Based on the relationships between the power of the public sector (α), profits, and social welfare, the following hold.*

(i) The higher the power of the public sector, α, the lower the private firm's profit,

[6]This can be explained in the same manner as in the above profit maximizing case. See Tomaru and Kiyono (2005).

(*ii*) *There exist certain levels of* $\alpha \in (0,1)$ *that maximize the partially privatized firm's profit and social welfare. These are given by*

$$\alpha_g = \frac{1-5c}{1-4c} \qquad and \qquad \alpha_p = \frac{1}{2},$$

respectively. They are the same only if $c = 1/6$.

As mentioned above, an increase in α lowers the market price. In addition, the market share of the private firm lowers because its output decreases while that of the partially privatized firm increases. Thus, the private firm's profit decreases with an increase in α. This is an intuition behind Proposition 1 (i).

Proposition 1 (ii) indicates that each owner's payoff becomes larger when the concerned firm has an objective function other than the owner's objective. Fershtman and Judd (1987) and Sklivas (1987) show that the separation of ownership and management can act as a type of commitment device to deviate the private firm's objective function from a function other than profit, which results in higher profits than in the case of the integration of ownership and management. In our mixed duopoly model, the weight α is this type of commitment device. If each owner can control α freely without the other owner's approval, then he gains a higher payoff than when he is the sole owner. However, it might be difficult for one owner to select α by ignoring the other owner's interest. In fact, Assumption 1 assures that both owners' desirable outcomes are different, which leaves some scope for bargaining between them over α as will be seen in the next section.

It may be regarded that welfare maximizing α_g is higher than profit maximizing α_p because α is the weight attached to welfare; however, the relationship between α_g and α_p is dependent on c. When c is relatively higher, an increase in α results in a large increase in total production costs, which may deteriorate social welfare. In effect,

$$\alpha_g - \alpha_p = \frac{1-5c}{1-4c} - \frac{1}{2} = \frac{1-6c}{2(1-4c)},$$

and this implies that α_p is higher than α_g if $1/6 < c < 1/4$. To exclude such a counterintuitive situation, we impose the following assumption.

Assumption 1. *The partially privatized firm's marginal cost is sufficiently low, that is,* $c < 1/6$.[7]

[7]As we will see later, this assumption is important when our bargaining problem is considered.

It should be noted that under this assumption, we have

$$W^{*\prime}(\alpha) \gtreqqless 0 \iff \alpha \lesseqqgtr \alpha_g \quad \text{and} \quad \Pi_0^{*\prime}(\alpha) \gtreqqless 0 \iff \alpha \lesseqqgtr \alpha_p. \tag{7}$$

The latter is obvious since Π_0^* is concave. W^* is also a hump-shaped curve whose maxima occurs at $\alpha = \alpha_g$.

3 Bargaining between the government and the private capitalist

In the previous section, we saw how a certain weight α influences the privatized firm's profits and social welfare. The results in the previous section demonstrate that the governmental owner of the firm prefers some intermediate value α_g to $\alpha = 1$ which implies that the government totally can control the firm. This leads to an important welfare implication, which has been already pointed out by Matsumura (1998) and Bennett and Maw (2003), that social welfare could be higher if the government partly loses its power in the management of the public firm. However, these studies do not explicitly consider the process of how this weight is determined. Thus, in this section and the following section, we establish a model wherein the governmental owner and the private capitalist engage in negotiations over the parameter α in the firm's objective function, which is assumed to represent the management policy of the firm, in order to answer (i) how each owner's share in the firm affects the bargaining outcome and (ii) whether or not the (partial) privatization of the public firm contributes to enhancing social welfare.

Before explaining the components of our bargaining model in detail, it is useful to confirm the reason why the government and the private capitalist have to bargain. We assume that the government owns a share of $s \in (0, 1)$ in the privatized firm 0 and that the private capitalist owns a share of $1 - s$. At the moment, the share s is assumed to be an exogenous parameter for the governmental owner and the private capitalist. In proportion to their shares, the two owners receive their dividends from the profit of the firm: $s\Pi_0^*(\alpha)$ and $(1 - s)\Pi_0^*(\alpha)$ for the government and the private capitalist respectively. Thus, both owners' payoffs are given by

$$U_g(\alpha) = W^*(\alpha) = \frac{(11 - 8\alpha)c^2 - 2(4 - 3\alpha)c + 8 - 10\alpha + 3\alpha^2}{2(3 - 2\alpha)^2},$$

$$U_p(\alpha; s) = (1 - s)\Pi_0^*(\alpha) = \frac{(1 - s)(1 - \alpha)(1 - 2c)^2}{(3 - 2\alpha)^2},$$

where subscripts g and p represent the government and the private capitalist respectively.

As mentioned in the previous section, the welfare maximizing level of α, i.e., $\alpha_g = (1 - 5c)/(1 - 4c)$, is higher than the profit maximizing level of α, i.e., $\alpha_p = 1/2$. Therefore, for $\alpha \in (0, 1/2)$, both the owners agree to an increase in α. Similarly, for $\alpha \in ((1-5c)/(1-4c), 1)$, they agree to a decrease in α. In contrast, when $\alpha \in [1/2, (1 - 5c)/(1 - 4c)]$, the government approves an increase in α, but the private capitalist opposes it. Thus, in this interval of the value of α, the owners' interests are conflicting, and thus, they have to agree on some value of α through bargaining in order to continue operating the firm. Through the negotiations between these owners, they decide on α in the range $[1/2, (1 - 5c)/(1 - 4c)]$.

We construct the following multistage game including the stage of bargaining between the government and the private capitalist over the management policy α.

Stage 1: The two parties engage in negotiation over weight $\alpha \in [0, 1]$. If they reach an agreement on the value of α, Stage 2a follows; otherwise, they play the game in Stage 2b.

Stage 2a: The partially privatized firm, with the agreed weight α in Stage 1, and the private firm compete in Cournot fashion.

Stage 2b: The two parties play a defund game in order to determine whether they should continue to operate the firm or defund and liquidate it.

We assume that the bargaining process in Stage 1 can be well described as the bargaining problem by Nash (1950, 1953) and thus characterized by two components: the feasible set of players' payoffs and their payoffs in the case of disagreement. The outcome of Stage 2a, which varies according to the value of α determined in Stage 1, defines the feasible payoffs of the players. This was solved in the previous section, and the outcomes were given in equaitions (1), (2), and (3). On the other hand, the payoff in the case of disagreement in the negotiation is determined through the defund game in Stage 2b, which is detailed in subsection 3.2.

In the following part of this section, we describe the bargaining situation in game-theoretic fashion. The bargaining model is characterized by the feasible set of their payoffs as well as the payoffs in the case of failure of negotiations. We assume that the bargaining environment satisfies Nash's four axioms. Thus, we use the Nash solution as a solution concept for the Stage 1 bargaining problem.

3.1 Feasible set

One of the essential components of the bargaining problem is the feasible set of the players' payoffs when all the possibilities of coordination has been considered. Here, we assume that the players can coordinate and negotiate the management policy of the firm, which is well represented by the value of α, and that they have full knowledge regarding the market outcome after agreeing on the management policy. Thus, with the basic assumption of the *free disposal of utility*, the feasible set of payoffs through bargaining is defined as follows:

$$\begin{aligned} A := & \{(u_g, u_p) \in \mathbb{R}^2 \ : \exists \alpha \in [0,1] \text{ such that } U_g(\alpha) \geq u_g \text{ and } U_p(\alpha, s) \geq u_p\} \\ = & \{(u_g, u_p) \in \mathbb{R}^2 \ : \exists \alpha \in [\alpha_p, \alpha_g] \text{ such that } U_g(\alpha) \geq u_g \text{ and } U_p(\alpha, s) \geq u_p\} \end{aligned} \tag{8}$$

The second equality holds because of the fact that the *strong* pareto frontier of the payoffs U_g and U_p is realized at $\alpha \in [\alpha_p, \alpha_p]$.

As the following lemma shows, the feasible set A has some desirable property in the context of the bargaining problem.

Lemma 1. *The feasible set of our bargaining problem, A, is a convex set under Assumption 1.*

Proof: See Appendix. $\square$

The frontier of the feasible set A is attained when $\alpha \in [1/2, (1-5c)/(1-4c)]$. The slope of the frontier is smooth not only in the interior of the interval $[1/2, (1-5c)/(1-4c)]$ but also at the endpoint of the interval since $dU_p/dU_g(1/2) = 0$ and $dU_p/dU_g \to -\infty$ as $\alpha \nearrow (1-5c)/(1-4c)$ (see Figure 1). Note that an increase in s contracts the feasible set A in the vertical direction. This is because an increase in s implies that the private capitalist receives less dividend whereas an allocation of dividends does not influence social welfare.

3.2 The defund game

The other component of the bargaining problem pertains to players' payoffs when the negotiation breaks down. These payoffs are determined in the defund game formulated as follows. After the breakdown of negotiations, the government and the private capitalist face a problem regarding whether they should continue operating the business of the partially privatized firm or defund and liquidate it. In the case of defunding the firm, the partially privatized firm is

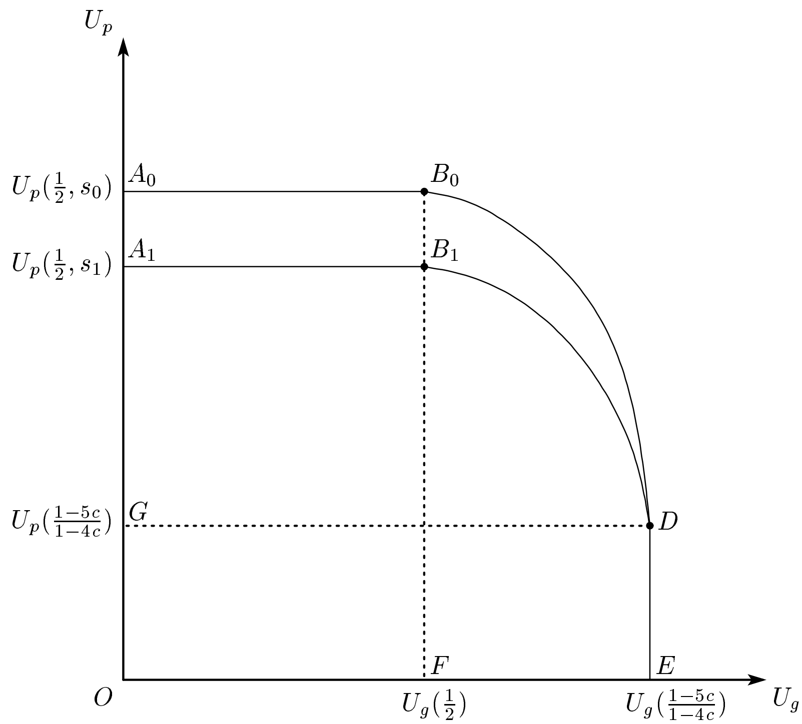

Figure 1: Feasible set A

wound up and the money invested is returned to both owners. Subsequently, the owners invest the refunded money in other investment avenues. In this case, the private capitalist obtains

$$b_p = \hat{b}_p(s) = r_p(1-s)K, \tag{9}$$

where K represents the total amount of investment in the firm and r_p, the return rate on other investments. Since the firm is liquidated, the remaining private firm 1 monopolizes the market. Therefore, social welfare is the sum of the welfare in private monopoly and the returns from the investments fo both parties. The government's payoff b_g is given by

$$b_g = \hat{b}_g(s) = W_M + r_g sK + r_p(1-s)K = \frac{3}{8} + [r_g s + r_p(1-s)]\,K, \tag{10}$$

where $W_M = 3/8$ represents welfare in private monopoly and r_g, the return rate on public investment. We do not assume that the two return rates, r_p and r_g, are the same.[8]

[8]Generally, public investment in infrastructure projects and public utilities is less profitable than private investment; however, it is important in facilitating industries or securing people's lives. Thus, even if the return rate on public investment r_g is lower than that on private investment, public investment must be persisted with as long as the government has funds for investment. Moreover, r_g can be higher than r_p, because people might

In the case that they decide to continue operating the firm, the majority party totally controls the management of the firm by resorting to the majority rule of the general shareholders meeting because he or she has already failed to coordinate with his opponent on the management of the firm. Thus, if the private capitalist is the majority party ($s < 0.5$), the payoffs e_i^p for party $i = p, g$ are

$$e_p^p = U_p(0) \quad \text{and} \quad e_g^p = U_g(0)$$

respectively. On the other hand, when the government is the majority party ($s < 0.5$), the payoffs e_i^g for party $i = p, g$ are

$$e_p^g = U_p(1) \quad \text{and} \quad e_g^g = U_g(1).$$

We now explain how the defund game is played between the two parties. In the defund game, each player simultaneously chooses either to continue operating the firm or to defund it. To simplify the exposition, we assume that when either of the players chooses to defund the firm, another player inevitably follows his partner's decision.[9]

Case I: $s < 0.5$

Table 1: Defund Game: $s < 0.5$

	Continue (C)	Defund (D)
Continue (C)	$e_p^p,\ e_g^p$	$b_p,\ b_g$
Defund (D)	$b_p,\ b_g$	$b_p,\ b_g$

The payoff matrix for the defund game in the case of $s < 0.5$ is described in Table 1. Only in the case when both parties choose to continue (C) do they obtain the payoffs of continuation of the firm; otherwise, they obtain the payoffs of defunding. For simplicity, we assume that

attach a higher value on a public investment, and this appraisal might raise the value measured in money, i.e., r_g.

[9]Suppose that one owner chooses to defund and the other chooses to continue. In this case, only the former owner has to start his business with only his share of the capital, because the money that the latter owner invested should be returned. This might make it impossible for his firm to produce goods with the same technology as before; in other words, the firm might encounter extremely high marginal costs. As a result, it might not be able to continue production anymore. In order to exclude such an extreme case, we adopt this assumption.

when a player is indifferent between selecting (C) and (D), he chooses (C). Then, (C, C) is an equilibrium when the following two conditions hold:

$$e_p^p \geqq b_p \iff \frac{1}{9}(1-2c)^2 \geqq r_p K \tag{11}$$

$$e_g^p \geqq b_g \iff \frac{1}{18}(8-8c+11c^2) \geqq \frac{3}{8} + (sr_g + (1-s)r_p)\, K \tag{12}$$

On the other hand, if either of the conditions is not satisfied, the equilibrium payoffs are (b_p, b_g).

Case II: $s > 0.5$

Table 2: Defund Game: $s > 0.5$

	Continue (C)	Defund (D)
Continue (C)	$e_p^g,\ e_g^g$	$b_p,\ b_g$
Defund (D)	$b_p,\ b_g$	$b_p,\ b_g$

Table 1 presents the payoff matrix for the defund game with $s > 0.5$. Similar to the defund game with $s < 0.5$, (C, C) is an equilibrium only if $e_p^g \geqq b_p$ and $e_g^g \geqq b_g$ hold. In fact, (C, C) does not become an equilibrium because when $e_p^g = 0 < b_p$ always holds (as long as $s < 1$), and one of the conditions is not satisfied. Thus, in this case, the equilibrium payoffs are (b_p, b_g).

Case III: $s = 0.5$

In the case of each party having an equal share, even when both the players choose to continue operating the firm, they cannot reach an agreement regarding the management of the firm and neither of the parties can enforce its objective. Thus, we assume that they inevitably defund the firm, and the equilibrium payoffs are (b_p, b_g).

Let the disagreement point of the bargaining be the equilibrium payoff of the defund game described above and denoted by $d = (d_p, d_g)$. From the observation of the above three cases, we obtain the following lemma.

Lemma 2. *A disagreement point $d = (d_g, d_p)$ of the bargaining is given as follows:*

$$(d_g, d_p) = \begin{cases} (e_p^p, e_g^p) & \text{if } s < 0.5 \text{ and if (11) and (12) are satisfied} \\ (b_p, b_g) & \text{otherwise.} \end{cases}$$

3.3 A bargaining problem and the Nash solution

The two parties bargain over which point in A they realize, where each point in the frontier of A has a one-to-one correspondence with the value of weight α, with disagreement payoff $d = (d_g, d_p)$ being their returns in the case of failure of negotiations. In other words, this is a situation where when each of them can enforce the payoffs of d, they explore a better outcome through their coordination. Thus, when there does not exist a bargaining outcome that is more beneficial to both as compared to their respective disagreement payoffs, there is no room for bargaining.

When the disagreement point is (e_p^p, e_g^p), it can be easily verified that $d \in A$ because A is a convex set and $0 < \alpha_p < \alpha_g < 1$. Thus, in this case, bargaining between the two parties takes place. On the other hand, when the disagreement point is (b_p, b_g), whether or not d is included in A depends on the selection of the parameters. However, the following lemma shows that this is achieved only by the restriction on the value of the capital K.

Lemma 3. *When K is relatively small in the sense that K is smaller than some upper bound $\bar{K} > 0$, $(b_p, b_g) \in A$.*

Proof: See Appendix. □

A pair (A, d) represents a *bargaining problem for the partially privatized firm's objective.* In order to make this bargaining problem plausible, we assume that the capital K is smaller than the upper bound $\bar{K}$ in Lemma 3. We use the Nash bargaining solution defined below as our solution concept for the bargaining problem.

Definition 1. *The Nash bargaining solution (U_g^*, U_p^*) is defined by the solution for the following maximization problem:*

$$\max\ (u_g - d_g)(u_p - d_p) \quad s.t.\ (u_g, u_p) \in A \text{ and } (u_g, u_p) \geq d. \tag{13}$$

Lemmas 1 and 3 on the feasible set and the disagreement point assure the existence and the uniqueness of the Nash solution. The Nash solution (U_g^*, U_p^*) is simply connected to the agreed value of α. Let α^* denote the solution of the following maximization problem:

$$\max\ (U_g(\alpha) - d_g)(U_p(\alpha, s) - d_p) \ \text{ s.t. } \alpha \in [1/2, (1-5c)/(1-4c)]. \tag{14}$$

Since (U_g^*, U_p^*) is located in the frontier of A due to the strong pareto efficiency of the Nash solution, $(U_g(\alpha^*), U_p(\alpha^*, s)) = (U_g^*, U_p^*)$ holds. Thus, α^* is the agreed value of the management policy of the firm through negotiations and is affected by the feasible set A and the disagreement point d.

In our setting, maximization problem (14) has an interior solution. Thus, the first-order condition yields

$$U_g'(\alpha)(U_p(\alpha, s) - d_p) + \frac{\partial U_p}{\partial \alpha}(U_g(\alpha) - d_g) = 0 \tag{15}$$

at $\alpha = \alpha^*(s)$.

3.4 The comparative statics of α^*

In this subsection, we examine how the agreed value α^* is affected by the variations in the feasible set A and the disagreement point d caused by the change in share s. The reason for focusing on the parameter s is that it is extensively considered in literature as the device that controls the objective of the partially privatized firm. Specifically, Matsumura (1998) demonstrates that partial privatization is better than both full privatization and full nationalization, and further shows that welfare maximization is attained by controlling the share s, under the assumption that s is positively correlated with α. Thus, the purpose of this subsection is to check the validity of the assumption of Matsumura (1998) in our bargaining model.

Recall that some parameters — r_p, r_g, K, and s — change the disagreement point d, as seen in Lemma 2. Then, we focus on how different the results are under two disagreement points $d = (e_p^p, e_g^p)$ and $d = (b_p, b_g)$. First, the result under the former disagreement point is presented as Proposition 2.

Proposition 2. *Under Assumption* 1, $s < 0.5$, *(11), and (12), there holds*

$$\alpha^{*\prime}(s) = 0,$$

and the agreed value of α is

$$\alpha_0 := \alpha^*(s) = \frac{31 - 146c - \sqrt{97 - 1084c + 3076c^2}}{2(18 - 76c)}.$$

Proof: This proposition can be easily derived. From Lemma 2 and the definitions of U_g, U_p, e_g^p, and e_p^p, the maximization problem for our bargaining can be rewritten as

$$\max \quad (1-s)(W^*(\alpha) - W^*(0))(\Pi_0^*(\alpha) - \Pi_0^*(0))$$
$$\text{s.t.} \quad \alpha \in [1/2, (1-5c)/(1-4c)].$$

The first-order condition for this problem is given as

$$0 = \frac{(1-2\alpha)^3\alpha\left\{2(9-38c)\alpha^2-(31-146c)\alpha+12(1-5c)\right\}}{18(3-2\alpha)^5}.$$

The agreed value α^*, which satisfies this equation and is included in $[\alpha_p, \alpha_g]$, is $\hat{\alpha}$. $\square$

We should note that α_0 is decreasing in c. Differentiating α_0 with respect to c,

$$\frac{d\alpha_0}{dc} = -\frac{2\left(-149+886c+17\sqrt{97-1084c+3076c^2}\right)}{(9-38c)^2\sqrt{97-1084c+3076c^2}} < 0, \qquad \text{for } \ c \in \left(0, \frac{1}{6}\right).$$

An improvement in the unit cost results in large marginal benefits from the expansion of the privatized firm's market share, as compared to the marginal loss. As a result, the private capitalist agrees to privatized firm's more aggressive actions.

Proposition 2 states that the government does not have the discretion to control α through buying or selling its shares if the size of its capital in the privatized firm is relatively small and the private capitalist still holds the majority of shares. Therefore, in this case, further privatization cannot influence the privatized firm's managerial policy and thus its profits and social welfare. This result stems from the fact that the capital received by government after the breakdown of the negotiations is reallocated to consumers in a lump-sum manner.

Indeed, the disagreement point need not be independent of s if this capital is used for another investment, and thereafter, the return is redistributed to consumers. Disagreement point $d = (b_p, b_g)$ corresponds to this situation. The business in which public or privatized firms engage is often strongly public in nature, and thus, it might be required that the size and scale of these firms be relatively large for some reasons such as sustaining perpetual business and securing universality of services. For such privatized firms, assumptions with respect to $d = (b_p, b_g)$ are satisfied. Further, the following proposition suggests that the government can have control over α.

Proposition 3. *Assume that* $(d_p, d_g) = (b_p, b_g)$*. Under Assumption* 1*, there holds*

$$r_g \gtreqless r_p \qquad \Longleftrightarrow \qquad \alpha^{*\prime}(s) \gtreqless 0.$$

Proof: See Appendix. $\square$

Proposition 3 shows that a buyback by the government (i.e., an increase in s or partial nationalization) raises the weight on welfare α when the return rate of public investment r_g is higher than that of investment by the private capitalist r_p. Conversely, if the public investment is less beneficial than the private investment, then partial nationalization lowers a government's influence on the objective function of the partially privatized firm. Matsumura (1998) assumes that α is positively related to s. However, our proposition implies that when negotiations between the government and the private capitalist are considered, the assumption need not hold.

To understand the intuition behind Proposition 3, we rewrite the Nash product as follows.

$$
\begin{aligned}
&(1-s)\left[W^*(\alpha)-\left(\frac{3}{8}+r_g sK+r_p(1-s)K\right)\right](\Pi_0(\alpha)-r_pK)\\
&\quad=(1-s)\left[\widehat{W}^*(\alpha)-s(r_g-r_p)K\right]\widehat{\Pi}_0^*(\alpha),
\end{aligned}
$$

where

$$
\widehat{W}^*(\alpha)=W^*(\alpha)-\frac{3}{8}-r_pK \quad \text{and} \quad \widehat{\Pi}_0^*(\alpha)=\Pi_0(\alpha)-r_pK.
$$

There are some parameters representing bargaining powers in the bargaining theory. One is the power of the differential between each player's payoff and the disagreement payoff. Another is the players' disagreement payoffs. The disagreement payoff is a kind of reservation utility for the player. A higher reservation utility implies a more advantageous position in negotiations. Thus, it appears sensible that the disagreement payoff itself tends to represent a bargaining power.[10]

Based on this, we employ our new Nash product to examine how a change in s affects the bargaining power of the government. We can interpret the disagreement point (d_g, d_p) as $(s(r_g - r_p), 0)$ in our new Nash product. Take the case where $r_g > r_p$. In this case, the bargaining power of the government is relatively large. If the government buys back the shares of the privatized firm and s is raised, the government's bargaining power becomes stronger. Thus, the weight on social welfare α becomes higher. In contrast, when $r_p > r_g$, the result is reversed since the government's reservation utility decreases as s increases.

In summary, small capital size in a privatized firm might allow the government control the firm's managerial policy by buying or selling its shares, that is, by privatization or nationalization. Furthermore, an effect of privatization or nationalization on the managerial policy, which is determined through bargaining, crucially depends on the rates of public and private investments.

[10] Without doubt, there exist some pathological examples that violate such monotonicity in Nash bargaining. Nevertheless, we can observe the monotone tendency; therefore, we apply this view.

4 Welfare implications

The results obtained in the previous section demonstrate that based on our bargaining model, it is difficult to support the assumption posed by Matsumura (1998) wherein α is positively related to s. It seems that our bargaining model merely allows the relationship between α and s to head in a different direction than that in Matsumura (1998). However, it plays an important role in examining the welfare implication. In our model, bargaining between the two parties occurs only when firm 0 is partially privatized, i.e., $s \in (0,1)$. This implies that the welfare function is discontinuous at $s = 0$ and $s = 1$. In this section, we argue whether or not this discontinuity changes the optimal privatization policy.

The model considered here is a multistage game similar to the one analyzed in the previous section. The difference is that we add a governmental choice stage of partial privatization before proceeding to the multistage game outlined in the previous section. Thus, in the first stage, the government chooses the portion of the share of the public firm that is sold to the private capitalist. In other words, the government chooses its ratio s in the partially privatized firm. Therefore, given the share s, the multistage game considered in the previous section follows. Thus, the government in the first stage selects some ratio of partial or full privatization instead of full nationalization, only when such a choice is beneficial with respect to social welfare. For analysis, we consider the following assumption.

Assumption 2. *The capital K satisfies the following condition:*

$$K \leq \frac{(3-14c)(1-2c)}{32\ \max\{r_p, r_g\}}.$$

This assumption implies that $U_p(\alpha_p) \geq b_p$ and $U_g(\alpha_p) \geq b_g$.[11] $U_p(\alpha_p) \geq b_p$ is an individual rationality condition. If this condition is violated, the private capitalist loses the incentive to hold any share in the privatized firm. On the other hand, $U_g(\alpha_p) \geq b_g$ is more restrictive than

[11] $U_p(\alpha_p) \geq b_p$ and $U_g(\alpha_p) \geq b_g$ are respectively given as

$$K \leq \frac{(1-2c)^2}{8r_p} \quad \text{and} \quad K \leq \frac{(3-14c)(1-2c)}{32\{sr_g + (1-s)r_p\}}.$$

Thus, Assumption 2 implies the latter condition. Moreover, simple calculation yields

$$\frac{(1-2c)^2}{8r_p} - \frac{(3-14c)(1-2c)}{32\ \max\{r_p, r_g\}} > 0.$$

the government's individual rationality condition. Nevertheless, the alleviation of competition accompanied by the liquidation of the privatized firm can deteriorate social welfare drastically, and the return of public investment might not be able to compensate this drastic welfare loss. Thus, it appears natural to consider that competition provides sufficient welfare even though production by the partially privatized firm is small due to a lower α.

In Figure 1, the disagreement point $d = (b_p, b_g)$ is included in A_0B_0FO (or A_1B_1FO) under Assumption 2. This area is involved with the private capitalist's higher payoffs. In this advantageous situation for the private capitalist, the optimal policy is given in Proposition 4.

Proposition 4. *Under Assumptions 1 and 2, the following hold:*

(*i*) *the government chooses partial privatization when* $1/10 < c < 1/6$*;*

(*ii*) *the government does not privatize the public firm at any level when* $\frac{\sqrt{33}-5}{8} < c \leq 1/10$*;*

(*iii*) *when* $c \leq \frac{\sqrt{33}-5}{8}$*, if (11) and (12) are satisfied, the government sells more than half its shares, whereas if not, then the government does not privatize the public firm at any level.*

Proof: See Appendix. □

Suppose that the unit cost of the privatized firm is relatively high. In this case, the marginal benefits from a decrease in price due to higher production by the privatized firm is lower than marginal losses from an increase in total costs. Hence, the government partially privatizes the firm and reallocates the output of the firm to that of the other firm, which, in turn, enhances the social welfare. In the case where the unit cost is low, it is possible that full nationalization is more welfare enhancing than certain levels of partial privatization. This is true if the firm has enough capital. If not, the government can achieve higher welfare by selling half its shares than by fully nationalizing the firm. In this case, the bargaining solution α^* is α_0. Since α_0 is decreasing in c, α^* is close to the most desirable level of the government α_g, when the unit cost is very low.

It may be plausible that this result relies largely on Assumption 2, since this assumption provides the private capitalist with some advantage in the disagreement payoff and thus in the bargaining; this, in turn, lowers α^*. Then, let us consider the following assumption.

Assumption 3. *The capital* K *satisfies the following condition:*

$$K \leq \frac{(1-4c)^2}{8\ \max\{r_p, r_g\}}.$$

This assumption implies that $U_p(\alpha_g) \geq b_g$ and $U_g(\alpha_g) \geq b_g$. In Figure 1, the disagreement point $d = (b_p, b_g)$ is in $GDEO$, which provides the government with an advantage in bargaining. However, we can obtain the same result as in Proposition 4 even if we impose Assumption 3, instead of Assumption 2. This is summarized in Proposition 5.

Proposition 5. *Under Assumptions 1 and 3, all the claims in Proposition 4 hold.*

Proof: See Appendix. □

As shown in Propositions 4 and 5, in contrast with Matsumura (1998) and other papers, except for Matsumura and Kanda (2005), partial privatization is not always desirable, depending on the partially privatized firm's marginal cost and the disagreement point. If the marginal cost is relatively high, then partial privatization is desirable. However, the government should not sell any shares in the public firm (fully nationalized firm) if the marginal cost is in the middle range. Further, provided that the marginal cost is relatively low, two possibilities can be considered. One is that the firm should be partially privatized, when the size of the firm's capital is low. The other is that it should be fully nationalized if the capital is not low.

5 Concluding remarks

In this paper, we examine the behavioral principle of a firm owned by different types of owners, and in particular, we analyze how this principle is determined. For this analysis, we utilize a mixed duopoly where a private firm competes against a partially privatized firm jointly owned by the welfare maximizing government and a profit maximizing private capitalist. This model is employed in many existing studies. Such studies usually assume that the government can control the objective function of the partially privatized firm by adjusting its shares in the firm, ignoring the possibility of the private capitalist opposing the government's claims and the process of determination of the firm's objective function. Further, existing studies also assume that if the government increases its shares, it can more strongly reflect its objective, that is, social welfare, in the objective function of the partially privatized firm. However, we show that these assumptions need not be adequate when both owners negotiate over the objective function of the firm. Specifically, the effect of an increment in the shares that the government holds on the objective function of the privatized firm relies on the difference between the return rates of public and private investments. If the former rate is higher than the latter, then the weight on social welfare in the privatized firm, α, becomes larger as the government's share s becomes large. Interestingly, if the return rate of public investment is lower than that of private investment, the result is reversed.

In addition, we find that in contrast with Matsumura (1998), partial privatization is not always desirable, depending on the partially privatized firm's marginal cost and the disagreement point. If the marginal cost is relatively high, then partial privatization is desirable. However, the government should not sell any shares in the public firm (fully nationalized firm) if the marginal cost lies in the middle range. Further, provided that the marginal cost is relatively low, two possibilities can be considered. One is that the firm should be partially privatized when the size of the firm's capital is low. The other is that it should be fully nationalized if the capital is not low.

Our model can be extended in many directions. The first direction pertains to the market structure. We assume that there is one private firm in the market. This is a slightly restrictive assumption. Matsumura and Kanda (2005) analyzes mixed oligopoly where the free entry of private firms is allowed and shows in their study that the government should fully nationalize the public firm. It would be interesting to examine how the results of Matsumura and Kanda (2005) change if bargaining between the government and a private capitalist is taken into consideration. Secondly, we neglect an incentive for the private capitalist to sell or buy shares in the privatized firm. The effectiveness of the privatization policy would be limited if the private capitalist does not want to acquire shares more than a certain level below a given price. This would require the introduction of a stock market and a model of how different owners may exchange shares in their firm. Finally, our model can be applied to the merger between a private firm owned by the profit maximizing private sector and a public firm owned by the welfare maximizing government.

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Appendix

Proof of Lemma 1

U_g and U_p satisfy

$$U_g'(\alpha) = \frac{(1-2c)\{1-\alpha-c(5-4\alpha)\}}{(3-2\alpha)^3} \quad \text{and} \quad \frac{\partial U_p}{\partial \alpha} = \frac{(1-2c)^2(1-s)(1-2\alpha)}{(3-2\alpha)^3}$$

respectively. The slope of the bargaining frontier is

$$\frac{dU_p}{dU_g} = \frac{\partial U_p/\partial\alpha}{dU_g/d\alpha},$$

if $dU_g/d\alpha \neq 0$. Moreover,

$$U_g''(\alpha) = \frac{(1-2c)(3-18c-4\alpha+16c\alpha)}{(3-2\alpha)^4} \quad \text{and} \quad \frac{\partial^2 U_p}{\partial \alpha^2} = -\frac{8(1-2c)^2(1-s)\alpha}{(3-2\alpha)^4}.$$

Then, as we know, U_g and U_p have the following relationship:

$$\frac{d^2U_p}{dU_g^2} = \frac{1}{(dU_g/d\alpha)^2}\left[d^2U_p/d\alpha^2 - \frac{(dU_p/d\alpha)(d^2U_g/d\alpha^2)}{dU_g/d\alpha}\right].$$

Based on the above relationships, we obtain

$$\frac{d^2U_p}{dU_g^2} = -\frac{1}{U_g'(\alpha)^3}\cdot\frac{(1-6c)(1-2c)^3(1-s)}{(3-2\alpha)^6}.$$

The sign of this second derivative is opposite to that of $dU_g/d\alpha$. Thus, from (7), dU_p/dU_g is positive and $\partial^2U_p/\partial U_g^2$ is negative if $\alpha \in [0,1/2)$. If $\alpha \in (1/2,(1-5c)/(1-4c))$, dU_p/dU_g and $\partial^2U_p/\partial U_g^2$ are negative. Finally, if $\alpha \in ((1-5c)/(1-4c),1]$, dU_p/dU_g and $\partial^2U_p/\partial U_g^2$ are positive. Moreover, $dU_p/dU_g \to \infty$ as $\alpha \searrow (1-5c)/(1-4c)$ and $dU_p/dU_g \to -\infty$ as $\alpha \nearrow (1-5c)/(1-4c)$, and $dU_p/dU_g(1/2) = 0$.

Define a function $f : (-\infty, U_g((1-5c)/(1-4c))] \to \mathbb{R}$ as follows. For $x \in (U_g(1/2), U_g((1-5c)/(1-4c))]$,

$$f(x) = U_p(\alpha(x)), \text{ where } \alpha(x) \text{ is such that } U_g(\alpha(x)) = x \text{ and } 1/2 \leqq \alpha(x) \leqq (1-5c)/(1-4c)$$

and for $x \in (-\infty, U_g(1/2)]$,

$$f(x) = U_p(1/2).$$

By the definition of A, the feasible set of the bargaining problem is characterized by the function f as follows:

$$A = \{(x, y) \in \mathbb{R} : x \leqq U_g((1-5c)/(1-4c)), y \leq f(x)\}$$

Since dU_p/dU_g and $\partial^2 U_p/\partial U_g^2$ are negative when $\alpha \in (1/2, (1-5c)/(1-4c))$, $f' < 0$ and $f'' < 0$ when $x \in (U_g(1/2), U_g((1-5c)/(1-4c)))$. Thus, we have the desired result. $\square$

Proof of Lemma 3

First, the disagreement payoffs must be less or equal to their maximum payoff. Thus, the following conditions hold:

$$b_p \leqq U_p(\alpha_p) = U_p(1/2) \iff 8r_pK \leqq (1-2c)^2 \tag{16}$$

$$b_g \leqq U_g(\alpha_g) = U_g(\frac{1-5c}{1-4c}) \iff 8\left(sr_g + (1-s)r_p\right)K \leqq 1 - 8c + 16c^2. \tag{17}$$

In addition to these, one of the loose sufficient conditions that d is included in A is that d is located at a position in the area under the line intersecting $(U_p(\frac{1-5c}{1-4c}), U_g(\frac{1-5c}{1-4c}))$ and $(U_p(1/2), U_g(1/2))$. Thus, we obtain

$$\begin{aligned} b_g <& \frac{U_g(\frac{1-5c}{1-4c}) - U_g(1/2)}{U_p(\frac{1-5c}{1-4c}) - U_p(1/2)}\left(b_p - U_p(1/2)\right) + U_g(1/2) \\ \iff & \quad 8\left(sr_g + (1-s)r_p\right)K + 2r_pK < 1 - 6c + 8c^2. \end{aligned} \tag{18}$$

$\square$

Proof of Proposition 3

Provided that $d = (b_p, b_g)$. For convenience, we define

$$\widehat{V}(\alpha, s) := (U_g(\alpha) - \hat{b}_g(s))(U_p(\alpha, s) - \hat{b}_p(s)).$$

Then, by implicit function theorem, we have $sgn\{\alpha^{*\prime}(s)\} = sgn\{\partial^2 \widehat{V}/\partial s \partial \alpha\}$. Notice that

$$\frac{\partial U_p}{\partial s} = -\Pi_0^*(\alpha) = -\frac{1}{1-s} \cdot U_p(\alpha, s) \quad \text{and} \quad \frac{\partial^2 U_p}{\partial s \partial \alpha} = -\Pi_0^{*\prime}(\alpha) = -\frac{1}{1-s} \cdot \frac{\partial U_p}{\partial \alpha}.$$

By using these, we can rewrite $\partial^2 \widehat{V}/\partial s \partial \alpha$ evaluated at $\alpha = \alpha^*(s)$ as follows:

$$
\begin{aligned}
\left.\frac{\partial^2 \widehat{V}}{\partial s \partial \alpha}\right|_{\alpha=\alpha^*(s)} &= U_g'(\alpha^*(s)) \left(\frac{\partial U_p}{\partial s} + \hat{b}'(s) \right) + \frac{\partial^2 U_p}{\partial s \partial \alpha} \left(U_g(\alpha^*(s)) - \hat{b}_g(s) \right) - \frac{\partial U_p}{\partial \alpha} \cdot \hat{b}_g'(s), \\
&= -\frac{1}{(1-s)} \Big\{ U_g'(\alpha^*(s)) \left[(1-s)\Pi_0^*(\alpha^*(s)) + (1-s) r_p K \right] \\
&\quad + (1-s) \cdot \Pi_0^{*\prime}(\alpha^*(s)) \left(U_g(\alpha^*(s)) - \hat{b}_g(s) \right) \Big\} + \frac{\partial U_p}{\partial \alpha} \cdot \hat{b}_g'(s), \\
&= -\frac{1}{1-s} \cdot \left[U_g'(\alpha^*(s)) \left(U_p(\alpha^*(s), s) - \hat{b}_p(s) \right) + \frac{\partial U_p}{\partial \alpha} \left(U_g(\alpha^*(s)) - \hat{b}_g(s) \right) \right] \\
&\quad + \frac{\partial U_p}{\partial \alpha} \cdot K(r_g - r_p), \\
&= \frac{\partial U_p}{\partial \alpha} \cdot K(r_g - r_p), \qquad \text{based on (15)}
\end{aligned}
$$

Since $\partial U_p/\partial \alpha \geq 0$ for $\alpha \in [\alpha_p, \alpha_g]$, we obtain $sgn\{\alpha^{*\prime}(s)\} = sgn(r_g - r_p)$. $\square$

Proof of Proposition 4

First of all, we prove that partial privatization is desirable in the case where $1/10 < c < 1/6$. Simple calculation yields

$$
\begin{aligned}
&W^*(\alpha_g) > W^*(1) \geq W^*(\alpha_p) > W^*(0), && \text{if } c \leq \frac{1}{10}, \\
&W^*(\alpha_g) > W^*(\alpha_p) > W^*(1) \geq W^*(0), && \text{if } \frac{1}{10} < c \leq \frac{1}{8}, \\
&W^*(\alpha_g) > W^*(\alpha_p) > W^*(0) > W^*(1), && \text{if } \frac{1}{8} < c < \frac{1}{6}.
\end{aligned} \tag{19}
$$

Thus, from the fact that $\alpha^*(s) \in [\alpha_p, \alpha_g]$ and $W^{*\prime}(\alpha) > 0$ for any $\alpha \in [\alpha_p, \alpha_g]$, we can find that partial privatization gives rise to higher welfare than does full nationalization or privatization.

However, for $c \leq 1/10$, we cannot conclude that partial privatization is desirable based on (19). Therefore, by using another approach, we show that full nationalization is the best policy for firm 0's marginal cost in the relevant range. Since the welfare function W^* is continuous, there exists $\hat{\alpha} \in [\alpha_p, \alpha_p]$ such that $W^*(1) = W^*(\hat{\alpha})$, and this α is equal to $(1-8c)/(1-6c)$. We prove the desirability of full nationalization (i.e., $W^*(\alpha^*(s)) < W^*(1)$) for $d = (b_p, b_g)$ by showing that $\alpha^*(s) < \hat{\alpha}$ for any $s \in (0,1)$. By the same procedure, we also prove the results in the case where $d = (e_p^p, e_g^p)$. For this purpose, we rearrange the first-order condition (15) and obtain

$$
A + B\alpha^*(s) + C\alpha^*(s)^2 + D\alpha^*(s)^3 = 0, \tag{20}
$$

where

$$A = 13 - 114c + 300c^2 - 248c^3 - 72(1-s)(2-7c)Kr_p - 72s(1-2c)Kr_g,$$
$$B = -30 + 252c - 648c^2 + 528c^3 + 8(1-s)(51-156c)Kr_p + 240s(1-2c)Kr_g,$$
$$C = 16 - 128c + 320c^2 - 256c^3 - 32(1-s)(11-31c)Kr_p - 224s(1-2c)Kr_g,$$
$$D = 32(1-s)(3-8c)Kr_p + 64s(1-2c)Kr_g.$$

For convenience, we define the following function:

$$F(\alpha, s) := A + B\alpha + C\alpha^2 + D\alpha^3.$$

(i) When $d = (b_p, b_g)$ and $r_g \geq r_p$

In this case, from Proposition 3, we know that $\alpha^{*\prime}(s) \geq 0$. Then, we now show that $\lim_{s\to 1} \ \alpha^*(s) < \hat{\alpha}$. Converging s in $F(\alpha, s)$ to 1 and evaluating this at $\alpha = \hat{\alpha}$, we obtain

$$\lim_{s\to 1} F(\hat{\alpha}, s) = \frac{E_0}{(1-6c)^3}, \tag{21}$$

where

$$E_0 = 8Kr_g(1-2c)^2(1-10c) - (1-6c)(1 - 18c + 336c^2 - 3400c^3 - 12048c^4 + 8960c^5).$$

The sign of (21) relies on the sign of the numerator E_0 since the denominator $(1-6c)^3$ is positive in the relevant range of c.

$$\begin{aligned}
E_0 &= 8Kr_g(1-2c)^2(1-10c) - (1-6c)(1 - 18c + 336c^2 - 3400c^3 - 12048c^4 + 8960c^5), \\
&\leq 8 \cdot \frac{(3-14c)(1-2c)}{32r_g} \cdot r_g(1-2c)^2(1-10c) \\
&\qquad - (1-6c)(1 - 18c + 336c^2 - 3400c^3 - 12048c^4 + 8960c^5), \qquad \text{(by Assumption 2)} \\
&= -\frac{1}{4}(1 - 28c + 1212c^2 - 19392c^3 + 124976c^4 - 319808c^5 + 212800c^6).
\end{aligned}$$

The right-hand side is negative for any $c \in (0, 1/10)$. Accordingly, from the second-order condition for our bargaining model, we have $\lim_{s\to 1} \ \alpha^*(s) < \hat{\alpha}$.

(ii) When $d = (b_p, b_g)$ and $r_p > r_g$

In this case, from Proposition 3, we know that $\alpha^{*\prime}(s) < 0$, and thus, we show that $\lim_{s\to 0} \alpha^*(s) < \hat{\alpha}$. Applying a procedure similar to that employed in (i), we find that

$$\lim_{s\to 0} F(\hat{\alpha}, s) = \frac{E_1}{(1-6c)^3},$$

where

$$E_1 = 8(1-2c)^3(1-11c)Kr_p - (1-6c)(1-18c+336c^2-3400c^3+12048c^4-8960c^5).$$

Further, based on Assumption 2, we find that

$$\begin{aligned} E_0 &\leq -\frac{1}{4}(1-25c+1174c^2-19208c^3+124544c^4+319312c^5-212576c^6), \\ &< 0. \end{aligned}$$

This implies that $\lim_{s\to 0} \alpha^*(s) < \hat{\alpha}$.

(iii) When $d = (e_p^p, e_g^p)$

Comparing $\hat{\alpha}$ and α_0 directly, we have

$$\begin{aligned} \hat{\alpha} - \alpha_0 = & \frac{5-108c+340c^2+(1-6c)\sqrt{97-1084c+3076c^2}}{4(1-6c)(9-38c)} \gtreqless 0 \\ & \iff \quad c \lesseqgtr \frac{\sqrt{33}-5}{8}. \end{aligned}$$

□

Proof of Proposition 5

We prove Proposition 5 using the same procedure as that used in the proof of Proposition 4. We only show the case where $d = (b_p, b_g)$, since the proof for the case wherein $d = (e_p^p, e_g^p)$ is independent of Assumptions 2 and 3.

(i) When $r_g \geq r_p$

Converging s in $F(\alpha, s)$ to 1 and evaluating this at $\alpha = \hat{\alpha}$, we obtain

$$\lim_{s\to 1} F(\hat{\alpha}, s) = \frac{E_2}{(1-6c)^3},$$

where

$$E_2 = 8(1-2c)^3(1-10c)Kr_g + (1-6c)(-1+18c-336c^2+3400c^3-12048c^4+8960c^5),$$

and based on Assumption 3,

$$E_2 \leq 4c^2(57 - 1114c + 7548c^2 - 19640c^3 + 13120c^4),$$
$$< 0.$$

This implies that $\lim_{s \to 1} \ \alpha^*(s) < \hat{\alpha}$.

(ii) When $r_p > r_g$

Converging s in $F(\alpha, s)$ to 0 and evaluating this at $\alpha = \hat{\alpha}$, we have

$$\lim_{s \to 1} F(\hat{\alpha}, s) = \frac{E_3}{(1 - 6c)^3},$$

where

$$E_3 = 8(1 - 2c)^3(1 - 11c)Kr_p - (1 - 6c)(1 - 18c + 336c^2 - 3400c^3 + 12048c^4 - 8960c^5),$$

and based on Assumption 3,

$$E_3 \leq -c(1 + 214c - 4380c^2 + 29992c^3 - 78304c^4 + 52352c^5),$$
$$< 0.$$

This implies that $\lim_{s \to 0} \ \alpha^*(s) < \hat{\alpha}$. $\square$

References

Aoki, M., 1980. A model of the firm as a stockholder-employee cooperative game. American Economic Review 70, 600–610.

Aoki, M., 1982. Equilibrium growth of the hierarchical firm: Cooperative game approach. American Economic Review 72, 1097–1110.

Bennett, J., Maw, J., 2003. Privatization, partial state ownership, and competition. Journal of Comparative Economics 31, 58–74.

Chao, C.C., Yu, E.S.H., 2006. Partial privatization, foreign competition, and optimum tariff, Review of International Economics 14, 87–92.

Crawford, V.P., Varian, H.R., 1979. Distortion of preferences and the Nash theory of bargaining. Economics Letters 3, 203–206.

DeFraja, G., Delbono, F., 1989. Alternative strategies of a public enterprise oligopoly. Oxford Economic Papers 41, 302–311.

Fershtman, C., 1985. Managerial incentives as a strategic variable in duopolistic environment. International Journal of Industrial Organization 3, 245–253.

Fershtman, C., Judd, K.L., 1987. Equilibrium incentives in oligopoly. American Economic Review 77, 927–940.

Fershtman, C., Judd, K.L., Kalai, E., 1991. Observable contracts: strategic delegation and cooperation. International Economic Review 32, 551–559.

Katz, M.L., 1991. Game-playing agents: Unobservable contracts as precommitments. Rand Journal of Economics 22, 307–328.

Matsumura, T., 1998. Partial privatization in mixed duopoly. Journal of Public Economics 80, 473–483.

Matsumura, T., Kanda, O., 2005. Mixed oligopoly at free entry markets. Journal of Economics 84, 27–48.

Matsumura, T., Matsushima, N., 2004. Endogenous cost differentiations between public and private enterprises. Economica 71, 671–688.

Megginson, W.L., Netter, J.M., 2001. From state to market: A survey of empirical studies on privatization. Journal of Economic Literature 39, 321–89.

Mizutani, F., 2004. Privately owned railway's cost function, organization size and ownership. Journal of Regulatory Economics 25, 297–322.

Nash, J.F., 1950. The bargaining problem. Econometrica 18, 155–162.

Nash, J.F., 1953. Two person cooperative games. Econometrica 21, 128–1240.

Nishimori, A., Ogawa,H., 2002. Public monopoly, mixed oligopoly, and productive efficiency. Australian Economic Papers 41, 185–90.

Schelling, T.C., 1980. The Strategy of Conflict. Harvard University Press.

Sklivas, S. D., 1987. The strategic choice of managerial incentives. Rand Journal of Economics 18, 452–458.

Sobel, J., 1981. Distortion of utilities and the bargaining problem. Econometrica 49, 597–619.

Tomaru, Y., 2007. Privatization, productive efficiency, and social welfare with a foreign competitor. Research in Economics 61, 224-232.

Tomaru, Y., Kiyono, K., 2005. Endogenous timing in a mixed duopoly with asymmetric cost functions. in Open Political-Economic Systems Globalization and Institutional Change Papers, pp. 456–71.

Vickers, J., 1985. Delegation and the theory of the firm. Economic Journal 95, 138–147.

White, M.D., 2001. Managerial incentives and the decision to hire managers in markets with public and private firms. European Journal of Political Economy 17, 877–896.