Capital Liberalization between the Exporting Countries
--Role of Location Choice in Strategic Export Subsidization--

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Working Paper No. 32

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Capital Liberalization between the Exporting Countries∗
– Role of Location Choice in Strategic Export Subsidization –

Kazuharu Kiyono †  Fang Wei ‡

Abstract

This paper presents an international capital liberalization model by allowing governments choose either to liberalize the domestic market for capital inflow or not. We examine the properties of the equilibrium in the export subsidization warfare when a single country opens the market for inward direct investment. We clarify that international coordination is not always necessary in the capital liberalization game. If the cost asymmetry of the two exporting firms is large enough, mutual capital restriction makes world welfare better off.

JEL Classification Numbers: F12, F13
Keywords: strategic export policy, location choice, inward direct investment, capital liberalization

1 Introduction

The theory of strategic export subsidization has made a remarkable progress towards the end of the 20th century in international trade since the pioneering work by Brander and Spencer (1985). Their main contribution lies in that export subsidization may enhance the exporting
country’s welfare in imperfect competition in the absence of interdependence with the other sectors in the economy. Their results soon led to the dispute on strategic subsidy theory.

Markusen and Venables (1988) indicated that the rent shifting effects of export subsidy become weak when Cournot markets are integrated. Under the same assumption of integrated markets, Horstman and Markusen (1986) showed that welfare enhancing export subsidy may bring the inefficient entry. Their result is also challenged by Eaton and Grossman (1986); the so-called rent extraction effects of export subsidization hinges on the market structure of quantity competition à la Cournot with zero conjectural variations. The optimal export subsidy may become negative with Bertrand-Competition. Another challenge comes from relaxing the assumption of entry restrictions. As for the lack of information for the government, it is also pointed out that free trade is the best policy instead of strategic subsidy by Dixit and Grossman (1986) when there are more than two oligopolistic export industries. However insofar as we are confined into the original Brander and Spencer (1985) framework and the long-run view of competition according to Kreps and Scheinkman (1983), one cannot neglect an exporting country’s incentive to subsidize its own domestic firms.

However such a view of export subsidization warfare has recently been challenged by Janeba (1998) once we take into account the firms’ opportunity of relocating their production bases. When the firms in the exporting countries can relocate their production bases, each exporting country is restrained from subsidization, for such high rates of subsidies also benefit the foreign firms relocating to the home country, leading to the outflow of rent. Janeba (1998) showed that the resulting equilibrium entails free trade, i.e., zero export subsidies, and that mutual capital liberalization dominates mutual capital restriction. ¹

However the previous studies have not explored the problem to a full extent, for the cost conditions are the same between the two exporting countries and each country’s capital liberalization policy is exogenously given. As we demonstrate in this paper, once we endogenize the governments’ decision on capital liberalization policy under asymmetric cost conditions, many results have different implications.

¹Peralta, Wauthy, and van Ypersele (2006) examined the firms’ location choice in view of the governments’ policy on corporate tax and the profit shifting control. Barros and Cabral (2000) analyzed subsidy competition to attract FDI from the third country by considering domestic employment gains.
The rest of our paper is organized as follows. In section 2 we build up the four-stage model of capital liberalization in which the governments of the exporting countries decide on its capital liberalization at the first stage. In section 3, we briefly summarize the standard strategic subsidization incentive in Brander and Spencer (1985) as the first subgame in the capital liberalization game. In section 4, we review the effects of relocatability of the firms following Janeba (1998) as the second subgame. In section 5, we discuss the subgame in which one exporting country liberalizes capital. In section 6, based on the discussion on the subgames, we explore the subgame perfect Nash equilibrium of our capital liberalization game and the implications of non-cooperative decisions by the exporting countries on the world welfare. Lastly, in section 7, some concluding remarks are summed up.

2 Model Setup

2.1 Structure of the Economies

We construct our model under the framework of Brander and Spencer (1985) (the BS model hereafter). Consider a world consisting of three countries, 1, 2 and 3. There is a firm residing in each of countries 1 and 2, producing a homogeneous product, and selling to country 3, which does not produce but only consume the product in question.

Let $x_i$ denote the output produced by firm $i$, $c_i$ its unit cost of production, and $s_i$ the unit export subsidy provided by country $i$’s government. Let $p$ denote the market price in country 3, an importing country, $X(= x_1 + x_2)$ its total consumption. The inverse import demand function in the third country is assumed to be linear throughout the paper:\footnote{The assumption of linear demand can be relaxed easily. See Kiyono and Wei (2002).}

$$p = a - X$$

where $a$ is a positive constant and $a > c_i$ ($i = 1, 2$). \footnote{This assumption ensures firm $i$ to have an incentive to produce even as a monopolist, for at the output level 0 under monopoly the marginal revenue is $a$ and its marginal cost is $c_i$.}
2.2 Structure of the Capital Liberalization Game

The game of our interest, which we call the capital liberalization game, incorporates the following four stages of decision.

1st stage The governments of both exporting countries decide simultaneously on whether to close or open the domestic market for capital inflow from abroad.

2nd stage After observing the decisions on capital liberalization, the governments of both exporting countries simultaneously decide on the production (=export) subsidy rate.

3rd stage If at least one country is ready to liberalize capital, the firms in the other countries decide simultaneously where to locate their production plants, either in country 1 or 2. If both countries have decided to refuse capital inflow, there follows the next stage.

4th stage After observing the locations of production plants, both firms simultaneously decide on how much to produce and export to country 3.

Each government has two policy instruments: (i) the capital liberalization policy \( \sigma_i (i = 1, 2) \in \{ C, O \} \) where \( C \) represents the policy of closing the domestic market against capital inflow from abroad and \( O \) the policy of opening the market, and (ii) the production subsidization policy \( s_i (i = 1, 2) \) where \( s_i \geq 0 \) denote the production subsidy per unit output. In view of the first-stage decisions for \( \sigma_i \), the present game can be divided into four subgames as shown in Table 1. A subgame associated with capital liberalization policy profile \( (\sigma_1, \sigma_2) (\in \{ C, O \} \times \{ C, O \}) \) is called subgame \( \sigma_1 \sigma_2 \). The payoff \( W_i^{\sigma_1 \sigma_2} (i = 1, 2) \) in the table denotes the equilibrium welfare of country \( i \) for subgame \( \sigma_1 \sigma_2 \). In terms of this terminology, subgame \( CC \) is the BS model in which both countries close their markets to restrain capital mobility, while subgame \( OO \) is the one analyzed by Janeba (1998) in which both countries are ready to liberalize capital. Therefore our model incorporates all the features of the previous studies and discuss endogenous determination of each exporting country’s capital liberalization policies.

For the succeeding discussion, let us first summarize the results of Brander and Spencer (1985) and Janeba (1998) as well as some other derivations necessary for our analysis.
3 The BS Model as Subgame CC

Subgame CC, i.e., the BS model explores governments’ incentives to subsidize the own exporting firms when each firm cannot relocate abroad. Given the subsidy rate \((s_i, s_j)\), each firm’s equilibrium output and profit in the market performance are expressed as below:

\[
x_i^*(s_i, s_j) = \frac{\beta_i + 2s_i - s_j}{3}
\]

\[
\pi_i^*(s_i, s_j) = \frac{(\beta_i + 2s_i - s_j)^2}{9}
\]

where \(\beta_i := a - 2c_i + c_j \geq 0 (i, j = 1, 2; j \neq i)\) for firm i’s output to be non-negative under duopoly. Throughout the rest of our paper, we use \(\beta_1/\beta_2\) as the indicator of the relative cost of firm 2 over firm 1, since \(\beta_1/\beta_2 = 1\) for \(c_1 = c_2\) and \(\beta_1/\beta_2\) is increasing in \(c_2\) and decreasing in \(c_1\).

Without firms’ mobility, each exporting country’s welfare is given by:

\[
W_i(s_i, s_j) := \pi_i^*(s_i, s_j) - s_i x_i^*(s_i, s_j) = \frac{(\beta_i + 2s_i - s_j)(\beta_i - s_i - s_j)}{9}.
\]

Each country’s reaction function denoted by \(R_i(s_j)\) is defined as a solution for maximizing net surplus in (3):\(^4\)

\[
R_i(s_j) := \arg \max_{s_i} W_i(s_i, s_j) = \frac{1}{4} (\beta_i - s_j)
\]

\(^4\)It is straightforward to verify:

(i) \(W_i(s_i, s_j)\) is strictly concave in \(s_i\) in view of (3), so that the standard second-order condition for welfare maximization is satisfied.

(ii) \(|R_i'(s_j)| < 1\) in view of (4), which assures stability of the non-cooperative equilibrium for the export subsidization game.
Country \(i\)'s reaction curve associated with (4) is shown by the curve \(R_iR_i'\) in Figure 1. The intersection labeled \(E_{CC}\) represents the equilibrium subsidy rate of country \(i\), \(s_{i}^{CC}\) which is given by:

\[
s_{i}^{CC} = \frac{4\beta_i - \beta_j}{15} \quad (i, j = 1, 2; j \neq i).
\]

(5)

The associated equilibrium welfare of each exporting country is expressed by

\[
W_{i}^{CC} := W_i\left(s_{1}^{CC}, s_{2}^{CC}\right) = 2\left(\frac{4\beta_i - \beta_j}{15}\right)^2 \quad (i, j = 1, 2; j \neq i).
\]

(6)

Figure 1: Export Subsidization Warfare Equilibrium in Subgame \(CC\) (the BS Model)

Depending on the parameters governing our model, it is possible to have a monopoly outcome. However, since the monopoly case is beyond the scope of our paper, we assume that the outputs of both firms are non-negative at the equilibrium, i.e., \(x_{i}^*(s_{1}^{CC}, s_{2}^{CC}) \geq 0\).  

This condition is equivalent to the following assumption.

**Assumption 1** \(\beta_1/\beta_2\) is satisfied as \(\frac{1}{4} \leq \beta_1/\beta_2 \leq 4\).

\(\text{Substituting } s_{i}^{CC} = \frac{4\beta_i - \beta_j}{15} \text{ into (1) yields } x_{i}^*(s_{1}^{CC}, s_{2}^{CC}) = \frac{2(4\beta_i - \beta_j)}{15}.\)
Thus the equilibrium subsidy of each country $s^{CC}_i (i = 1, 2)$ is non-negative which means that each country has a positive incentive to subsidize its own exports. For the later analysis, we found that there exists a unique rate of subsidy $\hat{s}_i$ in each country $i$ such that $\hat{s}_i := R_i(\hat{s}_i) = \beta_i/5$. We further have the following lemma:

**Lemma 1** For $\hat{s}_i \left( := \frac{\beta_i}{5} \right)$, there holds $s < R_i(s)$ if and only if $s < \hat{s}_i (i = 1, 2)$.

$\hat{s}_i$ defined in the above lemma is shown in Figure 1, which is determined by the intersection of the reaction curve $R_iR'_i$ and $45^\circ$ Line. In subgame CC, each country has an incentive to set relatively high subsidy rates due to the policy of banning inward direct investment from abroad. As we will discuss later otherwise, i.e., when allowing capital inflow, the governments lose the incentive to choose high subsidy rates, for such high subsidy rates lead the rent run out to the foreign firm having moved into the domestic market.

4 Subgame OO – Mutual Capital Liberalization

Subgame OO is the game explored by Janeba (1998), which is an extension of the BS model to the case in which both exporting countries liberalize capital, i.e., the two exporting firms can freely choose their location for production. The analysis makes sense only when both countries have already decided to accept inward direct investment from abroad. In our paper, we impose the following assumption as in Janeba (1998).

**Assumption 2** When a firm can relocate its production plant between countries 1 and 2, it must be subject to the following constraints.

(i) The firm cannot change the location of the headquarter for management.

(ii) The firm cannot undertake production simultaneously in both countries.

(iii) The same total production cost function is available whether in country 1 or 2.

(iv) The firm stays in the own country when the two countries set the same subsidy rates.\(^6\)

\(^6\)We impose the same tie-breaking rule for zero transportation cost as in Janeba (1998). Without this rule, the equilibria will involve more complicated mixed strategies.
When both exporting countries have liberalized capital, the firm’s strategic location choice depends on the subsidy rates chosen by the two countries, and the country offering a higher subsidy (or imposing a lower tax) can attract the both firms but suffer from the foreign rent outflow. Taxation can restrain this rent outflow but induces both firms to go abroad, leading to a loss of tax revenue. Therefore there can never exist an equilibrium with either strictly positive or negative subsidies. The strategic subsidization incentive of each country leads the equilibrium subsidy rates equal to zero for both exporting countries. Janeba (1998)’s result elucidates how the mutual capital liberalization by both countries (or the relocatability of both firms) affects the government’s subsidization incentives.

**Proposition 1 (Janeba (1998))** When the two exporting countries open their domestic markets allowing foreign capital inflow, the equilibrium subsidy of each exporting country becomes equal to zero.

The associated equilibrium welfare of each exporting country is expressed by

$$W_i^{OO} := \frac{\beta_i^2}{9} \quad (i = 1, 2).$$

(7)

Comparing the above equilibrium welfare in subgame OO with that in subgame CC in (6), we obtain:

$$W_i^{CC} - W_i^{OO} = \frac{7\beta_i^2 - 16\beta_i^2 + 2\beta_j^2}{225}.$$  

So that there holds the following proposition:

**Proposition 2** Mutual capital liberalization makes

i) exporting country 1 strictly better off for $\frac{\beta_1}{\beta_2} \in \left(\frac{8-5\sqrt{2}}{2}, \frac{8+5\sqrt{2}}{2}\right)$, and exporting country 2 strictly better off $\frac{\beta_1}{\beta_2} \in \left(\frac{8-5\sqrt{2}}{2}, \frac{8+5\sqrt{2}}{2}\right)$, and thus

ii) both exporting countries strictly better off for $\frac{\beta_1}{\beta_2} \in \left(\frac{8-5\sqrt{2}}{2}, \frac{8+5\sqrt{2}}{2}\right)$.

-use was made of the condition that exporting country i is made strictly better off if $\frac{\beta_i}{\beta_j} \in \left(\frac{8-5\sqrt{2}}{2}, \frac{8+5\sqrt{2}}{2}\right)$ where $i, j = 1, 2$ and $j \neq i$. 

8
Janeba (1998) demonstrates that the exporting countries are better off with mutual capital liberalization than when both ban inward direct investment. However his result depends on the assumption that both exporting countries have the same cost conditions, i.e., $\beta_1/\beta_2 = 1$. When the cost conditions differ sufficiently to have $\beta_1/\beta_2 \notin \left(\frac{8-5\sqrt{2}}{2}, \frac{8+5\sqrt{2}}{2}\right)$, both exporting countries will be worse off by mutual capital liberalization.

5 Subgames $OC, CO$ – Unilateral Capital Liberalization

Based on the above results of subgames $CC$ and $OO$, we next explore the other two subgames in which only one exporting country liberalizes capital, i.e., subgames $OC$ and $CO$. Since the two subgames are symmetric, we focus our attention on the analysis for subgame $OC$.

We have to explore the properties of each country’s reaction curve as well as its welfare function (i.e., the payoff) so as to obtain the equilibrium. We first deal with country 1’s best response.

5.1 Country 1’s Best Response

Since country 1’s choice of subsidy rate affects firm 2’s relocation incentive, we employ the following strategy to elucidate country 1’s best-response subsidy policy given $s_2$.

1st step Characterize country 1’s optimal subsidy given either (i) the policy of attracting firm 2 to the own country (hereafter the attracting policy) or (ii) the policy of refusing firm 2 (hereafter the non-attracting policy).

2nd step Choose the policy realizing the higher welfare between the attracting policy and the non-attracting policy.
5.1.1 Best Attracting Policy for Country 1

Let us consider country 1’s optimal decision on the subsidy rate when it succeeds in attracting firm 2 given \( s_2 \). Its associated welfare denoted as \( V_1^a \) can be expressed as:

\[
V_1^a(s_1) := W_1(s_1, s_1) - s_1 x_2^s(s_1, s_1) = \frac{(\beta_1 + s_1)^2}{9} - s_1 \frac{(\beta_1 + \beta_2 + 2s_1)}{3}.
\]  

(8)

Define \( s_1^a \) as country 1’s optimal subsidy rate for maximizing \( V_1^a(s_1) \) when firm 2 moves its production plant to country 1 and no longer relocates:

\[
s_1^a := \arg \max_{s_1} V_1^a(s_1) = -\frac{\beta_1 + 3\beta_2}{10} < 0. 
\]  

(9)

That is, since firm 2 never moves out of country 1, it is the best for country 1 to tax the duopoly rent of firm 2 through taxation. Thus country 1’s best-response subsidy given its policy of attracting firm 2, denoted by \( \Gamma_1^q(s_2) \) is \( s_1^q \) when \( s_2 < s_1^q \) and \( s_2 + \epsilon \) otherwise. The best-response subsidy and the corresponding maximized welfare level expressed by \( V_1^a(s_2) := \sup_{s_1} \{V_1^a(s_1)|s_1 > s_2\} \) are shown in Table 2.

5.1.2 Best Non-Attracting Policy for Country 1

Once country 1 bans any inward direct investment from abroad, its welfare is just the same as in the benchmark case of the BS model, i.e., \( W_1(s_1, s_2) \) and its best-response subsidy \( R_1(s_2) = \frac{\beta_1 - s_2}{4} \). However as shown in Lemma 1, this best-response subsidy of country 1 exceeds country 2’s subsidy rate if \( s_2 < s_1 \), so that country 1 is forced to accept firm 2. Given its non-attracting policy, country 1 cannot then employ \( R_1(s_2) \) but must match \( s_2 \) for its welfare maximization.

Therefore, country 1’s best-response subsidy against \( s_2 \) under the non-attracting policy, denoted by \( \Gamma_1^n(s_2) \) and the associated maximized welfare level denoted by \( V_1^a(s_2) := \max_{s_1} \{W_1(s_1, s_2)|s_1 \leq s_2\} \) are summarized in Table 2. 8

There is one remark concerning the equilibrium outputs of the firms here. In view of (1),

8In the table, \( \epsilon(>0) \) represents a sufficiently small positive number.
Table 2: Best-Response Subsidy and Welfare for Country 1

<table>
<thead>
<tr>
<th>Range of $s_2$</th>
<th>Best response subsidy $\Gamma_{1}^{b}(s_2)$</th>
<th>Maximum payoff $V_{1}^{b}(s_2)$</th>
<th>Range of $s_2$</th>
<th>Best response subsidy $\Gamma_{1}^{a}(s_2)$</th>
<th>Maximum payoff $V_{1}^{a}(s_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_2 &lt; s_1^a$</td>
<td>$s_1^a$</td>
<td>$-5s_2^2 + (\beta_1 + 3\beta_2)s_2 + \beta_1^2$</td>
<td>$s_2 &lt; \hat{s}_1$</td>
<td>$s_2$</td>
<td>$(\beta_1 - 2s_2)(\beta_1 + s_2)$</td>
</tr>
<tr>
<td>$s_2 \geq s_1^a$</td>
<td>$s_2 + \varepsilon$</td>
<td>$-5s_2^2 + (\beta_1 + 3\beta_2)s_2 + \beta_1^2$</td>
<td>$s_2 \geq \hat{s}_1$</td>
<td>$\beta_1 - s_2$</td>
<td>$(\beta_1 - s_2)^2$</td>
</tr>
</tbody>
</table>

(9) and the results in Table 2, duopoly obtains only if there holds $\beta_1/\beta_2 \geq 1/3$. For the reference in the succeeding discussion, we sum up in the following lemma.

**Lemma 2** When firm 2 locates its plant in country 1, the equilibrium outputs of both firms are non-negative only if $\beta_1/\beta_2 \geq 1/3$.

### 5.1.3 Policy Switch for Country 1

![Country 1's Payoff Curve](image)

Figure 2 shows the associated maximized welfare for country 1 summarized in Table 2.\textsuperscript{10}

The curve labeled $A_1A_2BA_3$ illustrates the welfare under the attracting policy, while the curve labeled $N_1'BN_2'N_3$ shows the welfare under the non-attracting policy.\textsuperscript{11}

\textsuperscript{9}We get $s_1^1(s_2^1, s_2^1) = \frac{2\beta_1 - 3\beta_2}{10}$ if $\beta_1/\beta_2 \geq 1/3$.

\textsuperscript{10}We set $\beta_1 = 1$ and $\beta_2 = \frac{5}{8}$ when drawing the welfare curves in Figure 2.

\textsuperscript{11}The curve $N_1'BN_2'N_3$ associated with the function $W_1 = \frac{(\beta_1 - 2s_2)(\beta_1 + s_2)}{9}$ is tangent to the curve $N_1N_2N_3$ associated with $W_1 = \frac{(\beta_1 - s_2)^2}{8}$ at $s_2 = \hat{s}_1 = \beta_1/5$. This is not a coincidence, for the best-response subsidy rates are the same both under the attracting and non-attracting policies.
Given country 2’s subsidy $s_2$, country 1 can choose whether to accept firm 2’s direct investment by strategically selecting its own subsidy rate. As shown in Figure 2, the two welfare curves for the two policies intersect at $s_2 = 0$, for country 1 cannot extract firm 2’s rent through zero subsidy rate. One can also prove that under Assumption 1 the curve $N_1'B$ is always below the curve $A_1A_2B$ assuring a unique intersection of the two payoff curves at $s_2 = 0$.

Therefore, country 1’s best-response subsidy against $s_2$ when taking into account its choice between the attracting and non-attracting policies, denoted by $\Gamma_1(s_2)$, is summarized in the following lemma.

**Lemma 3** Country 1’s best response $\Gamma_1(s_2)$ should satisfy

$$\Gamma_1(s_2) = \begin{cases} 
\Gamma^a_1(s_2) & \text{for } s_2 < 0 \\
\Gamma^n_1(s_2) & \text{for } s_2 \geq 0 
\end{cases}$$

Or more precisely, it can be expressed as

$$\Gamma_1(s_2) = \begin{cases} 
s_1^a & \text{for } s_2 \in (-\infty, s_1^a) \\
s_2^{\varepsilon} & \text{for } s_2 \in [s_1^a, 0) \\
0 & \text{for } s_2 = 0 \\
s_2 & \text{for } s_2 \in (0, \hat{s}_1) \\
R_1(s_2) & \text{for } s_2 \in (\hat{s}_1, +\infty) 
\end{cases}$$

where $\varepsilon(>0)$ is a sufficiently small positive number.

Country 1’s reaction curve is illustrated by the mixture of the thick real and broken curves, i.e., the curve labeled $A_1A_2A_3R_1$ in Figure 3.

### 5.2 Country 2’s Best Response

We turn to derive country 2’s best response as in the previous discussion for country 1’s.
5.2.1 Best Attracting Policy for Country 2

First consider the case in which given $s_1$ country 2 succeeds in attracting (or more precisely keeping) firm 2 at home. The welfare is just the same as in the benchmark case of the BS model, i.e., $W_2(s_1, s_2)$. The best-response subsidy is also given by the reaction function (4), i.e., $R_2(s_1)$. Likewise, as stated in Lemma 1, when $s_1$ is sufficiently high and greater than $\hat{s}_2$, country 2’s best-response subsidy $R_2(s_1)$ becomes lower than country 1’s subsidy $s_1$. In this case, country 2 is forced to match its subsidy with country 1’s so as to keep firm 2 at home.

Given country 2’s attracting policy, its best-response subsidy rate denoted by $\Gamma_2^n(s_1)$ and the maximized welfare denoted by $\bar{V}_2^n(s_1)$ are summarized in Table 3.

5.2.2 Best Non-Attracting Policy for Country 2

Next we consider the case in which country 2 has decided not to attract firm 2 (or more precisely decided to keep firm 2 away from home). In this case, the subsidy rate chosen by country 2 does not affect the market outcomes at all. Thus its maximized welfare denoted by $\bar{V}_2^n(s_1)$ depends only on country 1’s subsidy rate and exactly equals to firm 2’s profit, i.e., $\pi_2^n(s_1, s_1)$.

Since country 2 succeeds in keeping firm 2 away from home only with $s_2 < s_1$, its best-response subsidy against $s_1$ given the non-attracting policy, denoted by $\Gamma_2^n(s_1)$, is given by
\((-\infty, s_1)\) as shown in Table 3.

<table>
<thead>
<tr>
<th>Range of (s_1)</th>
<th>Best response subsidy (\Gamma_2^a(s_1))</th>
<th>Maximum payoff (V_2^a(s_1))</th>
<th>Range of (s_1)</th>
<th>Best response subsidy (\Gamma_2^n(s_1))</th>
<th>Maximum payoff (V_2^n(s_1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_1 &lt; \bar{s}_2)</td>
<td>(\frac{\beta_2 - s_1}{4})</td>
<td>(\frac{(\beta_2 - s_1)^2}{8})</td>
<td>all (s_1)</td>
<td>((\beta_2 - s_1)(\beta_2 + s_1))</td>
<td>(\frac{(\beta_2 + s_1)^2}{9})</td>
</tr>
</tbody>
</table>

5.2.3 Policy Switch for Country 2

In Figure 4, the curve named \(A_1B\) shows the maximized welfare of country 2 given the attracting policy and the curve named \(N_1B\) shows the maximized welfare of country 2 given the non-attracting policy.

![Figure 4: Country 2’s Payoff Curve](image)

Country 2 chooses the attracting policy only when there holds

\[
V_2^a(s_1) > V_2^n(s_1). \tag{10}
\]

In view of the results in Table 3, we have to deal with the following two cases for solving the above inequality.
Case 1: When \( s_1 \geq \hat{s}_2 \), (10) can be rewritten as below.

\[
\frac{(\beta_2 - 2s_1)(\beta_2 + s_1)}{9} > \frac{(\beta_2 + s_1)^2}{9}, \quad \text{or} \quad 0 > (\beta_2 + s_1)s_1.
\]

The above inequality never holds for \( s_1 > \hat{s}_2 (> 0) \), so that it is better for country 2 to employ the non-attracting policy, i.e., \((-\infty, s_1)\).

Case 2: When \( s_1 < \hat{s}_2 \), (10) now becomes

\[
\frac{(\beta_2 - s_1)^2}{8} > \frac{(\beta_2 + s_1)^2}{9}, \quad \text{or} \quad s_1^2 - 34\beta_2 s_1 + \beta_2^2 > 0.
\]

The inequality holds for \( s_1 < (17 - 12\sqrt{2})\beta_2 \) or \( s_1 > (17 + 12\sqrt{2})\beta_2 \). Since there holds (0 <) \( 17 - 12\sqrt{2} \beta_2 < \hat{s}_2 = \frac{\beta_2^2}{\beta_2^2} < (17 + 12\sqrt{2})\beta_2 \), we conclude that \( \bar{V}_2'(s_1) > V_2'(s_1) \) holds for \( s_1 < (17 - 12\sqrt{2})\beta_2 \). In the following discussion, we define:

\[
\bar{s}_1 := (17 - 12\sqrt{2})\beta_2 > 0
\]

(11)

for brevity of exposition.\(^{12} \) The best-response subsidization policy of country 2 can be summarized as follows.

\[
\Gamma_2(s_1) = \begin{cases} 
\Gamma_2(s_1) (= R_2(s_1)) & \text{for } s_1 < \bar{s}_1 \\
\{\Gamma_2'(\bar{s}_1)\} \cup \{\bar{R}_2'(\bar{s}_1)\} (= \{R_2(\bar{s}_1)\} \cup (-\infty, \bar{s}_1)) & \text{for } s_1 = \bar{s}_1 \\
\Gamma_2(s_1) (= (-\infty, s_1)) & \text{otherwise}
\end{cases}
\]

Therefore country 2’s best response curve is depicted as the segment \( R_2D \) and the shaded region excluding the dotted boundary in Figure 5 and 6.

5.3 Equilibrium under Unilateral Capital Liberalization

The results in the previous sections imply several possible equilibria. But they are roughly classified into the following two cases.

\(^{12}\bar{s}_1/\beta_2 = 17 - 12\sqrt{2} \propto \frac{17}{12} - \sqrt{2} > 0.\)
Figure 5: Pure Strategy Equilibrium when $\beta_1 / \beta_2 \leq \beta_{mix}$

Figure 6: Mixed Strategy Equilibrium when $\beta_1 / \beta_2 > \beta_{mix}$
• Case I: Nash equilibrium in pure-strategy (See Figure 5)

• Case II: Nash equilibrium in mixed-strategy (See Figure 6)

Comparison of the two figures indicates that the pure-strategy equilibrium is possible only if there holds \( \bar{s}_1 \geq s_1^{CC} \), i.e., \( \frac{\beta_1}{\beta_2} \leq \beta_{mix} \left( := 64 - 45\sqrt{2} \in (0, 1) \right) \). As in Krishna (1989), it is straightforward to prove the following proposition.

**Proposition 3** Depending on the value \( \beta_1/\beta_2 \), there emerge two types of equilibria for sub-game OC as follows.

(i) For \( \beta_1/\beta_2 \leq \beta_{mix} \left( := 64 - 45\sqrt{2} \in (0, 1) \right) \), there realizes the same pure-strategy equilibrium as in subgame CC.

(ii) Otherwise, there realizes a mixed-strategy equilibrium where country 1 (having employed O) chooses \( \bar{s}_1 \) with probability unity and country 2 (having employed C) randomizes over \( R_2(\bar{s}_1) \) and \((-\infty, \bar{s}_1)\).

Let \( \rho \) represent the equilibrium probability of country 2 choosing \( R_2(\bar{s}_1) \) and \( 1 - \rho \) the probability of its choosing other subsidy rates \( s_2 \) smaller than \( \bar{s}_1 \). The equilibrium expected welfare of country 1 in the mixed-strategy is denoted as \( W_1^{OC_{mix}}(s_1) \) where the superscript \( C_m \) represent that country 2 employs a mixed strategy on export subsidies. The equilibrium probability of country 2 choosing \( R_2(\bar{s}_1) \) can be obtained by analyzing country 1’s optimization

\[ s_1^{CC} - \bar{s}_1 = \frac{4\beta_2}{15} \left\{ \left( \frac{\beta_1}{\beta_2} \right) - \left( 64 - 45\sqrt{2} \right) \right\}. \]

One should also note \( \beta_{mix} > 1/3 \), as shown by

\[ \beta_{mix} - \frac{1}{3} = (64 - 45\sqrt{2}) - \frac{1}{3} \propto 191 - 135\sqrt{2} \propto \frac{191}{135} - \sqrt{2} > 0. \]

\[ \text{It is straightforward to derive} \]

\[ s_1^{CC} - \bar{s}_1 = \frac{4\beta_2}{15} \left\{ \left( \frac{\beta_1}{\beta_2} \right) - \left( 64 - 45\sqrt{2} \right) \right\}. \]
behavior. Given $\rho$, the expected welfare of country I choosing $s_1$ is given by

$$W_1^{OC m}(s_1) = \rho W_1(s_1, R_2(\bar{s}_1)) + (1 - \rho)V_1^a(s_1)$$

$$= \frac{\rho}{9} \cdot (\beta_1 + 2s_1 - R_2(\bar{s}_1)) \left( \beta_1 - s_1 - R_2(\bar{s}_1) \right)$$

$$+ (1 - \rho) \left( \frac{(\beta_1 + s_1)^2}{9} - s_1 \frac{(\beta_1 + \beta_2 + 2s_1)}{3} \right),$$

where use was made of (3) and (8). Differentiation with respect to $s_1$ yields

$$9 \frac{dW_1^{OC m}(s_1)}{ds_1} = \rho (\beta_1 - 4s_1 - R_2(\bar{s}_1)) + (1 - \rho) (-\beta_1 - 3\beta_2 - 10s_1).$$

Since there must hold $\lim_{s_1 \to \bar{s}_1} \frac{dW_1^{OC m}(s_1)}{ds_1} = 0$, $\rho$ should satisfy

$$\rho = \frac{4(\beta_1 + 3\beta_2 + 10\bar{s}_1)}{8\beta_1 + 11\beta_2 + 25\bar{s}_1} = \frac{\beta + 173 - 120\sqrt{2}}{2\beta + 109 - 75\sqrt{2}},$$

(12)

by virtue of $\bar{s}_1 = (17 - 12\sqrt{2})\beta_2$. Using $\rho$ in (12), the expected welfare of each country at the mixed-strategy equilibrium is given by:

$$W_1^{OC m} := \rho W_1(\bar{s}_1, R_2(\bar{s}_1)) + (1 - \rho)V_1^a(\bar{s}_1),$$

$$W_2^{OC m} := W_2(\bar{s}_1, R_2(\bar{s}_1)) = \frac{(\beta_2 + \bar{s}_1)^2}{9}. \quad (13)$$

$$W_1^{OC m} = \rho \frac{(4\beta_1 - \beta_2 - 3\bar{s}_1)(4\beta_1 - \beta_2 + 9\bar{s}_1)}{144} + (1 - \rho) \frac{\beta_2^2 - \bar{s}_1(\beta_1 + 3\beta_2) - 5\bar{s}_1^2}{9},$$

To examine the welfare implication for the production relocatability of firm 2, we should compare the above equilibrium welfare in mixed-strategy with those in pure-strategy, i.e., $W_1^{OC m}$ vs. $W_1^{CC}$ and $W_2^{OC m}$ vs. $W_2^{CC}$.

For country 2, it is easy to see that country 2's welfare is higher at point $D$ than at point $E$ along its best-response curve $R_2 R'_2$ in Figure 6. Thus $W_2^{CC} < W_2^{OC m}$ holds. For country 1, it can be demonstrated as follows. By using (13), country 1's expected welfare at the mixed-strategy equilibrium yields:
where \( \rho = \frac{4(\beta_1 + 3\beta_2 + 10\bar{s}_1)}{8\beta_1 + 11\beta_2 + 25\bar{s}_1} \) and \( \bar{s}_1 = (3 - 2\sqrt{2})^2\beta_2 \). Its comparison with \( W_1^{CC} \) yields:

\[
W_1^{CC} - W_1^{OC_m} = 2 \left( \frac{4\beta_1 - \beta_2}{15} \right)^2 - \rho \frac{(4\beta_1 - \beta_2 - 3\bar{s}_1)(4\beta_1 - \beta_2 + 9\bar{s}_1)}{144} - (1 - \rho) \frac{\beta_1^2 - \bar{s}_1(\beta_1 + 3\beta_2) - 5\bar{s}_1^2}{9}.
\]

\( W_1^{CC} - W_1^{OC_m} = 0 \) is isomorphic to a complicated cubic equation in \( \beta_1/\beta_2 \) with a positive coefficient for \((\beta_1/\beta_2)^3\). However, since \( \beta_1/\beta_2 = \beta_{mix} \in (1/3, 1) \) is a critical value yielding both a pure-strategy equilibrium and a mixed one in subgame \( OC \) as stated in Proposition 3, it should be one of the solutions. Besides, by using Mathematica, we can confirm that the equation should have three solutions, one of which is negative and thus can be precluded for consideration. Of the two positive solutions, \( \underline{\beta} \) and \( \overline{\beta} \) (\( \underline{\beta} < \overline{\beta} \)), we find \( \overline{\beta} \approx 0.27 < 1/3 \), so that we must have \( \overline{\beta} = \beta_{mix} \), which is easily confirmed by Mathematica, too. Thus in the range of \( \beta \in [1/3, 3] \), there holds

\[
W_1^{CC} > W_1^{OC_m} \quad \text{if and only if} \quad \frac{\beta_1}{\beta_2} > \beta_{mix}
\]
as established in the following Proposition.

**Proposition 4** \( W_1^{CC} > W_1^{OC_m} \) if and only if \( \beta_1/\beta_2 > \beta_{mix} \) in subgame \( OC \). Symmetrically \( W_2^{CC} > W_2^{OC_m} \) if and only if \( \beta_2/\beta_1 > \beta_{mix} \) in subgame \( CO \).

Therefore at the mixed-strategy equilibrium in subgame \( OC \), the production relocatability of firm 2 yields the following effects:

- It dampens the strategic subsidization incentive of the country liberalizing capital (country 1) and worsens its welfare.

- It strengthens the strategic subsidization incentive of the country not liberalizing capital (country 2) and enhances its welfare.

The following intuition underlies the above results. Due to firm 2’s unilateral relocatability, country 1 is reluctant to raise subsidy since the higher subsidy will attract firm 2 home and
lead the subsidization rent outflow to firm 2. Because of this rent outflow effect, country 1 gets worse off and has an incentive to lower the subsidy rate. On the contrary, country 2 becomes more aggressive with the greater subsidies to earn the larger rent in trade, since the rival country becomes weaker.  

6 Full Equilibrium for the Capital Liberalization Game

Since subgame $CO$ is symmetric to subgame $OC$, $\beta_1/\beta_2$ is constrained in the range of $[1/3, 3]$ in view of Lemma 2. To solve the first-stage capital liberalization game, we classify the equilibria depending on the value $\beta_1/\beta_2$ as shown in Figure 7 where $E_{\sigma_1\sigma_2}$ ($\sigma_i \in \{C, O\}$) denotes the equilibrium for subgame $\sigma_1\sigma_2$ and the subscript $m$ to $C$ represents that the player having chosen $C$ employs a mixed strategy at the subgame.  

![Table](image)

Figure 7: Classification of Equilibria for the Subgames

- **Type B**: The subgames in which unilateral capital liberalization yields mixed-strategy equilibria by the country closing the inward direct investment.

- **Type $M_i$ ($i = 1, 2$)**: The subgames in which country $i$‘s unilateral capital liberalization

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14We thank one anonymous referee for indicating the aggressive strategic behavior of country 2.

15Lemma 2 requires $\beta_1/\beta_2 \geq 1/3$ for both firms to produce non-negative outputs in subgame $OC$. Its counterpart for subgame $OC$ is $\beta_2/\beta_1 \geq 1/3$, i.e., $\beta_1/\beta_2 \leq 3$.

16Apply Proposition 3 to subgame $OC$ and $CO$. Then it is straightforward to get Figure 7. See also footnote 13 to confirm $\beta_{mix} \in (1/3, 1)$.
yields the same pure-strategy equilibrium as in subgame CC, while the other country 
\( j \neq i \)'s unilateral capital liberalization yields an equilibrium with a mixed strategy 
employed by country \( i \).

The figure also summarizes welfare comparisons among possible subgame equilibria by 
virtue of Propositions 2 and 4. 17 One should note that for any possible value of \( \beta_1/\beta_2 \), there 
is at least one country which employs mixed strategies in either subgame OC or CO.

**Proposition 5** For all the possible relevant values of \( \beta_1/\beta_2 \in [1/3, 3] \), there always exist 
subgames of unilateral capital liberalization having mixed-strategy equilibria.

In the rest of analysis, we focus on Types \( M_2 \) and Type \( B \) Equilibria.

### 6.1 Type \( M_2 \) Equilibria

With the parameter \( \beta_1/\beta_2 \in (1/\beta_{mix}, 3] \), our relevant payoff matrix at the first stage can be 
shown by the following Table 4.

<table>
<thead>
<tr>
<th>Country 1 ( \sigma_1 )</th>
<th>Country 2 ( \sigma_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_1 = C )</td>
<td>( \sigma_2 = C )</td>
</tr>
<tr>
<td>( \sigma_1 = O )</td>
<td>( \sigma_2 = O )</td>
</tr>
</tbody>
</table>

We demonstrate first that \( C \) strongly dominates \( O \) for country 1. As shown in Figure 7, 
when country 2 chooses \( C \), there holds \( W_1^{CC} > W_1^{OC} \). Similarly, when country 2 chooses 
\( O \), there holds \( W_1^{CC} > W_1^{OC} \). Thus \( O \) is a dominated strategy for country 1, so that we may 
delete it for consideration. As \( C \) and \( O \) are indifferent to country 2 given \( \sigma_1 = C \), we get two 
equilibria which yield the same payoff: (Close, Close) and (Close, Open). 18 The resulting 
payoff of each country is \( W_i^{CC} \) for \( i = 1, 2 \).

17There holds \( \beta_{mix} < \frac{5+\sqrt{2}}{2} < 1 \), which is given by:

\[
\beta_{mix} = \frac{8 - 5\sqrt{2}}{2} = 64 - 45\sqrt{2} - \frac{8 - 5\sqrt{2}}{2} = 60 \left( 1 - \frac{17\sqrt{2}}{24} \right) < 0.
\]

The result for their reverses further implies \( 1/\beta_{mix} > \frac{8-5\sqrt{2}}{8-\sqrt{2}} = \frac{8+5\sqrt{2}}{2} \). Thus \( \beta_1/\beta_2 \in (\frac{8-5\sqrt{2}}{8-\sqrt{2}}, \frac{8+5\sqrt{2}}{2}) \) is always 
in the range of Type B as shown in Figure 7.

18Likewise for Type \( M_1 \) with \( \beta_1/\beta_2 \in [1/3, \beta_{mix}) \), the equilibria are (Close, Close) and (Open, Close) yielding 
the same payoff.
6.2 Type B Equilibrium

With the parameter $\beta_1/\beta_2 \in (\beta_{mix}, 1/\beta_{mix})$, the relevant payoff matrix at the first stage can be shown by the following Table 5. As with country 1, there holds $W_1^{CC} > W_1^{OCm}$ against country 2’s choice of $C$, while there holds $W_1^{CmO} > W_1^{OO}$ against country 2’s choice of $O$. Thus we have established again that $C$ is the dominant strategy for country 1. Since the payoff structure is qualitatively symmetric, $C$ is also the dominant strategy for country 2. Thus (Close,Close) is the dominant strategy equilibrium.

<table>
<thead>
<tr>
<th>Country 1</th>
<th>$\sigma_1 = C$</th>
<th>$\sigma_1 = O$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_2 = C$</td>
<td>$W_1^{CC}$, $W_2^{CC}$</td>
<td>$W_1^{CmO}$, $W_2^{CmO}$</td>
</tr>
<tr>
<td>$\sigma_2 = O$</td>
<td>$W_1^{OCm}$, $W_2^{OCm}$</td>
<td>$W_1^{OO}$, $W_2^{OO}$</td>
</tr>
</tbody>
</table>

Lastly consider the cases for either $\beta_1/\beta_2 = \beta_{mix}$ or $\beta_1/\beta_2 = 1/\beta_{mix}$. Since the case is symmetric, we focus our attention on the case of $\beta_1/\beta_2 = \beta_{mix}$. In this case, since there holds $W_1^{CC} = W_1^{OCm}$, $C$ and $O$ are indifferent for country 1. But since $C$ is still the dominant strategy for country 2, the equilibrium is also still $CC$. The same logic applies to the case of $\beta_1/\beta_2 = 1/\beta_{mix}$.

In view of these results, the resulting equilibrium welfare of both countries are the same as at the equilibrium for subgame $CC$.

6.3 Welfare at Sub-game Perfect Nash Equilibria

In view of the above discussion, we have established

**Proposition 6** At the sub-game perfect Nash equilibria of the capital liberalization game, each country’s welfare is the same as when two exporting countries totally ban the inward direct investment from abroad.

As we have already shown in Proposition 2, mutual capital liberalization Pareto-dominates mutual capital restriction only when there holds $\beta_1/\beta_2 \in (\frac{8-5\sqrt{2}}{2}, \frac{8+5\sqrt{2}}{2})$. Thus unlike the

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19In fact, comparison between $W_2^{OO} = \frac{\bar{\beta}_2}{2}$ in (7) and $W_2^{OCm} = \frac{(\beta_2+\bar{\beta}_1)^2}{9}$ in (14) yields $W_2^{OCm} > W_2^{OO}$ where use was made of $\bar{\beta}_2 > 0$ by virtue of (11). Symmetrically, we can also obtain $W_1^{CmO} > W_1^{OO}$. 

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implication of Janeba (1998), coordination between the exporting countries will not always be required in the capital liberalization game.

**Proposition 7** In the capital liberalization game with firms’ mobility, both exporting countries need to coordinate and liberalize capital for welfare improvement only when $\frac{8-5\sqrt{2}}{2} < \beta_1/\beta_2 < \frac{8+5\sqrt{2}}{2}$.

For the third country, which is a country only importing the goods from Country 1 and 2, its welfare can be expressed as:

$$W^CC_3 = \frac{2(\beta_1 + \beta_2)^2}{25}, \quad W^{OO}_3 = \frac{(\beta_1 + \beta_2)^2}{18}.$$

Clearly $W^CC_3 > W^{OO}_3$, the third country is always worse off under mutual capital liberalization between the exporting countries than under their mutual capital restriction. This is because in subgame $CC$, the two countries subsidize their exports and thus expand their total sales to the third country, which means improvement of the importing country’s terms of trade. Furthermore, the world welfare under both cases yields:

$$\sum_{i=1}^{3} W^CC_i > \sum_{i=1}^{3} W^{OO}_i.$$

The result is in a sense obvious when the two exporting firms have the same cost conditions. For given the world social marginal cost of production, which is equal to the subsidy-exclusive marginal cost of each firm, the exporting countries’ subsidies expand the world output, leading to less distortion in oligopoly pricing.

When the exporting firms exhibit cost heterogeneity, de Meza (1986) shows that the country with the more efficient firm has the greater incentive to subsidize its exports. Then coupled with the gains from the total output expansion, the world also gains from the greater production efficiency, i.e., cost savings by the output expansion by the more efficient firm and the output contraction by the less efficient one.

**Proposition 8** The world welfare is higher under the exporting countries’ mutual capital
restrictions than under their mutual capital liberalization.

7 Conclusions

In this paper, we examine the strategic subsidy policies under a four-stage capital liberalization game by endogenizing the governments’ decision on capital liberalization policy and taking account of asymmetric cost conditions between the exporting countries.

As we have discussed, mutual capital restriction comes to be selected as a subgame-perfect Nash equilibrium. However, unlike Janeba (1998), mutual capital liberalization does not necessarily Pareto-dominates mutual capital restriction given asymmetric cost conditions, and furthermore the world is always better off under mutual capital restriction than under mutual capital liberalization.

To tell the truth, as free-trade supporters, we have tried to derive mutual capital liberalization as an equilibrium and a better outcome for the world, but in vain. It is our future task to find out what additional factors are necessary to advocate capital liberalization in imperfect competition.

REFERENCE


