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Stable Market Structures from Merger Activities in Mixed Oligopoly with Asymmetric Costs

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Abstract

This paper examines endogenous merger formations in mixed oligopoly. Applying the core as a solution concept, we analyze what market structure(s) remains stable when each firm is allowed to merge with any other in a mixed Cournot industry composed of three firms: two symmetric private firms and an inefficient public firm. We show that a state where a single private firm and a public firm merge is always stable although stable market structures vary according to the cost condition of the public firm. This suggests that both social welfare and the profit of each firm become relatively high in the duopoly resulting from a merger between a private and a public firm. Furthermore, this state can be achieved from an initial mixed oligopoly among the three firms.

JEL Classification: L11, L32, L41

Keywords: Mixed Oligopoly; Merger; Asymmetric Cost; Core

1 Introduction

It is empirically and theoretically known that a merger works as a means towards cost reduction for the merged firm. In this context, it has a positive effect on enhancing welfare in the market by reducing the cost of production. However, it also has a negative effect on welfare because it decreases the competitive pressure on the market. The latter effect deteriorates welfare so severely that the merger guidelines adopted in several countries prescribe that a merger must be proposed to and regulated by the anti-trust

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authority, even if the proposed and thus agreed merger is profitable for the participants.[1] However, in the case of a merger involving a public enterprise, this is not the case.[2] The owner of the public firm, *i.e.,* the government is concerned with social welfare. Because the owner of the public firm would never approve a merger that causes a deleterious change in social welfare, such a merger involving the public firm always improves social welfare.

In this paper, we analyze endogenous merger decisions in a mixed oligopoly where two private firms and one public firm compete in a homogeneous goods market. It is assumed that the public firm is less efficient in production than the two private firms.[3] As is mentioned above, since a merger between private and public firms never damages social welfare as long as the decision to merge is by their own incentives, our focus is on whether the public and private enterprises agree to merge. We also examine how the market structure evolves with a merger activity involving public firms.

Moreover, this study has some impact on the growing literature on the privatization policy of public firms. A merger between public and private firms can be seen as a means of introducing the private sector's technology to the public, and thus can be a partial privatization of the public enterprise. One advantage of our study is that the merger among private and public firms involves two main effects related to a privatization policy. The first is cost reduction through privatization. A public firm with inefficient technology improves its productivity after a merger with an efficient private firm. The second is a change in the objective of the firm after privatization. The firm, post-privatization is partially owned by a private owner who is concerned with ensuring the profit of the privatized firm. As a result, the partially privatized firm has to not only pursue social welfare but also consider both the profit alongside. In literature on a mixed oligopoly, however, privatization is seen as no more than a tool for changing the objective of the public firm, without any consideration of the effect of cost reduction through privatization.[4] Thus, we provide a model of privatization in which both cost reduction and change in the objective of the firm are considered, if a merger involving a public firm is interpreted as one of the forms of privatization policy.

A study on an endogenous decision to merge is useful in understanding "the evolution of industry structure over times as coalitions form and regroup"(Salant, Switzer, and Reynolds 1983, p187). A situation that prevails after a long fight of merger activities should satisfy a stability condition. Thus, we consider a stable merger where the owners of firms have no incentive to merge or break off the merger as the first characterization of our goal. In other words, we apply the core concept in cooperative game

[1]The effects of a horizontal merger on the profits of the insiders and the outsiders in the merger and social welfare are traditionally studied with equilibrium analysis. See Salant, Switzer, and Reynolds (1983); Perry and Porter (1985); Farrell and Shapiro (1990); Levin (1990).

[2]An example of mergers among private and public firms is observed among automobile manufacturers in Spain and France.

[3]Some theoretical works show that in mixed markets public firms are more inefficient than private ones. For example, see Matsumura and Matsushima (2004). In the paper, they show that the private firm's cost becomes lower than the public firm's cost since the private one engages in excessive strategic cost-reducing activities in a Hotelling-type spatial model. In the real world, it is well-known that the experiences of some countries, such as New Zealand and Great Britain provide an adequate explanation to the assumption that public firms have higher costs of production. See Mizutani and Uranishi (2003) for empirical studies.

[4]See DeFraja and Delbono (1989); Matsumura (1998).

theory to our merger problem in a mixed oligopoly. Similar to Barros (1998), who analyzed a merger in a private oligopoly market, we assume that participants for a merger have to agree with shareholding ratios in the merged firm. The merged firm's profit is distributed among them in proportion to their ratios. Because the shareholding ratios are a mere device for distributing the profits among the owners in the case of a merger between private firms, they do not influence the market outcome, such as price, profits, and welfare. In contrast, in a merger involving a public firm, the shareholding ratios affect the market outcomes because they have an impact on the objective of the merged firm in a way such that more the ratio owned by the public owner (private owner) increases, the more that firm reflects welfare (its profit) in its objective. This makes the analysis of a merger in a mixed oligopoly difficult in comparison with the one in a private oligopoly by Barros (1998). The second characterization is derived from a policy implication of a merger as privatization. Since the initial situation is a mixed oligopoly, we consider the achievablity of a stable state from the initial situation. The achievablity of a stable state is simply defined by the condition that some firms have an incentive to merge and induce the stable state from the initial mixed oligopoly.

In literature, Bárcena-Ruiz and Garzón (2003) show that in a mixed duopoly, *á la* Cournot with perfectly substitutable commodities, neither a private nor a public firm want to merge if there is no improvement in production efficiency after the merger. This is intuitively explained as follows. Because a merger weakens the competitive pressure in the market and there is no cost reduction through the merger, the owner of the public firm agrees to merge with the private one only if the shareholding proportion of the public owner in the merged firm is sufficiently high. On the other hand, the owner of the private firm also agrees to merge only in the case of a large shareholding proportion because a decrease in the proportion reduces not only the profit of the merged firm but also its allocation of the merged firm's profit. As a result, the interest of the private firm conflicts with that of the public one and they cannot agree to merge for any shareholding ratios in the merged entity. In contrast, if a merger works towards cost reduction for the less efficient public firm, this is not the case. the owner of the public firm has a strong incentive to merge with the efficient private firm; he will compromise on the shareholding ratio and accept a small proportion, making it possible for the owners of the public and private to agree with the merger decisions. In fact, we show that such a merger will happen and achieve a stable state if the cost reduction factor in a merger exists, irrespective of its value.

In this paper, we assume that the inverse demand function in the market is linear and that all firms have constant marginal costs of production technologies. In addition, a public firm is assumed to be inferior to a private one with respect to productivity. Our analysis revealed that the stable market structures change according to the value of the marginal cost of the public firm; however, we show that there always exists a domain of shareholding proportions of the government, which changes in response to the cost condition of the public firm, such that the state where a private and a public firm merge is in

the core and is stable. Thus, we find that the owner of the public firm has the incentive to merge with a private firm and the owners of private ones agree to the merger, irrespective of the marginal cost of the public firm.

The remainder of this paper is organized as follows. In Section 2, we formulate a model, and describe four regimes: (a) no mergers, (b) merger between two private firms, (c) merger between a private and a public firm, and (d) merger among all firms. Furthermore, we obtain the Cournot-Nash equilibrium values of various variables and the equilibrium payoffs of owners in each regime. In Section 3, we discuss the stability problem for mergers among firms in a mixed oligopoly by adopting the core as the solution concept. Moreover, we consider the achievablity of the stable market structures, *i.e.,* which of the structures is/are likely to emerge from the mixed oligopoly. Section 4 concludes the paper.

2 Model

We consider an industry consisting of two identical private firms and one public firm, both with a single homogeneous output. Let q_0 and q_i denote the quantities of the public firm and private firm i, respectively $(i = 1, 2)$ and let $Q = q_0 + \sum_{i=1}^{2} q_i$ denote the aggregate quantity of the market. The market price is determined by the inverse demand function $P = a - Q$, where we assume that a is sufficiently large. Let $c_j(q_j), j = 0, 1, 2$ denote a cost function of firm j. While the two private firms have constant and identical marginal costs of production normalized to 0, the public firm is assumed to be less efficient than them and its marginal cost is denoted by $c > 0$ and $c \leq a/3$.[5,6] Thus, $c_0(q_0) = cq_0$ and $c_i(q_i) = 0$ for $i = 1, 2$. Since entry decisions are not considered, we assume that there are no fixed costs.

The profit of firm i is given by

$$\pi_0 = q_0(a - Q) - cq_0, \quad \text{and} \quad \pi_i = q_i(a - Q), \quad i = 1, 2.$$

and social welfare W is defined as the sum of consumer surplus ($CS = Q^2/2$) and the profits of the firms (π_i) in the market.

Let the owner of firm i be referred to as owner i. By the definition, owner 0 is from the public sector and owners 1 and 2 are from the private sector. Following the literature, it is assumed that the public sector is concerned with social welfare and thus public firm 0 is a welfare maximizer. On the other hand, the private firms are assumed to be profit maximizers. When a public firm and private firms decide to merge, the merged entity m is owned by two parties which have different objectives. Thus, this merged firm m must consider both profit as well as social welfare. Let $s \in [0, 1]$ denote the public sector's

[5]There are studies on a mixed oligopoly with the constant marginal cost setting such as Mujumdar and Pal (1998), Pal (1998), Matsumura (2003) and Lu (2006).

[6]It implies that a is sufficiently large to the extent that $a \geq 3c > 0$. This assumption guarantees that each of the equilibrium outputs of the public firm in all four regimes is positive or equal to 0.

shareholding proportion in the merged firm and $(1-s) \in [0,1]$ denote the private sector's proportion. Then, following Matsumura (1998) and other studies related to partial privatization,[7] we assume that firm m maximizes

$$V(s, 1-s) = sW + (1-s)\pi_m$$

where q_m and $\pi_m = (a-Q)q_m - c_m(q_m)$ is the amount of output and the profit of firm m, respectively. Here the cost function of firm m, $c_m(q_m)$, is defined by

$$\min \sum_{i \in S} c_i(q_i) \quad \text{s.t.} \sum_{i \in S} q_i = q_m \text{ and } q_i \geq 0 \text{ for } i \in S,$$

where $S \subseteq \{0,1,2\}$ is a set of merger participants. Under the assumptions of constant marginal cost and inefficient cost condition of the public firm, $c_m(q_m) = 0$ because S always includes one of the private firms.

In what follows, we consider four types of regime classified by the merger: (a) no merger, *i.e.,* in the case where three firms compete in the market, (b) merger between two private firms, (c) merger between a private firm and a public firm, and (d) merger among all firms. Let us denote the merged entity by m_S, where S is a set of the participants in the merger, and denote the shares of owner i in the merged entity by $s_i \in [0,1]$, $i \in S$, $\sum_{i \in S} s_i = 1$.

(a) No merger

First, we consider a mixed triopoly with one public firm and two private firms, assuming that there is no merger (or before the merger activity). In this case, private firm i chooses q_i to maximize its own profit π_i while the public firm chooses q_0 to maximize social welfare $W = (q_0+q_1+q_2)^2/2 + \sum_{i=0}^{2} \pi_i$. Solving these three problems simultaneously, we obtain the Cournot-Nash equilibrium outcome for case (a) as follows:

$$q_0^a = a-3c, \quad q_i^a = c, \quad Q^a = a-c, \quad P^a = c,$$
$$\pi_0^a = 0, \quad \pi_i^a = c^2, \quad W^a = \frac{a^2-2ac+5c^2}{2}.$$

Here, we use the symbol a to denote the equilibrium corresponding to regime (a). In addition, in this case, the payoffs of owner 0 and the private owner i $(=1,2)$ are the equilibrium welfare and the equilibrium profit of firm i, respectively. Thus, the payoffs of owner 0 and private owner i $(=1,2)$ are given by

$$u_0^a = W^a = \frac{a^2-2ac+5c^2}{2}, \quad \text{and} \quad u_i^a = \pi_i^a = c^2, \quad (i=1,2).$$

[7] For example, Tomaru (2006) and Fujiwara (2006).

(b) Merger between two private firms

Second, we consider the case that two private firms (two private owners) decide to merge. The market structure after the merger is a mixed duopoly consisting of the public firm and the new 'merged' private firm m_{12}, where we use the notation m_{12} instead of $m_{\{1,2\}}$ for convenience. Social welfare, *i.e.,* the objective function of the public firm, is defined by $W = (q_0 + q_{m_{12}})^2/2 + \pi_0 + \pi_{m_{12}}$. According to the ownership of the firm, the objective of firm m_{12} is defined as the profit of the merged firm, $\pi_{m_{12}}$. In the equilibrium outcomes for regime (b), we obtain the following:

$$q_0^b = a - 2c, \quad q_{m_{12}}^b = c, \quad Q^b = a - c, \quad P^b = c,$$
$$\pi_0^b = 0, \quad \pi_{m_{12}}^b = c^2, \quad W^b = \frac{a^2 - 2ac + 3c^2}{2}.$$

In addition, the payoff of owner 0 is given by

$$u_0^b(s_1, s_2) = W^b = \frac{a^2 - 2ac + 3c^2}{2}.$$

Let $s_1 = t$ and $s_2 = 1 - t$. Then, the payoffs of the private owner 1 and 2 are given by

$$u_1^b(s_1, s_2) = s_1 \pi_{m_{12}}^b = tc^2, \quad \text{and} \quad u_2^b(s_1, s_2) = s_2 \pi_{m_{12}}^b = (1-t)c^2.$$

(c) Merger between a private firm and a public firm

Next, we consider the case in which a private and a public firm decide to merge. We assume that the public firm 0 merges with private firm i. Thus, firm $j = 1, 2$, $j \neq i$ is outside the merger. Social welfare is given by $W = (q_{m_{0i}} + q_j)^2/2 + \pi_{m_{0i}} + \pi_j$. While firm j maximizes π_j, the merged firm m_{0i} maximizes $V(s_0, s_i) = V(s, 1-s)$, where $s_0 = s$ and $s_i = 1 - s$ denote the ratios of the public owner 0 and private owner i in the merged firm's shareholding.

Solving maximization problem, given shares s, we now obtain the Cournot-Nash equilibrium in regime (c) as follows:

$$q_{m_{0i}}^c = \frac{a}{3-2s}, \quad q_j^c = \frac{a(1-s)}{3-2s}, \quad Q^c = \frac{a(2-s)}{3-2s}, \quad P^c = \frac{a(1-s)}{3-2s},$$
$$\pi_{m_{0i}}^c = \frac{a^2(1-s)}{(3-2s)^2}, \quad \pi_j^c = \frac{a^2(1-s)^2}{(3-2s)^2}, \quad W_{m_{0i}}^c = \frac{a^2(2-s)(4-3s)}{2(3-2s)^2},$$

The payoffs of owner 0 and private owner i, who is inside the merger, are given by

$$u_0^c(s_0, s_i) = W^c = \frac{a^2(2-s)(4-3s)}{2(3-2s)^2}, \quad \text{and} \quad u_i^c(s_0, s_i) = (1-s_0)\pi_{m_{0i}}^c = \frac{a^2(1-s)^2}{(3-2s)^2}.$$

In addition, since the payoff of private owner j who is outside the merger is the equilibrium profit of firm j, it is given by

$$u_j^c(s_0, s_i) = \pi_j^c = \frac{a^2 (1-s)^2}{(3-2s)^2}.$$

Here, we find an interesting property. The payoff of the private owner i, who is a participant in the merger, coincides with the payoff of another private owner j who is outside the merger. This property plays an important role when the stability problem is considered.[8]

(d) Merger among all firms

Finally, we consider the case where all firms merge. Thus, the market becomes a public monopoly.[9] In this regime, social welfare is defined by $W = (q_{m_{012}})^2/2 + \pi_{m_{012}}$. The merged firm m_{012} maximizes $V(s_0, s_1 + s_2) = V(s, 1-s)$, where s_i denotes the shares of owner i and $s = s_0$. Solving the maximization problem, we obtain the Cournot-Nash equilibrium in regime (d) as follows:

$$q_{m_{012}}^d = \frac{a}{2-s}, \quad P^d = \frac{a(1-s)}{2-s},$$
$$\pi_{m_{012}}^d = \frac{a^2(1-s)}{(2-s)^2}, \quad W_{m_{012}}^d = \frac{a^2(3-2s)}{2(2-s)^2}.$$

Given shares s, the payoff of owner 0 is defined as

$$u_0^d(s_0, s_1, s_2) = W_{m_{012}}^d = \frac{a^2(3-2s)}{2(2-s)^2}.$$

On the other hand, let $t = s_1/(1-s)$. Then, the payoffs of the private owner 1 and 2, are given by

$$u_1^d(s_0, s_1, s_2) = (1-s)t\pi_{m_{012}}^d = \frac{a^2(1-s)^2 t}{(2-s)^2},$$

and

$$u_2^d(s_0, s_1, s_2) = (1-s)(1-t)\pi_{m_{012}}^d = \frac{a^2(1-s)^2(1-t)}{(2-s)^2}.$$

3 Core and stable market structures

First, we define a "market structure" of an economy in our model. The market structure depends on the form of the merged firm. In particular, it is characterized as the following two things: which firms participate in the merger and proportions of the owners' shareholding in the merged firm. For notational

[8] This property holds as long as all private firms have symmetric constant marginal costs of production even if the number of private firms is more than two. To be more precise, when there are n private firms and one public firm, and the public firm and some of the private firms agree to merge, then, the distributed profits of the private owners of the merged firm coincide with the profits of each outsider of this merger.

[9] Barros (1998) excludes merger for monopoly by assuming that the antitrust authority would not give clearance to such a merger. However, we do not eliminate this case, because our focus is on stable states resulting from merger activities.

convenience, we denote the merged firm between firms 0 and 1 by m_{01} and the market structure, where firm 0 and 1 merge with the pair of proportions of shares, $(0.4, 0.6)$, by $(\{0,1\},(0.4,0.6))$. Thus, a set of all the market structures is defined as

$$A := \left\{ (T,s) :\ T \subseteq \{0,1,2\},\ |T| \geq 2,\ s = (s_i)_{i \in T},\ \sum_{i \in T} s_i = 1,\ s_i \geq 0,\ \forall i \in T \right\} \cup \{M\},$$

where $|T|$ represents the cardinality of set T, and M denotes the market structure of the mixed triopoly consisting of two symmetric private firms and one public firm. Note that based on the calculations in the previous sections, for any market structure α in A, the payoff u_i of owner i is determined as follows. When $\alpha = M$, $u_i(\alpha) = u_i^a$ for all $i \in N$. If $\alpha \neq M$, for owner 0,

$$u_0(\alpha) = \begin{cases} u_0^b(s_1, s_2) & \text{if } \alpha = (\{1,2\},(s_1,s_2)) \\ u_0^c(s_0, s_j) & \text{if } \alpha = (\{0,j\},(s_0,s_j)) \text{ for some } j = 1,2, \\ u_0^d(s_0, s_1, s_2) & \text{if } \alpha = (N,(s_0,s_1,s_2)), \end{cases} \tag{1}$$

and for owner $i, j = 1, 2$, $j \neq i$,

$$u_i(\alpha) = \begin{cases} u_i^b(s_1, s_2) & \text{if } \alpha = (\{1,2\},(s_1,s_2)) \\ u_i^c(s_0, s_i) & \text{if } \alpha = (\{0,i\},(s_0,s_i)) \\ u_i^c(s_0, s_j) & \text{if } \alpha = (\{0,j\},(s_0,s_j)) \\ u_i^d(s_0, s_1, s_2) & \text{if } \alpha = (\{0,1,2\},(s_0,s_1,s_2)). \end{cases} \tag{2}$$

Next, we define the inducement relation over the sets of all the market structures. A subset T of $N := \{0,1,2\}$ freely deviates from any market structure and then, the owners in T freely re-organize the following two things:

(1) which firms in T merge and

(2) the proportions of the owners' shareholding in the merged firm.

Thus, for instance, when all the owners deviate, they can induce any market structure from any other market structure. On the other hand, in case of deviation of a single owner from the merger between the three firms, we impose the following rule on the inducement relation for convenience: For instance, when the owner of firm 2 deviates from the market structure $(\{0,1,2\},(0.4,0.5,0.1))$, firms 0 and 1 remain merged and thus merged firm m_{01} exists with the pair of shares $(0.4/(0.4+0.5),\ 0.5/(0.4+0.5))$.

Formally, the inducement relations $\rightarrow_T$ over A are defined as follows:

- For $i \in N$,

  1. $M \rightarrow_i M$,

2. For any $(R, s) \in A$, if $|R| = 2$ and $i \in R$, then $(R, s) \rightarrow_i M$, and

3. For any $(N, s) \in A$, $(N, s) \rightarrow_i (N \backslash \{i\}, s')$, where $s' = (s'_j, s'_k)$ is such that $s'_j = \frac{s_j}{s_j + s_k}$ and $s'_k = 1 - s'_j$.

- For $T \subset N$ with $|T| = 2$,

1. For any $\alpha \in A$, $\alpha \rightarrow_T M$ and

2. For any $\alpha \in A$ and for any $(T, s) \in A$, $\alpha \rightarrow_T (T, s)$.

- When $T = N$ for any $\alpha \in A$ and any $\beta \in A$, $\alpha \rightarrow_T \beta$.

Market structure $\alpha \in A$ is said to be *blocked* by another market structure $\beta \in A$ via coalition $T \subseteq N$ if the following two conditions hold: (i) $\alpha \rightarrow_T \beta$ and (ii) $u_i(\beta) > u_i(\alpha)$ for all $i \in T$. Market structure α is said to be blocked via coalition T if there exists another market structure β by which α is actually blocked via coalition T. The *core* of A, denoted by $C(A)$, is defined to be the set of the market structures that are not blocked via any coalition.[10]

We now explore the stable market structures, *i.e.,* the core of A, $C(A)$. Our arguments proceed by proving several lemmas. The first lemma shows that there are no shares of two private owners so that the state where two private firms merge is in the core, $C(A)$.

Lemma 1. *For any $t \in [0, 1]$, market structure $(\{1, 2\}, (t, 1 - t))$ does not belong to the core, $C(A)$.*

Proof. As we described in Section 2, when private owners 1 and 2 decide to merge with a pair of share ratios of two private owners, $(t, 1 - t)$, they obtain tc^2 and $(1 - t)c^2$, respectively. On the other hand, each of the two private owners receives c^2 in the market structure M. Thus, either of them certainly prefers the market structure M to $(\{1, 2\}, (t, 1 - t))$, and is able to induce M from $(\{1, 2\}, (t, 1 - t))$ for himself. Therefore, market structure $(\{1, 2\}, (t, 1 - t))$ is blocked by M. □

Lemma 1 shows that the merger of the two private firms brings no advantage to their owners, since it only leads to the expansion of the market shares of the public firm.

Next, we argue whether the state where all the three firms merge, *i.e.,* where the market is monopolized by the firm m_{012} is in the core. The next lemma shows that there exist some shareholding proportions of the three owners such that the state with the merger among the three firms is in the core. This lemma is proved with the aid of Figures 1 and 2.

Lemma 2. *When $a(3 - \sqrt{3})/9 \leq c \leq a/3$, there exist $s \in [0, 1]$ and $t \in [0, 1]$ such that the market structure $(\{0, 1, 2\}, (s, (1 - s)t, (1 - s)(1 - t)))$ belongs to the core, $C(A)$. Such s is less than or equal to*

[10] In literature on applied economics, the core concept is broadly used, *e.g.,* Riezman (1985) in the context of customs unions in international economics, and Okada (2003) who analyzes international CO_2 emissions trading in environmental economics.

$\bar{s} = (5 - 3\sqrt{2})/7 \approx 0.11$ *and the shareholding proportion of private owner* 1 *is hardly different from the one of private owner* 2.[11]

Proof. First, we consider the deviation of the collusion of both the public owner 0 and private owner 1 (or private owner 2) from the market structure $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$. The following analysis are performed with the aid of Figure 1. Let curve AD in Figure 1 denote the locus between the payoff of owner 0 u_0^c (horizontal axis) and the payoff of private owner 1 u_1^c (vertical axis) in the market structure $(\{0,1\},(s',(1-s')))$ when s' varies in the interval $[0,1]$ (A and D represent the states where $s'=0$ and $s'=1$, respectively). In addition, let curve BD express the locus between the payoff of the public owner u_0^d (horizontal axis) and $u_{12}^d/2 := (u_1^d + u_2^d)/2$ (vertical axis), *i.e.,* the value when private owners 1 and 2 equally share $(1-s)\pi_{m_{012}}^d$ and s varies in the interval $[0,1]$ (B and D indicate the states where $s=0$ and $s=1$, respectively). Moreover, the share ratios of owner 0, *i.e., s*, at point E is $\bar{s} = (5-3\sqrt{2})/7 \approx 0.11$.

We present the following claims:

Claim 1. Market structure $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$ is blocked through the deviation of coalition of two players, which consists of owner 0 and one of the private owners if $s > \bar{s}$.

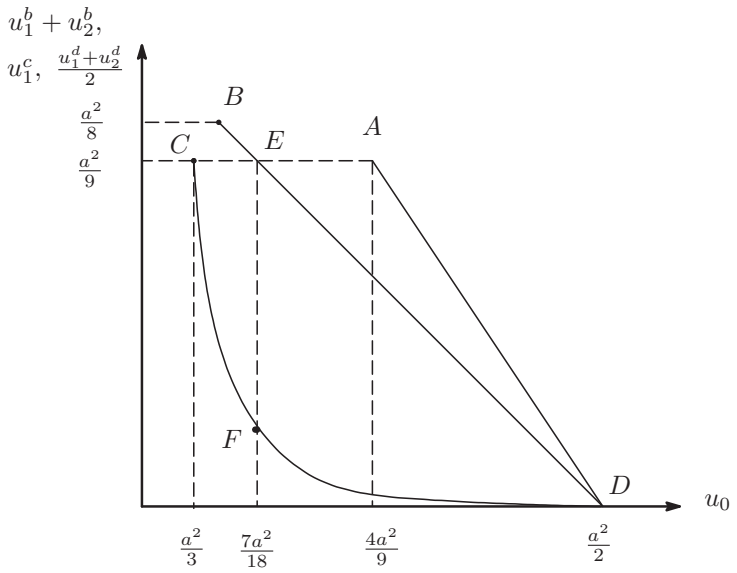

Figure 1: Relationship between the payoffs of the three owners in regimes (b), (c), and (d)

In Figure 1, the market structure $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$ with $s > \bar{s}$ corresponds to the region on curve ED except for point E (when the two private owners obtain equal payoffs). From the figure, for any point on ED, there exist points on the curve AD, which is located in the region upper right from the point. Thus, in the case where the proportions of two private owners' shareholding are equal, both public owner 0 and private owner 1 (or 2) can obtain better payoffs by estab-

[11] The concrete region of s and t in which the market structure $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$ is present in the core, $C(A)$ is given in Figure 2.

lishing the firm m_{01} and adjusting the proportions of their shareholding in the firm, after deviating from the market structure $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$. Hence when $s > \bar{s}$, market structure $(\{0,1,2\},(s,0.5(1-s),0.5(1-s))) \in A$ is blocked. In addition, in the case where the shareholding proportions of the two private owners are different, we select a coalition of the public owner and a private owner who has lower shares in the firm than the other. Then, the locus between the payoffs of the public owner and the private owner is located under the curve ED. After all, the state is blocked for the same reason as the case where the share ratios of the two private owners are equal. Therefore, the market structure $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$ is blocked if $s > \bar{s}$.

On the other hand, the next claim asserts that there exist pairs (s,t) such that the state where the three firms merge is not blocked by the one where a private and a public firm merge, if $s \leq \bar{s}$.

Claim 2. There exist $s \leq \bar{s}$ and $t \in [0,1]$ such that the market structure $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$ is not blocked through the deviation of coalition of two players, which consists of the public owner and one of the two private owners.

If $s \leq \bar{s}$, from Figure 1, we find that owner 0 prefers the market structure with the merger between firm 0 and 1 (or 2) to the market structure where all the three firms merge, irrespective of the proportion of its shares. Therefore, we investigate only whether there is an incentive for the private owner 1 (or 2) to merge his own firm with the public firm by deviating with the public owner. The payoff of the private owner 1 (or 2) is $a^2/9$ at the maximum when firm 1 (or 2) merges with the public firm. Thus, for $s \leq \bar{s}$, if either

$$u_1^d = (1-s)t\pi_{m_{012}}^d < \frac{1}{9}a^2 \quad \text{or} \quad u_2^d = (1-s)(1-t)\pi_{m_{012}}^d < \frac{1}{9}a^2,$$

holds, the market structure $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$ is blocked by the deviation of coalition of the two players of the public owner and the private owner, and the state is not in the core. On the other hand, if the following two conditions hold:

$$u_1^d \geq \frac{1}{9}a^2 \quad \text{and} \quad u_2^d \geq \frac{1}{9}a^2,$$

then $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$ is unblocked by the deviation of the collusion of the two.[12] The pairs of s and t that satisfy the above conditions are found in the shaded area of Figure 2.

Finally, we show that the states corresponding to the shaded area of Figure 2 are present in the core $C(A)$ in the next claim if the marginal cost of the public firm c is relatively high, thus proving this lemma.

Claim 3. If $a(3-\sqrt{3})/9 \leq c \leq a/3$, there exists a (s,t) pair in the shaded area of Figure 2 such that the market structure $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$ is in the core.

[12] These two conditions also hold since each of the two private owners cannot obtain the payoff more than $a^2/9$ by assuming the marginal cost of the public firm, $0 < c \leq a/3$, even if the public owner 0 and private owner 1 (or 2) induce the state M after deviating from $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$, as in the following analysis.

In order to prove this claim, by the claim 2, it is enough to show that neither a coalition of two private owners, a single player coalition of a private owner, nor a single player coalition of owner 0 block the market structure with the pair (s,t) in the shaded area.[13]

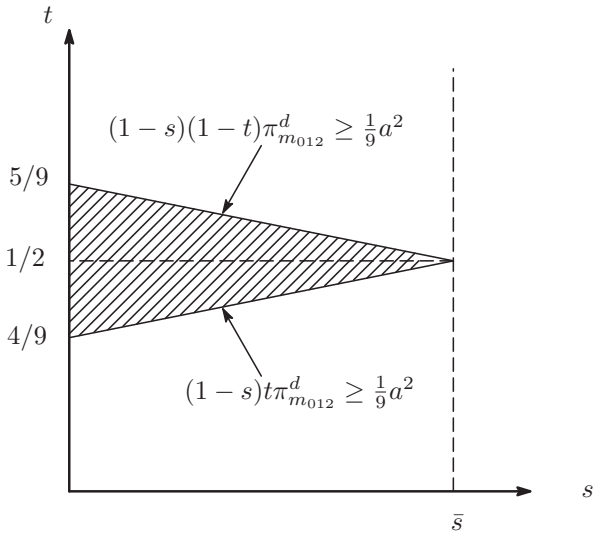

Figure 2: The domain of s and t such that the state where the three firms merge is in the core

First we have to check the possibility of deviation such that all the owners in the merged firm simply change their shareholding ratios. It is easily verified that u_0^d is increasing in s and u_i^d $(i = 1, 2)$ is decreasing in s. Moreover, u_1^d is increasing in t and u_2^d is decreasing in t. Thus, such a deviation does not succeed.

Next, we explore the possibility of the deviation of the coalition of two private owners. Since both private owners 1 and 2 obtain c^2 in the market structure M and the payoffs are less than or equal to c^2 in the market structure $(\{1,2\},(t,1-t))$, it blocks the market structure of the merger among all the three firms only if $u_1^d < c^2$ or $u_2^d < c^2$. However, in the corresponding area of the market structure $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$, both private owners 1 and 2 can obtain the payoffs more than or equal to $a^2/9$ $(\geq c^2)$ by the assumption of the marginal cost of the public firm, $a/3 \geq c > 0$. Thus, it cannot block the market structure of the merger among all the three firms with the shares in the area.

Furthermore, if a private owner deviates, he obtains a payoff which is equal to the payoff of another private owner.[14] Thus, the private owner cannot acquire a payoff of more than $a^2/9$. Therefore, it is impossible for the deviation of a single private owner to block the market structure of the merger among the three firms in the corresponding area.

Finally, we investigate whether the market structure $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$ is blocked via a single player coalition of the public owner. Social welfare depends on the marginal cost of the public

[13]Note that if the state where all the firms merge with some pair of shares of the three owners belongs to the core $C(A)$, it is not blocked by redistributing the shares of their owners in the merger among the three firms as is clear from Figure 1.

[14]We have already explained this property in regime (c) in Section 2.

firm c after it ceases the merger with two private firms. Let the curve CD in Figure 1 denote the locus between the payoff of owner 0 u_0^b and the sum of those of two private owners $u_{12}^b := u_1^b + u_2^b = c^2$ in the market structure $(\{1,2\},(t,1-t))$, when the marginal cost of the public firm c changes in the interval $(0, a/3]$ (C and D represent the states where $c = a/3$ and where $c = 0$, respectively). When the value of c varies on the curve FD (note that F is not included), we find that the state in the shaded area in Figure 2 is blocked by the market structure $(\{1,2\},(t,1-t))$ induced by the public owner. On the contrary, when the value of c changes on the curve CF, the state where the public owner's shareholding proportion, s is near $\bar{s}$ (*e.g.,* $s = \bar{s}, t = 0.5$) is unblocked and is in $C(A)$. We obtain $c = a(3-\sqrt{3})/9$ as the value of c on F by solving $(a^2 - 2ac + 3c^2)/2 = 7a^2/18$. That is, if $c \geq a(3-\sqrt{3})/9$, there exist combinations of values of s and t such that the market structure $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$ is not blocked by any other state.

□

Lemma 2 describes that the shareholding proportions of the three owners in the firm m_{012}, such that the market structure $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$ is in the core, vary depending on the value of the marginal cost of the public firm, c. The larger the value of $c \in [a(3-\sqrt{3})/9,\ a/3]$ is, the weaker the incentive for deviation of the public owner is. As the value of c approaches to $a/3$, the domain of the pairs of s and t in which the market structure $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$ is in the core is broader than the one in which c is relatively small compared to $a/3$. Note that the public owner prefers the merger between a single private and a public firm to the merger of three firms because the addition of a private firm to the two firms' merger only results in deteriorating the competitive pressure without improving the production efficiency. However, once the market structure $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$ with the values of s and t in Figure 2 is established, both private owners have no incentives to accept the proposal of the deviation by the public owner, since they obtain relatively high payoffs. On the other hand, social welfare is non-incremented if the public owner deviates on its own from $(\{0,1,2\},(s,(1-s)t,(1-s)(1-t)))$ with the values of s and t in Figure 2 (the domain of s and t broadens as the value of c become higher), when $a(3-\sqrt{3})/9 \leq c \leq a/3$, because the negative effect of the inefficiency of the public firm overshadows the positive effect of the increase of the competitive pressure in the market.

Next, we explore whether the state where a single private and a public firm merge is in the core $C(A)$. The following lemma holds, and we obtain a positive result.

Lemma 3. *When* $0 < c \leq a/3$, *there exists* $s \in [0,1]$ *such that the market structure* $(\{0,i\},(s,1-s)),(i = 1,2)$ *belongs to the core* $C(A)$.

Proof. Without loss of generality, we assume $i = 1$ in advance of the proof of this lemma. First, we show that the state where a single private and a public firm merge is not blocked by the market structure of the merger among all the three firms. Let us consider a point on curve AD in Figure 3 and call it F.

Note that F corresponds to a pair of payoffs of public owner 0 and private owner 1 in regime (c) for some of their shares. By the definition of blocking, the payoff of the public owner has to be strictly improved by the merger of the three firms. Note again that the curve BD denotes the locus between the payoff of the public owner u_0^d and $u_{12}^d := (u_1^d + u_2^d)/2$ in the market structure of the merger of the three firms. Thus, to enhance the payoff of the public owner through the merger of the three firms, its shares and payoff in the merged firm must correspond to the point on the curve BD, which is lower right to point F (see Figure 3). Let us consider such a point and call it point G.

Here, we find that the value of the vertical axis of point G is always lower than that of point F. This implies that the payoff of the private owner 1 in the initial situation (on point F) is lower than the average of the payoffs of private owners 1 and 2 in the state of the three firms' merger corresponding to point G. In the market structure $(\{0,1\},(s,1-s))$, since the private owner 2 obtains the same payoff as that of the private owner 1 (see regime (c) in Section 2), the payoff of one of the two private owners certainly decreases by the deviation from F to G. Therefore, the market structure $(\{0,1\},(s,1-s))$ is not blocked in the state where all the three firms merge.

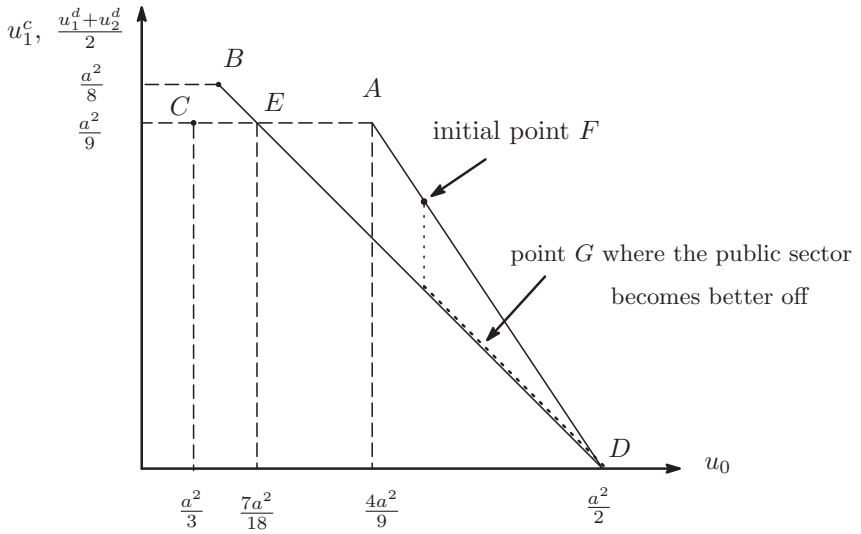

Figure 3: Relationship between the payoffs of the three owners in the regimes (c) and (d)

Next, we investigate whether the market structure $(\{0,1\},(s,1-s))$ is blocked by the single player deviation of the public owner 0 or private owner 1. The arguments presented below proceed with the aid of Figure 4. Let the upper right curve connecting A and D in Figure 4 denote the locus between the payoff of the public owner and that of private owners in regime (c) when s varies from 0 to 1. On the other hand, let the left under curve connecting A and D express the locus between the payoff of the public owner and that of private owners in regime (a) when the marginal cost c varies from 0 to $a/3$ (A represents the state where $c = a/3$ and D is the one where $c = 0$). Here, note that $c^2 \leq a^2/9$ by the assumption of the marginal cost of the public firm.

From Figure 4, we find that for any point on the left under curve AD, there exist points on the upper

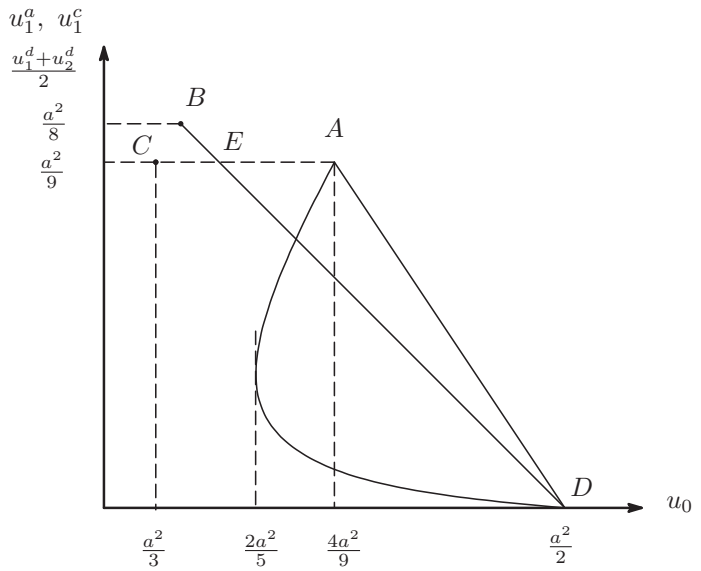

Figure 4: Relationship between the payoffs of the three owners in the regimes (a), (c), and (d)

right of the curve AD, which is located upper right to or at the same point. This means that for any value of c, there exist some $s \in [0,1]$ such that the market structure $(\{0,1\},(s,1-s))$ is not blocked by a single player deviation of the public owner 0 or private owner 1. In addition, it is obvious that the state of the merger between a private and a public firm with such shareholding proportions is also unblocked by the market structure $(\{1,2\},(t,1-t))$ via the coalition of the two private owners.[15]

We consider the deviation of the coalition of two players of the public owner 0 and private owner 2 from the merger between the public owner 0 and private owner 1. Note that by observing regime (c) in the previous section, the private owner 2 obtains the same payoff as that of the private owner 1 in the market structure $(\{0,1\},(s,1-s))$. This fact shows that blocking by such deviation does not succeed.[16]

Finally, we investigate the deviation such that the public and private owners in the merged firm simply change their shareholding ratios. As is described in the curve AD in Figure 4, each owner wants his larger proportion in the merged firm. Thus, the deviation of the two owners in the merged firm does not succeed. □

From Lemma 3, we can infer that there exist pairs of share proportions of the public owner 0 and private owner i such that the market structure $(\{0,i\},(s,1-s))\,,\ (i=1,2)$ always belongs to the core $C(A)$, as long as $c>0$. When the marginal cost of the public firm c is higher, the states with the merger, which is of advantage to a private owner, are in the core. On the other hand, as c decreases, the states where the two owners' shareholding proportions in the merged firm are favorable to the public owner are introduced in the core $C(A)$. The payoff of the public owner (or social welfare) in regime (c) is higher

[15] See the relation between the curve AD and CD in Figure 1.

[16] Consider that curve AD, which denotes the state in the merger between a private and a public firm, is downward-sloping in Figure 3 and 4. Thus, the state where a private and a public firm merge is not blocked by re-allocating the shareholding proportions of the public owner and the one private owner.

than or equal to the one in regime (d) to the same value of s.[17] Therefore, to improve the payoff of the public owner by inducing the state where all the three firms merge, the value of s must be higher than those of the original state, *i.e.,* the state where a private and a public firm merge. However, this results in decreasing the payoff of at least one private owner. This is the reason that the market structure $(\{0,i\},(s,1-s)),(i=1,2)$ is unblocked through the state where the three firms merge. On the other hand, in a mixed triopoly, the existence of the public firm whose productivity is worse compared to that of the two private firms incurs the state with comparatively low social welfare. In addition, in the state where a private and a public firm merge, both private owners can acquire the payoffs that are equal to and higher than the ones in regime (a) by coordinating the shareholding proportions of the public owner and the private owner in the merged firm. Thus, the market structure $(\{0,i\},(s,1-s)),(i=1,2)$ is not blocked by the deviation of a single player, *i.e.,* the public owner or the private owner.

Finally, we confirm whether the market structure M belongs to the core.

Lemma 4. *When $c=a/3$, market structure M belongs to the core, $C(A)$.*

Proof. When $0<c<a/3$, from Figure 4, we find that the market structure M is blocked by $(\{0,i\},(s,1-s))$ via the coalition consisting of the public owner 0 and private owner i, $(i=1,2)$. On the contrary, when $c=a/3$, Point A in Figure 4 denotes each of the owners' payoffs in the mixed triopoly. Thus, the payoff of the public owner is $4a^2/9$, whereas that of the private owner is $a^2/9$. It is easily verified that the state of the mixed triopoly on A is unblocked by any other market structure induced by all deviations of owners. □

In the mixed triopoly, when $c=a/3$, each of the two private owners obtains the highest payoff equal to $a^2/9$, since the public firm exits from the market. Thus, to enhance the payoffs of the two private owners, it is necessary that the shares of the public owner in the three-firm merger is low enough. However, it is inevitable that the decrease of s results in deteriorating social welfare, *i.e.,* the payoff of the public owner. Therefore, the mixed triopoly is not blocked by any other state.

To state the following proposition, we must prepare some words and notions. If a market structure is in the core, we call it a stable market structure. In addition, merger $T\,(\subseteq N)$ is *stable* if there exist some proportions of each owner's shareholding $s=(s_i)_{i\in S}$ such that

$$(T,s)\in C(A).$$

[17] This property is confirmed by the following easy calculation.

$$u_0^c-u_0^d=\frac{a^2(1-s)^3(5-3s)}{2(6-7s+2s^2)^2}\geq 0,\ \forall s\in[0,1].$$

Note that $u_0^c-u_0^d$ is equal to zero when $s=1$.

Moreover, the state of the mixed triopoly is stable if

$$M \in C(A).$$

We obtain the next proposition according to the above-mentioned analysis.

Proposition 1. *Stable mergers are classified into three cases depending on the value of the marginal cost of the public firm c.*

1. *When $c = a/3$, four states of M, merger $\{0, i\}$, $(i = 1, 2)$, and $\{0, 1, 2\}$ are stable.*
2. *When $a(3 - \sqrt{3})/9 \leq c < a/3$, three states of merger $\{0, i\}$, $(i = 1, 2)$, and $\{0, 1, 2\}$ are stable.*
3. *When $0 < c < a(3 - \sqrt{3})/9$, two states of merger $\{0, i\}$, $(i = 1, 2)$ are stable.*

We explain the intuitions behind Proposition 1 as follows. First, the market structure M is stable, only if the marginal cost of the public firm is the highest, *i.e.*, $c = a/3$ to the degree that the public firm exits from the market. In this state, the two private owners obtain relatively high payoffs because of the existence of the public firm whose productivity is low. Thus, the public owner wishes a merger with a private firm for improving its productivity, however, when $c = a/3$, both private owners disagree on the merger, no matter how much the public owner makes concession to the shareholding proportion in the merged firm. As the marginal cost of the public firm is lower than $c = a/3$, the market structure M is blocked by $(\{0, i\}, (s, 1 - s))$ via coalition of two players consisting of the public owner 0 and private owner i, $(i = 1, 2)$ and is not in the core. In addition, from the viewpoint of the public owner, the productivity of the public firm is higher when it merges with a single private firm, because the merger with two private firms deteriorates social welfare by reducing the competitive pressure. Considering this fact, we can find the reason why the market structure $(\{0, 1, 2\}, (s, (1 - s)t, (1 - s)(1 - t)))$ is in the core when the marginal cost of the public firm is under the high and middle level. If the sum of the two private owners' shareholding proportions is high, a single private owner can share a relatively high payoff with another private owner. Thus, there are no incentives for both private owners to deviate with the public owner and induce the state where a private and a public firm merge, although the public owner wants to merge with a single private firm. Moreover, when the productivity of the public firm is bad, social welfare deteriorates even if the public owner deviates on its loan from the state where the three firms merge. That is, when c is relatively high and the shares of the public owner in the three-firm merger are low, it is a clever choice for the public owner to leave the situation as it stands. In other words, the public owner is too inefficient to deviate on its own from the state where all the three firms merge. On the contrary, the market structure $(\{0, 1, 2\}, (s, (1 - s)t, (1 - s)(1 - t)))$ does not belong to the core when the marginal cost of the public firm is under the low level, since the public owner does not have any incentive to merge with the two private firms because of the cutdown on the improvement of its productivity.

Finally, we check which states in the core are most likely to occur. Although it is quite a difficult question theoretically, we can provide a reasonable answer because we have additional information on an initial situation by viewing a merger as "privatization." Since the initial situation is a mixed triopoly of one public firm and two private firms, we focus on the elements of the core that block the initial mixed triopoly. A market structure $\alpha \in A$ is said to be *achievable* from initial situation M if M is blocked by α via some coalition T. A merger among $T \subseteq N$ is said to be stable and achievable from M if there exists a market structure $\alpha = (T, s) \in A$, such that it is stable as well as achievable from the initial situation M. The following proposition shows that only the two-firm merger between a public and private firm is likely to occur as a result of the merger activity from the mixed triopoly.

Proposition 2. *For any $c \in (0, a/3)$, only the two-firm merger $\{0, i\}$, $i = 1, 2$, is stable and achievable from an initial mixed triopoly.*

Proof. By the proof of Lemmas 3 and 4, if $(\{0, i\}, s) \in A$ belongs to the core, it blocks M for any $c \in (0, a/3)$.

Next we show that $(N, s) \in C(A)$ is not achievable from M. By Lemma 2, the stable market structure (N, s) is on the curved segment BE of Figure 4. Because the pair of payoffs of the public owner and the private owner in the initial mixed triopoly M is expressed by the under left curve AD, the public owner always prefers M to (N, s) on the curved segment BE. □

4 Conclusion

In this paper, we investigate what market structures are stable using the core theory when firms are allowed to merge with each other in a mixed oligopoly where one public and two symmetric private ones exist. First, we show that the stable market structures in the core are classified into three cases depending on the degree of the marginal cost of the public firm. Then, although multiple types of merger are stable for some range of the marginal cost of the public firm ($a(3 - \sqrt{3})/9 \leq c \leq a/3$), a stable and achievable merger from an initial mixed oligopoly is only the merger between one public and one private firm. Thus, this state tends to occur after the merger activity of the firms.

We discuss three features of stable market structures belonging to the core. First, the state where a single private and a public firm merge belongs to the core in all three cases. According to this fact, we can recognize the state where the public firm is built into the private firm at a constant rate, *i.e.,* absorbed into the private one as stable. We succeed to show "the (partial) acquisition of an inefficient public firm by a private one" is certainly adequate. Second the state where all the three firms merge is in the core when the marginal cost of the public firm is under the high and middle level. This is because the public owner whose payoff is social welfare wishes to enhance the productivity of the public firm through the

merger at the expense of deteriorating consumer surplus by declining the aggregate quantity, resulting in the monopoly of the merged firm. This clearly demonstrates that the enhancement of the cost condition of the public firm is more important for the public owner than the disadvantage of the monopoly, when the marginal cost of the public firm is relatively high. Finally, the state in the mixed triopoly belongs to the core only if the marginal cost of the public firm is the highest, *i.e.,* $c = a/3$, to the extent that the public firm exits from the market. Therefore, the state that actually occurs, is a private duopoly composed of firms 1 and 2. We find that each owner of two private firms does not assent to all deviations from the mixed triopoly by considering that they obtains comparatively high payoffs due to the disparity of productivity between a private and a public firm. As long as we address the stability problem from the viewpoint of the core theory, under the assumption that the public firm is more inefficient than the private ones, this problem occurs. In this state, we must pay attention about the fact that any privatization policy is meaningless because the public firm actually exits from the production market. Extremely inefficient firms should be discarded even though they are public firms.

Two interesting extensions remain. First is the analysis in which the number of firms is generalized. In this paper, we assume that there exist one public and two private firms. Thus, we must confirm what incentives of mergers for firms' owners occur in the model where more than three generalized numbered firms exist in the market. However, it is conjectured that the results of the generalized model are almost the same as ours. For example, when we consider a mixed oligopoly model with one public and $n \geq 1$ symmetric private firms analogous to Pal (1998), the state where a public and a single private firm merge tends to be in the core, and the one where all firms merge and no merger (*i.e.,* an original mixed oligopoly) probably belongs to the core when a public firm has relatively low productivity.

More importantly, second is how a cost function of a merged firm should be treated. We assume that in advance of our analysis when a private and a public firm merge into a new firm, the merged firm can produce with the technology of the private firm whose productivity is higher, independent of the public owner's shareholding proportion in the merged firm, similar to many existing works in the context of mergers in a private oligopoly. This assumption on the cost function of the merged firm may be adequate when it denotes the production technology of the firm itself (for example, the performance of machines in the plant, *etc*). On the contrary, we also assume that a public firm is more inefficient than a private one. If the inefficiency of a public firm expresses the intrinsic one such as X-inefficiency, the assumption that the merged firm can use the more efficient technology, may not be appropriate. In this case, one may consider that the assumption that the productivity of the merged firm has deteriorated in response to the increase in the proportion of the public owner's shareholding is adequate. Extending our model in these directions remains a subject for future research.

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