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Interregional Mixed Duopoly, Location and Welfare

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Interregional Mixed Duopoly, Location and Welfare*

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Abstract

This paper investigates the effect of a local public enterprise on locations of firms and welfare in an interregional mixed duopoly. We employ a spatial model (linear city model) by dividing a linear city into two districts and assume that there are two firms each of which has different home district. One of them is a local public enterprise owned by the local government which reigns over one of the districts, while the other is a private firm. The local public enterprise is characterized as the one which maximizes welfare of its own district. We show that our two-stage game composed of the location choice and the price competition has two types of equilibria. One is that the two firms are located in the different districts and the other is that they are in the same district whose local government owns the local public enterprise. We consider the equilibrium selection problem. Moreover, we examine the changes in ownership of firms as the central or local government policy.

JEL classification: L13; L32; R32; R59

Keywords: local public enterprise; interregional mixed duopoly; spatial model

1 Introduction

This paper investigates the effect which a local public enterprise has on a mixed market in an *interregional mixed duopoly*. More precisely, we consider the market in which a private firm competes against a public firm owned by a local government which aims at maximizing its own welfare (local welfare). We employ a spatial model and examine location patterns in equilibrium. Besides, we explore the effect which the decentralization of government in public enterprise has on social welfare.

The studies of mixed markets with private and public firms began with Merrill and Schneider (1966), and in response to the global trend of privatization of state-owned enterprises in the 1980s,[1] the number of the researches which analyze the mixed markets has increased over the last two decades. The seminal work of De Fraja and Delbono (1989) introduced game theory into the study and many researchers have taken into account the strategic interaction between public and private firms when they analyze the markets. However, little attention has been directed at local public enterprises. Although many local public enterprises exist in reality, most of the studies assume that public firms are state-owned.

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[1] Examples of such privatization are; British Petroleum[1979] and British Aerospace[1981] in UK; Nippon Telegraph and Telephone Public Corporation[1985] and Japan Monopoly Corporation[1985] in Japan; Deutsche Bundespost[1989, 1994] in Germany.

In this paper we consider the mixed market in which a local public firm and a private firm compete. We employ a Hotelling (1929) type spatial model in order to explain clearly the difference in the region over which the central government and the local government reign. We suppose that the firms make decision on their locations and prices of their products along with other literature which adopts the spatial model.

Some works on mixed oligopoly employed the spatial model (see *e.g.*, Cremer et al., 1991; Matsumura and Matsushima, 2003, 2004; Matsushima and Matsumura, 2003; Li, 2006) and Cremer et al. (1991) is the pioneering work in this field. They assume that state-owned and private firms exist in a linear city and the firms decide their locations and prices.[2] We extend their model by dividing the linear city into two symmetric districts, Region A and B, each of which is reigned over by a local government, and thus the firm owned by the government is regarded as a local public enterprise.

The critical difference between Cremer et al. (1991) and this paper is the owner of the public enterprise. In Cremer et al. (1991) the owner is the central government while in this paper it is the local government. We suppose that the local government of Region A owns the public firm and the owners of the private firm reside in Region B. In addition, we also assume that the public firm aims at maximizing local welfare in Region A and the local welfare does not include the profit of the private firm. Since the public firm and the private firm are related to a different region, we describe the situation as an interregional mixed duopoly.

In this interregional mixed duopoly, we construct a two-stage game which consists of location choice stage and price setting stage. We show that there exist two types of equilibria in the game. In one equilibrium E_1, each firm is located in its home region (*i.e.*, the local public firm is in Region A and the private firm is in Region B) and, in the other equilibrium E_2, both firms are located in Region A. We show that E_2 payoff dominates E_1 while social welfare in E_2 is lower than in E_1, *viz.*, E_1 is the socially desirable equilibrium whereas E_2 is the payoff dominant equilibrium. Thus, we consider the equilibrium selection by means of *risk* dominance criterion. Under this criterion, E_1 is more realized than E_2 by the decisions of rational agents.

We also consider a decentralization problem on the ownership of the public enterprise in order to clarify the nature of vertical relationship between the central and local government in the mixed duopoly. Comparing three policies of the central government: nationalization, decentralization and privatization of the ownership of the firm in the view of maximizing social welfare, we show that the government selects the nationalization. In addition, we investigate the horizontal relationship between local governments by analyzing the decision of each government on whether to privatize its own firm when there is strategic interaction between governments.[3]

The rest of this paper is organized as follows. In the next section, we explain our basic framework of the spatial model. In Section 3, we explore the subgame perfect equilibrium for the two-stage game: In the first stage, each firm chooses its location and, in the second, the firms compete in price. Section 4 discusses the properties of two types of equilibria and further discussions about the effects of the decentralization and privatization are done in Section 5. We conclude in Section 6.

[2] In their models, the state-owned firm indicates the firm which maximizes welfare of the whole city (social welfare). For instance, Cremer et al. (1991) analyzes how social welfare is affected by the number of state-owned and private firms.

[3] Bárcena-Ruiz and Garzón (2005) also considers the strategic privatization of public firms in the context of international trade.

2 Model

We consider the spatial model where there is a linear city of length one represented by the interval $[0, 1]$ and consumers are uniformly distributed on that city with density one. This city consists of two districts, $[0, 1/2)$ and $[1/2, 1]$, which are called Region A and Region B respectively.

There are two firms which produce a homogeneous product at zero cost. One firm, indexed by A, is owned by the local government of Region A and the other, indexed by B, is owned by private shareholders in Region B. Let $a \in [0, 1]$ and $b \in [0, 1]$ denote the locations of Firm A and B respectively. We assume that the consumers purchase one unit of the product irrespective of its price. To purchase the product, the consumers must incur the transportation cost in addition to the mill price. Thus, for instance, a consumer living at point $y \in [0, 1]$ incurs the transportation cost of $t(y - a)^2$ and the mill price P_A when she purchases the product from Firm A.[4] Since both firms produce a homogeneous product, each consumer patronizes the firm with the lower full price, *i.e.*, the sum of the transportation cost and the mill price.

For any $a \in [0, 1]$ and $b \in [0, 1]$, let x denote the location of the consumer who bears the same full price from either of the two firms. This marginal location x is determined as the solution of the following equation:

$$P_A + t(x - a)^2 = P_B + t(x - b)^2,$$

where P_i $(i = A, B)$ is the price of Firm i. Thus, when $a \neq b$, x is given by

$$x = \frac{a + b}{2} + \frac{P_A - P_B}{2(a - b)t}. \tag{1}$$

If Firm A is located in the left of the location of Firm B, *i.e.*, $a < b$, the consumers who live in the left side of x purchase from Firm A while the ones, living in the right side of x, purchase from Firm B, and vice versa. Accordingly, the demand for Firm A's product is given by

$$D_A(P_A, P_B, a, b) = \begin{cases} x & \text{if} \quad a < b, \\ 1 - x & \text{if} \quad a > b. \end{cases}$$

The demand for Firm B's product is defined by the total demand minus the demand for Firm A's, that is, $D_B = 1 - D_A$. Only when $a = b$, the demand for each firm drastically changes. Thus, we mention the case when $a = b$ after the explanation of $a \neq b$.

Since there is no fixed cost, the profit of each firm is expressed by

$$\Pi_i = P_i D_i \quad i = A, B. \tag{2}$$

Social welfare W is defined by

$$W = \begin{cases} -\int_0^x t(a - z)^2 dz - \int_x^1 t(b - z)^2 dz & \text{if} \quad a \leq b, \\ -\int_0^x t(b - z)^2 dz - \int_x^1 t(a - z)^2 dz & \text{otherwise.} \end{cases} \tag{3}$$

Individual demands being perfectly inelastic, so positive prices (*i.e.*, the prices above marginal costs) do not create distortions in the allocation of resources. Thus, maximizing social welfare is equivalent to minimizing the total transportation cost. The sum of the transportation costs of all the consumers depends on the locations of two firms, and so does social welfare. Note that, when $a = b$, social welfare is also represented by Equation (3).

[4] The value of t (> 0) does not matter to the analysis in this paper.

Table 1: the classification of four cases ($a \neq b$)

<table>
<tr><th></th><th>location</th><th>marginal consumer</th><th>W_A</th><th>Π_A</th><th>T_A</th><th>C_A</th></tr>
<tr><td>Case 1</td><td rowspan="2">$a < b$</td><td>$x \geq 1/2$</td><td>F_1</td><td rowspan="2">$P_A x$</td><td>$\int_0^{1/2} t(a-z)^2 dz$</td><td>$P_A/2$</td></tr>
<tr><td>Case 2</td><td>$x < 1/2$</td><td>F_2</td><td>$\int_0^{x} t(a-z)^2 dz$
$+\int_x^{1/2} t(b-z)^2 dz$</td><td>$P_A x + P_B(1/2 - x)$</td></tr>
<tr><td>Case 3</td><td rowspan="2">$a > b$</td><td>$x \geq 1/2$</td><td>F_3</td><td rowspan="2">$P_A(1-x)$</td><td>$\int_0^{1/2} t(b-z)^2 dz$</td><td>$P_B/2$</td></tr>
<tr><td>Case 4</td><td>$x < 1/2$</td><td>F_4</td><td>$\int_0^{x} t(b-z)^2 dz$
$+\int_x^{1/2} t(a-z)^2 dz$</td><td>$P_B x + P_A(1/2 - x)$</td></tr>
</table>

Next, we consider local welfare of Region A. In contrast to social welfare, local welfare is affected by the firm's profit and the prices of the products which are borne by local residents. Accordingly, local welfare of Region A is expressed by

$$W_A = \Pi_A - T_A - C_A,$$

where T_A and C_A respectively denote the total transportation cost and the total mill price which are borne by the residents in Region A. Note that Π_A, T_A and C_A vary along with the locations of the two firms and the corresponding marginal location x. These relations are summarized in Table 1. Let F_i denote W_A in Case i ($i = 1, 2, 3, 4$). In other words, $W_A = F_i$ when Case i.

Since $W = W_A + W_B$, local welfare of Region B is given by

$$W_B = W - W_A. \tag{4}$$

We consider the following standard two-stage game: In the first stage, each firm chooses its location simultaneously and in the second stage, the firms choose their prices simultaneously. We assume that each firm can locate any point in the interval $[0, 1]$ without any restriction. In other words, we do not assume $a < b$.

While Firm B maximizes its profit, Firm A, which is owned by the government of Region A, aims at maximizing local welfare of Region A, *i.e.*, W_A.

In the remainder of this section, we consider the case when $a = b$. In this case, every consumer incurs the same transportation cost, and thus the demands for both firms' products are affected only by the prices they set. We assume that the total demand is equally divided to both firms in the case of $P_A = P_B$. The demand function for Firm A is given by

$$D_A(P_A, P_B, a, b) = \begin{cases} 0 & \text{if} \quad P_A > P_B, \\ \frac{1}{2} & \text{if} \quad P_A = P_B, \\ 1 & \text{if} \quad P_A < P_B, \end{cases}$$

when $a = b$. Besides, $D_B = 1 - D_A$. The profit of each firm is represented by Equation (2) again. In this case, the equilibrium price of each firm is zero by the above particular demand function. We explain it here to simplify the analysis in the next section.

Firm B, which is a private firm and thus maximizes its own profit, always chooses the price which is slightly lower than P_A.[5] For this reason, Firm B does not have the optimal strategy in price setting game as long as $P_A > 0$. Consequently, $P_A = P_B = 0$ is a unique equilibrium when $a = b$.

[5]In the case of $P_A < P_B$, Firm B can capture the entire demand of the market. Thus, such price setting has a substantial effect on the profit of Firm B.

In addition, when $a = b$, $W_A = T_A$ since $P_A = P_B = 0$ in equilibrium. Thus, if $a = b$,

$$W_A = -\int_0^{1/2} t(a-z)^2 dz.$$

Local welfare of Region B is represented by Equation (4) again.

3 Results

3.1 *Price competition*

Given firms' locations a, b chosen at the first stage, the two firms compete on price at the second stage of our two-stage game. Since the objectives of two firms vary along with their locations, we must separate two cases to analyze the equilibrium: (I) $a > b$ and (II) $b < a$. At this moment, we do not consider the case of (III) $a = b$ because we have know that $P_A = P_B = 0$ is a unique equilibrium.

Let us denote the objectives of two firms in (I) and (II) as $U_i^j(P_A, P_B),\ i = A, B,\ j = I, II$. Then, the first order conditions for the maximizations are as follows.

$$\frac{\partial U_A^j}{\partial P_A} = 0, \quad \frac{\partial U_B^j}{\partial P_B} = 0.$$

The reaction functions of firm A and B, denoted by $r_A^j(P_B)$ and $r_B^j(P_A)$ respectively, satisfy

$$\frac{\partial U_A^j}{\partial P_A}(r_A^j(P_B), P_B) = 0, \quad \frac{\partial U_B^j}{\partial P_B}(P_A, r_B^j(P_A)) = 0.$$

The equilibrium price of the second stage is defined by the fixed point of the composite function of the two reaction functions. In other words, $P_A^* = r_A^j(r_B^j(P_A^*))$ holds.

(I) $\boldsymbol{a < b}$. This case corresponds to Case 1 and 2 in Table 1. Since $a < b$, we obtain

$$F_1 - F_2 = -\frac{(P_A - P_B + \alpha)^2}{4(b-a)t} \leq 0,$$

where F_i is local welfare of Region A in Case i and $\alpha := (b-a)(1-a-b)t$. Thus $F_1 < F_2$ holds unless $P_A = \widetilde{P}_A := P_B - \alpha$. In addition, it is easily verified that F_1 and F_2 are the concave functions in P_A, and moreover,

$$\left.\frac{\partial F_1}{\partial P_A}\right|_{P_A=\widetilde{P}_A} = \left.\frac{\partial F_2}{\partial P_A}\right|_{P_A=\widetilde{P}_A} = \frac{P_B - \alpha}{2(a-b)t} = -\frac{\widetilde{P}_A}{2(b-a)t}.$$

We have Figure 1 which represents the relationship of F_1 and F_2. The thin and thick curves denote F_1 and F_2 respectively. Since the signs of the slopes of F_1 and F_2 at $\widetilde{P}_A$ are changed according to the sign of $\widetilde{P}_A$, we have to separate two cases.

Further, Equation (1) and Table 1 imply that, under $a < b$,

$$\begin{cases} P_A \leq \widetilde{P}_A \iff x \geq \frac{1}{2} & \Longrightarrow \quad \text{Case 1 and } W_A = F_1, \\ P_A > \widetilde{P}_A \iff x < \frac{1}{2} & \Longrightarrow \quad \text{Case 2 and } W_A = F_2, \end{cases}$$

and $P_A = \widetilde{P}_A \iff x = 1/2$. The curves with shaded portion in Figure 1 represent local welfare W_A. Thus, the maximum of W_A is attained by the maximization of F_2 when $\widetilde{P}_A < 0$ and by the maximization of F_1 when $\widetilde{P}_A \geq 0$. By the first order conditions for maximization of F_1 and F_2, we obtain

$$r_A^I(P_B) = \begin{cases} \frac{P_B - \alpha}{2} & \text{if} \quad \widetilde{P}_A \geq 0 \quad (P_B \geq \alpha), \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

In contrast with Firm A, the objective of Firm B is $U_B^I = \Pi_B = P_B(1-x)$ irrespective of Case 1 or 2. Thus, we have, by the first order condition of maximizing U_B^I,[6]

$$r_B^I(P_A) = \frac{P_A - (a-b)(2-a-b)t}{2}. \tag{6}$$

Equations (5) and (6) yield the equilibrium prices as follows:

$$P_A^1(a,b) = -\frac{(a-b)(a+b)t}{3}, \quad P_B^1(a,b) = -\frac{(a-b)(3-a-b)t}{3}.$$

Superscript 1 indicates that the equilibrium holds for the range of Case 1 ($x \geq 1/2$).

(II) $\boldsymbol{a > b}$. This corresponds to Case 3 and 4 in Table 1. We obtain, by $a > b$,

$$F_3 - F_4 = -\frac{(P_A - P_B + \alpha)^2}{4(a-b)t} \leq 0.$$

Thus, $F_3 < F_4$ holds except for the case that $P_A = \widetilde{P}_A$. Further, both F_3 and F_4 are concave in P_A and

$$\left.\frac{\partial F_3}{\partial P_A}\right|_{P_A = \widetilde{P}_A} = \left.\frac{\partial F_4}{\partial P_A}\right|_{P_A = \widetilde{P}_A} = -\frac{P_B - \beta}{2(a-b)t},$$

where $\beta := (a-b)(a+b)t > 0$. We have Figure 2 which represents the relationship of F_3 and F_4. In this figure, the thin and thick curves represent F_3 and F_4 respectively. Note that, the signs of the slopes of F_3 and F_4 at $P_A = \widetilde{P}_A$ vary according to the sign of $P_B - \beta$, and so we have to separate two cases.

Equation (1) and Table 1 imply that under $a > b$,

$$\begin{cases} P_A \geq \widetilde{P}_A \iff x \geq \frac{1}{2} & \Longrightarrow \quad \text{Case 3 and } W_A = F_3, \\ P_A < \widetilde{P}_A \iff x < \frac{1}{2} & \Longrightarrow \quad \text{Case 4 and } W_A = F_4, \end{cases}$$

and $P_A = \widetilde{P}_A \iff x = 1/2$. The curves with shaded portion represent local welfare W_A. Thus, we have, by the maximization of F_3 and F_4,

$$r_A^{II}(P_B) = \begin{cases} \frac{P_B + (a-b)(2-a-b)t}{2} & \text{if} \quad P_B < \beta, \\ (a-b)t & \text{otherwise.} \end{cases} \tag{7}$$

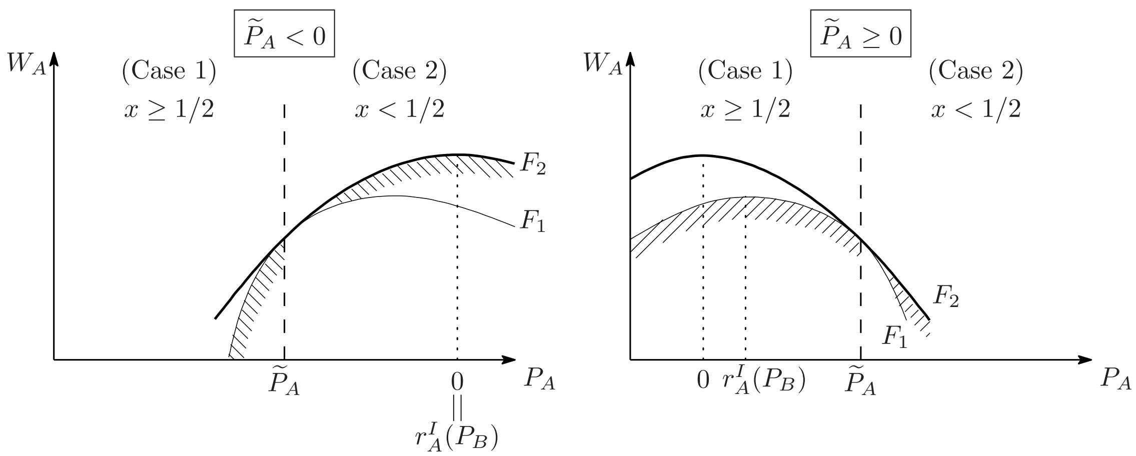

Figure 1: local welfare of Region A in (I) $a < b$

[6]The second order condition is satisfied.

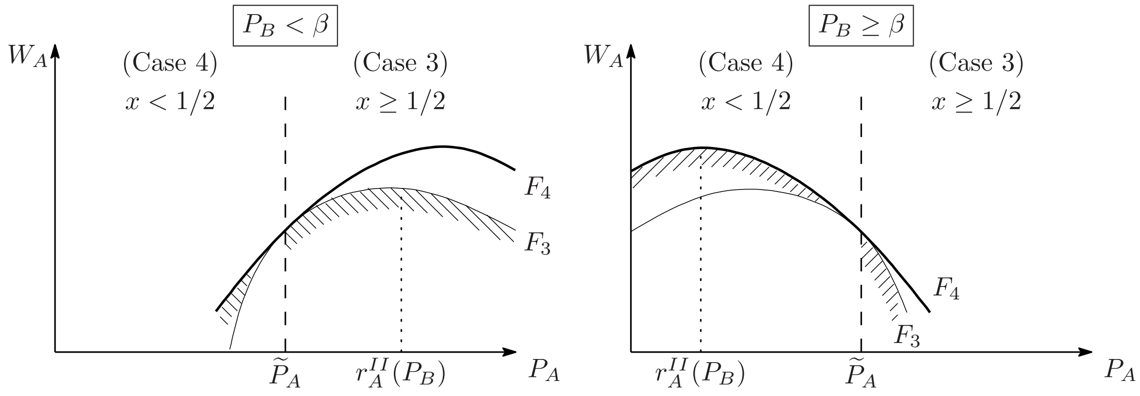

Figure 2: local welfare of Region A in (II) $a > b$

Finally, the objective of firm B is $U_B^{II} = \Pi_B = P_B x$ and we have, by the maximization of Π_B,[7]

$$r_B^{II}(P_A) = \frac{P_A + (a-b)(a+b)t}{2}. \tag{8}$$

By Equations (7) and (8), we have Figure 3 which represents the relationship of the two reaction curves when $a > b$. Since the reaction curve of firm A is kinked at $P_B = \beta$, there are two kinds of possible intersection. As is shown in Figure 3, when $\beta > (a-b)t \iff a+b > 1$, the intersection is attained in the area of $P_A < \beta$ and $P_B < \beta$. On the other hand, when $\beta \leq (a-b)t \iff a+b \leq 1$, the intersection is in $P_A \geq \beta$ and $P_B \geq \beta$. Thus, we obtain the equilibrium prices as follows:

$$\begin{cases} P_A^3(a,b) = \frac{(a-b)(4-a-b)t}{3}, & P_B^3(a,b) = \frac{(a-b)(2+a+b)t}{3} \quad \text{if} \quad a+b > 1, \\ P_A^4(a,b) = (a-b)t, & P_B^4(a,b) = \frac{(a-b)(1+a+b)t}{2} \quad \text{otherwise.} \end{cases}$$

Superscripts 3 and 4 indicate that the equilibria hold for the ranges of Case 3 ($x \geq 1/2$) and Case 4 ($x < 1/2$) respectively.

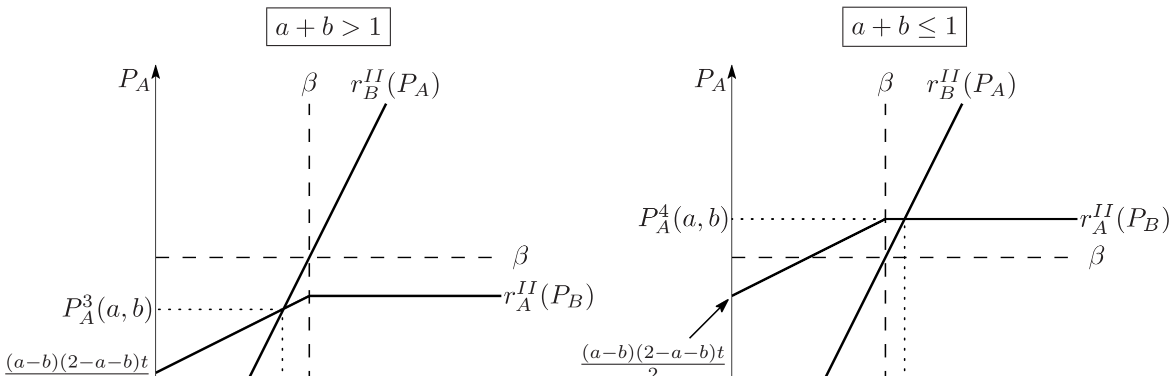

Figure 3: the reaction curves in price competition (in (II) $a > b$)

These results are summarized as the following lemma.

Lemma 1. *The relation between the locations a, b and the equilibrium prices in the second stage is as follows:*

[7] The second order condition holds.

(I) $a < b$

$$P_A^1(a,b) = -\frac{(a-b)(a+b)t}{3}, \quad P_B^1(a,b) = -\frac{(a-b)(3-a-b)t}{3}.$$

(II) $a > b$

$$\begin{cases} P_A^3(a,b) = \frac{(a-b)(4-a-b)t}{3}, & P_B^3(a,b) = \frac{(a-b)(2+a+b)t}{3} \quad \textit{if} \quad a+b>1, \\ P_A^4(a,b) = (a-b)t, & P_B^4(a,b) = \frac{(a-b)(1+a+b)t}{2} \quad \textit{otherwise.} \end{cases}$$

(III) $a = b$.

$$P_A(a,b) = 0, \quad P_B(a,b) = 0.$$

The superscript j of the equilibrium price $P_i^j(a,b)$ corresponds to the Case j in Table 1.

Let $\mathbf{P}^j(a,b)$ denote the pair of the equilibrium price for Case j ($j = 1, 3, 4$), $(P_A^j(a,b), P_B^j(a,b))$. By Lemma 1, we have Figure 4 showing the range in which $\mathbf{P}^j(a,b)$ holds. The marginal location, in which the consumer is indifferent to either of two firms which she purchases the product from, in each case, is obtained as follows:

$$\begin{cases} x^1(a,b) = \frac{3+a+b}{6} & \text{if} \quad \mathbf{P} = \mathbf{P}^1(a,b), \\ x^3(a,b) = \frac{2+a+b}{6} & \text{if} \quad \mathbf{P} = \mathbf{P}^3(a,b), \\ x^4(a,b) = \frac{1+a+b}{4} & \text{if} \quad \mathbf{P} = \mathbf{P}^4(a,b). \end{cases}$$

We finish the preparation of analyzing the location choices in the first stage and in the next subsection, we consider the location problem.

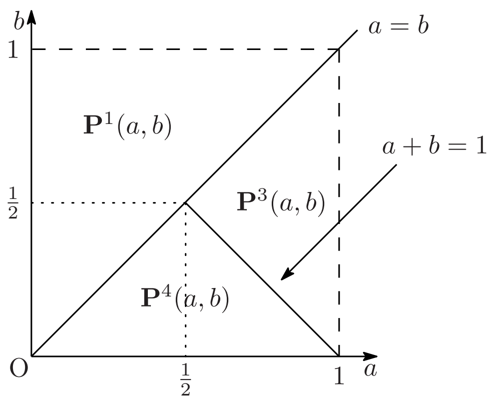

Figure 4: the ranges of equilibrium prices

3.2 *Location choice*

We now consider the location choices of two firms in the first stage. We see from Figure 4 that if Firm B is located in $[0, 1/2)$, the equilibrium price of Firm A changes from $P_A^1(a,b)$ to $P_A^4(a,b)$ at $a = b$, and from $P_A^4(a,b)$ to $P_A^3(a,b)$ at $a = 1-b$ as a increases. Similarly, if Firm B is located in $[1/2, 1]$, it changes from $P_A^1(a,b)$ to $P_A^3(a,b)$ at $a = b$ as a increases. Accordingly, in order to obtain the reaction function of Firm A in the first stage, we need to distinguish the two cases.

First, when $b \in [0, 1/2)$, the objective of Firm A is given by[8]

$$W_A = \begin{cases} F_1 = P_A^1(x^1 - \frac{1}{2}) - \int_0^{1/2} t(a-z)^2 dz & \text{if} \quad a \le b, \\ F_4 = \frac{P_A^4}{2} - P_B^4 x^4 - \int_0^{x^4} t(b-z)^2 dz - \int_{x^4}^{1/2} t(a-z)^2 dz & \text{if} \quad b < a \le 1-b, \\ F_3 = P_A^3(1-x^3) - \frac{P_B^3}{2} - \int_0^{1/2} t(b-z)^2 dz & \text{otherwise,} \end{cases} \tag{9}$$

[8]Henceforth, we omit the argument of the objective functions to simplify the description.

and when $b \in [1/2, 1]$,

$$W_A = \begin{cases} F_1 = P_A^1(x^1 - \frac{1}{2}) - \int_0^{1/2} t(a-z)^2 dz & \text{if} \quad a \le b, \\ F_3 = P_A^3(1-x^3) - \frac{P_B^3}{2} - \int_0^{1/2} t(b-z)^2 dz & \text{otherwise.} \end{cases} \tag{10}$$

Thus, we can obtain the reaction function of Firm A as follows:

$$R_A(b) = \begin{cases} \frac{10-b-\sqrt{73-20b+4b^2}}{3} & \text{if} \quad b < \bar{b}, \\ \frac{-18-2b+\sqrt{378+72b+16b^2}}{6} & \text{otherwise,} \end{cases} \tag{11}$$

where $\bar{b} \approx 0.3656$. See Appendix for the derivation of Equation (11).

If Firm A is located in $[0, 1/2)$, the equilibrium price of Firm B changes from $P_B^4(a,b)$ to $P_B^1(a,b)$ at $b = a$ as b increases. Similarly, if Firm A is located in $[1/2, 1]$, it changes from $P_B^4(a,b)$ to $P_B^3(a,b)$ at $b = 1-a$, and from $P_B^3(a,b)$ to $P_B^1(a,b)$ at $b = a$ as b increases. Thus, we need to classify the objective of Firm B into two cases. When $a \in [0, 1/2)$, the objective is given by

$$\Pi_B = \begin{cases} G_4 = P_B^4 x^4 & \text{if} \quad b < a, \\ G_1 = P_B^1(1-x^1) & \text{otherwise,} \end{cases} \tag{12}$$

and when $a \in [1/2, 1]$,

$$\Pi_B = \begin{cases} G_4 = P_B^4 x^4 & \text{if} \quad b \le 1-a, \\ G_3 = P_B^3 x^3 & \text{if} \quad 1-a < b < a, \\ G_1 = P_B^1(1-x^1) & \text{otherwise,} \end{cases} \tag{13}$$

where G_i $(i = 1, 3, 4)$ is the profit of Firm B corresponding to each equilibrium price. We can derive the reaction function of Firm B as follows:

$$R_B(a) = \begin{cases} 1 & \text{if} \quad a < \bar{a}, \\ 0 & \text{otherwise,} \end{cases} \tag{14}$$

where $\bar{a} \approx 0.3799$. See Appendix for the derivation of Equation (14).

The reaction functions $R_A(b)$ and $R_B(a)$ are described in Figure 5. $R_A(b)$ is jumped at $b = \bar{b}$ and $R_B(a)$ is jumped at $a = \bar{a}$. Therefore, this model has two subgame perfect equilibria, E_1 and E_2. Let (a_i^*, b_i^*) denote the pair of equilibrium location points in E_i $(i = 1, 2)$. We have the following theorem.

Theorem 1. *In the first stage, there are two equilibria E_1 and E_2. Each location point is as follows:*

$$E_1 : \begin{cases} a_1^* = \frac{-20+\sqrt{466}}{6} \approx 0.2645 \\ b_1^* = 1 \end{cases} \qquad E_2 : \begin{cases} a_2^* = \frac{10-\sqrt{73}}{3} \approx 0.4853 \\ b_2^* = 0 \end{cases}$$

4 Further discussions on the properties of the two equilibria

4.1 *Equilibrium comparison*

Theorem 1 shows that our two-stage game has two types of equilibria. While in the equilibrium E_1, each firm is located in its home district, in the equilibrium E_2, both firms are located in Region A whose government owns Firm A. Figure 6 describes the locations of two firms in the interval $[0, 1]$ at the two equilibria. Here, x_k^* $(k = 1, 2)$ denotes the position of the marginal consumer who is indifferent to which firm he purchases from.

The equilibrium payoffs are as follows:

$$E_1 : \begin{cases} \text{Firm } A : & W_A(a_1^*, b_1^*) = \frac{(-9953+466\sqrt{466})t}{1944} \approx 0.0548t, \\ \text{Firm } B : & \Pi_B(a_1^*, b_1^*) = \frac{(26-\sqrt{466})(32-\sqrt{466})^2 t}{3888} \approx 0.1231t, \end{cases}$$

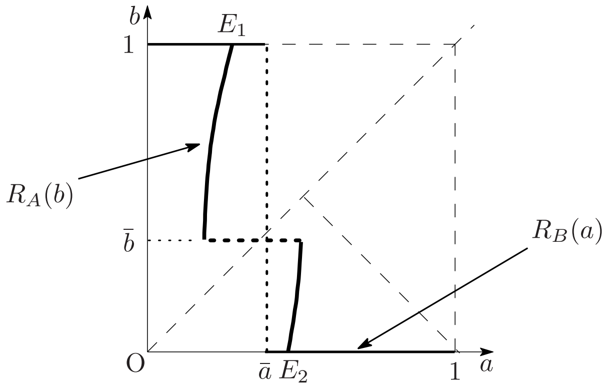

Figure 5: the reaction curves in location choice

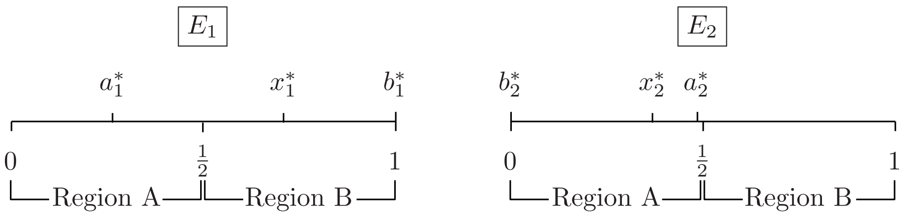

Figure 6: the locations of firms in two types of equilibria

$$E_2 : \begin{cases} \text{Firm } A: \quad W_A(a_2^*, b_2^*) = \frac{(-604+73\sqrt{73})t}{216} \approx 0.0913t, \\ \text{Firm } B: \quad \Pi_B(a_2^*, b_2^*) = \frac{(10-\sqrt{73})(13-\sqrt{73})^2 t}{216} \approx 0.1338t. \end{cases}$$

The above equations show that the equilibrium E_2 payoff dominates the equilibrium E_1 since both $W_A(a_2^*, b_2^*) > W_A(a_1^*, b_1^*)$ and $\Pi_B(a_2^*, b_2^*) > \Pi_B(a_1^*, b_1^*)$ hold. We explain the intuition behind this result below.

First, we consider the equilibrium E_1. It is helpful for understanding the intuition behind the equilibrium that we compare the equilibrium E_1 with the equilibrium for location model of two private firms with $a < b$. In this case, there is an equilibrium such that private firm, Firm A, is located in point 0 and the other private firm, Firm B, is in point 1. This is because each firm maximizes its profit, and thus both the firms are unwilling to get up close to each other to cause a severe price competition (see d'Aspremont et al., 1979).

This maximum differentiation is explained by two effects in Matsumura and Matsushima (2004). One is that the demand elasticity for Firm A is reduced as Firm B gets away.[9] Then, both the firms can set higher prices and enjoy higher profits. This is the direct effect. The other is that the higher price of Firm B raises the price of Firm A by the strategic complementarity in price-setting between the firms. This second effect is the indirect effect (strategic effect). These two effects induce both the private firms to depart from each other. Therefore, both firms have the incentives for *stepping away* from each other.

This incentive remains in Firm A even when it is owned by the local government of Region A. However, its incentive for stepping away and price-raising gets lower than when it is owned privately, because its profit from the residents of Region A and the purchases costs of them always cancel each other in local welfare. In addition to this, the local public firm also takes the transportation costs of consumers in Region A into account, and thus it has an incentive to move from point 0 to the right direction to decrease the transportation costs of the consumers. Moreover, it has another incentive to move right for

[9]The following explanation about Firm A is also applicable to Firm B.

capturing the demand of the consumers in Region B. Since local welfare of Region A consists of the profit of Firm A and the sum of the transportation costs and the purchases costs which are borne by the residents in Region A, the increase in sales to the residents in Region B contributes to the increase in local welfare of Region A. Thus, Firm A has the incentive to move toward Region B for selling its residents.

The a_1^* is a position such that the above three incentives, one of which has the power to move Firm A left (stepping away and price-raising), another one (reducing transportation costs) induces it to locate the center of Region A and the other (capturing the demand of Region B) do it to move to right, balance and Firm A does not want to move further. In private duopoly, the last incentive is dominated by the first incentive (stepping away and price-raising), but in our interregional mixed duopoly, this dominance relationship inverts at point 1/4. Thus, a_1^* is slightly right from point 1/4. On the other hand, Firm B keeps in point 1 because he has only the preference for stepping away and price-raising.

Second, we explain the equilibrium E_2. For facilitating understanding, we compare the equilibrium with one when Firm A is a private firm and it is located to the right of Firm B, *viz.*, $a > b$. Then, the outcome that firm A is located in point 1 and firm B is located in point 0 is supported by an equilibrium. When Firm A becomes a local public firm for Region A, as is explained in a few paragraphs above, there are two forces (reducing the transportation costs of residents in Region A and capturing the demand of residents in Region B) to make the firm moving left from point 1 and one force (stepping away from Firm B and price-raising) to keep it being that position. Therefore, when $a > b$, Firm A has the incentive to move left, and as a result the firm is located in a_2^* whereas Firm B keeps in point 0. Moreover, in this equilibrium, Firm A sets the relatively high price[10] ($P_A \approx 0.4853t$) since most of the customers for the firm are residents in Region B and it can obtain the profit without imposing majority of the residents in Region A on the relatively high purchases costs. On the other hand, Firm B sets the relatively low price ($P_B \approx 0.3604t$) and gets the large demand from Region A (see Figure 6, $x_2^* = (13-\sqrt{73})/12 \approx 0.3713$).

The structure mentioned above well explains why the equilibrium E_2 is preferable for both firms to the equilibrium E_1. This is because in equilibrium E_2, both firms obtain the benefits at the sacrifice of consumer surplus in Region B. On the one hand, Firm A receives the *direct* benefit from Region B by selling the products at the relatively high price. On the other hand, Firm B receives the *indirect* benefit from Region B because thanks to the high price which Firm A sets, Firm B can sell much to consumers in Region A.

In the meanwhile, the residents in Region B face the dual hardships. One is, as is explained above, that they face the high price which Firm A sets. The other is that the transportation costs they incur are very high since both two firms are located in Region A. Thus, local welfare of Region B in E_2 is lower than in E_1. In fact, these hardships for Region B are so serious that, in equilibrium E_2, not only local welfare of Region B but also social welfare decrease compared to the equilibrium E_1, as the following calculations show:

$$E_1 : \begin{cases} W_B(a_1^*, b_1^*) = -\frac{(-20056+935\sqrt{466})t}{1296} \approx -0.0987t, \\ W(a_1^*, b_1^*) = -\frac{(-40262+1873\sqrt{466})t}{3888} \approx -0.0439t, \end{cases}$$

[10] The equilibrium prices are as follows:

$$E_1: \quad (P_A(a_1^*, b_1^*), P_B(a_1^*, b_1^*)) = \left(\frac{(-14+\sqrt{466})(26-\sqrt{466})t}{108}, \frac{(32-\sqrt{466})(26-\sqrt{466})t}{108} \right) \approx (0.3100t, 0.4255t),$$

$$E_2: \quad (P_A(a_2^*, b_2^*), P_B(a_2^*, b_2^*)) = \left(\frac{(10-\sqrt{73})t}{3}, \frac{(13-\sqrt{73})(10-\sqrt{73})t}{18} \right) \approx (0.4853t, 0.3604t).$$

$$E_2 : \begin{cases} W_B(a_2^*, b_2^*) = -\frac{7(-337+40\sqrt{73})t}{216} \approx -0.1543t, \\ W(a_2^*, b_2^*) = -\frac{(-195+23\sqrt{73})t}{24} \approx -0.0630t. \end{cases}$$

Thus, as the summary of this subsection, we obtain the following proposition.

Proposition 1. *The equilibrium E_2 Pareto dominates equilibrium E_1 with respect to the payoffs of both firms. However, social welfare in E_2 is lower than that in E_1 because the efficiency of E_2 stands on top of the sacrifice of Region B.*

4.2 *Equilibrium selection*

When a game has several Nash equilibria, a question arises which equilibrium they actually play. The 2×2 bimatrix normal form game in Table 2 describes a situation that each player has to choose one from two actions which corresponds to the two equilibria respectively. For instance, if their choice is (a_1^*, b_1^*) (resp. (a_2^*, b_2^*)), they succeed in coordinating their actions which achieve E_1 (resp. E_2). However, if they have different prospects each other, neither of equilibria occur.

Table 2: the game of equilibrium selection

Firm B / Firm A	b_1^*	b_2^*
a_1^*	$(0.0548t,\ 0.1231t)$	$(0.0645t,\ 0.0529t)$
a_2^*	$(0.0250t,\ 0.0656t)$	$(0.0913t,\ 0.1338t)$

As Proposition 1 suggests, both players prefer (a_2^*, b_2^*) to (a_1^*, b_1^*), and thus from the *payoff dominance* criterion, they likely choose their actions which constitute equilibrium E_2. This seems to be considered as a negative result by welfare economists because as Proposition 1 also suggests, social welfare of the whole area in E_2 is worse than that in E_1. The payoff dominance criterion, however, passes over the players' consideration for the risk that they may fail to coordinate their actions. A notion of *risk dominance* by Harsanyi and Selten (1988) is the very concept that captures players' risk consideration. In the language of Harsanyi and Selten (1988), in a 2×2 bimatrix game with two strict Nash equilibria, one equilibrium risk dominates another if the product of deviation losses at the former is greater than that at the latter. In the game of Table 2, the product of deviation losses of (a_1^*, b_1^*) is

$$(0.0548t - 0.0250t)(0.1231t - 0.0529t) \approx 0.0021t^2,$$

and that of (a_2^*, b_2^*) is

$$(0.0913t - 0.0645t)(0.1338t - 0.0656t) \approx 0.0018t^2.$$

Hence, we obtain the following proposition.

Proposition 2. *While the equilibrium E_2 payoff dominates the equilibrium E_1, the equilibrium E_1 risk dominates the equilibrium E_2.*

5 Decentralization of government and privatization

Our analysis has some implications on a world wide trend toward decentralization of government to local jurisdictions. In such a case, the central government turns over the ownership of a public firm to the local government. The transfer of the ownership changes the objective of that firm, and thus changes social welfare. Although there are many researches which consider the changes of firm's ownership from the central government to the private sector (*viz.*, privatization), the changes from the central government to the local one have been paid less attention by the researchers who concern the recent wave of privatization.

We provide the two scenarios on decentralization of government and changes in firms' ownership. These two scenarios are considered in order to capture essential situations in terms of the decentralization of the central government. One is a decision problem for the central government. On promotion of the decentralization, the government faces the decision of appropriately allocating ownership of a public firm. It must decide whether to give the ownership of the public firm to the local government, privatize the firm, or keep the ownership.

The central government reigns over the linear city $[0, 1]$ which consists of the two districts $[0, 1/2)$ and $[1/2, 1]$. In each of them, a local government exists. In an initial situation, a public firm owned by the central government and a private firm compete in this linear city. They play the two-stage game composed of the location choice and the price competition. Such a situation was analyzed by Cremer et al. (1991). According to their results, the two firms are located at points $1/4$ and $3/4$, and set the same price of $t/2$. Further, social welfare is $-t/48 \approx -0.0208t$, which is the maximum value because the total transportation cost in the market is minimized.

If the central government gives the ownership of the public firm to one of the local governments (we define this change in ownership as *localization*), competition between the local public firm and the private firm occurs. As we showed in previous sections, in equilibrium, social welfare is $-0.0439t$ (we adopt the risk dominance equilibrium E_1). Finally, if the central government privatizes the public firm, this is the location choice model analyzed by d'Aspremont et al. (1979). One firm is located at point 0 and the other is at point 1, and social welfare is $-t/12 \approx -0.0833t$.

In summary, the decision problem faced by the central government can be described as Table 3. Thus, we have the following proposition.

Table 3: the choice of the central government and social welfare

	nationalization	localization	privatization
social welfare	$-0.0208t$	$-0.0439t$	$-0.0833t$

Proposition 3. *Assume that the central government concerns social welfare and it has the three choices on the ownership of the public firm: nationalization, localization and privatization. Then, he selects the nationalization of the firm.*

The other is a decision problem for the local government. In this case, there are two local governments which reign over the neighboring regions and each of them owns a firm. Each government has the option of changing the ownership of the firm: nationalize, privatize, and holding the status quo. After the choice, the two firms play the two-stage game considered in the previous sections.

The local governments A and B reign over the two districts $[0, 1/2)$ and $[1/2, 1]$, respectively. If both governments keep their ownership of the firms, then two local public firms compete in the market considered in our companion paper (Inoue et al., 2007). We find that the two-stage game composed of the location choice and the price competition stages has a unique subgame perfect equilibrium in which each local public firm is located at the center of its own district (Firm A is at point $1/4$, and Firm B is at point $3/4$), and each local welfare is $-t/96 \approx -0.0104t$. Note that, in this case, social welfare (W

$\approx -0.0208t$) is equal to the case where a state-owned firm and a private firm compete (see Cremer et al., 1991).

If one government decides to nationalize its firm and the other does not, there are two types of firms in the market, a state-owned firm and a local public firm. Solving the two-stage game, we obtain a unique subgame perfect equilibrium which completely coincides with the equilibrium by the above competition of local public firms. Moreover, if both governments nationalize their firms, there are two state-owned firms in the market. In this case, both the firms aim at maximizing social welfare, in equilibrium, Firm A is located at $1/4$ and Firm B is at $3/4$ (where $a < b$). Thus, the equilibrium in this case also coincides with the former case.

On the other hand, if one government decide to privatize its public firm and the other does not, this is a situation considered in this paper. Further, if both governments privatize their firms, the competition between the two private firms appears. This case is analyzed by d'Aspremont et al. (1979).

Thus, the decision situation faced by the two local governments are described as the following payoff matrix. This game has four Nash equilibria such that both firms select the "status quo" or "nationalize." The following proposition holds.

Table 4: the game of privatization

Region A \ Region B	privatize	status quo	nationalize
privatize	$(-0.0417t,\ -0.0417t)$	$(-0.0987t,\ 0.0548t)$	$(-0.0104t,\ -0.0104t)$
status quo	$(0.0548t,\ -0.0987t)$	$(-0.0104t,\ -0.0104t)$	$(-0.0104t,\ -0.0104t)$
nationalize	$(-0.0104t,\ -0.0104t)$	$(-0.0104t,\ -0.0104t)$	$(-0.0104t,\ -0.0104t)$

Proposition 4. *When each local government decides the ownership of its public firm, they both do not privatize the firm. Then, social welfare is maximized.*

6 Conclusion

In this paper we investigate a mixed duopoly involving a private firm and a local public firm which is owned by a local government. We construct a two-stage game and show that the game has two subgame perfect equilibria (E_1 and E_2). One of them (E_1) is that each firm is located in its home district, and the other (E_2) is that both firms are located in the same region, Region A.

We also find that equilibrium E_2 payoff dominates E_1, but social welfare in E_2 is lower than in E_1 because the burden of Region B in E_2 is much heavier than that in E_1. On the other hand, equilibrium E_1 risk dominates E_2. In these two types of equilibria, three following incentives balance at the location point of Firm A. One is the incentive for departing from Firm B for avoiding the severe price competition. Second one is that the firm wishes to decrease the transportation costs of the residents in Region A. The last one is the inducement of capturing the demand of residents in Region B. Only the first one makes Firm A be located far from Firm B and the other two make Firm A get close to Firm B. On the other hand, the private firm B has only this incentive, and thus the firm is located in the corner point in both equilibria.

We introduce a local public enterprise into the analysis of mixed markets while most of the literature on mixed oligopoly treat a public firm as state-owned. In addition to this, we analyze the strategic decisions of each government by considering multiple regions as Bárcena-Ruiz and Garzón (2005). Our setting can be applied in the context of an international relationships such as the location choice of multinationals. In that context, equilibrium E_2 indicates a foreign firm's direct investment. As pointed by Bárcena-Ruiz and Garzón (2005), particularly in the EU, although the Single Market was introduced, the decision whether to privatize firms or not is a national issue. Thus, there are strategic interactions among member countries in the market. We consider the privatization game between two governments of local districts in Section 5. In consequence, both governments does not privatize their own firms. This result goes against the recent privatization trend. If we take a cost improvement into account in the effect of the privatization, the trend might be shown. Accordingly, the analysis of the situation which both firms select the production costs endogenously such as Matsumura and Matsushima (2004) is a further subject for future research.

This model is restrictive since we employ a simple duopoly model. However, This paper has a significant impact on the argument of the mixed oligopoly for the above reason. Moreover, this research might extend to the cases of asymmetric regions and a quantity-setting oligopoly, to consider a variety of conditions. It is necessary to analyze the mixed oligopoly involving local public firms in order to clarify the desirable ownership patterns of firms.

Appendix

Derivation of Equation (11)

In Figure 7, we show local welfare of Region A described in Equations (9) and (10). If $b < 1/2$ (left figure), W_A is maximized when $a \in [0, b]$ ($W_A = F_1$) or $a \in (b, 1-b]$ ($W_A = F_4$). If $b \geq 1/2$ (right figure), W_A is maximized when $a \in [0, b]$ ($W_A = F_1$). In $b < 1/2$, whether the maximum value of W_A exists in $a \in [0, b]$ or $a \in (b, 1-b]$ depends on the value of b. To derive this condition, we calculate following equation:

$$\begin{aligned} &W_A(R_A^1(b), b) - W_A(R_A^4(b), b) \\ &\qquad = -\frac{[9(215 + 73\gamma - 42\delta) + 18b(213 - 10\gamma - 4\delta) + 4b^2(27 - 34b + 9\gamma - 4\delta)]t}{1944}, \\ &\qquad\qquad \gamma \equiv \sqrt{73 - 4b(5-b)}, \quad \delta \equiv \sqrt{378 + 8b(9 + 2b)}, \end{aligned}$$

where $R_A^j(b) := \text{argmax}_a\ F_j$ $(j = 1, 4)$. This equation is a monotonically increasing function of b. In addition, when $b = \bar{b} \approx 0.3656$, the equation equals to zero. Thus, when $b \leq \bar{b}$, $R_A^4(b)$ maximizes W_A, otherwise $R_A^1(b)$ maximizes W_A. Hence, in the first stage, the reaction function of Firm A is expressed by

$$R_A(b) = \begin{cases} \frac{10-b-\sqrt{73-20b+4b^2}}{3} & \text{if} \quad b < \bar{b}, \\ \frac{-18-2b+\sqrt{378+72b+16b^2}}{6} & \text{otherwise.} \end{cases}$$

Derivation of Equation (14)

In Figure 8, we show the profit of Firm B described in Equations (12) and (13). If $a < 1/2$ (left figure), Π_B is maximized when $b = 0$ ($\Pi_B = G_4$) or $b = 1$ ($\Pi_B = G_1$). If $a \geq 1/2$ (right figure), Π_B is maximized when $b = 0$ ($\Pi_B = G_4$). In $a < 1/2$, whether the maximum value of Π_B exists at $b = 0$ or $b = 1$ depends

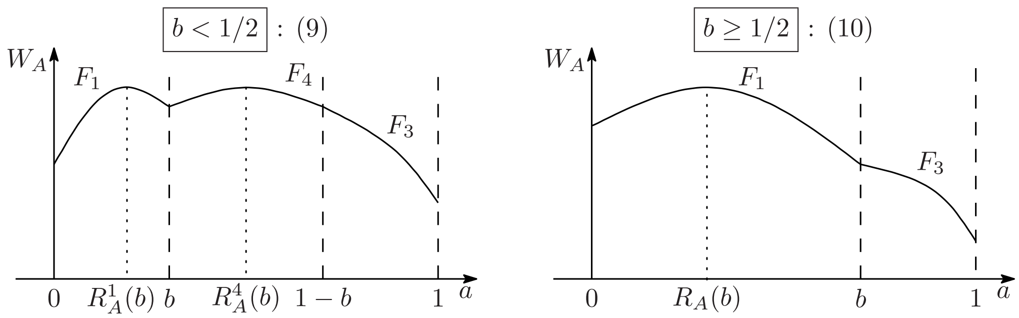

Figure 7: derivation of Equation (11)

on the value of a. To derive this condition, we calculate following equation:

$$\Pi_B(a, R_B^1(a)) - \Pi_B(a, R_B^4(a)) = \frac{(16 - 41a + 2a^2 - 13a^3)t}{72},$$

where $R_B^j(b) := \text{argmax}_b\ G_j\ (j = 1, 4)$. This equation is a monotonically decreasing function of a. In addition, when $a = \bar{a} \approx 0.3799$, the equation equals to zero. Thus, when $a < \bar{a}$, $R_B^1(a)$ maximizes Π_B, otherwise $R_B^4(a)$ maximizes Π_B. Hence, in the first stage, the reaction function of Firm B is expressed by

$$R_B(a) = \begin{cases} 1 & \text{if} \quad a < \bar{a}, \\ 0 & \text{otherwise.} \end{cases}$$

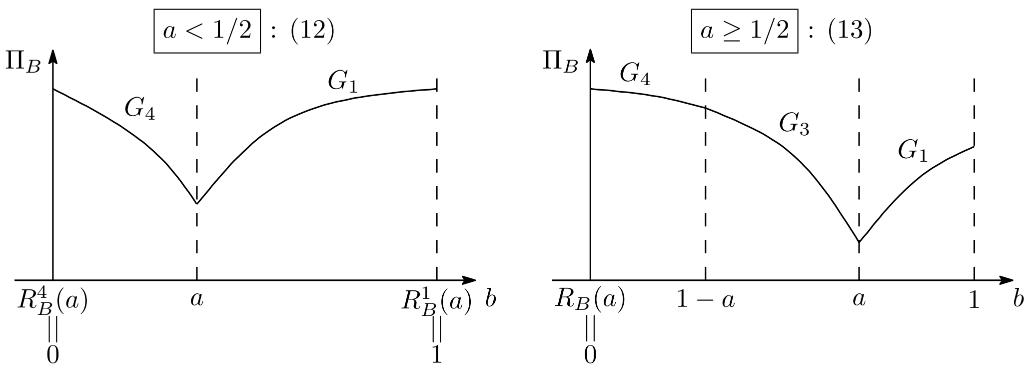

Figure 8: derivation of Equation (14)

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