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Risk Shifting Incentive of the Firm when its Value is Correlated with Interest Rates

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Risk-shifting Incentive of the Firm when its Value is Correlated with Interest Rates

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Abstract

Without appealing to taxes, bankruptcy cost or other frictions, we demonstrate equityholders' risk-shifting incentive and the optimal risk level of asset are not necessarily monotonically increasing in leverage in a stochastic interest rates environment. Equityholders will substitute less risky negative-net-present-value projects for riskier positive-net-present-value projects if the correlation coefficient between firm value process and interest rate process is sufficiently negative. Our counter-intuitive results are mainly due to the difference between asset volatility in real world probability measure and risk-neutral volatility under stochastic interest rates. Contrary to the current theoretical consensus, it is also shown shortening debt maturity may actually increase agency cost. Comparative statics analysis and associated numerical illustration reveal changes in parameters of interest rate process have complex and non-trivial effects on risk-shifting incentive and agency cost of debt. They depend critically upon capital structure and the correlation coefficient between firm value and interest rates.

JEL Classification: G32

Keywords: risk-shifting, asset substitution, agency cost of debt, stochastic interest rates

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1. Introduction

Jensen and Meckling (1976) first discussed importance of agency cost in determining the optimal capital structure, arguing such costs arising from conflicts between different groups within a firm increase as the firm employs more debt financing. Galai and Masulis (1976) formalized their idea in the classical option pricing framework of Black and Scholes (1973), regarding equity as call option on firm value. Adopting the project of the highest volatility will maximize equity value possibly at the sacrifice of debt value. Assuming managers act in the interests of equityholders, managers are able to transfer wealth to equityholders from bondholders by taking excessive risk. Such an incentive problem is referred to as risk-shifting or asset substitution.[1]

The fairly large literature on risk management and capital structure theories is developed under the widely accepted assumption that an increase in leverage increases equityholders' risk-shifting incentive and associated agency cost. However, a little thought will reveal such a naive point of view lacks theoretical justification. Since standard call option premium is a monotone increasing function of volatility, the optimal asset volatility for equityholders is infinite irrespective of leverage level. The optimal asset volatility, then, should be independent of leverage. Furthermore, firms in reality do not seem to seek for projects with infinite volatility.[2]

Gavish and Kalay (1983), in their pioneering study, demonstrates equityholders' gains from an unexpected increase in variance of the investment do not increase monotonically with the firm's leverage. The gains are expected to decrease when leverage is high. Their analysis raises a serious question whether the agency cost of asset substitution really increases monotonically in leverage. In a similar setting, Green and Talmor (1986) confirm Gavish and Kalay's result and further formalize the optimal risk policy of the firm. They show as the promised debt payment increases, equityholders' risk-shifting incentive increases monotonically, although equityholders' gains do not. Although these studies provide profound insights regarding the nature of agency cost due to asset substitution, it should be pointed out they are based on the restrictive models in

1 These terms are used interchangeably in this article.

2 Recent papers by Chesney and Ashner（1999, 2001）clarified this point, and they introduce a downside knock-out barrier for equity so that optimal volatility level of project becomes finite. Their model can explain why only highly levered firms involves in risk-shifting behavior.

which all agents are risk neutral and the risk-free interest rate is zero. Their assumption on the future cash flows or asset returns is fairly general, however, it precludes important probability distributions such as log-normal, which we will adopt in the following analysis.[3]

In a continuous time setting, Leland (1998) and Ericsson (2000) investigate the magnitude of agency cost in analyzing the optimal capital structure problem. These authors assume a firm can change its asset volatility in a dynamic framework. The risk choice of the firm is endogenized in their models, and bankruptcy costs and tax advantage of debt are traded off to yield an interior optimum of capital structure. In interpreting numerical results, Leland suggests agency cost of debt may not be positively associated with optimally chosen levels of leverage, whereas in Ericsson's model equityholders always prefer the higher risk level. Both papers assume the firm has the choice of only two exogenous possible levels for the asset volatility parameter, and the relationship between leverage and equityholders' risk-shifting incentive does not seem fully explored yet.

To sum up, though some doubt has been cast on Jensen and Meckling's argument, existing literature overall agrees with conventional wisdom that the incentive of equityholders for asset substitution and the resultant agency cost increase monotonously in leverage. A few non-monotonic results have been suggested only in the models, which appeal to taxes, bankruptcy costs, and other market frictions. These models, however, do not explicitly consider the real investment cost associated with deviation from value maximization investment policy.

This paper explicitly treats any investment to fulfill risk-shifting incentive to have less NPV than the first best choice for firm value maximization. We do not assume any market frictions such as taxes and bankruptcy costs. In our model, the real investment cost due to sub-optimal investment is traded off for an increase in call option value of equity, and finite optimal risk level is determined endogenously for equityholders. Taking advantage of the first order condition in the stochastic interest rate environment, we will show the agency cost borne by equityholders do not necessarily increase in leverage.

When the correlation is non-negative, we confirm Green and Talmor's result that risk-

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3 In these studies, the end of period cash flow or firm value is assumed to be distributed as $\widetilde{A}(T) = a + b\,\widetilde{\varepsilon}$, where $E[\widetilde{\varepsilon}] = 0$, and parameters *a* and *b* represent mean and volatility, respectively. This specification is fairly general, however, it obviously excludes log-normal distribution.

shifting incentive is a monotone increasing function of leverage. However, our proof is original in assuming lognormal firm value process, for which existing literature has not examined yet. When the correlation is negative, we show equityholders have an incentive to substitute *less risky* assets for riskier investment. Such an incentive is contrary to the risk-shifting actions under usual asset substitution and is expected to exist only in the stochastic interest rate setting. We use the term "*the reverse asset substitution*" for such an incentive problem to distinguish from the normal asset substitution shifting into riskier projects. Secondly, we find the agency cost of asset substitution can be a decreasing function of the maturity of the debt. Contrary to a widely accepted view in risk management literature, shortening debt maturity is shown to often increase agency cost. Our result is consistent with empirical literature, such as Barclay and Smith (1995) and Stohs and Mauer (1996), which has reported mixed results on relation between debt maturity and measures of agency cost related to growth opportunities. Sharing the view presented by Gavish and Kalay (1983), our analysis raises a serious doubt concerning the validity of some agency literature that naively assumes less leverage and shortening maturity of debt can mitigate the agency problem between the equityholders and debtholders. Our analysis also indicates the possibility of eliminating the agency conflicts by carefully choosing the debt maturity given other environmental parameter values.

Finally, by employing comparative static analysis, we examine the effects of changes in exogenous parameter values describing the stochastic process of the interest rate, which is assumed to follow Ornstein-Uhlenbech diffusion. Numerical illustration indicates unexpected changes in the parameters are shown to have complex and significant effects on risk-shifting incentive of equityholders. We find the signs of comparative statics depend crucially on the capital structure and the correlation coefficient between firm value and interest rate process.

The paper is organized as follows. The next section presents the framework to explore the asset substitution problem when the interest rates are stochastic. Valuation formula of equity and debt are reported in closed form. Section 3 examines the risk-shifting incentive of equityholders by comparative statics analysis. The relation between risk-shifting incentive and leverage, debt maturity and interest rate parameters are explored. Section 4 illustrates and confirms our theoretical results by numerical illustration. The last section concludes.

2. The Model

2.1. The Market Setting

A single period model is employed. The beginning of the period is time 0 and the end of the period is denoted as T. The firm we consider does not pay any dividends until the end of the period, and they liquidate at T. Asset markets are competitive and frictionless, and no taxes or bankruptcy costs are existent.

We assume the capital structure decision is already made at the beginning of the period. The firm is financed by equity and zero coupon debt in the amount B, bearing a promised payment F . Debtholders receive their contractual payment F at a maturity date T, and equityholders receive the residual value of the firm as long as the firm is solvent.[4] To apply option pricing methodology, we assume the firm value represents the value of net cash flows generated by the firm's activities and these cash flows are dynamically spanned by the cash flows of marketed securities. Hence, there exists an equivalent martingale measure under which discounted price processes are martingales. We use the term firm's asset and firm value interchangeably.

Managers are assumed to behave in equityholders' interests and conflicts between management and equityholders are not considered. After the debt is in place, managers can alter risk of investment ex post. This implies the risk strategy followed by the management cannot be pre-contracted in the debt covenants or any type of contracts, and renegotiations among debtholders and equityholders are too costly to implement. In equilibrium, debtholders will correctly anticipate the effect of risk strategy initiated by equityholders and managers. The effect of risk strategy will be reflected in the pricing of corporate securities under rational expectations. In this paper, however, we follow Gavish and Kalay's methodology to examine risk-shifting as a wealth redistribution mechanism, and debt and equity are priced assuming a given investment policy. Then an experiment is performed in which the investment's risk is shifted to maximize equity value, and the resulting changes in firm value is examined to evaluate the magnitude of agency cost ultimately borne by equityholders.

[4] In our model, given exogenous variables F and T, the market price of debt B is endogenously.

To analyze equityholders' incentive for asset substitution, we modify Green and Talmor's (1986) scale parameter α, which summarizes the risk policy of the firm. Their model assumes α lies in the set [0, 1]. Here we extend the set to $[-1, 1]$. If no incentive to change firm risk exists, such a situation is represented by $\alpha = 0$. The manager on behalf of equityholders controls α, and if they have incentive to increase the firm risk represented by asset volatility, it is described by an increase in α in the region [0, 1]. Thus, the variable α represents the level of equityholders' incentive for asset substitution. When α is positive, the firm substitutes riskier investment projects for the less risky first best project at the expense of decreasing firm value. $\alpha = 1$ represents the highest possible incentive level of asset substitution.

In our model, the set for α is extended to take on negative values. We consider the possibility of the firm's substituting the *less* risky negative-net-present value projects for the first best project. The negative region of α corresponds to such a new type of asset substitution, which we refer to as the *reverse asset substitution*. $\alpha = -1$ represents the highest level of such a risk-shifting activity. At the beginning of the period, with F being given, equityholders choose the investment policy α that maximizes the present value of equity.

$A(\alpha)$ represents the current firm value if the risk policy α is adopted. For brevity, time subscript 0 for the current values is suppressed throughout the analysis. If the firm is financed with all equity, obviously no conflicts between equityholders and debtholders exist. All investments are then carried out to maximize firm value, and only the projects with positive net present value are adopted. With debt in place, however, it is natural to assume any firm activities originated from conflicts among claimants have less net present value, because investment decisions are distorted away from firm value maximization. Thus, we assume

$$A'(\alpha) \lesseqqgtr 0 \quad \text{when } \alpha \gtreqqless 0. \tag{1}$$

For $A(\alpha)$ to have a maximum at $\alpha = 0$, $A''(\alpha) < 0$ is also assumed. In other words, a levered firm controlled by equityholders chooses the investment policy that maximizes equity value rather than total firm value. As pointed out, any deviations from net present value maximizing policies are reflected in a decline in firm value in our model. The decline is a real investment cost ultimately borne by equityholders. Accordingly, the levered firm value is reduced compared with no debt outstanding.

Once the level of α is optimally chosen by equityholders, firm value evolves as

$$\mathrm{d}A(\alpha)/A(\alpha)=\mu(\alpha)\mathrm{d}t+\beta(\alpha)\mathrm{d}W_A^{(\alpha)}, \tag{2}$$

where $\mu(\alpha)$ and $\beta(\alpha)$ are positive constants and $W_A^{(\alpha)}$ represents a Brownian motion driving the stochastic process. It should be noted, in this section, $\{A(\alpha)\}$ is the only exogenous source of risk, and the effect of normal and reverse asset substitutions on asset volatility is simply captured by

$$\beta'(\alpha)>0. \tag{3}$$

Any changes in volatility may affect the drift of the firm value process through a general equilibrium, however, the drift is always equal to the instantaneous risk free rate under the equivalent martingale measure. Thus, no restrictions on $\mu(\alpha)$ are imposed in the following analysis. Thanks to option pricing methodology we employ, no restrictions on investors' preferences are required, either.

As for the risk free interest rates, they are assumed to follow Ornstein-Uhlenbeck diffusion process, which is given by

$$\mathrm{d}r=\kappa(\theta-r)\mathrm{d}t+\sigma\,\mathrm{d}W_r. \tag{4}$$

For brevity, the current risk free interest rate is denoted r without a time subscript. The process has a property of mean reversion to the long-run mean θ, and κ describes the speed of adjustment to θ. Both θ and κ are given as positive constants. s is a volatility parameter associated with the Brownian motion W_r , which drives the stochastic process. We assume interest rate process is correlated with firm value process such that

$$\mathrm{d}W_r\,\mathrm{d}W_A^{(\alpha)}=\rho\mathrm{d}t. \qquad (-1\leqq\rho\leqq1)$$

Note that each firm in this economy is characterized by a different value of the correlation coefficient ρ, representing the firm's investment policy. In our setting, ρ is exogenous and it summarizes the assets structure of the firm.

2.2. Valuation Formula in Closed Form

With the interest rate process specified, Vasicek (1977) obtains an equilibrium price of a default free discount bond with maturity of T and face value of unity. It is given as

$$P = a(T)\,\mathrm{e}^{-rb(T)} \tag{5a}$$

where

$$b(T) = \frac{1-\mathrm{e}^{-\kappa T}}{\kappa}\ (>0), \tag{5b}$$

$$a(T) = \exp\{(\theta + \frac{\sigma\lambda}{\kappa} - \frac{\sigma^2}{2\kappa^2})[b(T) - T] - \frac{\sigma^2 b(T)^2}{4\kappa}\}, \tag{5c}$$

$$\lambda = \frac{\mu_P(T) - r}{\sigma_P(T)}, \quad \sigma_P(T) = b(T)\,\sigma \tag{5d}$$

With κ and T being positive, $b(T)$ is algebraically positive. $\mu_P(T)$ and $\sigma_P(T)$ satisfy a stochastic differential equation

$$\mathrm{d}P/P = \mu_P\,\mathrm{d}t - \sigma_P\,\mathrm{d}W_r.$$

Thus, they are interpreted as the instantaneous expected return and the standard deviation of the default free discount bond. λ is the market price of risk, which represents the preference of investors toward interest rate risk.[5]

When the firm issues a zero coupon bond with face value of F, as shown by Black and Scholes (1973), maturity payoff patterns for equity resembles a call option on firm value. Payoffs for equity and debt at maturity are given by

$$\widetilde{S}(\alpha)_T = \mathrm{Max}[\widetilde{A}(\alpha)_T - F, 0] \tag{6a}$$

$$\widetilde{B}(\alpha)_T = \widetilde{A}(\alpha)_T - \widetilde{S}(\alpha)_T = \mathrm{Min}[F, \widetilde{A}(\alpha)_T], \tag{6b}$$

where subscripts represent maturity of debt. To derive the present values of these payoffs in the stochastic interest rate setting, expectations are taken with respect to the forward probability measure first suggested by Jamshidian (1990), under which asset prices discounted by default-free discount bond become martingale. The closed form results for equity and debt values are as follows.[6]

$$S(\alpha) = A(\alpha)\,\Phi(h) - PF\,\Phi(h - v), \tag{7a}$$

5 For the exact functional form of μ_P, which is irrelevant to the following exposition, see the Vasicek paper.

6 The closed form expression for European call option with Ornstern-Uhlenbeck interest rate process is obtained by Rabinovitch (1989) and Simko et al.(1993).

$$B(\alpha)=A(\alpha)\Phi(-h)+PF\Phi(h-v). \tag{7b}$$

where

$$h=\frac{\ln[A(\alpha)/PF]}{v}+\frac{v}{2}, \tag{7c}$$

$$v^2=\int_0^T[\beta(\alpha)^2+\sigma_P(s)^2+2\rho\beta(\alpha)\sigma_P(s)]\mathrm{d}s$$

$$=\beta(\alpha)^2T+[T-2b(T)+\frac{1-\mathrm{e}^{-2\kappa T}}{2\kappa}]\frac{\sigma^2}{\kappa^2}+\frac{2\rho\beta(\alpha)\sigma[T-b(T)]}{\kappa}. \tag{7d}$$

$\Phi(\cdot)$ is a distribution function of standard normal. As noted, $\sigma_P(s)$ is given by (5d), representing the instantaneous standard deviation of the return on $P(s)$, a default free discount bond with maturity of $s\in(0,T)$. $b(T)$ is defined by (5d), which takes on only positive value. v^2 is a variance of firm value expressed in the unit of default free bond, rather than the dollar invested. In fact, v^2 is risk-neutral volatility, which is a proper measure of variability of firm value in a stochastic interest rate economy.[7] The closed form result enables us to examine the nature of optimal risk policy analytically. The next section presents our main results.

3. Risk-shifting Incentive under Stochastic Interest Rates

3.1. Equity Value and Incentive for Normal/Reverse Asset Substitution

To explore the relation between risk-shifting incentive and equity value, a partial derivative is calculated. We have

$$\frac{\partial S}{\partial \alpha}(\alpha)=A'(\alpha)\Phi(h(\alpha))+A(\alpha)\phi(h(\alpha))\beta'(\alpha)\frac{\partial v}{\partial \beta}(\alpha). \tag{8a}$$

where

$$\frac{\partial v}{\partial \beta}(\alpha)=\frac{1}{v}\{\beta(\alpha)\beta'(\alpha)T+\frac{\rho\beta'(\alpha)\sigma[T-b(T)]}{\kappa}\}. \tag{8b}$$

$\phi(\cdot)$ stands for the density function of standard normal distribution. To see equityholders' incentive when the first best investment policy is initially assumed, setting $\alpha=0$ and noting $A'(0)=0$ yield

[7] Counter-intuitive results reported in this article are mainly due to the difference between asset volatility in real world probability measure and risk-neutral volatility is under stochastic interest rates. By the same line of argument, Ikeda (1995) demonstrates default premium of fixed rate debt is larger than that of floating rate debt issued by the same firm in the analysis of interest rate swaps.

$$\frac{\partial S}{\partial \alpha}(0) \gtreqless 0 \Leftrightarrow \frac{\partial v}{\partial \beta}(0) \gtreqless 0 \Leftrightarrow \rho \gtreqless -\frac{\kappa^2 T}{e^{-\kappa T}+\kappa T-1} \cdot \frac{\beta(0)}{\sigma} \equiv \rho^*. \tag{9}$$

The above inequality indicates the firm with the correlation coefficient ρ greater (smaller) than the critical value ρ^* can increase (decrease) equity value by increasing (decreasing) α. Furthermore, if $\rho=\rho^*$ holds, equity value becomes independent of α.[8] This result leads to our first proposition.

Proposition 1. When the risk free interest rates evolve stochastically, equityholders of the firm, whose correlation coefficient with the risk free interest rates is greater than the critical value ρ^* given by eq.(9), *have incentive to increase risk of investment; asset substitution takes place. If the firm's correlation coefficient is lower than the critical value, reverse asset substitution arises, i.e., equityholders will substitute less risky negative-net-present-value projects for riskier positive-net-present-value projects. If the correlation coefficient is equal to the critical value, equityholders have no incentive to alter the risk of the firm. No conflicts between equityholders and debtholders exist in such a case.*

It is easy to demonstrate ρ^* is algebraically negative. Thus, it should be noted that equityholders of the firms characterized by the non-negative ρ value have incentive for asset substitution regardless of parameter choices.

For $S(\alpha)$ to have an interior maximum, the second order condition must hold. The second partial derivative is calculated as

$$\frac{\partial^2 S}{\partial \alpha^2}=A''\Phi(h)+\frac{Av'^2}{v}\phi(h)[h^2-\frac{2\varepsilon+v^2}{v}h+\frac{\varepsilon^2}{v^2}+2\varepsilon+\frac{vv''}{v'^2}] \tag{10a}$$

where

$$v'=\frac{\partial v}{\partial \alpha}=\frac{\partial v}{\partial \beta}\beta'(\alpha) \quad (10b), \qquad v''=\frac{\partial^2 v}{\partial \beta^2}\beta'(\alpha)^2+\frac{\partial v}{\partial \beta}\beta''(\alpha) \tag{10c}$$

$$\varepsilon=\frac{A'v}{}=\frac{A'(\alpha)v(\alpha)}{} \tag{10d}$$

8 Simko et al (1993) point out the possibility that the credit spread of corporate debt can be a decreasing function of interest rate volatility. Such counter-intuitive result appears when the correlation between firm value and interest rates is negative. Our result is thus consistent with their analysis.

$Av' \quad A(\alpha)(\partial v/\partial \beta)\beta'(\alpha)$

The argument α is often suppressed when obvious. Note that ε represents the elasticity of asset value with respect its risk-neutral volatility when α is marginally increased. The sign of ε depends the sign of $\partial v/\partial \beta$, which will be discussed shortly. The above expression includes the functions $A''(\alpha)$ and $\beta''(\alpha)$, of which signs can be given exogenously. If we assume $A''(\alpha) < 0$, and/or assign an appropriate functional form for $\beta(\alpha)$ so that the quadratic equation of h appearing in the second term of eq.(10a) takes negative values for all α, then $\partial^2 S / \partial \alpha^2 < 0$ holds universally. In the next section, we will assign specific functions for $A(\alpha)$ and $\beta(\alpha)$ to show the second order condition indeed holds with plausible parameter values. An interior optimal solution for α and corresponding asset volatility will be presented as numerical illustration.

3.2. Leverage and Risk-shifting Incentive

In this section, we explore the relationship between leverage and asset substitution problem in the stochastic interest rate setting by examining the first order condition given by eq.(8). The next Lemma shows there exists an upper bound for the optimal leverage expressed as what Merton (1974) calls quasi-debt ratio.

Lemma 1 *If the level of risk-shifting incentive is determined so as to maximize the equity value of the firm, its quasi-debt ratio is bounded from above as follows;*

$$\frac{PF}{A(\alpha^*)} < \exp\{-\varepsilon(\alpha^*) + \frac{v(\alpha^*)^2}{2}\}, \tag{11}$$

where ε *and* v^2 *are elasticity measure and risk neutral variance at optimum, which are given by* (10d) *and* (7d), *respectively.*

Proof From the first order condition evaluated at $\alpha = \alpha^*$, we have

$$A'(\alpha^*)\Phi(h(\alpha^*)) + A(\alpha^*)\phi(h(\alpha^*))\beta'(\alpha^*)\frac{\partial v}{\partial \beta}(\alpha^*) = 0$$

$$\Leftrightarrow -\frac{\varepsilon(\alpha^*)}{v(\alpha^*)} = \frac{\phi(h(\alpha^*))}{\Phi(h(\alpha^*))}, \quad \varepsilon(\alpha^*) \equiv \frac{A'(\alpha^*)v(\alpha^*)}{A(\alpha^*)(\partial v/\partial \beta)\beta'(\alpha^*)}. \tag{12}$$

Now we use a convenient property of the normal distribution such that $\phi(x)/\Phi(x) > -x \ \ \forall x$. Together with eq.(12) , we have

$$\varepsilon(\alpha^*)/v(\alpha^*) < h(\alpha^*). \tag{13}$$

Eq.(7b) is then substituted for $h(\alpha^*)$, and the desired result is obtained. □

This Lemma is used to establish the next proposition, which demonstrates equityholders' risk-shifting incentive and the optimal risk level of asset are not necessarily increasing in leverage, contrary to the current theoretical consensus.

Proposition 2 *Assume the equity of the firm is currently maximized at* $\alpha=\alpha^*$. If *the risk free interest rates are stochastic and correlate with firm value with a coefficient* ρ, *which is higher than the critical value* $\rho^*(\alpha^*)$, *given by*

$$\rho^*(\alpha^*) = -\frac{\kappa^2 T}{e^{-\kappa T}+\kappa T-1}\cdot\frac{\beta(\alpha^*)}{\sigma},$$

equityholders have incentive to increase volatility of investment as the promised debt payment increases. If the firm is characterized by the coefficient ρ, *which is lower than* $\rho^*(\alpha^*)$, *an increase in leverage causes equityholders to decrease volatility of the investment by substituting less risky investment with negative-net-present-value for riskier investment with positive-net-present-value. If the coefficient* ρ *is equal to* $\rho^*(\alpha^*)$, *the optimal volatility level is independent of leverage. In particular, if the firm is characterized by* $\rho = \rho^*(0)$, *which is given by eq.*(9), *the firm is free from the normal/reverse asset substitution problem, and leverage does not provide any risk-shifting incentive for equityholders.*

Proof. For notational brevity we denote the first order condition by $H\equiv\partial S/\partial\alpha=0$. The implicit function theorem assures the existence of an implicit function $\alpha=\alpha^*(F)$, and $d\alpha^*/dF = -(\partial H/\partial F)/(\partial H/\partial \alpha)$. As discussed, since the second order condition $\partial H/\partial\alpha < 0$ is assumed to hold, an interior optimum exists at $\alpha=\alpha^*$, and $\mathrm{sign}(d\alpha^*/dF)=\mathrm{sign}(\partial H/\partial F)$ holds. With a little algebra, we obtain

$$\frac{\partial H}{\partial F}=\frac{\phi(h)}{Fv}[A(\alpha)\beta'(\alpha)h(\alpha)\frac{\partial v}{\partial \beta}(\alpha)-A'(\alpha)].$$

Thus, it is easy to show

∂H $>$ ∂v $>$ $A'(\alpha)$ $\varepsilon(\alpha)$ ∂v

$$\frac{}{\partial F} \gtreqless 0 \Leftrightarrow \quad h(\alpha)\frac{}{\partial \beta}(\alpha) \gtreqless \frac{}{A(\alpha)\beta'(\alpha)} = \frac{}{v(\alpha)}\frac{}{\partial \beta}(\alpha)$$

$$\Leftrightarrow \quad [h(\alpha) - \frac{\varepsilon(\alpha)}{v(\alpha)}]\frac{\partial v}{\partial \beta}(\alpha) \gtreqless 0. \tag{14}$$

When $\alpha = \alpha^*$, eq.(13) of Lemma 1 states $h(\alpha^*) - [\varepsilon(\alpha^*)/v(\alpha^*)] > 0$, and we have

$$\frac{\partial H}{\partial F} \gtreqless 0 \Leftrightarrow \frac{\partial v}{\partial \beta}(\alpha^*) \gtreqless 0 \quad \Leftrightarrow \rho \gtreqless \rho^*(\alpha^*),$$

which is the desired result. If $\alpha^* = 0$, Proposition 1 shows in such a situation equityholders have no risk-shifting incentive, and optimal volatility becomes independent of leverage. □

It is to be noted that when the firm engages in reverse asset substitution by reducing asset volatility, it aggravates agency cost. As Jensen and Meckling (1976) clarified, agency cost is measured by the decline in firm value due to conflicts of interests among stakeholders. We follow Green and Talmor (1986)'s definition of agency cost of debt, that is;

$$AC(F) = A(\alpha(0)) - A(\alpha^*(F)). \tag{15}$$

The first term of the right hand side represents the firm value when no leverage is employed. Since the firm's decisions in the absence of debt are made to maximize firm value, existence of debt in capital structure will, in general, lead to sub-optimal decisions. In contrast, no conflicts exist when $F=0$, and $A(\alpha(0)) = A(0)$ represents the highest possible firm value given the investment policy. The second term of the right hand side is the firm value under the optimal risk policy maximizing equity value when the face value of debt is F. Thus, the difference of these terms represents the real investment cost due to potential conflicts between equityholders and bondholders. With this definition and Proposition 2, the following Corollary obtains immediately.

Corollary 1. *When the risk free interest rates are stochastic and correlate with firm value with a correlation coefficient $\rho \neq \rho^*(\alpha^*)$, agency cost of debt increase monotonically in the promised debt payment.*

Proof. Differentiating eq. (15) with respect to F yields

$$AC'(F) = -A'(\alpha^*)\frac{d\alpha^*}{dF}.$$

By Proposition 2 , under the condition of $\rho \neq \rho^*(\alpha^*)$, for the normal asset substitution case, $\alpha^* >$

0, $A'(\alpha^*) < 0$ and $d\alpha^*/dF > 0$ hold, whereas for the reverse asset substitution case, $\alpha^* < 0$, $A'(\alpha^*) > 0$, and $d\alpha^*/dF < 0$ are established. In both cases, $AC'(F) > 0$ holds, implying agency cost of debt being an increasing function of leverage. □

The above result is in accordance with a widely accepted view in agency cost literature except for the case of $\rho \neq \rho^*(\alpha^*)$, when agency cost of debt is independent of leverage. In fact, our Corollary is a generalization of Green and Talmor (1986)'s result in two aspects. First, their analysis is conducted in the risk-neutral economy where risk free interest rate is a fixed constant of zero, whereas we allow risk free interest rates to be stochastic without restricting investors' preference. Secondly, their analysis is based on the normal asset substitution while our result is obtained based on both normal and reverse asset substitutions.

3.3. Maturity of debt and Risk-shifting Incentive

In agency cost literature, it seems to be a theoretical consensus that shortening maturity of debt contributes to lessen the agency cost of debt. Such a dominant view, however, is based on a naive call option argument, that the option premium is an increasing function of maturity. It implies the equityholders have incentives to prolong the debt maturity to infinite in order to transfer wealth from debtholders to themselves. In this section, with a closed form formula of equity available, we can examine the effect of maturity changes on risk-shifting incentive, again by applying the implicit function theorem. We have

$$\frac{\partial H}{\partial T} = \frac{\phi(h)}{v}\Big[\frac{\partial v}{\partial \beta}\Big\{\Big[\frac{\partial v}{\partial T}(h - v) + \frac{1}{P}\frac{\partial P}{\partial T}\Big]A\beta'(\alpha)\Big(h - \frac{\varepsilon}{v}\Big) - \frac{\partial v}{\partial T}\Big\} + \{\beta + \rho\,\sigma\, b(T)\}\Big] \gtreqless 0 \quad (16)$$

In the above expression, it is easy to verify $\partial v/\partial T$ is positive. Lemma 1 states the sign of the factor $h-(\varepsilon/v)$ is also positive. The signs of $\partial v/\partial \beta$ and $\partial P/\partial T$ are, however, analytically indeterminate. As discussed, the sign of $\partial v/\partial \beta$ depends on the value of ρ, which characterizes the firm in question. The possibility of $\partial P/\partial T$ being negative is a well known defect of employing Ornstein-Uhlenbech process for the instantaneous interest rate dynamics, and we will assume $\partial P/\partial T$ being always negative hereafter.[9] As for the factor $h-v$, the sign depends on

[9] In the numerical analysis, we will only use parameter choices which preclude $\partial P/\partial T$ from being non-positive.

the capital structure, since it is easy to show

$$h-v \gtreqless 0 \quad \Leftrightarrow \quad \frac{PF}{A} \lesseqgtr \exp\{-\frac{v^2}{2}\} \tag{17}$$

holds. Notice the left-hand side of inequality is the quasi-debt ratio, and the right-hand side is less than but close to unity if v^2 is of plausible value. Accordingly, highly levered firms, which are characterized by high quasi-debt ratios, are likely to have a negative sign for $h-v$, while ordinary levered firms are likely to have a positive sign.

Recalling that sign($d\alpha^*/dT$)$=$sign($\partial H/\partial T$), eq. (16) indicates possibility of α^* being a decreasing function of T, that is an important departure from the widely accepted view. From eq.(16), we predict $d\alpha^*/dT$ can be negative in two cases. The first case is when $\partial v/\partial \beta > 0$ and $h-v<0$, that is when the firm is highly levered and characterized by the ρ greater than ρ^*, namely (normal) asset substitution. If the last term in the bracket, $\{\beta+\rho\sigma b(T)\}$, is non-positive, or positive but insignificant compared with previous terms in eq.(16), $\partial H/\partial T$, thus $d\alpha^*/dT$ becomes unambiguously negative. In such a case equityholders have incentive to *increase* asset volatility if maturity of debt is shortened. The second case arises when $\partial v/\partial \beta < 0$ and $h-v>0$. Such a situation is characterized by the moderately levered firm with ρ being less than ρ^*, implying the reverse asset substitution case. This case is in line with a dominant view that shortening debt maturity reduces agency cost, however, the reasoning is different. Agency cost is reduced since increased asset volatility reduces reverse asset substitution problem.

An interesting case arises when $\partial v/\partial \beta < 0$ and $d\alpha^*/dT > 0$. The firm in this case aggravates reverse asset substitution when debt maturity is shortened. Inspection of eq. (16) reveals such situation takes place when $h-v$ is negative, namely when the firm is highly levered. We summarize the result as the next proposition.

Proposition 3 *When the risk free interest rates are stochastic and correlate with firm value, equityholders of highly levered firms currently engaging in asset substitution are likely to have incentive to increase asset volatility if the debt maturity is shortened. The equityholders of firms engaging in reverse asset substitution, on the other hand, will further decrease asset volatility if debt maturity is shortened. In both cases, agency cost of debt is likely to increase if the maturity is shortened.*

The sign of $d\alpha^*/dT$ is not analytically determined in general, suggesting risk-shifting incentive is not a simple monotone function of debt maturity. This result has an important implication for risk management. As will be illustrated in the next section, by choosing debt maturity appropriately, we can eliminate or at least minimize agency cost of debt for the reverse asset substitution case.

3.4. Risk free Interest Rates, Other Environmental Parameters and Risk-shifting Incentive

The sensitivity of α^* to changes in the level of risk free interest rates and other variables can be examined by comparative statics analysis. When a firm faces unexpected changes in environmental variables such as risk free interest rates, it is natural to assume equityholders will try to maintain equity value being maximized by adjusting the level of α^*, which may result in altering firm value volatility.[10] As the environmental variables to investigate here, we consider the current level of instantaneous risk free interest rate and parameters characterizing its Ornstein-Uhlenbeck diffusion process. The correlation coefficient between firm value and risk free interest, and market price of risk are also examined.

Current level of Instantaneous Risk free Interest Rate/ Long-Run Mean/ Market Price of Risk

These parameters share the common property of appearing only in P, the risk free pure discount bond price. If we represent these parameters as $\Theta(\ni\{r, \theta, \lambda\})$, we have

$$\frac{\partial H}{\partial \Theta} = \phi(h)\ \frac{A\beta'}{vP}\ \frac{\partial v}{\partial \beta}\ \frac{\partial P}{\partial \Theta}\left(h - \frac{\varepsilon}{v}\right). \tag{18}$$

Recalling the sign of $h-(\varepsilon/v)$ is positive at $\alpha=\alpha^*$, it can be established that

$$\mathrm{sign}\left(\frac{d\alpha^*}{d\Theta}\right)=\mathrm{sign}\left(\frac{\partial H}{\partial \Theta}\right)=\begin{cases}\mathrm{sign}\left(\dfrac{\partial P}{\partial \Theta}\right) & \text{if } \dfrac{\partial v}{\partial \beta} \geqq 0 \\ \mathrm{sign}\left(-\dfrac{\partial P}{\partial \Theta}\right) & \text{if } \dfrac{\partial v}{\partial \beta} < 0.\end{cases} \tag{19}$$

Straightforward calculation reveals that for these parameters,

10 In this article, the perfect market assumption enables equityholders to know exact parameter values of risk free interest rate process.

$$\frac{\partial P}{\partial r}<0,\quad \frac{\partial P}{\partial \theta}<0,\quad \frac{\partial P}{\partial \lambda}<0$$

hold. Thus, we can summarize the comparative statics result as the next proposition.

Proposition 4 *In an economy where the risk free interest rates are stochastic and correlate with firm value, assume the current level of instantaneous risk free interest rate rises* (*falls*) *unexpectedly. In such a situation, the equityholders of the firms engaging in asset substitution have incentive to decrease* (*increase*) *asset volatility. The equityholders of the firm engaging in reverse asset substitution, on the other hand, have incentive to increase* (*decrease*) *asset volatility. For both types of asset substitution, agency cost of debt is reduced* (*increased*). *The same comparative statics results hold for long-run mean and market price of risk parameters.*

Volatility of Interest Rate Process/ Speed of Mean Reversion/ Correlation Coefficient

Correlation coefficient parameter appears only in v, the risk neutral volatility of firm value process, however, volatility of interest rate process and speed of mean reversion parameter also appear in P, the risk free pure discount bond price. Accordingly, the partial derivatives for these parameters are highly complicated. We have

$$\frac{\partial H}{\partial \sigma}=\frac{\phi(h)A\beta'}{v}\left(\left\{\frac{\partial v}{\partial \beta}\left[\frac{1}{P}\frac{\partial P}{\partial \sigma}+\frac{\partial v}{\partial \sigma}(h-v)\right]\left(h-\frac{\varepsilon}{v}\right)+\frac{v}{\sigma}(1-\eta_{v\sigma})\right\}-\frac{\beta T}{\sigma}\right), \tag{20}$$

$$\frac{\partial H}{\partial \kappa}=\frac{\phi(h)A\beta'}{v}\left(\left\{\frac{\partial v}{\partial \beta}\left[\frac{1}{P}\frac{\partial P}{\partial \kappa}+\frac{\partial v}{\partial \kappa}(h-v)\right]\left(h-\frac{\varepsilon}{v}\right)+\frac{v}{\kappa}(c-\eta_{v\kappa})\right\}-\frac{\beta T}{\kappa}c\right). \tag{21}$$

$$\frac{\partial H}{\partial \rho}=\frac{\phi(h)\,A\beta'\beta\,\sigma\,(T-b)}{v^{2}\kappa}\cdot\frac{\partial v}{\partial \beta}\left[\left(h-\frac{\varepsilon}{v}\right)(h-v)-\frac{1}{A\beta'}+\frac{\varepsilon}{A'\beta}\right] \tag{22}$$

where

$$\eta_{v\sigma}=\frac{\partial v}{\partial \sigma}\cdot\frac{\sigma}{v},\qquad \eta_{v\kappa}=\frac{\partial v}{\partial \kappa}\cdot\frac{\kappa}{v},$$

$$b=\frac{1-e^{-\kappa T}}{\kappa}>0,\qquad c=-\frac{(2+\kappa T)e^{-\kappa T}+\kappa T-2}{e^{-\kappa T}+\kappa T-1}<0.$$

The signs of these comparative statics depend on the sign of $(h-v)$ that describes whether the firm is moderately or highly levered, as well as the sign of $(\partial v/\partial \beta)$ that depends on whether the firm is involved in normal or reverse asset substitution. In addition, the partial derivative of risk free discount bond and elasticity of risk neutral volatility with respect to respective parameters enter in the comparative statics. Thus, the signs of above partial derivatives are analytically indeterminable in general, and we expect changes in these parameters cause extremely complex effects on risk-shifting incentive. The direction and magnitude of the effect will be evaluated in the next section by numerical analysis assuming plausible parameter values.

4. Numerical Illustration

4.1. Risk-shifting Incentive, Asset Value and Volatility

In this section, to illustrate the results in the previous section, specific functional forms for $A(\alpha)$ and $\beta(\alpha)$ are provided. Specifically, we assume

$$A(\alpha) = -75\alpha^4 - 15\alpha^2 + 100 \tag{23}$$

$$\beta(\alpha) = 0.15\times 3^{\alpha} + 0.05 \tag{24}$$

in the following illustration. With this specification, $A(0)=100$, $A(-1)=A(-1)=10$, and $A'(\alpha)=-300\alpha^3-30\alpha$, which makes $A'(\alpha)<0$ for $\alpha\in(0,1)$, $A'(\alpha)>0$ for $\alpha\in[-1,0)$, and $A'(0)=0$ as desired. We can also confirm $A''(\alpha)<0$ for $\alpha\in[-1,1]$, which is one of sufficient conditions for an existence of interior maximum of equity value. For asset volatility, the above specification leads to $\beta(0)= 0.2$, $\beta(-1)= 0.1$, $\beta(1)= 0.5$, and $\beta'(a)=(0.15)(\ln 3)3^{\alpha}$ which is positive, insuring β being monotone increasing function of α.

As the base case, we assume $F=100$ and $T=12$ (years) for the firm's debt. The parameter values for the Ornstein-Uhlenbeck interest rate process are; $r=0.05$, $\kappa=0.01$, $\theta=0.06$, and $\sigma=0.05$.[11] To determine the risk free discount bond price, we set the market price of risk as $\lambda=0.1$.[12] Sensitivity analysis is then conducted by altering these values. For the correlation

11 These parameter values are chosen based on the empirical work of Chan, Karolyi, Longstaff, and Sanders (1992), Tse (1995), Stanton (1997), Babbs and Nowman (1998), and Aït-Sahalia(1999).

12 We follow Vasicek's definition of market price of risk, whose sign is required to be positive for the instantaneous expected return on risk free bond to be higher than the instantaneous risk free interest rate. Empirical literature, however, usually follow the notational convention in accordance with other interest rate models, CIR model for example, so that market price of risk must take negative values to ensure excess return to be positive.

coefficient between firm value and interest rates we consider three cases, namely, $\rho=-0.8$, $\rho=\rho^*$, and $\rho=0.3$, where ρ^* is the critical value given by eq. (9) discussed in the previous section. With other parameter values assumed, the critical correlation value is calculated as

$$\rho^*=-\frac{\kappa^2 T}{e^{-\kappa T}+\kappa T-1}\cdot\frac{\beta(0)}{\sigma}\fallingdotseq-0.69360.$$

It is well known that the Ornstein-Uhlenbeck process can take negative value, and the yield to maturity of risk free bond is not guaranteed to remain positive mathematically. To cope with such defects in conducting sensitivity analysis, we made the following restrictions on respective parameter values; $r\geqq 0.02398$, $\sigma\leqq 0.06289$, $\lambda\geqq 0.01496$, $T\leqq 15.29960$. These restrictions are made since they are sufficient to prevent the discount bond price being more than unity in the following numerical analysis.

4.2. Leverage

(Fig.1A, 1B, 1C)

(Fig.2A, 2B)

Figure 1A plots equity value for asset volatility chosen by the firm for the case of $\rho=0.3$. As for the face value of debt, we examined nine cases from $F=0$ (no leverage) to $F=200$. For each case, we first calculate asset volatility $\beta(\alpha)$ for $\alpha\in[-1,1]$. Horizontal axis runs from $\beta(-1)=0.1$ to $\beta(1)=0.5$, and equity values calculated by eq. (7) are plotted. The maximum equity value for each face value of debt is numerically calculated so that eq. (8) equals zero, satisfying the first order condition.[13]

The figure shows equity value is given by concave function of asset volatility, confirming our choice of functions $A(\alpha)$ and $\beta(\alpha)$ indeed satisfies the second order condition. For each level of face value, we can observe equity value has a unique global maximum that is designated by a large black circle. As face value is increased, equity value is depicted by the lower curves. Note that when $F=0$, equity value maximized at $\beta=20\%$ amounts to 100, indicating the first best

13 We do not report the level of α here, since β is a monotone increasing function of α, and the level of α itself does not have any economical implication beyond β.

investment project is adopted without any agency conflicts. The figure shows the optimal asset volatility level, marked with black circles, increases and departs from β＝20% as face value of debt increases. This situation clearly describes well-known (normal) asset substitution.

Figure 1B and 1C show the relationship between asset volatility and equity value for the case of ρ=－0.8 and ρ＝ρ*, respectively. When ρ＝－0.8, which is less than ρ* in our example, Figure 1B indicates the optimal asset volatility for equityholders decreases as face value of debt increases. As the firm employs more leverage, equityholders prefer less volatile asset. This is what we term as reverse asset substitution, since the equityholders have incentive to substitute *less* volatile asset for more volatile one. When ρ＝ρ*, as is obvious from Figure 1C, the optimal asset volatility is 20% irrespective of leverage level. Note that such a level of asset volatility is the same as the one chosen when the firm is unlevered. Thus, firm value maximization is consistent with equity value maximization in this case, and no conflicts between equityholders and debtholders exist. These figures confirm Proposition 1.

Figure 2A then plots optimal asset volatility chosen by equityholders for face value of debt. Three types of risk-shifting behavior can be observed. It is obvious optimal asset volatility as well as underlying risk-shifting incentive is not necessarily an increasing function of leverage. It can be either increasing, independent, or decreasing function of face value of debt. When ρ=0.3, for example, optimal asset volatility for F=200 is 28.05％and risk-shifting parameter is numerically calculated as α=0.3912. For the ρ=－0.8 case, the corresponding asset volatility and risk-shifting parameter are 16.85％ and α=－0.2146, respectively. If the firm is characterized by ρ=ρ*≒－0.6936, equityholders' wealth is maximized at β=20% which corresponds to α=0 irrespective of leverage level.

In Figure 2B, agency cost of debt is calculated for three cases of ρ, and plotted for face value of debt. In our formulation, any deviation from the first best investment policy, which is characterized by β＝20%, incurs a decrease in asset value. For the ρ=0.3 case, the firm with F＝200 suffers from agency cost of 4.05%, meaning the firm value is reduced from 100 to 95.95, compared with unlevered firm value. For the firm with ρ=－0.8, agency cost of reverse asset substitution amounts to 0.85% of unlevered firm value, which is less severe than normal asset substitution. For the firm with ρ=ρ*≒－0.6936, we can confirm Proposition 2 and Corollary 1 that agency cost does not exist in this special case irrespective of leverage level.

4.3. Debt Maturity

(Fig.3A, 3B)

With the base case parameters, Figure 3A plots optimal asset volatility for debt maturity ranging from 0 to 15 years. We consider the two cases, namely, the firm with $\rho = 0.3$, and the firm with $\rho = -0.8$. [14] We can observe for debt maturity less than 2 years, two curves representing optimal asset volatility appear almost identical in the figure, and both increase rapidly in maturity. Optimal asset volatility increases in debt maturity for both cased, implying the incentive of asset substitution.

The optimal volatility for the ρ=0.3 case reaches the highest value when T=5(years), with β=26.29%. It then declines very slowly, and β=25.56% when T=15(years). The optimal volatility for the ρ=−0.8 case starts to decline at T=3(years), and the graph crosses the β =20% level at $T \fallingdotseq 10.34788$(years), that is numerically calculated. Optimal asset volatility decreased in debt maturity, that is to say equityholders have incentive for reverse asset substitution when debt maturity is extended in this region. The graph for the ρ=−0.8 case has an important implication for controlling the agency problem. If debt maturity is agreed to be contracted as about 10.35 years, the first best investment policy of β=20% is realized with the base case parameters. If debt maturity is properly contracted, agency conflicts are completely eliminated. We can also predict that if maturity is set less than 10.35 years, the firm will engage in asset substitution, whereas reverse asset substitution will take place if the maturity is longer that critical level.

Figure 3B exhibits the corresponding agency cost. For the ρ=0.3 case, agency cost become highest at about five years of maturity, amounting to 2.30% of unlevered firm value. In contrast, in the ρ=−0.8 case, agency cost becomes highest for about three years of maturity, amounting to 2.05% of unlevered firm value. As noted, if maturity is contracted as 10.35 years, agency conflicts are completely eliminated. We can observe that for the debt maturity longer than 10.35 years, agency cost increases in maturity again. This agency cost is due to reverse asset substitution. At T=15 years, agency cost amounts to 0.6958%. This sensitivity analysis reveals

[14] In Figure 3A and 3B, the case for $\rho=\rho^*$ is not exhibited since the value of ρ^* depends on debt maturity T.

that if the firm's asset value is characterized by the negative correlation with risk free interest rates, the firm has a possibility to eliminate agency cost of debt by carefully choosing debt maturity.

From Figure 3A and 3B, it is obvious that optimal asset volatility is not a monotone increasing function of debt maturity. In our examples, contrary to conventional wisdom, shortening debt maturity may incur heavier agency cost if the current debt maturity is longer than three years for the $\rho=-0.8$ case, and five years for the $\rho=0.3$ case, respectively. These figures, thus, confirm validity of Proposition 3. In both cases, however, if the debt maturity is shortened sufficiently to take advantage of initial upward sloping curves for shorter maturity, agency cost is drastically reduced as conventional wisdom suggests.

4.4. Risk free Interest Rates and Other Environmental parameters

Risk free Interest Rates

(Fig.4A, 4B)

Figure 4A and 4B are illustrated for the current level of risk free interest rates ranging from 2.4 % to 20%.[15] Figure 4A shows for the case of $\rho=0.3$, optimal asset volatility declines monotonically in current risk free interest rates level. Optimal asset volatility is 26.74% at r=2.4 %, and it declines to 21.08% at r=20%. For the case of $\rho=-0.8$, on the other hand, optimal asset volatility increases in leverage, from 17.62% to 19.97%. In both cases, normal/reverse risk-shifting incentive declines as current level of risk free interest rates increases. For the case of $\rho=\rho^*\fallingdotseq-0.6936$, it is clear that optimal asset volatility is independent of current risk free interest rates level, indicating no agency conflicts exist for such a firm even with the existence of debt. These figures are consistent with Proposition 4.

In Figure 4B, we can see for the firm characterized by $\rho=\rho^*$, agency cost of debt is zero irrespective of interest rates level. For the firms with $\rho\neq\rho^*$, however, agency cost decreases as

[15] As mentioned in the previous section, to preclude negative yield of risk free discount bond, $r\geqq 2.39\%$ is assumed in the base case examples.

the current risk free interest rates level increases. Furthermore, we can see agency cost of normal asset substitution is higher than that of reverse asset substitution in general. When $r = 2.4\%$, for example, agency cost of normal asset substitution (ρ=0.3) amount to 2.69% of unlevered firm value, whereas for the reverse asset substitution (ρ=−0.8), agency cost is only 0.42%.

Long-Run Mean

(Fig.5A, 5B)

As Proposition 3 predicts, in Figure 5A, optimal asset volatility is found to be a monotone decreasing function of long-run mean for $\rho = 0.3$, and a monotone increasing function for $\rho = -0.8$, respectively. For $\rho = \rho^*$, optimal volatility is independent of long-run mean. If long-run mean of the interest rates increases, both types of asset substitution become less active, which results in decline in agency cost as shown in Figure 5B. These figures indicate, however, changes in the long-run mean do not provide significant effects on optimal asset volatility and associated agency cost compared with the current level of interest rates.

Interest Rate Volatility

(Fig.6A, 6B)

Recall eq.(20) suggests the sign of comparative statics regarding interest rate volatility is too complex to determine. However, with the base case parameters, Figure 6A reveals optimal volatility is simply a monotone decreasing function of interest rate volatility. For the ρ=0.3 case, as interest rate volatility increases, optimal asset volatility is seen to declines very slowly. For the ρ=−0.8 case, in contrast, it decreased rapidly and eventually becomes less than 20%, indicating the occurrence of reverse asset substitution for high interest volatility level. In fact, at σ=6%, optimal volatility is 17.26%, which is less than the first best volatility of 20% in our example.

In Figure 6B, it is noteworthy that for the $\rho = -0.8$ case, agency cost of debt becomes zero at $\sigma \fallingdotseq 4.33499\%$, which is calculated numerically. At this level, $\partial v / \partial \beta$ is zero with other parameter values assumed, implying that risk neutral volatility becomes independent of asset volatility. Accordingly, risk-shifting incentive is not existent if the interest rate volatility is given at this level.

Speed of Mean Reversion

(Figure 7A, 7B)

As with interest rate volatility, it is too complex to analytically determine the sign of eq.(20) that evaluates the comparative statics with respect to the speed of mean reversion parameter κ. In Figure 7A, however, with base case parameter values, optimal volatility increases monotonically in κ for the $\rho=0.3$ case, whereas it monotonically increases in for the $\rho=-0.8$ case. In Figure 7B, we can observe agency cost is not lowered much for the $\rho=0.3$ case. For the $\rho=-0.8$ case, in contrast, agency cost exhibits the U shape graph in the speed of mean reversion. It should be noted in the latter case agency cost is zero at the critical value of $\kappa \fallingdotseq 0.0478$. If κ is smaller than that critical value, the firm is considered to engage in reverse asset substitution. As κ increases, such incentive is weakened, and higher asset volatility is preferred by equityholders. If κ is larger than the critical value, the firm now engages in normal asset substitution. Agency cost of debt, thus, starts to increase in κ.

Market Price of Risk

(Fig.8A, 8B)

Consistent with Proposition 4 , Figure 8A shows optimal asset volatility decreases in market price of risk λ for the $\rho=0.3$ case, and increases for the $\rho=-0.8$ case. As also predicted, optimal asset volatility is independent of λ when $\rho=\rho^*$.[16] In Figure 8B, agency cost of debt is seen to decrease very rapidly in λ. It becomes negligibly small if λ is larger than 0.3 for the $\rho=-0.8$ case, and if λ is larger than 0.7 for the $\rho=0.3$ case, respectively. Obviously these results apply only to the base case parameter choices, however, the market price of risk plays a non-trivial role in determination of agency cost.

5. Conclusions

Contrary to the prevailing view in the agency literature, we demonstrate equityholders' optimal level of incentive for asset substitution is not necessarily a monotone increasing function

[16] To avoid negative yields of risk free bond, market price of risk in our illustration is restricted as $\lambda \geqq 0.01496$. Figure 8A and 8B are, thus, depicted for $0.015 \leqq \lambda \leqq 1$.

of leverage. It can be decreasing in or independent of leverage when risk free interest rates are stochastic and correlate with firm value.

We show, depending on the correlation coefficient between firm value and the risk free interest rates, equityholders may have incentive to substitute *less* risky investment for riskier one. Such incentive is contrary to well known normal asset substitution and is expected to exist only in the stochastic interest rates environment. We refer to this as *the reverse asset substitution*.

In a stochastic interest rate economy, effective firm volatility relevant to pricing is represented under the risk-neutral measure under which every asset price discounted by the money market account becomes martingale. Because the interest rate process is given by Ornstein-Uhlenbeck process in our setting, the relevant risk-neutral volatility turns out to be a product of asset volatility, correlation coefficient, interest rates volatility, and speed of mean reversion parameter. If the correlation coefficient characterizing the firm is sufficiently negative, a decrease in firm value volatility caused by reverse asset substitution increases the risk-neutral volatility, which results in an increase in equity value.

We also find agency cost of debt can be a decreasing function of debt maturity. Thus, contrary to the dominant view, shortening debt maturity may aggravate agency problem between equityholders and debtholders when risk free interest rates are stochastic. The numerical analysis also suggests the possibility that agency conflicts can be completely eliminated by carefully choosing the debt maturity given other parameter values.

Overall, unexpected changes in the environmental parameter values have complex and non-trivial effects on risk-shifting incentive of equityholders. Especially, uncertainty created by stochastic interest rates adds great complexity to the agency problem. However, as the choice of debt maturity indicates, it also provides a new possibility for risk management of the firm. We leave this topic for future research.

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Figure 1A
Asset Volatility and Equity Value: ρ=0.3

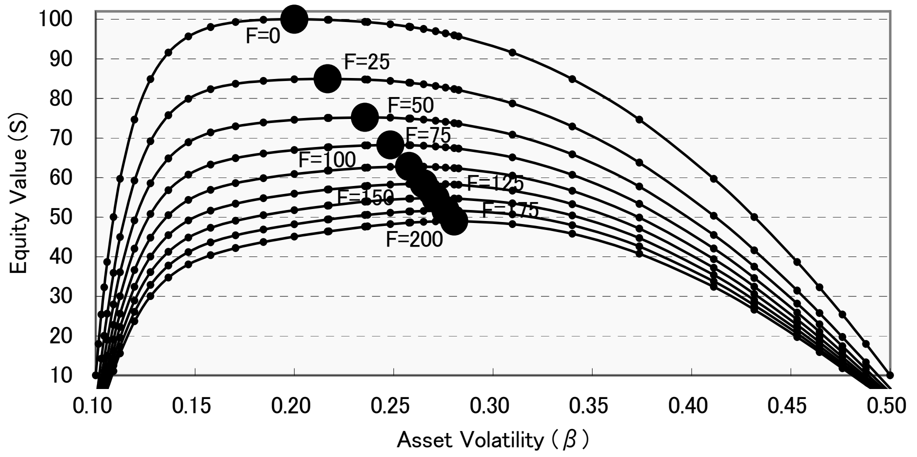


Figure 1B
Asset Volatility and Equity Value: ρ =－0.8

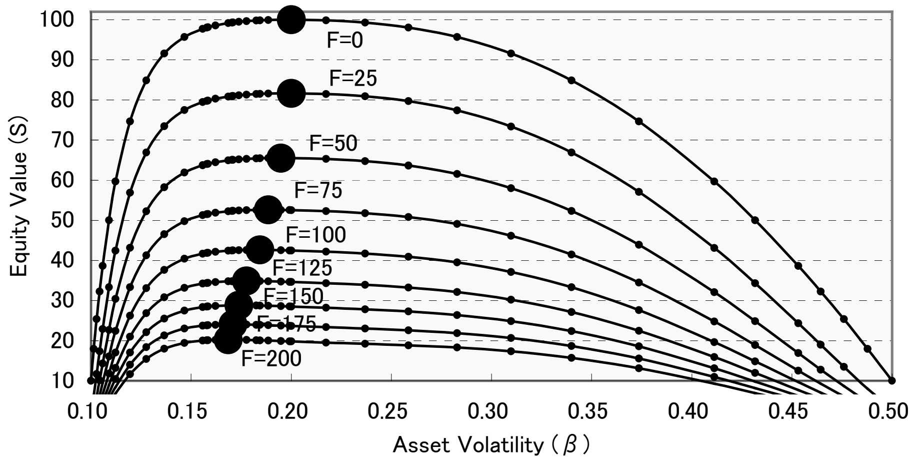
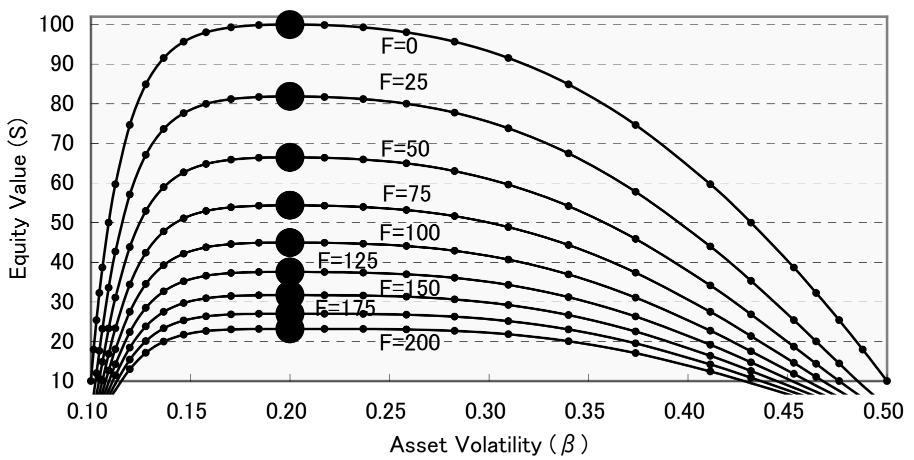


Figure 1C
Asset Volatility and Equity Value: $\rho=\rho*$



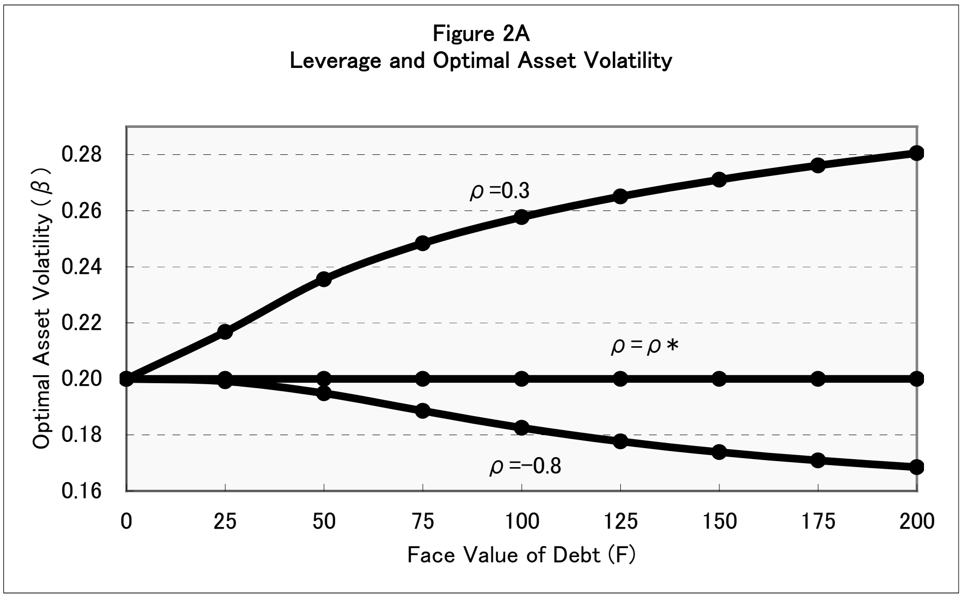


Figure 2A
Leverage and Optimal Asset Volatility
Optimal Asset Volatility (β)
0.28
0.26
0.24
0.22
0.20
0.18
0.16
ρ=0.3
ρ=ρ*
ρ=−0.8
0
25
50
75
100
125
150
175
200
Face Value of Debt (F)

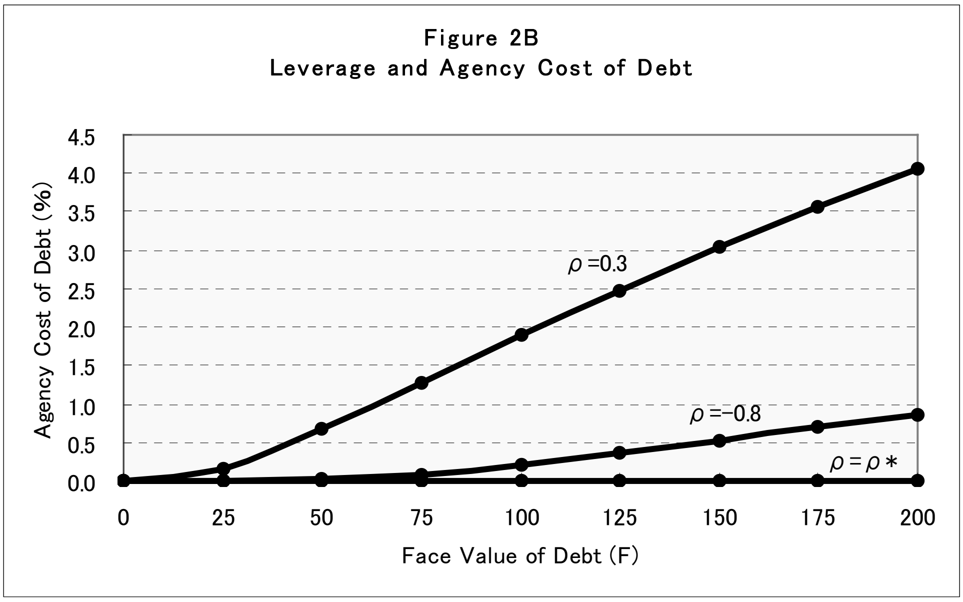


Figure 2B
Leverage and Agency Cost of Debt
Agency Cost of Debt (%)
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
ρ=0.3
ρ=−0.8
ρ=ρ*
0
25
50
75
100
125
150
175
200
Face Value of Debt (F)

Figure 3A
Debt Maturity and Optimal Asset Volatility

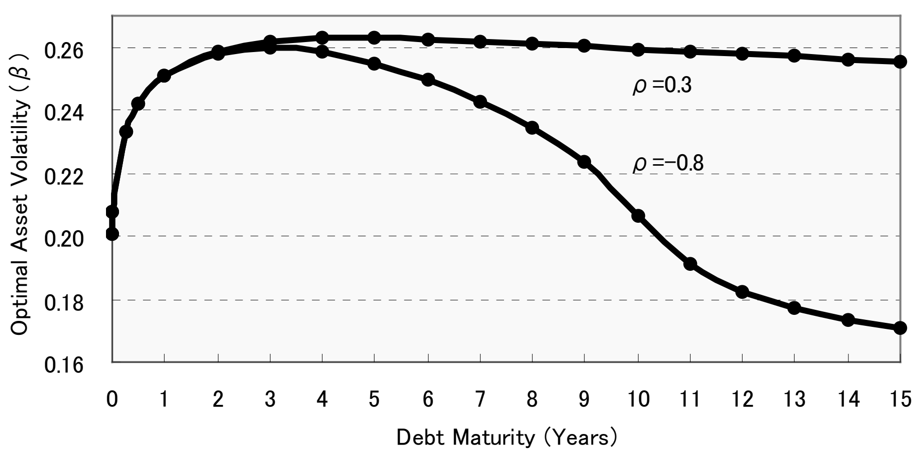


Figure 3B
Debt Maturity and Agency Cost of Debt

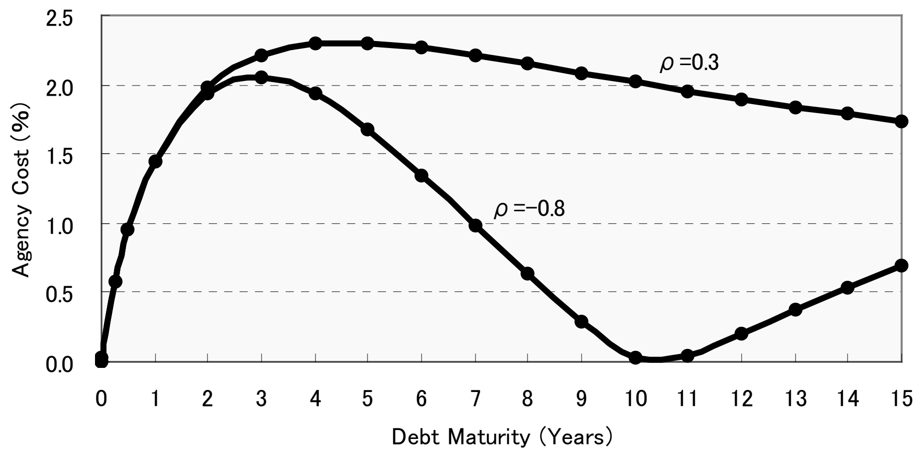


Figure 4A
Risk Free Interest Rate and Optimal Asset Volatility

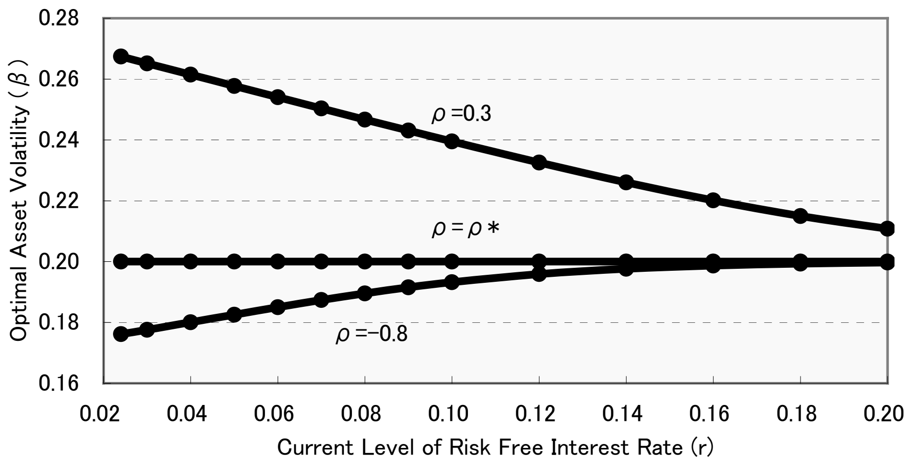


Figure 4B
Risk Free Interest Rate and Agency Cost of Debt

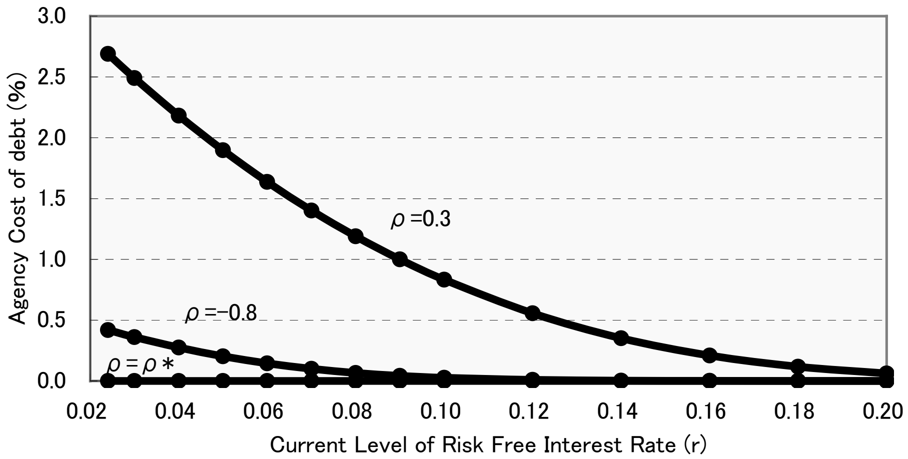


Figure 5A
Long-Run Mean and Optimal Asset Volatility

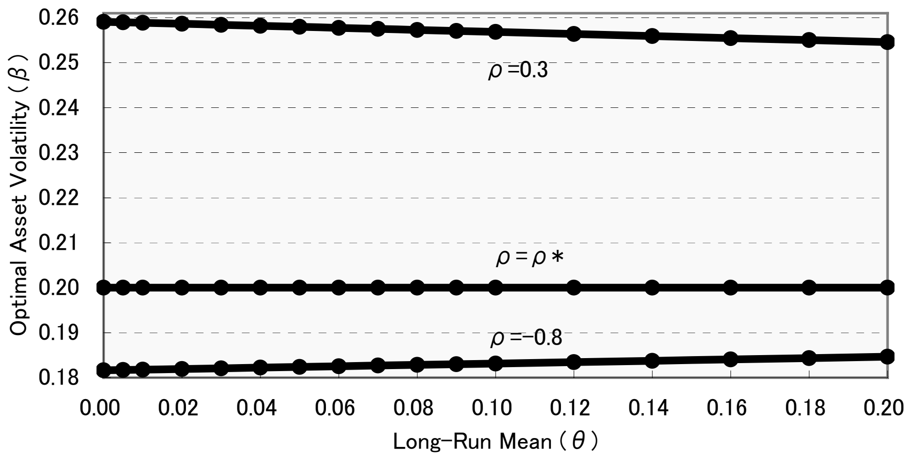


Figure 5B
Long-Run Mean and Agency Cost of Debt

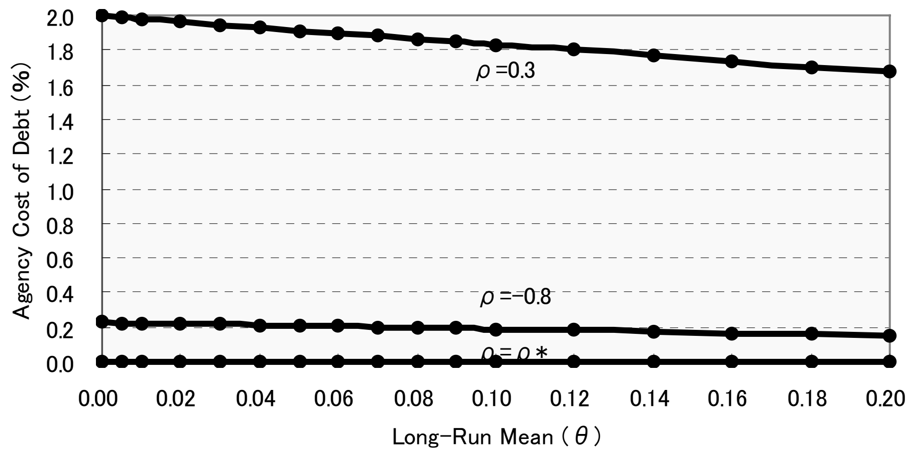


Figure 6A
Interest Rate Volatility and Optimal Asset Volatility

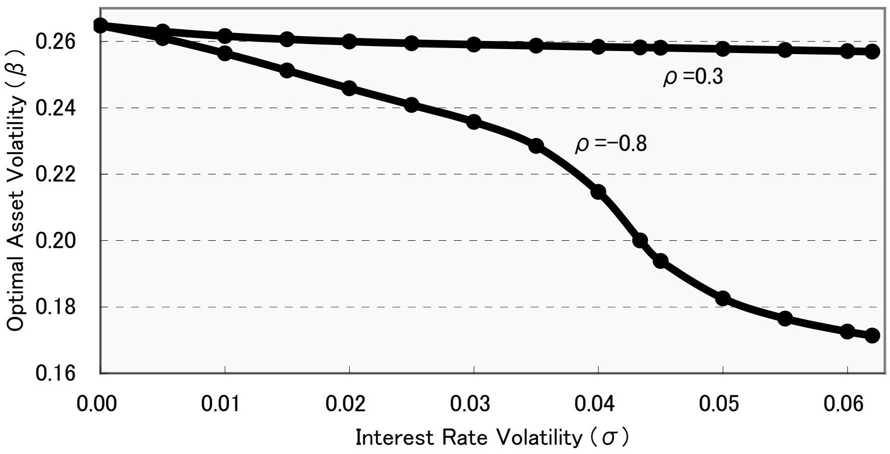


Figure 6B
Interest Rate Volatility and Agency Cost of Debt

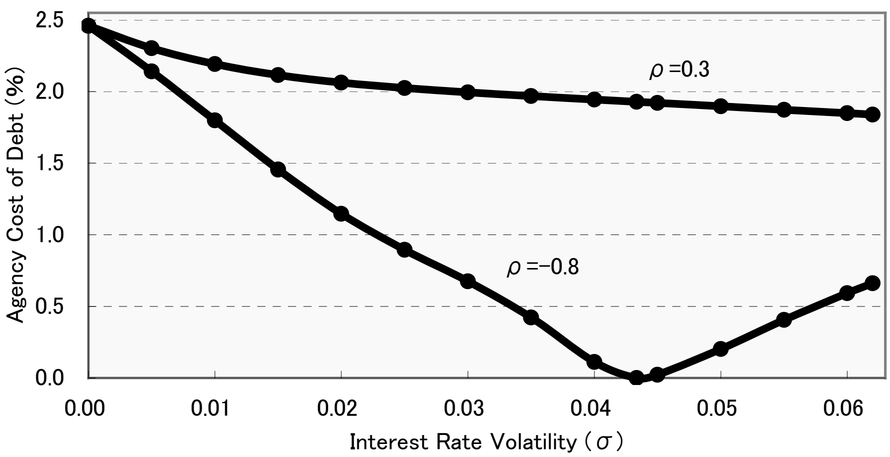


Figure 7A
Speed of Mean Reversion and Optimal Asset Volatility

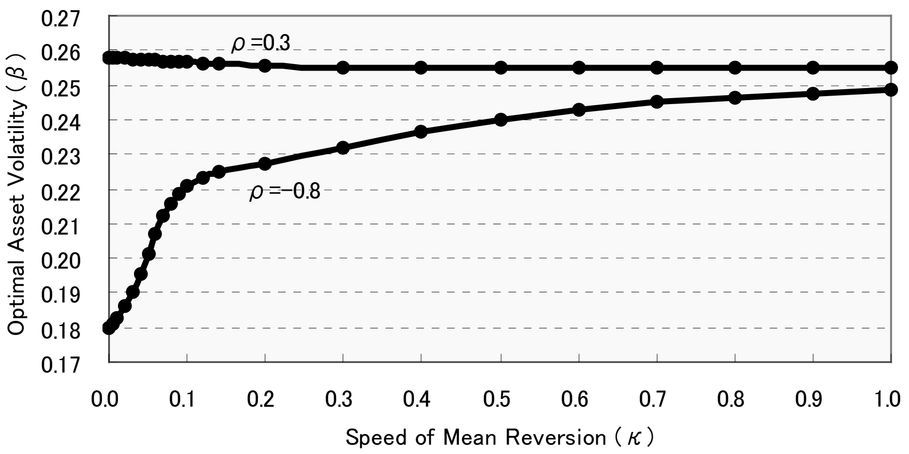


Figure 7B
Speed of Mean Reversion and Agency Cost of Debt

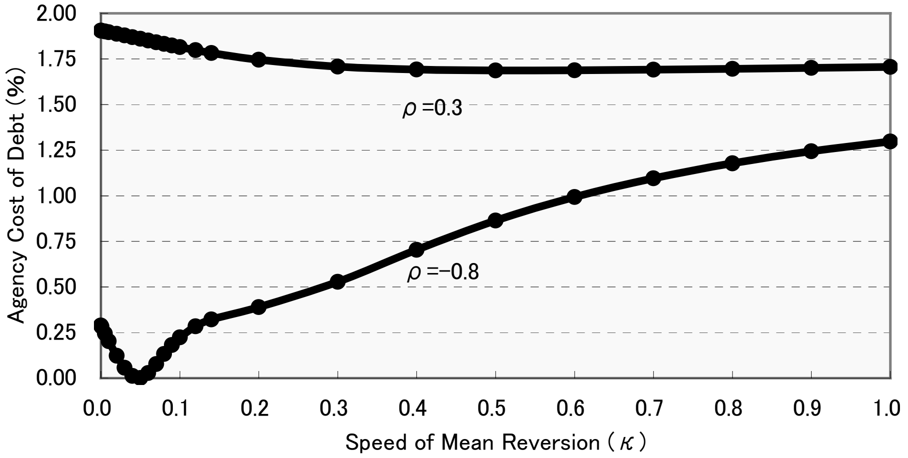


Figure 8A
Market Price of Risk and Optimal Asset Volatility

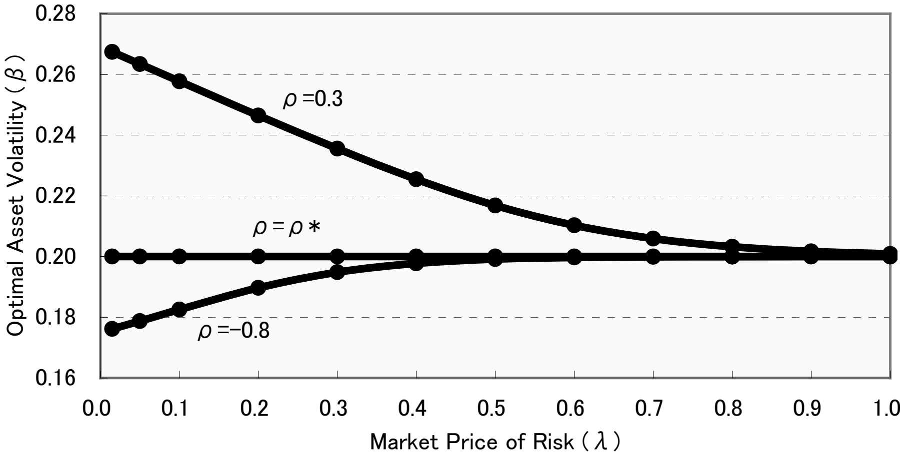


Figure 8B
Market Price of Risk and Agency Cost of Debt