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ABSTRACT

This paper investigates a deposit contract model with a liquidation cost. The main result shows that efficient consumption is uniquely implementable in strictly dominant strategies without deposit insurance under any liquidation cost and any relative risk aversion of consumers.

1 Introduction

This paper investigates a *runs preventing contract* for deposit contracts, a concept introduced by Cooper and Ross (1998), that ensures that self-fulfilling bank runs do not occur in equilibrium. Cooper and Ross (1998) characterize the set of runs preventing contracts in the generalized model of Diamond and Dybvig (1983). They show that runs preventing contracts are

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efficient if both the relative risk aversion of consumers and the liquidation cost for a long-term investment are sufficiently small.

This paper revisits the characteristic of runs preventing contracts by focusing on the mechanism that give contracts the run-preventing property. The point of a runs preventing contract is to prevent bank runs. Hence, the mechanism is more important than a contract itself. From this point of view, I divide runs preventing contracts examined in previous work into the following three categories, according to their mechanisms: 1) contracts involving a *deposit-freeze policy*;[1] 2) contracts following Cooper and Ross's framework, which I refer to as the *Cooper–Ross doctrine*; and 3) contracts used in Diamond and Dybvig's deposit insurance, although I focus on the mechanism rather than the deposit insurance itself, and I refer to the mechanism as the *Diamond–Dybvig doctrine*.[2]

The Cooper–Ross doctrine requires any contract to "specify a consumption level for each type of consumer *independent* of the number of consumers claiming to be each type" (Cooper and Ross, 1998, p.31), which is slightly demanding for deposit contract design. In fact, because the Cooper–Ross doctrine is too demanding, efficient consumption levels fail to be achieved whenever either the relative risk aversion of consumers or the liquidation cost for productive investment is large. In addition, both the deposit-freeze policy and the Diamond–Dybvig doctrine violate the Cooper–Ross doctrine, even though they ensure that contracts are not only run-preventing but also efficient.

However, the Diamond–Dybvig doctrine (without deposit insurance) can be seen as a weak version of the Cooper–Ross doctrine. The main difference between them is that the Diamond–Dybvig doctrine allows several payments in one period, whereas the Cooper–Ross doctrine excludes them a priori.

[1] Diamond and Dybvig (1983) show that a deposit-freeze policy, which they refer to as involving the *suspension of convertibility*, can prevent bank runs while achieving ex ante efficiency. However, Engineer (1989) shows that such a deposit-freeze policy fails to work when there is uncertainty as to each consumer's type. In addition, Ennis and Keister (2009) show that a bank cannot ensure the prevention of bank runs if the bank's deposit-freeze policy takes into account *ex post* efficiency.

[2] Regarding deposit insurance, some research papers point out the relationship with moral hazard. See Cooper and Ross (2002) and Martin (2006), for instance.

I investigate the design of a deposit contract under the Diamond–Dybvig doctrine. The main result of this paper is to show that there exists a runs preventing contract that follows the Diamond–Dybvig doctrine and which implements an efficient consumption level for each type of consumer in dominant strategy equilibrium. This result requires no deposit insurance and is obtained independent of the relative risk aversions and liquidation costs.

2 Definitions

2.1 Banking model

The model is almost the same as that of Cooper and Ross (1998). Consider an economy with a single consumption good, a representative bank, and a continuum of consumers. The bank implicitly faces competition with other banks, and its profit is assumed to be zero. The economy has three periods: t_0, t_1, and t_2.

For the consumption good, there are two technologies. A *storage* technology yields one unit of the consumption good in t_1 whereas a *production* technology yields $R > 1$ units of the good in t_2, per unit of input in t_1. The bank can liquidate a productive investment before t_2, but this yields $1 - \kappa$ in t_1 per unit of input, where the parameter $\kappa \in [0, 1]$ denotes a *liquidation cost*.

Let $I := [0, 1]$ denote the set of consumers. Each consumer has one unit of the consumption good as an endowment. At the commencement of period t_1, a fraction of the consumers $\theta \in (0, 1)$ face a liquidity shock and they can obtain utility from consumption only in period t_1, whereas the remaining consumers can obtain utility in both periods t_1 and t_2. We refer to the former type as *early* consumers and the latter as *late* consumers. We use $\theta_i \in \{e, l\}$ to denote the *type* of consumer i, where e and l indicate early and late consumers, respectively. Throughout this paper, we assume that θ_i is the private information of consumer i. Note that the parameter θ is equal to the proportion of early consumers.

Let u denote the consumers' utility function over consumption. We assume that u is a von Neumann–Morgenstern utility function $u : \mathbf{R}_+ \to \mathbf{R}$, which is strictly increasing, strictly concave, and twice differentiable, and which satisfies $\lim_{c \to 0} u'(c) = \infty$.[3] Furthermore, for consumption in period t_k, say c_k, we assume that consumer i has the following form of utility function: $u(c_1 + c_2 \mathbf{1}_{\{\theta_i = l\}})$, where $\mathbf{1}_{\{\theta_i = l\}}$ is an indicator function that equals one if $\theta_i = l$ and is otherwise zero.

Suppose that θ is public information and that all consumers deposit their endowments with the bank. Then, competitive pressure makes the bank offer a consumption plan to consumers that maximizes their expected welfare. The ex ante efficient consumption is defined as the solution of the following optimization problem:

$$\begin{aligned} &\max_{c_e, c_l, \psi} \ \theta u(c_e) + (1-\theta) u(c_l) \\ &s.t. \ \ \theta c_e \leq 1 - \psi, \quad (1-\theta) c_l \leq R\psi, \end{aligned} \tag{1}$$

where $\psi \in [0, 1]$ is the ratio of productive investment. In the optimum, we obtain: $\psi^* = 1 - \theta c_e^*$, $c_l^* = R(1 - \theta c_e^*)(1-\theta)^{-1}$, and the first-order condition implies that $c_e^* < c_l^*$.

2.2 Individual provision functions

We introduce an important concept of this paper, called the *individual provision function*. The individual provision function in period t_k is a function $\chi^k : I \to \mathbf{R}_+$. We identify $\chi^k(i)$ as the consumption of consumer i in period t_k.

We see here that an individual provision function corresponds to a *deposit contract* that the bank offers to consumers. Suppose that θ is public and let $c_j^* = c_j(\theta)$ denote the optimal consumption level for a type j consumer that constitutes the solution of problem (1).

Whenever we refer to the pair (c_e^*, c_l^*) as a contract, it implicitly implies

[3] Diamond and Dybvig (1983) assume that $-cu''(c)/u'(c) > 1$ for all $c > 0$, that is, the degree of relative risk aversion is greater than one, which makes an optimal consumption level greater than one. Cooper and Ross (1998), and our paper, do not make this assumption.

the existence of an individual provision function such that $\chi^1(i) = c_e^*$ or $\chi^2(i) = c_l^*$ for any consumer $i \in I$ who wishes to consume in period t_1 or t_2, respectively.

2.3 Deposit-freeze policy

Whenever the bank offers (c_e^*, c_l^*) to consumers as a contract, there is a danger of the contract breaking down due to bank runs when $c_e^* > 1$ because the total amount of available assets in t_1 is generally less than one. In such a case, the bank uses a *deposit-freeze* policy, which is described by the following individual provision function: for any consumer $i \in I$ who wishes to consume in period t_1:

$$\chi^1(i) = \begin{cases} c_e^* & \text{if } i \in [0, \theta] \\ 0 & \text{if } i \in (\theta, 1] \end{cases} \tag{2}$$

and $\chi^2(i) = 0$; for any consumer $i \in I$ who wishes to consume in period t_2, $\chi^1(i) = 0$, and $\chi^2(i) = c_l^*$.[4]

Diamond and Dybvig (1983) show that this deposit-freeze policy prevents bank runs and achieves the efficient consumption in equilibrium, but it is noteworthy that the deposit-freeze policy assumes that the bank can use the information on how many consumers have been served previously when the bank provides a consumer with a consumption good.

2.4 The Cooper–Ross doctrine

Cooper and Ross (1998) exclude such information availability in deposit contracts. They focus on the contracts that provide consumption equally in each period independent of the number of withdrawals. For convenience, we refer to their criterion as the *Cooper–Ross doctrine*, which can be summarized as follows.

Cooper–Ross doctrine: *Any contract should satisfy the following property: for each k, there exists $c^k \in \mathbf{R}_+$ such that $\chi^k(i) = c^k$ for any $i \in I$ who is qualified to be served in period t_k.*

[4] For a more detailed description, see the Appendix.

Cooper and Ross (1998) refer to the contract that prevents bank runs as a *runs preventing contract* (hereafter, *RPC*). By definition, a contract with a deposit-freeze policy is an RPC. A contract that follows the Cooper–Ross doctrine is an RPC if $c^1 < c^2$. In fact, if $c^1 < c^2$, it is optimal for each early and late consumer to consume c^1 and c^2 in period t_1 and t_2, respectively. Furthermore, this action constitutes a *strictly dominant strategy equilibrium*, that is, any consumer worsens by changing his/her consumption period regardless of other consumers' actions.

If a contract (c_e, c_l) is an RPC that follows the Cooper–Ross doctrine, it must satisfy:

$$c_e \leq 1 - \psi + \psi(1 - \kappa) = 1 - \kappa\psi, \tag{3}$$

where ψ represents the productive investment for c_l. The meaning of inequality (3) is clear: the bank can provide c_e for all consumers in period t_1 with the liquidation of productive investment ψ.

Clearly, an efficient contract may fail to be RPC under the Cooper–Ross doctrine because our problem (1) is set independently of inequality (3). This is easy to see. As $\kappa \geq 0$ and $\psi \geq 0$, an efficient RPC is required to satisfy $c_e^* \leq 1$ from inequality (3). Hence, the efficient contract in the Diamond–Dybvig (1983) model cannot be an RPC under the Cooper–Ross doctrine because we always have $c_e^* > 1$ in the Diamond–Dybvig model. This is because the relative risk aversion is greater than unity in their model. In addition, inequality (3) with efficient investment $\psi^* = 1 - \theta c_e^*$ is equivalent to:

$$c_e^* \leq \frac{1 - \kappa}{1 - \kappa\theta}. \tag{4}$$

As $c_e^* > 0$ whenever $\theta > 0$, efficient contracts are no longer RPCs if $\kappa = 1$, regardless of the level of relative risk aversion. Therefore, for any $\theta \in (0, 1)$, the efficient contract (c_e^*, c_l^*) fails to be an RPC under the Cooper–Ross doctrine for some level of relative risk aversion and some liquidation cost κ.

2.5 The Diamond–Dybvig doctrine

Diamond and Dybvig (1983) propose *deposit insurance* as an idea for the prevention of bank runs while achieving efficient consumption. We first briefly explain the mechanism of their deposit insurance model.

The deposit insurance in the Diamond–Dybvig (1983) model comprises two communication phases in t_1 and one such phase in t_2. We refer to the former as Day 1 and Day 2 and the latter as Day 3, for convenience. Let $\chi(\cdot, k)$ denote an individual provision function at Day k, where $\chi^1(i) = \chi(i,1)+\chi(i,2)$ and $\chi^2(i) = \chi(i,3)$ for each $i \in I$. For any consumer $i \in I$ who wishes to consume on Day 1, $\chi(i,1) = r$, where $r \geq 0$ is a fixed consumption provision. On Day 2, for any consumer i who is served on Day 1, $\chi(i,2) = -r\tau(\theta_1)$, where θ_1 is the number of consumers who wish to consume on Day 1 and $\tau : [0,1] \to [0,1]$ is the *tax rate* of the government such that:

$$\tau(\theta_1) = 1 - r^{-1}c_e(\theta_1).$$

For other consumers, $\chi(i,2) = 0$. On Day 3, $\chi(i,3) = c_l(\theta_1)$ for any consumer $i \in I$ who has not been served yet and wishes to consume on Day 3.

In plain words, the government gives r to the bank on Day 1 and the bank holds $1 + r - r\theta_1 > 1$ at the end of Day 1.[5] The government then collects $\theta_1 r\tau(\theta_1)$ as a tax from early consumers and requires the bank to repay $(1-\theta_1)r + \theta_1 c_e^*(\theta_1)$ on Day 2. Hence, the bank holds only $1 - \theta_1 c_e^*(\theta_1)$ and the government's budget is balanced at the end of Day 2.

Diamond and Dybvig (1983) show that $\theta_1 = \theta$ is realized in strictly dominant strategy equilibrium under this deposit insurance mechanism. Hence, early and late consumers consume c_e^* and c_l^*, respectively. Note that this result is independent of parameters such as the relative risk aversion and liquidation cost.

Although this mechanism violates the Cooper–Ross doctrine, it is important for us to note that this mechanism follows a "weak" version of the Cooper–Ross doctrine, which is described as follows.

[5] This implies that the bank holds assets equal to or more than one at the end of Day 1 in period t_1, which is a result of the government's financial support.

Diamond–Dybvig doctrine: *Any contract should satisfy the following property: for each k, there exists c^k such that $\chi(i,k)=c^k$ for any $i \in I$ who is qualified to be served on* ***Day*** *k.* (The bold text represents the difference between the Diamond–Dybvig doctrine and the Cooper–Ross doctrine.)

Further, the efficient outcome from deposit insurance is because the information θ_1 is available when completing the provision in period t_1. In the next section, we construct a mechanism that implements an efficient allocation by using this information availability of the Diamond–Dybvig doctrine *without* the government's financial support.

3 The main result

The goal of this section is to show that efficient consumption is uniquely implementable under the Diamond–Dybvig doctrine without the government's financial support regardless of the relative risk aversions and liquidation costs.

Suppose that $\theta \in (0,1)$ is public and let $c_j(\theta) = c_j^*$ for $j = e, l$, where $(c_e(\theta), c_l(\theta))$ is the solution of problem (1). We design an RPC that achieves the efficient consumption (c_e^*, c_l^*) in equilibrium.

First, we consider the following problem: under an arbitrary $\hat{\theta} \in (0,1)$:

$$\begin{aligned} &\max_{c_e, c_l} \hat{\theta} u(c_e) + (1-\hat{\theta}) u(c_l) \\ &s.t.\ \hat{\theta} c_e \leq \theta c_e^* \\ &\qquad (1-\hat{\theta}) c_l \leq R(1-\theta c_e^*) + \theta c_e^* - \hat{\theta} c_e. \end{aligned} \tag{5}$$

Let $(\hat{c}_e(\hat{\theta}), \hat{c}_l(\hat{\theta}))$ denote the solution of problem (5).

Lemma 1 *We obtain $\hat{c}_e(\hat{\theta}) < c_e^* < c_l^* < \hat{c}_l(\hat{\theta})$ for all $\hat{\theta} \in (\theta, 1)$.*

Proof. See the Appendix. ■

Next, we consider the following consumption provision functions:

$$\tilde{c}_e(\hat{\theta}) = \begin{cases} c_e^* & \text{if } \hat{\theta} \leq \theta \\ \hat{c}_e(\hat{\theta}) & \text{if } \hat{\theta} > \theta \end{cases}, \quad \tilde{c}_l(\hat{\theta}) = \begin{cases} c_l'(\hat{\theta}) & \text{if } \hat{\theta} \leq \theta \\ \hat{c}_l(\hat{\theta}) & \text{if } \hat{\theta} > \theta \end{cases}$$

where: $(1-\hat{\theta})c_l'(\hat{\theta}) = (1-\theta)c_l^* + (\theta-\hat{\theta})c_e^*$. We define $\hat{c}_e(1) \equiv \lim_{\hat{\theta}\to 1} \hat{c}_e(\hat{\theta})$ and $\hat{c}_l(1) \equiv \lim_{\hat{\theta}\to 1} \hat{c}_l(\hat{\theta}) \in [-\infty, \infty]$. Then, with Lemma 1, we can easily see that $\tilde{c}_e(\hat{\theta}) < \tilde{c}_l(\hat{\theta})$ for all $\hat{\theta} \in [0,1]$. To confirm this, we only have to show that $c_l'(\hat{\theta}) > c_e^*$ for all $\hat{\theta} \leq \theta$. The fact that $c_l^* > c_e^*$ implies that:

$$\begin{aligned}(1-\hat{\theta})c_l'(\hat{\theta}) &= (1-\theta)c_l^* + (\theta-\hat{\theta})c_e^* \\ &> (1-\theta)c_e^* + (\theta-\hat{\theta})c_e^* \\ &= (1-\hat{\theta})c_e^*.\end{aligned}$$

The result follows from this inequality because $1-\hat{\theta} > 0$.

Theorem 1 *There exists a contract that implements efficient consumption (c_e^*, c_l^*) in strictly dominant strategy equilibrium for all relative risk aversions and liquidation costs.*

Proof. Let $\underline{c}_e \equiv \inf_{\hat{\theta}\in(\theta,1]} \hat{c}_e(\hat{\theta})$. Then, $\underline{c}_e > 0$ because $\lim_{c\to 0} u'(c) = \infty$. We consider the following three-day mechanism:
Day 1. For any consumer $i \in I$, $\chi(i,1) = \underline{c}_e$.
Day 2. For any consumer $i \in I$ who is served on Day 1, $\chi(i,2) = \tilde{c}_e(\theta_1) - \underline{c}_e$, where θ_1 is the number of consumers who are served on Day 1. For other consumers, $\chi(i,2) = 0$.
Day 3. For any consumer $i \in I$ who has not been served yet, $\chi(i,3) = \tilde{c}_l(\theta_1)$.

The individual provision functions can provide $\tilde{c}_e(\theta_1)$ and $\tilde{c}_l(\theta_1)$ to θ_1 and $1-\theta_1$ consumers in periods t_1 and t_2, respectively. As $\tilde{c}_e(\theta_1) < \tilde{c}_l(\theta_1)$ for all $\theta_1 \in [0,1]$, all late consumers choose to be served in period t_2. Hence, we obtain $\theta_1 = \theta$ and the efficient outcome (c_e^*, c_l^*) in strictly dominant strategy equilibrium. ■

This three-day mechanism has three important characteristics. First, it ensures that the contract is an RPC. Second, the unique equilibrium outcome is efficient. Third, the mechanism is "detail free" in the sense that it is independent of the relative risk aversions and liquidation costs.[6]

[6] The concept of the "detail-free" mechanism is introduced by Matsushima (2005).

Our three-day mechanism realizes the same outcome in Diamond–Dybvig's deposit insurance model if $r = \underline{c}_e$. Then, we have $\tau(\theta) = 1 - \underline{c}_e^{-1}\tilde{c}_e(\theta) < 0$ and, hence, our mechanism does not tax consumers but pays an *interest*. This interest payment implies the existence of a *refund cap* for Day 1's withdrawals. In particular, the constraint in problem (5) implies that $\underline{c}_e < 1$, which implies that early consumers cannot withdraw all their deposits at once.

In order to achieve an efficient allocation with our detail-free RPC, however, such a refund cap is necessary. Thus, the demand deposit contract turns out to be like a *time deposit* contract in the sense that it is more illiquid than demand deposit contracts, if we seek to achieve both the efficiency and the run-preventing property regardless of the relative risk aversions and liquidation costs under the Diamond–Dybvig doctrine.

3.1 Example

Suppose that consumers have the following utility function:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \tag{6}$$

where $\gamma > 0$ is their relative risk aversion coefficient. It is easily derived that the solution of (1) is:

$$c_e^* = \frac{1}{\theta + (1-\theta)R^{\frac{1-\gamma}{\gamma}}}, \qquad c_l^* = R^{\frac{1}{\gamma}} c_e^*.$$

In the optimum of problem (5) with utility function (6), we obtain:

$$\hat{c}_e(\hat{\theta}) = \frac{1-\psi^*}{\hat{\theta}}, \quad \hat{c}_l(\hat{\theta}) = \frac{R\psi^*}{1-\hat{\theta}},$$

where:

$$1 - \psi^* = \theta c_e^*, \quad \psi^* = (1-\theta)R^{\frac{1-\gamma}{\gamma}} c_e^*.$$

Then:

$$\begin{aligned}
\hat{c}_l(\hat{\theta}) - \hat{c}_e(\hat{\theta}) &= \frac{c_e^* \left(\theta(1-\theta)R^{\frac{1}{\gamma}} - \theta(1-\theta)\right)}{\hat{\theta}(1-\hat{\theta})} \\
&> \frac{c_e^* \theta(1-\theta)\left(R^{\frac{1}{\gamma}} - 1\right)}{\hat{\theta}(1-\hat{\theta})} \\
&> 0.
\end{aligned}$$

In order to signify that efficient consumption depends on the relative risk aversion level, we denote it as $c_e^* = c_e^*(\gamma)$. It is easy to see that $\underline{c}_e = 1 - \psi^* = \theta c_e^*(\gamma) \in (0,1)$ and $c_e(\hat{\theta}) - \underline{c}_e = (1-\hat{\theta})\theta c_e^*(\gamma)\hat{\theta}^{-1} > 0$ for all $\gamma > 0$ and $\hat{\theta} \in [\theta, 1)$.

Suppose that $\gamma > 1$. Then, it is equivalent to $R^{\frac{1-\gamma}{\gamma}} < 1$, and hence we have $c_e^* > 1$. As pointed out, if the bank follows the Cooper–Ross doctrine, it cannot be an RPC in this case. On the other hand, our three-day mechanism works well in this case: every early consumer gets $\underline{c}_e = \theta c_e^*$ on Day 1 and $(1-\theta)c_e^*$ on Day 2; every late consumer gets c_l^* on Day 3. As $\theta c_e^* < 1$ and consumers deposit 1 in period t_0, our three-day mechanism does indeed have a refund cap in period t_1.

Suppose that $\gamma < 1$ but κ is large enough to break inequality (4); that is, (γ, κ) satisfies:

$$\frac{\theta + (1-\theta)R^{\frac{1-\gamma}{\gamma}} - 1}{\theta + (1-\theta)R^{\frac{1-\gamma}{\gamma}} - \theta} < \kappa \leq 1.$$

As pointed out, any RPC that follows the Cooper–Ross doctrine cannot be efficient in this case. On the other hand, under the Diamond–Dybvig doctrine, our three-day mechanism makes the contract in Theorem 1 an RPC and implements the efficient allocation in strictly dominant strategy equilibrium.

4 Appendix

4.1 Description of the deposit-freeze policy

Let $\hat{\theta} : I \to \{0, 1\}$ denote a function that describes a premature withdrawal, where $\hat{\theta}(i) = 1$ implies that consumer i wishes to withdraw in period t_1 and $\hat{\theta}(i) = 0$ implies that he/she does not. Let $\omega : I \to I$ denote a bijection that determines the *order* of being served, where $i = \omega(p) < \omega(q) = j$ implies that consumer p is served earlier than q if $\hat{\theta}(p) = \hat{\theta}(q) = 1$. As ω is a bijection, we can identify the magnitude relationship on I with the order of being served without loss of generality. Let us define the number $\hat{\theta}_i$ as: $\hat{\theta}_i = \mathcal{L}(\{j \in [0, i) \mid \hat{\theta}(j) = 1\})$, where $\mathcal{L}$ is the Lebesgue measure. Then, the deposit-freeze policy is described as the following binary function χ^1:

$$\chi^1(i) = \begin{cases} c_e^* & \text{if } \hat{\theta}_i \leq \theta \\ 0 & \text{if } \hat{\theta}_i > \theta. \end{cases}$$

Clearly, function (2) is insufficient to describe the deposit-freeze policy; if $\theta = 1/2 < i < 2/3$ and $\hat{\theta}(i) = 1$ but $\hat{\theta}(j) = 0$ for all $j \leq 1/2$, then consumer i can be served as $\hat{\theta}_i \leq 1/2$. However, function (2) is justified as follows. Let $A^{\hat{\theta}} = \{j \in I \mid \hat{\theta}(j) = 1\}$ and consider a bijection $\omega : I \to I$ as an order function such that $\omega(i) < \omega(j)$ implies that $\omega(i) \in A^{\hat{\theta}}$ and $\omega(j) \in I \setminus A^{\hat{\theta}}$. If we identify I with $\omega(I)$, then we can say that function (2) describes the deposit-freeze policy.

4.2 Proof of Lemma 1

For notational convenience, we let $\hat{c}_j = \hat{c}_j(\hat{\theta})$, $j = e, l$. The Lagrangian of the problem (5) is:

$$L = \hat{\theta}u(c_e) + (1-\hat{\theta})u(c_l) + \lambda(\theta c_e^* - \hat{\theta}c_e) + \mu(R(1-\theta c_e^*) + \theta c_e^* - \hat{\theta}c_e - (1-\hat{\theta})c_l),$$

with some $\lambda \geq 0$ and $\mu \geq 0$. Using the Kuhn–Tucker theorem, we obtain:

$$u'(\hat{c}_e) = u'(\hat{c}_l) + \lambda, \quad u'(\hat{c}_l) = \mu, \tag{7}$$

with complementary slackness conditions $\lambda(\theta c_e^* - \hat{\theta}\hat{c}_e) = 0$ and $\mu(R(1-\theta c_e^*) + \theta c_e^* - \hat{\theta}\hat{c}_e - (1-\hat{\theta})\hat{c}_l) = 0$. Equation (7) implies that we obtain $\hat{c}_e \leq \hat{c}_l$ in the optimum. If $\hat{c}_e = \hat{c}_l = \hat{c}$, that is, $\lambda = 0$, then the complementary slackness condition for μ implies $\hat{c} = R(1-\theta c_e^*) + \theta c_e^*$. As $R(1-\theta c_e^*) = (1-\theta)c_l^*$ and $(1-\theta)c_l^* + \theta c_e^* > c_e^*$, we must have $\hat{c} > c_e^*$. Then, the first inequality constraint of (5) implies that $\hat{\theta} < \theta$. Hence, we obtain $\hat{c}_e < \hat{c}_l$ whenever $\hat{\theta} > \theta$, which implies that $\lambda > 0$. Then, the complementary slackness condition for λ implies that $\theta c_e^* = \hat{\theta}\hat{c}_e$. As $\mu > 0$, the complementary slackness condition for μ implies that $\hat{c}_l = (1-\hat{\theta})^{-1}(R(1-\theta c_e^*)) = (1-\hat{\theta})^{-1}(1-\theta)c_l^* > c_l^*$. Thus, we obtain $\hat{c}_e < c_e^* < c_l^* < \hat{c}_l$. As $\hat{\theta}$ is arbitrary, the result follows.

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