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Incorporation of Identity into Deposit Contract Design*

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ABSTRACT

This paper investigates a deposit contract model featuring the *identity* of depositors. Using this approach, we show that efficient contracts are uniquely implementable with iterative deletions of strictly dominated strategies if some depositors have a mild identity-based preference. We also show that no efficient contract is uniquely implementable in ex post equilibrium without identity-based preferences.

Keywords: identity; mechanism design; implementation; financial intermediation; bank run; psychological cost
JEL Classification: C72, D82, G21, Z13

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1 Introduction

In a seminal banking study, Diamond and Dybvig (1983) show that a deposit contract achieves a socially efficient allocation, but may also bring about a *bank-run problem*, which brings with it a Pareto inefficient allocation. Following the Diamond–Dybvig model, a number of studies in the extant literature have addressed bank runs.[1] Of these, at least some have modeled the causes of bank runs.[2]

The present paper tackles the bank-run problem with a new approach: namely, the introduction of a depositor's *identity*, as originally proposed by Akerlof and Kranton (2000). Our main finding then shows that the identity of depositors solves the bank-run problem and the socially efficient allocation is then uniquely implementable in equilibrium.

Akerlof and Kranton (2000) define "identity" as a person's sense of self. According to their work, people belong to "social categories". These social categories in turn cultivate the *identity* of people, and most importantly, this identity influences their decisions. In doing so, Akerlof and Kranton's (2000) identity model yields a plausible explanation for many economic and social situations, including gender discrimination in the workplace, the economics of poverty and social exclusion, and other situations.

Following Akerlof and Kranton (2000), we divide depositors into one of two categories—a *socially conscious category* and a *self-interested category*. Depositors belonging to the former, whom we refer to as *socially conscious depositors*, are assumed to desire a socially efficient outcome for any deposit contracts.[3] Hence, if they do not need funds immediately, they hesitate to withdraw their deposits before maturity because their early withdrawal acts against social efficiency. In order to represent such preferences, we assume

[1]This body of research splits bank runs into two types: *self-fulfilling bank runs* and *information-based bank runs*. The current analysis and Diamond and Dybvig (1983) address the former. For suitable studies of information-induced bank runs, see Gorton (1985), Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), and Gorton and Pennacchi (1990).

[2]See, for example, Postlewaite and Vives (1987), Engineer (1989), Nicoló (1996), Cooper and Ross (1998), Green and Lin (2003), and Goldstein and Pauzner (2005).

[3]We implicitly assume that socially conscious depositors have a deep knowledge of banking systems and understand the fragility of these systems.

that the payoff of socially conscious depositors comprises their own monetary interests *and their identities*, where we represent identity as a *psychological cost* that indicates a loss in identity with early withdrawals. Alternatively, depositors belonging to the self-interested category, whom we refer to as *self-interested depositors*, have a preference that only represents their own monetary interests.

Along with the basic Akerlof and Kranton (2000) framework, we consider a novel aspect of identity, that is, the *reduction of motive*. In the social psychology literature, using field experiments, Keizer et al. (2008) show that a norm violation in a society by some people causes subsequent norm violations by other people.[4] We incorporate this phenomenon into the concept of identity as a reduction of psychological cost: that is, the psychological cost of a depositor in early withdrawal *declines if he/she recognizes that some other depositors have already withdrawn their deposits before his/her first attempt at early withdrawal*. This is the key to our model. By utilizing this cost-reduction property, we can design a payment mechanism that uniquely implements efficient contracts with iterative deletions of strictly dominated strategies.[5]

It is noteworthy that we can obtain our affirmative result with "small identity," that is, all we require are *a few* socially conscious depositors and *arbitrarily small* psychological costs. Further, we also show that no mechanism uniquely implements an efficient deposit contract in ex post equilibrium if there are only self-interested depositors. Thus, the results lead us to consider the use of identity in designing bank deposit contracts.

The remainder of the paper is organized as follows. Section 2 describes our basic framework. Section 3 defines the feasible set of outcomes available to banks. We also discuss that efficient contracts fail to be implementable in ex

[4]Wilson and Kelling (1982) propose the "broken windows theory," which states that people tend to become vandals when they observe small signs of social disorder. Keizer et al. (2008) test their theory in field experiments and conclude that subjects are likely to violate other norms once they observe the violation of one norm.

[5]Other than Akerlof and Kranton (2000), several studies of game-theoretic decision making also take into account psychological preferences. See, for example, Geanakoplos et al. (1989), Bernheim (1994), Dufwnberg and Lundholm (2001), Battigalli and Dufwnberg (2009), Matsushima (2009), and Dufwenberg et al. (2011).

post equilibrium. Section 4 defines the psychological cost of socially conscious depositors. Section 5 presents the main result and Section 6 provides the concluding remarks. The Appendix presents the proof of the propositions.

2 Banking model

The model setting follows Diamond and Dybvig (1983). Consider an economy with a single consumption good, a representative bank, and a continuum of depositors. The economy comprises three periods: t_0, t_1, and t_2. The set of depositors are represented by $D = [0, 1]$. Each depositor has an endowment of one unit of the consumption good, which he/she deposits in the representative bank in t_0. At the commencement of period t_1, some depositors face a liquidity shock and obtain utility from consumption only in period t_1, while the remaining consumers can obtain utility from consumption in both period t_1 and period t_2. We refer to the former type as *early* depositors and the latter type as *late* depositors. We denote with $\omega_i \in \{e, l\}$ a *type* of depositor i, where e and l indicate early and late, respectively. Throughout this paper, we assume that ω_i is the private information of depositor i. We define the set of early and late depositors respectively as

$$D(e) = \{i \in D \mid \omega_i = e\}, \quad D(l) = \{i \in D \mid \omega_i = l\}.$$

By definition, $D = D(e) \cup D(l)$ and $D(e) \cap D(l) = \emptyset$. We then introduce a parameter θ, defined as $\theta = \mathcal{L}(D(e))$, where $\mathcal{L}$ is the Lebesgue measure. Our assumption implies that the bank does not know the accurate value of θ in advance. For simplicity, we exclude nonmeasurable profiles of the types.

Assumption 1 *$D(l) \neq \emptyset$ implies $\mathcal{L}(D(l)) > 0$.*

Assumption 1 implies that if depositor i finds him/herself to be of type l, then he/she knows that there are so many late depositors as to be $\theta < 1$. Indeed, Assumption 1 ensures that $\theta = 1$ is equivalent to $D(l) = \emptyset$.[6]

[6] $\mathcal{L}(D(l)) = 0$ implies $D(l) = \emptyset$, which means $\theta = 1$. On the other hand, $\theta = 1$ implies $\mathcal{L}(D(e)) = 1$, which means $\mathcal{L}(D(l)) = 0$, hence $D(l) = \emptyset$.

Let u denote depositors' utility function over consumption. We assume that u is a von Neumann–Morgenstern utility function $u : \mathbf{R}_+ \to \mathbf{R}_+$, which is strictly increasing, strictly concave, and twice differentiable, and satisfies $u(0) = 0$, $\lim_{c \to 0} u'(c) = \infty$, $\lim_{c \to \infty} u'(c) = 0$, and $-u''(c)/u'(c) > 1$ for all $c > 0$.[7]

In relation to the consumption good, there are two investment technologies available to the bank. The first is a *storage* technology, which in t_1 yields one unit of the consumption good per unit input in t_0. The second is a *production* technology, which in t_2 yields $R > 1$ units of the good per unit input in t_0. Premature liquidation of the productive investment in t_1 is possible without any cost.[8]

We here define efficient deposit contracts. Suppose that the bank knows θ in t_0. Then, the bank can offer an *ex ante efficient allocation* to depositors, which is defined as the solution of the following optimization problem:

$$\begin{aligned} &\max_{c_e, c_l} \theta u(c_e) + (1-\theta)u(c_l) \\ &\ s.t.\ (1-\theta)c_l \le R(1-\theta c_e), \end{aligned} \tag{1}$$

where the subscripts e and l denote early and late depositors, respectively. In the optimum, the solution $(c_e^*(\theta), c_l^*(\theta))$ exists uniquely for each $\theta \in (0,1)$ and satisfies $1 < c_e^*(\theta) < c_l^*(\theta) < R$ for all $\theta \in (0,1)$.[9] Note that Equation (1) shows that $c_e^*(1) = 1$ and $c_l^*(0) = R$, while $c_e^*(0)$ and $c_l^*(1)$ are indeterminate. We define a *contract* as a pair of contingent consumptions on a realized state θ and denote it as $\boldsymbol{c} = (c_e(\cdot), c_l(\cdot))$. A contract is said to be *efficient* if for all $\theta \in (0,1)$, $\boldsymbol{c}(\theta)$ is the solution to the problem in Equation (1) and if for $\theta \in \{0,1\}$, $c_e(1) = 1$ and $c_l(0) = R$.

[7]Diamond and Dybvig (1983) assume $u : \mathbf{R}_{++} \to \mathbf{R}_+$, but not $u(0) = 0$. These represent the only differences between their model and ours.

[8]Without liquidation costs, Cooper and Ross (1998) show that our bank-run problem does not occur if depositors are not very risk-averse, that is, $-u''/u' \le 1$.

[9]For these derivations, see Diamond and Dybvig (1983).

3 Material environment

3.1 Formalization

This section formally describes the environment in which we define our banking model. First, we define the set of all feasible outcomes, say Y. In order to do so, we consider an *individual provision function* $\chi^k : D \to \mathbf{R}_+$, which is Lebesgue measurable, and we identify $\chi^k(i)$ as the consumption of depositor i in period t_k. Let $D(\chi^k > x)$ denote a subset of depositors served by more than $x \geq 0$ in period t_k. Then, we define the set of feasible outcomes in t_1 by

$$Y^1 \equiv \left\{ \chi^1 \;\middle|\; 0 \leq \int_{i \in D} \chi^1 di \leq 1 \right\},$$

where $\int_{i \in D} \chi^1 di = \int_0^\infty \mathcal{L}(D(\chi^1 > x))dx$. The set of feasible outcomes in t_2 depends on the amount of withdrawn consumption in t_1, or χ^1:

$$Y^2(\chi^1) \equiv \left\{ \chi^2 \;\middle|\; 0 \leq \int_{i \in D} \chi^2 di \leq R(1-z) \right\},$$

where $z = \int_{i \in D} \chi^1 di$. Let $Y^2 = \bigcup_{\chi^1 \in Y^1} Y^2(\chi^1)$. We define the set of feasible outcomes Y as

$$Y = \left\{ (\chi^1, \chi^2) \in Y^1 \times Y^2 \;\middle|\; \chi^2 \in Y^2(\chi^1) \right\}.$$

By definition, any contract $\boldsymbol{c}$ is subject to the feasible set Y. We refer to the environment stated above and with Assumption 1 as the *material environment* $\mathcal{E}$.

Here, we make an assumption on the environment $\mathcal{E}$.

Assumption 2 *If $\hat{\chi}^1 \in Y^1$ gives $z = 1$, then*

$$Y^2(\hat{\chi}^1) = \{\chi^2 \mid \forall i, \chi^2(i) = 0\}.$$

This assumption implies that the bank cannot pay anything in t_2 if it returned all assets to depositors in t_1. Later in this section, we explain the significance of this assumption.

In order to implement a contract, the bank selects a *mechanism*. The mechanism asks each depositor in t_1 (after he/she knows his/her own type) to identify when he/she intends to consume. Let m_i be the *message* of depositor i and M_i be the set of all messages available to i. A profile of all depositors' messages is described as $m \in M \equiv \prod_{i \in D} M_i$. We write $m = (m \backslash m_i)$ or $m = (m_i, m_{-i})$ for tractability. Let $g = (g^1, g^2) : M \to Y$ be the *outcome function* of a mechanism, where g^k is a mapping such that $g^1 : M \to Y^1$ and $g^2 : M \to Y^2(g^1)$. A *mechanism* is defined by a tuple of a message space M and an outcome function g, denoted by $\mathcal{M} = (M, g)$.

A *direct mechanism* relative to an efficient contract $\mathbf{c}^*$, denoted by $\mathcal{M}^{\mathbf{c}^*} = (M^d, g^d)$, is such that $M_i^d = \{e, l\}$ for all $i \in D$, and $g^{d1}(m)$ and $g^{d2}(m)$ determine individual provision functions respectively as

$$\chi^1(i) = \begin{cases} c_e^*(\hat{\theta}) & \text{if } m_i = e \\ 0 & \text{if } m_i = l \end{cases}, \quad \chi^2(i) = \begin{cases} 0 & \text{if } m_i = e \\ c_l^*(\hat{\theta}) & \text{if } m_i = l, \end{cases}$$

where $\hat{\theta} = \mathcal{L}(\{i \in D \mid m_i = e\})$. Note that we assume "no sequential service," that is, in each period, the mechanism can determine the consumption provision to depositors after receiving messages.[10]

In the material environment $\mathcal{E}$, we denote with $U_i(g(m), \omega_i)$ the payoff function of a type ω_i depositor i under a message profile m, or

$$U_i(g(m), \omega_i) = u(\chi^1(i) + \chi^2(i)\mathbf{1}_\ell), \tag{2}$$

where $\mathbf{1}_\ell$ is an indicator function such that $\mathbf{1}_\ell = 1$ if $\omega_i = l$, and $\mathbf{1}_\ell = 0$ if $\omega_i = e$. Let $G = (\{U_i\}_{i \in D}, \mathcal{M}, \mathcal{E})$ denote a game with a mechanism $\mathcal{M}$ on the environment $\mathcal{E}$. Let s_i denote the *pure strategy* of depositor i in G, or $s_i : \{e, l\} \to M_i$. We denote by ω the *type profile* of depositors, or $\omega \in \{e, l\}^D$.

Definition 1 *A strategy profile s is an* ex post equilibrium *in G if*

$$\forall i, \forall \omega, \forall m_i : U_i(g(s(\omega)), \omega_i) \geq U_i(g(s(\omega) \backslash m_i), \omega_i).$$

[10] Although Diamond and Dybvig (1983) assume sequential service, some subsequent studies do consider deposit contract models without sequential service. See, for example, Allen and Gale (1998, 2000).

Definition 2 *An efficient contract* $\boldsymbol{c}^* = (c_e^*(\cdot), c_l^*(\cdot))$ *is* implementable *in a solution concept* $\mathcal{S}$ *if there exists a mechanism* $\mathcal{M} = (M, g)$ *such that for any* $\theta \in [0,1]$ *and for any profile* s^* *of* $\mathcal{S}$*-solution concept, outcome* $g(s^*(\omega))$ *provides* $c_e^*(\theta)$ *for all early depositors and* $c_l^*(\theta)$ *for all late depositors.*

Note that this definition requires *full implementability* in the sense that for all θ and for all depositors i, if i is an early depositor, then $\chi^1(i) = c_e^*(\theta)$; if i is late, then $\chi^2(i) = c_l^*(\theta)$.[11]

Let $G^{\boldsymbol{c}^*}$ denote a game with a direct mechanism $\mathcal{M}^{\boldsymbol{c}^*}$ in the material environment $\mathcal{E}$. We can easily check that the truth-telling strategy profile, say $s^*(\omega) = \omega$, constitutes an ex post equilibrium in $G^{\boldsymbol{c}^*}$. To confirm this, we first note that the truth-telling strategy profile realizes $\hat{\theta} = \theta$. Then, we have $U_i(g(s^*(\omega)\backslash e), e) = u(c_e^*(\theta)) > u(0) = U_i(g(s^*(\omega)\backslash l), e)$ and $U_i(g(s^*(\omega)\backslash l), l)$ $= u(c_l^*(\theta)) > u(c_e^*(\theta)) = U_i(g(s^*(\omega)\backslash e), l)$.[12]

However, there exists an inefficient ex post equilibrium in $G^{\boldsymbol{c}^*}$, known as the *bank-run* equilibrium, such that $s_i'(\omega_i) = e$ for all i and ω_i. Under the profile s', we have

$$U_i(g(s'(\omega)\backslash e), \omega_i) = u(c_e^*(1)) = u(1) > u(0) = U_i(g(s'(\omega)\backslash l), \omega_i)$$

for each $\omega_i \in \{e, l\}$. Hence the game $G^{\boldsymbol{c}^*}$ defined with a direct mechanism relative to any efficient deposit contract $\boldsymbol{c}^*$ has such an inefficient equilibrium outcome. In fact, we can show that any game G defined with any (indirect) mechanism cannot uniquely implement efficient deposit contracts in ex post equilibrium.

Proposition 1 *In the material environment* $\mathcal{E}$ *with Assumption 2, any efficient contract cannot be implementable in ex post equilibrium.*

Proof. See the Appendix. ■

[11] Note that Assumption 1 excludes the case where there are infinitely many zero-measured late depositors while $\theta = 1$. Nicoló (1996) investigates a run-proof mechanism that uniquely achieves *approximately efficient* outcomes with self-interested depositors.

[12] Owing to Assumption 1, any late depositor i need not wonder if $\theta = 1$.

That is, it is *impossible* to eliminate inefficient ex post equilibrium outcomes in our material environment $\mathcal{E}$.[13] Note that we obtain this impossibility result even if we assume an ideal environment with no sequential service.

Our impossible result is owing to Assumption 2, according to which nothing can be provided from a zero-asset holding. One may consider that the bank can offer any positive consumption in period t_2, even if $\hat{\theta} = 1$ is observed, because the measure of unserved depositors is zero. However, if we assume that the bank can offer positive consumption to zero-measured depositors, the bank-run problem itself disappears, including that in Diamond and Dybvig (1983). In the Appendix, we derive the following statement as a corollary to Proposition 1.

Corollary 1 *Suppose that we put aside Assumption 2 in material environment $\mathcal{E}$ and suppose that an efficient contract $\boldsymbol{c}^*$ can provide $c_l^*(1) > 1$ for a single depositor i in t_2. Then, the efficient contract $\boldsymbol{c}^*$ is implementable in ex post equilibrium with the direct mechanism relative to $\boldsymbol{c}^*$.*

Hence Assumption 2 is necessary for the models analyzing bank-run problems.

3.2 The suspension of convertibility and deposit insurance

Diamond and Dybvig (1983) show how to deal with the bank-run problem. This section restates their idea with our notion of individual provision functions. We use the essence of this section to show our main theorem.

If the bank knows θ exactly in advance, a mechanism with the following individual provision functions implements an efficient allocation in ex post equilibrium (more precisely, in dominant strategies): Let $M_i = \{e, l\}$ for all

[13] Green and Lin (2003) analyze a finite-trader version of the Diamond–Dybvig model. They show that efficient contracts are uniquely implementable in strictly dominant strategies. Their affirmative result, however, hinges on the finiteness of depositors. If depositors are finite, the behavior of a single depositor can influence the outcome of a contract. As a result, the bank can incentivize a depositor to reveal his/her true type.

$i \in D$.

$$\chi^1(i) = \begin{cases} c_e^*(\theta) & \text{if } m_i = e \text{ and } \hat{\theta} \le \theta \\ 0 & \text{if } m_i = l \text{ or } \hat{\theta} > \theta \end{cases}$$
$$\chi^2(i) = \begin{cases} 0 & \text{if } m_i = e \\ c_l^*(\theta) + \frac{(\theta-\hat{\theta})c_e^*(\theta)}{1-\hat{\theta}} & \text{if } m_i = l \text{ and } \hat{\theta} < \theta \\ c_l^*(\theta) & \text{if } m_i = l \text{ and } \hat{\theta} \ge \theta \end{cases} \tag{3}$$

The individual provision functions in (3) constitute a mechanism with *suspension of convertibility*, as referred to in Diamond and Dybvig (1983). Indeed, Diamond and Dybvig (1983) show that this mechanism can implement efficient allocations if θ is known.

If the bank does not know θ in advance, Diamond and Dybvig (1983) propose the idea of using *deposit insurance* provided by the government. They show that an efficient contract is implementable in dominant strategies if the government supports the bank with deposit insurance. The key is that deposit insurance definitely expands the feasible set more than Y.

Here we briefly explain the mechanism with deposit insurance as in Diamond and Dybvig (1983). Their mechanism, say $\mathcal{M}^*$, comprises two communication phases in t_1, say, Day 1 and Day 2, and one such phase in t_2, say Day 3.

Depositor i's message takes a form $m_i = (m_i^1, m_i^2, m_i^3) \in \{0,1\}^3 = M_i$, where *action* $m_i^k = 1$ implies that depositor i tenders a full withdrawal at Day k, while $m_i^k = 0$ implies that i does not tender any withdrawal at Day k. Let $\chi(i,k)$ denote an individual provision function at Day k, where $\chi^1(i) = \chi(i,1) + \chi(i,2)$ and $\chi^2(i) = \chi(i,3)$. In Day 1,

$$\chi(i,1) = \begin{cases} r & \text{if } m_i^1 = 1 \\ 0 & \text{if } m_i^1 = 0, \end{cases}$$

where $r \geq 0$ is a fixed provision of consumption. In Day 2,

$$\chi(i,2) = \begin{cases} -r\tau(\theta_1) & \text{if } m_i^1 = 1 \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta_1 = \mathcal{L}(\{i \in D \mid m_i^1 = 1\})$ and $\tau : [0,1] \to [0,1]$ is the *tax rate* of the government such that

$$\tau(\theta_1) = 1 - r^{-1} c_e^*(\theta_1).$$

In Day 3,

$$\chi(i,3) = \begin{cases} c_l^*(\theta_1) & \text{if } m^1 = 0 \text{ and } m_i^3 = 1 \text{ and if } \theta_1 < 1 \\ R & \text{if } m^1 = 0 \text{ and } m_i^3 = 1 \text{ and if } \theta_1 \geq 1 \\ 0 & \text{otherwise.} \end{cases}$$

In plain words, the government gives the bank r on Day 1 and the bank holds $1 + r - r\theta_1$ at the end of Day 1. The government then collects $\theta_1 r\tau(\theta_1)$ as a tax from early depositors and requires the bank to repay $(1-\theta_1)r + \theta_1 c_e^*(\theta_1)$ on Day 2. Hence the bank holds only $1 - \theta_1 c_e^*(\theta_1)$ and the government's budget is balanced at the end of Day 2.

This mechanism gathers information about how many depositors wish to withdraw early and uses it to determine the tax rate. Note that the tax rate and the property of $c_e^*(\cdot)$ imply $1 < c_e^*(\theta_1) \leq r$ for all $\theta_1 \in (0,1)$. Hence it is obvious that the feasible set of outcomes is larger than our Y. The after-tax values of consumption are $c_e^*(\theta_1)$ in t_1 and $c_l^*(\theta_1)$ in t_2. Because $c_e^*(\theta_1) < c_l^*(\theta_1)$ for all $\theta_1 \in (0,1)$ and $c_e^*(1) = 1 < R = \chi(i,3)$ for $\theta_1 = 1$, the efficient contract $(c_e^*(\cdot), c_l^*(\cdot))$ is implementable in dominant strategies.[14]

Finally, we make a remark on the mechanism explained above. It appears that the action m_i^2 is irrelevant. This is due to the assumption that the range of τ is $[0,1]$. To see this, suppose that $r = 1$ and consider the following individual function: $\chi'(i,2) = -\tau(\theta_1)$ if $m_i^1 = m_i^2 = 1$ and otherwise $\chi'(i,2) = 0$. The mechanism defined with $\chi(i,1)$, $\chi'(i,2)$, and $\chi(i,3)$

[14] Note that we ignore Assumption 2 because there is financial support from the government, hence $\chi(i,3) = R$ is possible.

achieves the same outcome we stated above. We can interpret this modified mechanism as a *current account* on Day 1 but that it pays *interest* on Day 2. However, we should recall that this interest-payment mechanism works well because the government supports the bank with deposit insurance. Without deposit insurance, Proposition 1 implies that the mechanism cannot prevent bank runs.

4 Psychological environment

4.1 Identity

We now introduce the "identity" of depositors into the material environment $\mathcal{E}$. Identity is based on *social categories*, $\mathbf{C}$. In addition, there is a *prescription*, $\mathbf{P}$, for each social category that determines personal principles.[15] We let $\mathbf{C} = \{\alpha, \beta\} \ni \boldsymbol{c}_j$, where α is the "socially conscious category" of depositors and β is the "self-interested category" of depositors. The socially conscious depositor desires a socially efficient outcome from the deposit contract. He/she also knows the fragility of the efficient deposit contract in the sense that a synchronized early withdrawal breaks the contract and appreciates that such a breakdown implies inefficiency in society. Thus, the socially conscious depositors hesitate to withdraw their deposits before maturity if they are late depositors.[16] We give a prescription $\mathbf{P}^*$ on our material environment $\mathcal{E}$ such that a person in category α should avoid early withdrawals when his/her type is late and that a person in category β should behave self-interestedly. Hereafter, we refer to socially conscious depositors as α *depositors* and self-interested depositors as β *depositors*. Let

[15] "Prescriptions $\mathbf{P}$ indicate the behavior appropriate for people in different social categories in different situations. The prescriptions may also describe an ideal for each category in terms of physical characteristics and other attributes." (Akerlof and Kranton (2000, p. 718)).

[16] Bankers, financial scholars, and those that have attended a lecture on the "microeconomics of banking" at a university are typical persons included in our socially conscious category of depositors. They are therefore sufficiently educated to be aware that synchronized early withdrawal from a bank brings about a bad result as far as society is concerned.

$D(\alpha) = \{j \in D \mid \mathbf{c}_j = \alpha\}$ denote the set of α depositors (the description of the set of β depositors is similar). By definition, $D = D(\alpha) \cup D(\beta)$ and $D(\alpha) \cap D(\beta) = \emptyset$. Here, we make an assumption concerning the existence of depositors to avoid a trivial issue.

Assumption 3 *Let* $D(l, \alpha) = D(l) \cap D(\alpha)$. *(i) If* $\theta \in [0, 1)$, *then* $\mathcal{L}(D(l, \alpha)) > 0$. *(ii) For all* $\eta \in \{\alpha, \beta\}$, *if* $D(l, \eta) \neq \emptyset$, *then* $\mathcal{L}(D(l, \eta)) > 0$.

Assumption 3(i) implies that if there are late depositors, some of them are α-late. Assumption 3(ii) implies that if an α depositor turns out to be type-l, then he/she is sure that there are many α-late depositors. Similar to Assumption 1, Assumption 3 excludes a trivial case where the set of α-late depositors has a zero Lebesgue measure. Notice that Assumption 3 also implies that if a β depositor finds him/herself to be of type-l, then he/she is sure not only there are many type-l depositors, but also some of them are α-late.

Given a mechanism $\mathcal{M}$, the payoff function of depositor j is denoted as

$$\hat{U}_j(g(m), \omega_j, I_j),$$

where $I_j = I_j(m, \omega_j; \mathbf{c}_j, \mathbf{P})$, which we refer to as the *identity function*.[17] Here we assume that the payoff function takes the following form:

$$\hat{U}_j(g(m), \omega_j, I_j) = U_j(g(m), \omega_j) + I_j(m, \omega_j; \mathbf{c}_j, \mathbf{P}), \tag{4}$$

where $U_j(\cdot, \cdot)$ is the same as in (2). We refer to the material environment in which preferences are defined by (4) as the *psychological environment* $\mathcal{E}_p$.

4.2 Psychological costs

We now introduce the result in Keizer et al. (2008) into our identity function. The point is that *once people observe a signal of norm violations, they are*

[17]This modeling follows Akerlof and Kranton (2000). They additionally consider another characteristic ϵ_j for the description of how close to (far from) the ideal of person j's category, but we ignore such a characteristic because our main result is unchanged with or without it.

likely to violate norms. We model this phenomenon as a reduction of the *psychological cost* against violating norms. In our banking model, the norm corresponds to the prescription of categories. Here, we consider the following psychological preference: if an α-late depositor i attempts early withdrawal, then the depositor i incurs a *cost* against early withdrawal, but *this cost lessens if the depositor i recognizes that some other depositors have already violated the norm.*

In order to state this property formally and to use it effectively, we consider a three-phase mechanism $\mathcal{M}^*$ such as in Section 3.2. Then, we introduce the number τ_i such that

$$\tau_i \equiv \min\left\{k \in \{1,2,3\} \mid m_i^k = 1\right\},$$

which is the smallest number of days for depositor i such that i tenders a withdrawal. If $m_i^k = 0$ for all $k \in \{1,2,3\}$, we let $\tau_i = 3$.

Assumption 4 *Given the prescription $\mathbf{P}^*$ and $\mathcal{M}^*$, for all $\theta \in [0,1]$, $j \in D$, and $m \in M$, the identity function of depositor j is as follows.*

1. *If m is such that $\tau_j = 3$, then $I_j(m, l; \alpha, \mathbf{P}^*) = 0$.*

2. *If m is such that $\tau_j \leq \tau_i < 3$ for all $i \in D$, then $I_j(m, l; \alpha, \mathbf{P}^*) = -d_1 < 0$.*

3. *If m is such that there exists $i \in D \setminus \{j\}$ such that $\tau_i < \tau_j < 3$, then $I_j(m, l; \alpha, \mathbf{P}^*) = -d_2 > -d_1$ where $d_2 \geq 0$.*

4. *$I_j(m, \cdot; \beta, \mathbf{P}^*) \equiv 0$.*

Properties 1 and 2 in Assumption 4 imply that a late depositor i is better off with respect to the identity function if he/she does not tender an early withdrawal. In addition, Properties 2 and 3 imply that a late depositor i is better off with respect to the identity function if he/she waits to tender his/her early withdrawal until some other depositors tender a withdrawal. Property 3 represents the cost reduction referred to above. Property 4 implies that β depositors do not feel any psychological motive. Later, we will note that the costs (d_1, d_2) can be taken as being *arbitrarily small* as long as $d_1 > d_2 \geq 0$.

5 Main results

5.1 Solution concepts

Given a prescription $\mathbf{P}$ and a mechanism $\mathcal{M}$, we consider a game $\hat{G} = (\{\hat{U}_i\}_{i \in D}, \mathcal{M}, \mathcal{E}_p)$ under the psychological environment $\mathcal{E}_p$. Let $s_i(\cdot) \equiv s_i(\cdot; \mathbf{c}_i) : \{e, l\} \to M_i$ denote a pure *strategy* of depositor i. We denote by S_i the set of pure strategies of depositor i.

A message m'_i is *strictly dominated* against $\hat{M}_{-i} \subset M_{-i}$ if, for all $m_{-i} \in \hat{M}_{-i}$, there exists a message m''_i that satisfies

$$\hat{U}_i(g(m''_i, m_{-i}), \omega_i, I_i) > \hat{U}_i(g(m'_i, m_{-i}), \omega_i, I_i).$$

A message m_i is *generated* by $\hat{S}_i$ if there exist $s_i \in \hat{S}_i$ and $\omega_i \in \{e, l\}$ such that $m_i = s_i(\omega_i)$. Similarly, the set of messages $\hat{M}_i$ is generated by $\hat{S}_i$ if

$$\hat{M}_i = \{m_i \in M_i \mid \exists s_i \in \hat{S}_i,\ \exists \omega_i \in \{e, l\},\ m_i = s_i(\omega_i)\}.$$

In the same way, we can define a message profile m that is generated by $\hat{S}$ and the set $\hat{M}_{-i}$ that is generated by $\hat{S}_{-i}$.

A strategy $s'_i \in S_i$ is *strictly dominated* against $\hat{S}_{-i} \subset S_{-i}$ if, for some ω_i, the message $s'_i(\omega_i)$ is strictly dominated against $\hat{M}_{-i}$ that is generated by $\hat{S}_{-i}$. Consider a sequence of sets indexed by $k \in \mathbf{N} \equiv \{0, 1, 2, \cdots\}$, or $(S_i^k)_{k \in \mathbf{N}}$, such that (i) $S_i^0 = S_i$, (ii) $S_i^k \subseteq S_i^l$ if $k \geq l$, and (iii) any $s'_i \in S_i^l \backslash S_i^k$, if it exists, is strictly dominated against S_{-i}^l. We refer to the sequence that satisfies properties (i) – (iii) as a *deletion sequence.* A strategy s_i^* is *iteratively undominated* if there exists a deletion sequence such that $s_i^* \in \bigcap_{k \in \mathbf{N}} S_i^k$.

In the next section, we show that an efficient contract is implementable in *iteratively undominated strategies* under psychological environment $\mathcal{E}_p$, while in the Appendix, we show that any efficient contract cannot be implementable in *ex post equilibrium* under material environment $\mathcal{E}$.

5.2 Implementation of efficient contracts

We employ the three-phase mechanism $\mathcal{M}^*$ referred to in Section 4.2. Let $\hat{G}^* = (\{\hat{U}_i\}_{i \in D}, \mathcal{M}^*, \mathcal{E}_p)$ denote the game that we consider in this section. The goal is to show that efficient contracts are implementable in iteratively undominated strategies on $\hat{G}^*$. The outcome function of $\mathcal{M}^*$ we define plays the following roles. On Day 1, the mechanism gathers information about θ, say θ_1; on Day 2, it provides efficient consumption on the basis of θ_1 for a certain number of depositors; and on Day 3, it provides efficient consumption under θ_1 for late depositors, but only as long as consumption remains. The key in our mechanism is not to return all deposits at one time.

Theorem 1 *In the psychological environment $\mathcal{E}_p$ with Assumptions 2, 3, and 4, any efficient contract $(c_e^*(\cdot), c_l^*(\cdot)) \in C$ is implementable in iteratively undominated strategies, where*

$$C = \{\boldsymbol{c}^* \mid c_e^*(0) \in (0, R)\}.$$

Remark. The result is valid when the psychological costs d_1 and d_2 are *arbitrarily small* as far as $d_1 > d_2 \geq 0$.

Proof. We consider the following three-phase mechanism $\mathcal{M}^*$.

Day 1: Let θ_1 denote the number of depositors such that $m_i^1 = 1$, or $\theta_1 = \mathcal{L}(\{i \in D \mid m_i^1 = 1\})$. For depositor i with $m_i^1 = 1$, the individual provision function is

$$\chi(i,1) = \begin{cases} c_e^*(\theta_1) & \text{with probability } p \\ 0 & \text{with probability } 1-p, \end{cases}$$

where $p \in (0,1)$ is such that $2pu(R) < d_1 - d_2$. For depositor i with $m_i^1 = 0$, $\chi(i,1) = 0$.

Day 2: Let θ_2 denote the number of depositors such that $m_i^2 = 1$. For any depositor i who is not served on Day 1, if $m_i = (1, 1, \cdot)$, then

$$\chi(i,2) = \begin{cases} c_e^*(\theta_1) & \text{if } \theta_2 \leq (1-p)\theta_1 \\ 0 & \text{otherwise;} \end{cases}$$

if $m_i = (0, 1, \cdot)$, then

$$\chi(i,2) = \begin{cases} c_e^*(\theta_1) - \epsilon & \text{if } \theta_2 \leq (1-p)\theta_1 \\ 0 & \text{otherwise,} \end{cases}$$

where $\epsilon > 0$ is small enough such that $(1-p)u(c_e^*(\theta_1)) < u(c_e^*(\theta_1) - \epsilon)$. For the other depositors, $\chi(i, 2) = 0$.

Day 3: For any depositor i who is not served on Days 1 and 2, if $m_i^3 = 1$,

$$\chi(i,3) = \begin{cases} c_l^*(\theta_1) & \text{if } \theta_1 < 1 \\ 0 & \text{if } \theta_1 = 1. \text{ (Because of Assumption 2.)} \end{cases}$$

For the other cases, $\chi(i, 3) = 0$.

We show that the strategy s_i^* such that $s_i^*(l;\alpha) = s_i^*(l;\beta) = (0, 0, 1)$ and $s_i^*(e;\alpha) = s_i^*(e;\beta) = (1, 1, 0)$ is an iteratively undominated strategy. The profile of s_i^* realizes $\theta_1 = \theta$ and each early and late depositor consumes $c_e^*(\theta)$ and $c_l^*(\theta)$, respectively.

Let $\Gamma(m)$ be an indicator function such that

$$\Gamma(m) = \begin{cases} 1 & \text{if } \theta_2 \leq (1-p)\theta_1 \\ 0 & \text{otherwise.} \end{cases}$$

(1. α-late depositors): Suppose that a depositor i finds him/herself α-late. First, suppose that depositor i expects $\theta_1 > 0$, that is, a lot of other depositors tender a withdrawal on Day 1. If $m_i = (1, 1, \cdot)$, his/her payoff is

$$\hat{U}_i(g(m), l, I_i) = pu(c_e^*(\theta_1)) + (1-p)\Gamma(m)u(c_e^*(\theta_1)) - d_1.$$

If depositor i changes his/her message to $\tilde{m}_i = (0, 1, \cdot)$, then

$$\hat{U}_i(g(m\backslash\tilde{m}_i), l, I_i) = \Gamma(m\backslash\tilde{m}_i)u(c_e^*(\theta_1) - \epsilon) - d_2.$$

Because of the continuum depositors, $\Gamma(m) = \Gamma(m\backslash\tilde{m}_i)$ and m_{-i} determines θ_1 and θ_2. Hence, for all $m_{-i} \in M_{-i}$, we obtain

$$\begin{aligned}
&\hat{U}_i(g(m), l, I_i) - \hat{U}_i(g(m\backslash\tilde{m}_i), l, I_i) \\
&\quad = p(1 - \Gamma(m))u(c_e^*(\theta_1)) + \Gamma(m)\big(u(c_e^*(\theta_1)) - u(c_e^*(\theta_1) - \epsilon)\big) - (d_1 - d_2) \\
&\quad < pu(c_e^*(\theta_1)) + u(c_e^*(\theta_1)) - u(c_e^*(\theta_1) - \epsilon) - (d_1 - d_2) \\
&\quad < 2pu(c_e^*(\theta_1)) - (d_1 - d_2) \\
&\quad < 0,
\end{aligned} \tag{5}$$

where the first inequality is obtained by letting $\Gamma(m) = 0$ in the first term and $\Gamma(m) = 1$ in the second term of the second line in Equation (5); the second inequality is owing to the assumptions on p and ϵ in the Day 1 and 2 rules, respectively, and the third inequality results from the assumption on p and $d_1 - d_2$, and the presumption that $c_e^*(\theta_1) < R$ for all $\theta_1 \in [0, 1]$. Thus, the message $m_i = (1, 1, \cdot)$ cannot be optimal in this case.

Next, suppose that depositor i expects $\theta_1 = 0$, that is, no other depositors or at most zero-measured depositors tender a withdrawal on Day 1. Then, depositor i believes that there is no provision on Day 2 and that he/she can be definitely served $c_l^*(0) = R$ on Day 3, which is greater than $c_e^*(0)$. Thus, the message $m_i = (1, \cdot, \cdot)$ of any α-late depositor i is strictly dominated against M_{-i}. We define a subset of strategies $S_i^1 \equiv \{s_i \in S_i \mid s_i(l; \alpha) = (0, \cdot, \cdot)\}$.

(2. β-late depositors): Suppose that $S^1 \equiv \prod_{i \in D} S_i^1$ is given and depositor i finds him/herself β-late. Then, by Assumption 3, depositor i deduces that there are α-late depositors and $\theta_1 < 1$ is certain. Hence, he/she can definitely be served $c_l^*(\theta_1)$ on Day 3. Thus, for any β-late depositor i, the messages other than $m_i = (0, 0, 1)$ are strictly dominated against M_{-i}^1 generated by S_{-i}^1. We define a subset of strategies $S_i^2 \equiv \{s_i \in S_i^1 \mid s_i(l; \beta) = (0, 0, 1)\}$.

(3. α-late depositors): Suppose that S^2 is given and depositor i finds him/herself α-late. Then, using the same reasoning as for β-late depositors, the messages of α-late depositor i other than $m_i = (0,0,1)$ are strictly dominated against M^2_{-i}. We define a subset of strategies $S_i^3 \equiv \{s_i \in S_i^2 \mid s_i(l;\alpha) = (0,0,1)\}$.

(4. Early depositors): Suppose that S^3 is given and depositor i finds him/herself early. If $m_i = (1,1,\cdot)$, then his/her payoff is

$$pu(c_e^*(\theta_1)) + (1-p)\Gamma(m)u(c_e^*(\theta_1)).$$

If depositor i changes his/her message to $\tilde{m}_i = (0,1,\cdot)$, then his/her payoff is $\Gamma(m\backslash\tilde{m}_i)u(c_e^*(\theta_1) - \epsilon)$. Then,

$$\begin{aligned} &U_i(g(m),e) - U_i(g(m\backslash\tilde{m}_i),e) \\ &\qquad = pu(c_e^*(\theta_1))\,(1-\Gamma(m)) + \Gamma(m)\big(u(c_e^*(\theta_1)) - u(c_e^*(\theta_1)-\epsilon)\big). \end{aligned} \tag{6}$$

Equation (6) is strictly positive regardless of $\Gamma(m)$ because $c_e^*(\theta_1) > 0$ for all $\theta_1 \in [0,1]$. Thus, the message $\tilde{m}_i = (0,1,\cdot)$ of early depositor i is strictly dominated by a message $m_i = (1,1,\cdot)$ against M^3_{-i} generated by S^3_{-i}. Also, it is easy to see that message $m_i' = (0,0,\cdot)$ is strictly dominated by $m_i = (1,1,\cdot)$. We define a subset of strategies $S_i^4 \equiv \{s_i \in S_i^3 \mid s_i^1(e;\cdot) = 1\}$. Any message profile generated by S^4 implies that (a) all early depositors choose $m_i^1 = 1$; (b) all late depositors choose $m_i^1 = m_i^2 = 0$; and hence, (c) $\Gamma(m) = 1$ is certain for all $m \in M^4$. Hence, message $m_i = (1,1,\cdot)$ brings payoff $u(c_e^*(\theta_1))$ to early depositor i. Given the set S^4, if depositor i changes his/her message to $\tilde{m}_i = (1,0,\cdot)$, then his/her payoff is $pu(c_e^*(\theta_1))$. Thus, for early depositor i, the message $m_i = (1,0,\cdot)$ is strictly dominated against M^4_{-i}.

We define the set $S_i^5 \equiv \{s_i \in S_i^4 \mid s_i(e;\cdot) = (1,1,\cdot)\} = \{s_i(e;\cdot) \in \{(1,1,1),(1,1,0)\}, s_i(l;\cdot) = (0,0,1)\}$. Consider the sequence of the sets of strategies, say $(S_i^k)_{k\in\mathbf{N}}$, such that $S_i^0 = S_i$; S_i^1, S_i^2, S_i^3, S^4, and S^5 are the same as defined above; $S_i^k = S_i^5$ for all $k \geq 5$. Then, $(S_i^k)_{k\in\mathbf{N}}$ is a deletion sequence and any $s_i \in \bigcap_{k\in\mathbf{N}} S_i^k = S_i^5$ is an iteratively undominated strategy. For any $\theta \in [0,1]$ and any $s \in S^5$, $g(s(\omega))$ uniquely realizes the consumption

$c_e^*(\theta)$ for all early depositors and $c_l^*(\theta)$ for all late depositors. ■

An early depositor i is indifferent as to whether $s_i(e) = (1, 1, 1)$ or $s_i(e) = (1, 1, 0)$ if all late depositors follow the undominated action. Hence we obtain the following fact as a corollary of Theorem 1.

Corollary 2 *In the same environment as Theorem 1, any efficient contract* $\boldsymbol{c}^* \in C = \{\boldsymbol{c}^* \mid c_e^*(0) \in (0, R)\}$ *is implementable in ex post equilibrium.*

Remember that any efficient contract cannot be implementable in ex post equilibrium if there are no socially conscious depositors.

One may wish to model the situation we consider as a *dynamic game* because Keizer et al. (2008) report that people tend to violate norms if they *observe* a sign of norm violations. It is easy to expand our model to a dynamic situation. Suppose that at the commencement of Days 2 and 3 in the mechanism $\mathcal{M}^*$, the bank makes public θ_1 and θ_2, respectively. In the game with this mechanism, we can show that efficient contracts are implementable in *subgame perfect equilibrium.*

Corollary 3 *In the same environment as Theorem 1, any efficient contract* $\boldsymbol{c}^* \in C = \{\boldsymbol{c}^* \mid c_e^*(0) \in (0, R)\}$ *is implementable in subgame perfect equilibrium.*

Proof. See the Appendix. ■

5.3 Discussion

Our affirmative result is valid for *any* $d_1 > 0$. When $d_1 > 0$, we can select a $p \in (0, 1)$ that meets $0 < 2pu(R) < d_1 - d_2$. Hence the infimum of cost d_1 is zero. This observation implies that the psychological costs can be *arbitrarily small* when compared with the consumption of the initial endowment, that is,

$$u(1) - d_1 > u(0) = 0. \tag{7}$$

Equation (7) excludes a trivial case where α-late depositors do not tender any early withdrawal before period t_2 owing to a large psychological cost, which appears to be a plausible assumption in our psychological model. This "small cost allowance," however, is not observed in the prototype model of Akerlof and Kranton (2000, Section III), in which identity changes the decision of economic agents only if identity occupies a large part of the utilities.

It is important for our affirmative result to take p such that $p \in (0,1)$. If $p = 1$, Day 2 has no meaning; the mechanism provides $c_e^*(\theta_1)$ on Day 1 and $c_l^*(\theta_1)$ or 0 on Day 3. In this case, however, the strategy profile s', such that $s'_i = (1,1,1)$ for all $i \in D$, constitutes an ex post equilibrium if the psychological costs are sufficiently small to satisfy Equation (7). If $p = 0$, depositors are not served on Day 1. In this case, Equation (6) can be equal to zero if $\Gamma(m) = 0$, where $\Gamma(m) = 0$ is possible for some message profiles m generated by S^3. Hence, we fail to obtain deletion sequences. The nondegenerate probability p therefore plays a crucial role in our mechanism. In plain words, the nondegenerate probability means that the bank should not return the deposits at one time. Even if we use identity, a one-time whole payback may cause self-fulfilling bank runs without large psychological costs.

Finally, we comment on the two-day separation in our payments. The two-day separation can be interrupted as a "moratorium." This moratorium results from the assumption that banks do not know θ exactly in advance. Because our mechanism is independent of the distribution of θ, this moratorium is required for all cases where the bank does not know θ exactly, even if the bank has its expectation. The only exception, as we stated in Section 3.2, is where the bank knows θ exactly in advance.

6 Concluding remarks

This paper introduces the identity of agents into a deposit contract model á la Diamond and Dybvig (1983) in an environment with uncertain states without sequential service constraints. We establish that bank runs can be prevented while achieving a fully efficient outcome, which has been hitherto thought to be impossible. Our affirmative result is also obtained without any form of government intervention, such as deposit insurance. In addition, we formalize the environment in which bank runs can occur and show that any efficient deposit contract cannot be implementable in ex post equilibrium if all depositors are self-interested.

Identity plays a crucial role in our main result. This paper newly sheds light on *motivation loss* in human psychology and introduces it into an identity model in the form of a reduction in psychological costs. By using this motivation loss effectively, our affirmative result holds even if their utilities in the identity are arbitrarily small (as long as they are positive). Hence our result establishes an example in which "small identity" drastically changes a classic known result. This fact and our impossible result in the material environment suggests that it is beneficial to consider the psychological aspect of depositors when designing a deposit contract.

7 Appendix

7.1 An impossibility result in material environment $\mathcal{E}$

Pick an arbitrary efficient contract $\boldsymbol{c}^*$ and consider a game with the direct mechanism relative to $\boldsymbol{c}^*$, that is, $G^{\boldsymbol{c}^*}$. As shown in Section 3, the truth-telling strategy profile $s^*(\omega) = \omega$ constitutes an ex post equilibrium in $G^{\boldsymbol{c}^*}$. Then, the efficient contract $\boldsymbol{c}^*$ is said to be *ex post incentive compatible.* Because a profile of messages determines $\hat{\theta}$, we hereafter denote an allocation as $\boldsymbol{c}^*(s(\omega)) = (c_e^*(s(\omega)), c_l^*(s(\omega)))$.

We know that there exists an inefficient ex post equilibrium in $G^{\boldsymbol{c}^*}$, but such an equilibrium profile may be eliminated by considering a game with some *indirect* mechanism. If $\boldsymbol{c}^*$ is implementable in ex post equilibrium (with some indirect mechanism), then it is known that $\boldsymbol{c}^*$ necessarily satisfies the following *ex post monotonicity* condition (Bergemann and Morris, 2008).

Definition 3 *An efficient contract $\boldsymbol{c}^*$ satisfies* ex post monotonicity *if, for all type reporting strategy profile s in $G^{\boldsymbol{c}^*}$, if $\boldsymbol{c}^* \circ s \neq \boldsymbol{c}^* \circ s^*$, then there exist*

$i \in D$, $\omega \in \{e, l\}^D$, and $y_i \in Y$, such that

$$U_i(y_i, \omega_i) > U_i(\boldsymbol{c}^* \circ s(\omega), \omega_i), \tag{8}$$

while for all $\omega_i' \in \{e, l\}$,

$$U_i(\boldsymbol{c}^*(s^*(\omega)\backslash\omega_i'), \omega_i') \geq U_i(y_i, \omega_i'). \tag{9}$$

Claim 1. *The efficient contract $\boldsymbol{c}^*$ does not satisfy ex post monotonicity.* To confirm this, consider an inefficient strategy profile s' such that $s_i'(\omega_i) = e$ for all $i \in D$ and $\omega_i \in \{e, l\}$, which realizes $\hat{\theta} = 1$ for all ω. Then, $\boldsymbol{c}^*(s'(\omega)) = (1, 0)$ for all ω because of Assumption 2. Thus the outcome y_i in Equation (8) must be $y_i = \chi^1(i) \in Y^1$ and $y_i > 1$. However, there exists a state θ' such that $\theta' < 1$ and $y_i > c_e^*(\theta') > 1$ because $\lim_{\theta \to 1} c_e^*(\theta) = c_e^*(1) = 1$. Hence, depositor i prefers y_i to $c_e^*(\theta')$ whenever his/her type is early, which contradicts Equation (9) when $\omega_i' = e$ and $\omega = (\omega_i', \omega_{-i})$ realizes θ'.

Claim 1 implies that any efficient contract cannot be ex post implementable, hence we obtain a proof of Proposition 1. It is obvious that this impossibility hinges on Assumption 2. However, without Assumption 2, any banking model in the material environment $\mathcal{E}$, including Diamond and Dybvig (1983), does not have inefficient bank-run outcomes. To see this, suppose that we discard Assumption 2 and consider the following assumption.

Assumption 5 *For all $A \subset D(l)\backslash\emptyset$ and for any provision function $\hat{\chi}^1 \in Y^1$ that gives $z = 1$, if $\mathcal{L}(A) = 0$, then there exists $\chi^2 \in Y^2(\hat{\chi}^1)$ such that $\chi^2(r) > 1$ for all $r \in A$.*

Claim 2. *If we make Assumption 5 instead of Assumption 2, then there is no bank-run equilibrium in material environment $\mathcal{E}$.* To see this, we pick an individual provision function χ^2 such that $\chi^2(i) > 1$ for any depositor $i \in A$ and identify $\chi^2(i)$ with y_i of Equation (8). In order to make y_i satisfy Equation (9) also, we must have $c_l^*(1) \geq \chi^2(i)$. Hence, an efficient contract $\boldsymbol{c}^*$ must satisfy $c_l^*(1) > 1$. Because $1 = c_e^*(1)$, the efficient contract $\boldsymbol{c}^*$ itself incentivizes depositor i to delay his/her withdrawal whenever his/her type is late, even if all the depositors other than i rush into the bank in t_1. Because we can take A arbitrarily, any late depositor has an unilateral deviation incentive from inefficient strategy profiles in $G^{\mathbf{c}^*}$. Thus, any inefficient strategy profile in $G^{\mathbf{c}^*}$ under Assumption 2 is no longer ex post equilibrium.

7.2 Proof of Corollary 3

The strategies in our mechanism $\mathcal{M}^*$ with public information announcements depends on the information θ_1 and θ_2. For an arbitrary depositor i, given $\mathbf{c}_i$ and ω_i, consider functions $s_i^k : \mathcal{H}^k \to M_i$ for $k = 1, 2, 3$, where $\mathcal{H}^k$ denotes a set of *histories* such that $\mathcal{H}^1 = \{\emptyset\}$ and $\mathcal{H}^{k+1} = \mathcal{H}^k \times (M_i \times [0,1])$, where $M_i \times [0,1] \ni (m_i, \theta_k)$ for $k = 1, 2$. Let $s_i : \{e, l\} \times \cup_{k=1}^3 \mathcal{H}^k \to M_i$ denote the *pure strategy* of depositor i. We define functions $s_i|_{h^1} = s_i^1$, $s_i|_{h^2} : \cup_{k=1}^2 \mathcal{H}^k \to M_i$, and $s_i|_{h^3} : \mathcal{H}^1 \to M_i$, which induce an action plan after an arbitrary history is observed.

Definition 4 *A strategy profile s is a* subgame perfect equilibrium *in $\hat{G}$ if*

$$\forall i, \forall \omega, \forall h^k, \forall s_i'|_{h^k} : U_i(g(s|_{h^k}), \omega_i) \geq U_i(g(s|_{h^k} \backslash s_i'|_{h^k}), \omega_i).$$

We consider a state-dependent strategy σ_i such that (1) if i is late, then $m_i^1 = 0$ and on Days 2 and 3, $m_i^2 = 0$ and $m_i^3 = 1$ respectively if and only if $\theta_1 < 1$ is observed, otherwise $m_i^2 = 1$; (2) if i is early, then $m_i^1 = m_i^2 = 1$. We first show that σ constitutes a subgame perfect equilibrium.

(Day 3): There remain only late depositors who have not been served until now. If a depositor i observes $\theta_1 < 1$, then $m_i^3 = 1$ is a dominant strategy. If $\theta_1 = 1$, then it is indifferent for depositor i to choose $m_i^3 = 1$ or $m_i^3 = 0$. Hence $s_i|_{h^3}(\emptyset) = 1$ is optimal for all h^3 if i is late and it brings his/her a payoff $u_i(c_l^*(\theta_1))$.

(Day 2): For an early depositor j, $m_j^2 = 1$ is a (weakly) dominant strategy regardless of θ_1. For a late depositor i, if $\theta_1 < 1$, then $m_i^2 = 0$ is optimal; if $\theta_1 = 1$, then $m_i^2 = 1$ is optimal.

(Day 1): For an early depositor j, if $m_j^1 = 1$, then he/she gets $pu(c_e^*(\theta_1)) + (1-p)\Gamma(m)u(c_e^*(\theta_1))$; if $m_j^1 = 0$, $pu(c_e^*(\theta_1)) + (1-p)\Gamma(m)u(c_e^*(\theta_1) -$

ϵ). Hence $m_j^1 = 1$ is a (weakly) dominant strategy. A late depositor i knows $\theta < 1$ because of Assumption 3. Given σ_{-i}, $m_i^1 = 0$ is optimal.

These observations imply that σ constitutes a subgame perfect equilibrium. Next we show that there is no other subgame perfect equilibrium in this game. This is quite simple. Note that late depositors obtain the consumption $c_l^*(\theta_1)$ that is preferable to $c_e^*(\theta_1)$ as long as $\theta_1 < 1$. Hence we only have to show that there is no optimal strategy such that a late depositor chooses $m_i^1 = 1$. In order to show that, we can apply the reasoning in the proof of Theorem 1, where $m_i^1 = 1$ is strictly dominated for any α-late depositor i, which implies $\theta_1 < 1$ is assured.

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