# Provision Scheme Design for Run-Preventing Contracts

OHASHI Yoshihiro*
Center for Finance Research, Waseda University
5th Floor, Nihombashi 1-chome Bldg.,
1-4-1 Nihombashi, Chuo-ku, Tokyo, 103-0027, Japan
Tel: +81-3-3272-6782
Fax: +81-3-3272-6789

October 13, 2011

**ABSTRACT**

This paper shows that it is possible to uniquely implement the efficient consumption of a financial intermediation model with a liquidation cost. While taking into account a sequential service constraint, I formulate a bank provision scheme to achieve the efficient outcome while preventing bank runs. The provision scheme and the obtained contract depend on neither liquidation costs nor the relative risk aversion coefficients of depositors.

## 1 Introduction

The prevention of bank runs is practically and theoretically the central issue in the context of financial intermediation by banks. In the model of financial

*This work was supported by MEXT Grant-in-Aid for Research Activity Start-up (22830103). yohashi@aoni.waseda.jp

intermediation, we are interested in whether an efficient outcome can be achieved while preventing bank runs. However, many studies have shown negative results in this regard.[1]

This paper revisits a contract design problem of deposit contracts by focusing on a payment policy by banks. The main result states that there exists a deposit contract that can achieve an ex ante efficient outcome while preventing bank runs *without deposit insurance*[2].

The key for this affirmative result lies in *provision scheme* design, where a provision scheme corresponds to a bank payment policy. While taking into account a sequential service constraint (also known as a "first-come, first-served" constraint), I construct a provision scheme that corrects the information on the number of depositors that have been served so far and reflects the information in current payments. In a game induced by the provision scheme, I show that an efficient outcome is uniquely achieved in a strictly "self-selective" strategy equilibrium.

The provision scheme I propose imposes a *refund cap* on one-time withdrawals. Such a cap allows banks to acquire information on how many depositors urgently need consumption, without fear for bankruptcy. On the basis of this the information, banks provide efficient consumption. As a result, the demand deposit contract must be similar to a *time deposit* contract in that it has a refund cap when we seek to achieve the efficient outcome while preventing bank runs.

This paper contributes to the literature on *run-preventing contracts*, a concept introduced by Cooper and Ross (1998). A run-preventing contract ensures incentive compatibility for all depositors. Cooper and Ross (1998) extend the Diamond–Dybvig (1983) model and investigate when a contract has the run-preventing property. They state that an efficient contract can

[1]For example, Diamond and Dybvig (1983) show that a *deposit freeze* policy can prevent bank runs. However, Engineer (1989) shows that such a deposit freeze policy fails to work when there is uncertainty on depositors' type. Also, Ennis and Keister (2009) show that a bank cannot ensure the prevention of bank runs if the bank's deposit freeze policy takes into account *ex post* efficiency.

[2]Diamond and Dybvig (1983) show that *deposit insurance* solves the unique implementation problem. However, the deposit insurance causes a moral hazard problem. See Cooper and Ross (2002) and Martin (2006), for instance.

be a run-preventing contract if both the relative risk aversion coefficient of depositors and a liquidation cost for a long-term investment are sufficiently small. These restriction can be attributed to the fact that they only consider a "constant" contract, that is, a contract that aims to provide a certain outcome regardless of the number of withdrawal tenders. In contrast, in this paper, I consider a wider class of contracts and show that a well-designed provision scheme can implement a contract that achieves the efficient outcome in equilibrium, regardless of the relative risk aversion coefficients and liquidation costs.

This paper is organized as follows. Section 2 sets up the basic framework of the model on the basis of Cooper and Ross (1998). Section 3 introduces several definitions, including that of a provision scheme and a sequential service constraint. Section 4 presents the main result. I also refer to a robustness property on the obtained contract with our provision scheme by comparing it with an equilibrium outcome under a "deposit freeze" policy. Section 5 presents the concluding remarks. All proofs are presented in the Appendix.

# 2 Preliminaries

The model used in the paper is based on the Cooper–Ross (1998) model. Consider an economy with a single consumption good, a representative bank, and depositors. The bank implicitly faces competition with other banks, and its profit is assumed to be zero. The economy has three event phases: $E_0, E_1$, and $E_2$. The consumption good has two opportunities: storage and investment. Storage yields one unit of the consumption good in $E_1$ per unit input in $E_0$. Investment yields $R > 1$ units of the consumption at a maturity time in $E_2$ per unit input in $E_0$. Premature liquidation of the investment yields $1 - \kappa$ in $E_1$ per unit input in $E_0$, where $\kappa \in [0, 1]$.

Depositors are identical in $E_0$ and are represented by the continuum $I := [0, 1]$. Each depositor has one unit consumption good as an endowment, which it deposits in the representative bank in $E_0$. Some time after depositing and investing, say, at the beginning of $E_1$, a fraction of the depositors, $\theta \in$

$(0, 1)$, which we call *early depositors*, face a liquidity shock. The remaining depositors, $1 - \theta$, which we call *late depositors*, do not face a liquidity shock. We refer to $\theta$, which I assume to be non-stochastic, as a *type state*. Early depositors must exit from the market by a time $T$ in $E_1$, but late depositors can exist in the market after $T$ and the maturity time in $E_2$.

Whether a depositor is early or late is his private information. Early depositors value consumption in $E_1$, while late depositors value consumption in both $E_1$ and $E_2$. Let $c_n$ denote the consumption of depositors in $E_n$ $(n = 1, 2)$. The payoffs of the depositor $i$ are given by

$$U_i(c_1, c_2) = \begin{cases} u(c_1) & \text{if } i \text{ is an early depositor} \\ u(c_1 + c_2) & \text{if } i \text{ is a late consumer,} \end{cases} \tag{1}$$

where $u$ in (1) is a von Neumann–Morgenstern utility function $u : \mathbb{R}_+ \to \mathbb{R}_+$, which is strictly increasing, strictly concave, and twice differentiable, and which satisfies $u(0) = 0$ and $\lim_{c \to 0} u'(c) = \infty$.

The bank chooses a portfolio $(\psi_E, \psi_L)$ that maximizes the expected social welfare, where subscript $E$ $(L)$ means early (late). The ex ante efficient consumption is defined as the solution of the following optimization problem:

$$\begin{aligned} \max_{c_E, c_L, \psi_E, \psi_L} \quad & \theta u(c_E) + (1 - \theta) u(c_L) \\ s.t. \ & \psi_E + \psi_L = 1 \\ & \theta c_E \leq \psi_E \\ & (1 - \bar{\theta}) c_L \leq R \psi_L \end{aligned} \tag{2}$$

In the optimum, we obtain $\psi_E^* = \theta c_E^*$, $c_L^* = R(1 - \theta c_E^*)(1 - \theta)^{-1}$, and $c_E^* < c_L^*$.

# 3 Definitions

Our goal is to show that the optimal outcome in (2) is uniquely achieved. We focus on the incentives of depositors in $E_1$ without loss of generality. For simplicity, let us denote $E_1 = [0, T)$ and $E_2 = [T, T']$, where $t = 0$ refers to the time of liquidity shock and $t = T$ refers to the time by which early depositors have completely exited from the market *and* the maturity time of the investment.

### Provision scheme

A provision scheme describes when and how much depositors are served by the bank. To define such a scheme, I first describe the action of the depositors. We denote the action of a depositor $i$ by a function $m_i : [0, T'] \to \{0, 1\}$, where $m_i(t) = 1$ (0) means that depositor $i$ tenders (does not tender) a full withdrawal at time $t$. Let us denote $m_i = (m_i(t))_{t \in [0,T']}$ and $m = (m_i)_{i \in I}$. We denote by $M_i$ the set of all possible actions of depositor $i$. Let $g_i(m, t)$ denote the provision of the good for depositor $i$ at time $t$, where $g_i : \prod_{i \in I} M_i \times [0, 1] \to \mathbb{R}_+$. We refer to $\mathcal{M} = (M_i, g_i(\cdot, \cdot))_{i \in I}$ as a *provision scheme.* A provision scheme induces a game among depositors. Let $s_i : \{E, L\} \to M_i$ denote a strategy of depositor $i$. Strategy $s_i^*$ is a *strictly self-selective strategy* if depositor $i$'s type is $x \in \{E, L\}$, then

$$U_i(g_i(s_i^*(x), m_{-i}, t)) \geq U_i(g_i(m_i, m_{-i}, t)) \tag{3}$$

for all $m_i \in M_i$, $m_{-i} \in M_{-i}$, and $t \in [0, T']$, and if $m_i = s_i^*(y)$, where $y \in \{E, L\} \backslash \{x\}$, then (3) holds strictly for all $m_{-i} \in M_{-i}$ and $t \in [0, T']$. A strategy profile of strictly self-selective strategies is said to be a *strictly self-selective strategy profile.* By definition, a strictly self-selective strategy profile constitutes an (ex-post) Nash equilibrium.

### Sequential service constraint

*Sequential service* in this paper means that the provision from the bank at time $t$ can only depend on the information on what amount the bank has served by $t$ and not on any future information. To begin with, I make the following assumption.

**Assumption 1** *For each $t \in [0, T']$, if there exists a depositor $i$ such that $g_i(\cdot, t) > 0$, then $g_j(\cdot, t) = 0$ for all $j \in I \backslash \{i\}$.*

This assumption states that the bank can pay to at most a single depositor at each time. In addition, I prohibit any "retroactive levy" of banks; that is, banks cannot make depositors pay back any served consumption.

**Assumption 2** *The bank is subject to a* no-retroactive-levies constraint*: If $g_i(\cdot, t) = c_t > 0$, depositor $i$ consumes $c_t$ at time $t$.*

I describe the history (i.e., a sequence of messages) of a message $m_i$ from time 0 to $k$ as $m_i[k] \equiv (m_i(t))_{t\in[0,k]}$. Let $\Psi(t)$ denote the set of the depositors who have already been served positive provision from the bank by time $t$:

$$\Psi(t) \equiv \{j \in I \mid \exists t' < t, \quad g_j(\cdot, t') > 0\}.$$

We introduce a sequential service constraint into provision functions $\{g_i\}_{i\in\mathcal{I}}$ such that the provision at time $t$ does not depend on any messages after time $t$.

**Definition 1** (Sequential service constraint) *For each $i \in I$, $m \in \prod_{i\in I} M_i$, and $t \in [0, T']$,*

$$g_i(m, t) = g_i(m_i[t], (m_j[t])_{j\in\Psi(t)}, t). \tag{4}$$

This sequential service constraint implies that the bank should provide consumption contingent only on the number of withdrawals that have occurred so far.

We should note that the sequential service constraint with Assumption 1 implies that the bank cannot know immediately how many depositors tender at time 0. Suppose that many depositors tender a full withdrawal at time 0. Because of Assumption 1, the bank can provide consumption to a single depositor at time 0 and all the information available to the bank at time 0 is that there is one depositor who tenders a full withdrawal. Furthermore, Assumption 2 makes it impossible for the bank to get the consumption back from early depositors.

## 3.1 Run-preventing contracts and efficiency

Let $\boldsymbol{c} = (c_E(\cdot), c_L(\cdot))$ denote a *contract*. We refer to $\boldsymbol{c}(\theta) = (c_E(\theta), c_L(\theta))$ as *consumption* for the realization $\theta$. Let $c_q^*$ denote the solution of (2) for

$q \in \{E, L\}$ for tractability. Diamond and Dybvig (1983) consider the contract such that $c_E(\hat{\theta}) = c_E^*$ and $c_L(\hat{\theta}) = \max\{R(1-\hat{\theta}c_E^*)(1-\hat{\theta})^{-1}, 0\}$ for any realization $\hat{\theta} \in [0,1]$, where

$$\hat{\theta} = \int_{\{j|m_j(0)=1\}} \mathcal{L}(dj)$$

with the Lebesgue measure $\mathcal{L}$. As is widely known, this contract has an inefficient *bank run* equilibrium.

Cooper and Ross (1998), on the other hand, introduce the concept of *run-preventing contracts.* A run-preventing contract is supposed to ensure incentive compatibility for all depositors under all realizations of type states. In this paper, I define a run-preventing contract as follows.

**Definition 2** *A contract* $\boldsymbol{c}$ *is said to be a* run-preventing contract (RPC) *if there exists a provision scheme* $\mathcal{M}$ *that makes* $\boldsymbol{c}(\theta)$ *a unique equilibrium outcome.*

By the revelation principle (e.g., Mas-Colell et al.(1995)), any RPC $\boldsymbol{c}$ should satisfy $c_E(\hat{\theta}) \leq c_L(\hat{\theta})$ for all $\hat{\theta} \in [0,1]$.

Cooper and Ross (1998) solely consider a "constant" contract. They show that a constant RPC can achieve the efficient outcome only if the liquidation cost and the relative risk aversion coefficient of depositors are sufficiently small. To see this, we first note that a constant RPC $\bar{\boldsymbol{c}} = (\bar{c}_E, \bar{c}_L)$ requires $\bar{c}_E \leq 1 - \kappa\psi_L$ for all $\psi_L \in [0,1]$, which is equivalent to $\psi_E + (1-\kappa)\psi_L \geq \bar{c}_E$. Hence, an efficient constant RPC $\bar{\boldsymbol{c}}^* = (\bar{c}_E^*, \bar{c}_L^*)$ must satisfy

$$\bar{c}_L^* = \frac{R(1-\theta\bar{c}_E^*)}{1-\theta}, \qquad \bar{c}_E^* \leq \frac{1-\kappa}{1-\kappa\theta}. \tag{5}$$

For a given $\kappa \in (0,1)$ and $\theta \in (0,1)$, (5) implies that $\bar{c}_E^* < 1$, which is satisfied only if the relative risk aversion coefficient of depositors is sufficiently small.[3] Furthermore, even if the relative risk aversion coefficient is less than unity, efficient contracts are no longer RPCs whenever $\kappa = 1$, because the efficient

[3]Diamond and Dybvig (1983) show that $c_E^* > 1$ whenever the relative risk aversion coefficient is larger than unity.

outcome must satisfy $c_E^* > 0$.[4] This result is pessimistic because the efficient consumption of the Diamond–Dybvig model must satisfy $c_E^* > 1$. Hence their optimal constant RPC cannot coincide with any efficient outcome in the Diamond–Dybvig model.[5]

# 4 Results

## 4.1 RPC with efficient outcome

We design an RPC that achieves the efficient consumption in equilibrium. First, we consider the consumption bundle $(\hat{c}_E(\hat{\theta}), \hat{c}_L(\hat{\theta}))$, which is the solution of the following problem:

$$\begin{aligned} &\max_{c_E, c_L} \hat{\theta} u(c_E) + (1-\hat{\theta}) u(c_L) \\ &\quad s.t.\ \hat{\theta} c_E \leq \theta c_E^* \\ &\qquad (1-\hat{\theta}) c_L \leq R(1-\theta c_E^*) + \theta c_E^* - \hat{\theta} c_E. \end{aligned} \tag{6}$$

**Lemma 1** *We obtain* $\hat{c}_E(\hat{\theta}) < c_E^* < c_L^* < \hat{c}_L(\hat{\theta})$ *for all* $\hat{\theta} > \theta$.

**Proof.** See the Appendix. ■

Next, we define a contract by using $(\hat{c}_E(\cdot), \hat{c}_L(\cdot))$. Let $\boldsymbol{c}^{**} = (c_E^{**}(\cdot), c_L^{**}(\cdot))$ denote the contract defined as

$$c_E^{**}(\hat{\theta}) = \begin{cases} c_E^* & \text{if } \hat{\theta} \leq \theta \\ \hat{c}_E(\hat{\theta}) & \text{if } \hat{\theta} > \theta \end{cases}, \quad c_L^{**}(\hat{\theta}) = \begin{cases} c'_L(\hat{\theta}) & \text{if } \hat{\theta} \leq \theta \\ \hat{c}_L(\hat{\theta}) & \text{if } \hat{\theta} > \theta \end{cases} \tag{7}$$

where $(\hat{c}_E(\hat{\theta}), \hat{c}_L(\hat{\theta}))$ is the same one in (6) and $(1-\hat{\theta})c'_L(\hat{\theta}) = (1-\theta)c_L^* + (\theta - \hat{\theta})c_E^*$. This contract $\boldsymbol{c}^{**}$ satisfies $c_E^{**}(\hat{\theta}) < c_L^{**}(\hat{\theta})$ for all $\hat{\theta} \in [0,1]$. To see

[4] This statement is Proposition 2 of Cooper and Ross (1998).

[5] Indeed, Cooper and Ross (1998) state that the optimal RPC in the Diamond–Dybvig model is $\boldsymbol{c} = (1, R)$, which is an efficient outcome for the case of the relative risk aversion coefficient being 1. When considering a finite-depositors version of the Diamond–Dybvig model, Green and Lin (2003) show that the optimal RPC uniquely achieves the efficient outcome if we ignore sequential service constraints.

this, we only have to show that $c'_L(\hat{\theta}) > c^*_E$ for all $\hat{\theta} \leq \theta$. The fact $c^*_L > c^*_E$ implies that

$$\begin{aligned}(1-\hat{\theta})c'_L(\hat{\theta}) &= (1-\theta)c^*_L + (\theta-\hat{\theta})c^*_E \\ &> (1-\theta)c^*_E + (\theta-\hat{\theta})c^*_E \\ &= (1-\hat{\theta})c^*_E.\end{aligned}$$

The contract $\boldsymbol{c}^{**}$ proves to be an RPC.

**Proposition 1** *The contract $\mathbf{c}^{**}$ is an RPC such that the efficient consumption $\mathbf{c}^{**}(\theta) = (c^*_E, c^*_L)$ can be achieved in strictly self-selective strategies for all relative risk aversion coefficients and liquidation costs.*

**Proof.** See the Appendix. (I provide a sketch of proof in the subsequent paragraph.) ■

The contract $\boldsymbol{c}^{**}$ has three important characteristics. First, this is an RPC. Second, the unique equilibrium outcome is efficient. Third, the contract is "parameter-free"; that is, it is independent of the relative risk aversion coefficients and liquidation costs. Although the third property directly follows from the definition of $\boldsymbol{c}^{**}$, we can show that a provision scheme for $\boldsymbol{c}^{**}$ to be implemented is also parameter-free.

I provide an intuitive explanation of our provision scheme. In order to avoid bankruptcy, the provision scheme sets a *refund cap* on one-time withdrawals during some time span $[0, K)$ with $K < T$. Let $\underline{c}_E$ denote such a refund cap. Early depositors tender a full withdrawal at time 0 but almost all of them must stand in line waiting to be served $\underline{c}_E < 1$ due to Assumption 1. Any early withdrawal tender can be served $\underline{c}_E$ in $[0, K)$ since there is a bijection from $I$ to $[0, K)$. The bank counts the number of early depositors while providing $\underline{c}_E$. Then, in $[K, T)$, the scheme provides $c^{**}(\hat{\theta}) - \underline{c}_E$ to the early withdrawal tenders who get $\underline{c}_E$, where $\hat{\theta}$ is the counted number of early depositors in $[0, K)$. Hence all $\hat{\theta}$ early withdrawal tenders can be served $c^{**}(\hat{\theta})$ by time $T$. Since $c^{**}_E(\hat{\theta}) < c^{**}_L(\hat{\theta})$, any late depositor does not tender a full withdrawal in $E_1$.

The most important role in our provision scheme is the refund cap $\underline{c}_E$. We can show that there exists $\underline{c}_E > 0$ such that $c^{**}(\hat{\theta}) - \underline{c}_E \geq 0$ for all $\hat{\theta} \in [0,1]$,[6] so that our provision scheme is consistent with Assumption 2. In practice, this refund cap corresponds to the view that the bank should not accept a large number of full withdrawal tenders at one time lest the bank goes bankrupt. The refund cap $\underline{c}_E$ makes deposit contracts have the run-preventing property *and* helps the bank provide efficient consumption. Thus, our result shows that the demand deposit contract must be similar to a *time deposit* contract when we seek to achieve both the efficiency and the run-preventing property.

### 4.1.1 Example

Suppose that the depositors have the following utility function:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \tag{8}$$

where $\gamma > 0$ is their relative risk aversion coefficient. In this case, it is easily derived that the solution of (2) is

$$c_E^* = \frac{1}{\theta + (1-\theta)R^{\frac{1-\gamma}{\gamma}}}, \qquad c_L^* = R^{\frac{1}{\gamma}} c_E^*. \tag{9}$$

In the optimum of problem (6) with utility function (8), we obtain

$$\hat{c}_E = \frac{\psi_E^*}{\hat{\theta}}, \qquad \hat{c}_L = \frac{R\psi_L^*}{1-\hat{\theta}},$$

where

$$\psi_E^* = \theta c_E^*, \quad \psi_L^* = (1-\theta)R^{\frac{1-\gamma}{\gamma}} c_E^*.$$

[6] See the Appendix.

Then,

$$\begin{aligned}
\hat{c}_L - \hat{c}_E &= \frac{c_E^* \left( \theta(1-\theta) R^{\frac{1}{\gamma}} - \theta(1-\theta) \right)}{\hat{\theta}(1-\hat{\theta})} \\
&> \frac{c_E^* \theta(1-\theta) \left( R^{\frac{1}{\gamma}} - 1 \right)}{\hat{\theta}(1-\hat{\theta})} \\
&> 0.
\end{aligned}$$

Further, since $\underline{c}_E = \psi_E^* > 0$ and $c_E(\hat{\theta}) - \underline{c}_E = (1-\hat{\theta})\psi_E^* \hat{\theta}^{-1} > 0$ for all $\hat{\theta} \in [\theta, 1)$, our provision function ensures the incentive compatibility for early depositors. Note also that we obtain $\hat{c}_E < c_E^* < c_L^* < \hat{c}_L$ when $\hat{\theta} > \theta$ in this example.

## 4.2 Robustness of RPC $\boldsymbol{c}^{**}$

We see that our provision scheme enable the contract $\boldsymbol{c}^{**}$ to uniquely achieve the efficient outcome in equilibrium. Here, we should remember that the provision scheme, known as *deposit freeze* or the *suspension of convertibility*, can also uniquely achieve the efficient outcome.[7] I would like to emphasize that the contract $\boldsymbol{c}^{**}$ with our provision scheme is more robust than a contract with deposit freeze policies.

To elucidate this point, consider the following contract $\tilde{\boldsymbol{c}} = (\tilde{c}_E(\cdot), \tilde{c}_L(\cdot))$:

$$\tilde{c}_E(\hat{\theta}) = \begin{cases} c_E^* & \text{if } \hat{\theta} \leq \tilde{\theta} \\ 0 & \text{if } \hat{\theta} > \tilde{\theta} \end{cases}, \quad \tilde{c}_L(\hat{\theta}) = \begin{cases} c_L'(\hat{\theta}) & \text{if } \hat{\theta} \leq \theta \\ c_L''(\hat{\theta}) & \text{if } \theta < \hat{\theta} \leq \tilde{\theta} \\ c_E^* & \text{if } \hat{\theta} > \tilde{\theta} \end{cases} \tag{10}$$

where $c_L'$ is the same one in (7),

$$c_L''(\hat{\theta}) = \frac{R}{1-\hat{\theta}} \left( 1 - \theta c_E^* - \frac{(\hat{\theta} - \theta) c_E^*}{1-\kappa} \right),$$

[7] See Diamond and Dybvig (1983). However, Engineer (1989) shows an example in which a deposit freeze policy fails to achieve an efficient outcome. See also Ennis and Keister (2009).

and $\tilde{\theta}$ is such that $c''_L(\tilde{\theta}) = c^*_E$. This contract represents a deposit freeze policy. We can easily check that $\tilde{\boldsymbol{c}}$ is an RPC.[8] However, we can also see that whether or not $\tilde{\boldsymbol{c}}$ is well-defined depends on $\kappa$; $\tilde{\boldsymbol{c}}$ is no longer defined when $\kappa$ is sufficiently large. Hence, the contract $\tilde{\boldsymbol{c}}$ is sensitive for the parameter $\kappa$.

Next, consider the following contract $\boldsymbol{c}^1 = (c^1_E(\cdot), c^1_L(\cdot))$:

$$c^1_E(\hat{\theta}) = \begin{cases} c^*_E & \text{if } \hat{\theta} \leq \theta \\ 0 & \text{if } \hat{\theta} > \theta \end{cases}, \quad c^1_L(\hat{\theta}) = \begin{cases} c'_L(\hat{\theta}) & \text{if } \hat{\theta} \leq \theta \\ b_L(\hat{\theta}) & \text{if } \hat{\theta} > \theta \end{cases} \tag{11}$$

where $c'_L$ is the same one in (7) and

$$b_L(\hat{\theta}) = \frac{R(1-\theta c^*_E)}{1-\hat{\theta}}.$$

The contract $\boldsymbol{c}^1$ is an RPC that is consistent with a deposit freeze policy for $\kappa = 1$. Here, suppose that a type state $\theta$ is stochastic and $\theta' = \theta + \epsilon$ is realized, where $\epsilon > 0$ is sufficiently small. Then, the outcome of (11) is $c^1_E(\theta') = 0$ while $c^{**}_E(\theta') = \hat{c}_E(\theta') \approx c^*_E$, and $c^1_L(\theta') = b_L(\theta') = \hat{c}_L(\theta') = c^{**}_L(\theta')$. Hence, contract $\boldsymbol{c}^1$, compared with contract $\boldsymbol{c}^{**}$, distorts the consumption for $\epsilon$ early depositors, whereas both contracts provide the same consumption for late depositors. This shows that $\boldsymbol{c}^{**}$ is more robust than $\boldsymbol{c}^1$ in that $\boldsymbol{c}^1$ drastically changes the consumption for early depositors with respect to a small shock for the type state.

# 5 Concluding remarks

This paper shows that the efficient consumption of a deposit contract can be achieved even if there exists a liquidation cost for a long-term investment technology. Compared with Cooper and Ross (1998), this paper shows that a provision scheme design can expand the set of incentive-compatible run-preventing contracts. Further, we can provide efficient consumption regardless of the liquidation cost and the relative risk aversion coefficient of

[8]The bank provides $c^*_E$ to a depositor for all $t' \in [0, T)$ if $\int_{j \in \Psi(t')} \mathcal{L}(dj) \leq \tilde{\theta}$; otherwise 0 for all $t \in [t', T)$. When $t \geq T$, the bank knows $\hat{\theta}$ and hence provides $\tilde{c}_L(\hat{\theta})$ to each $i \in \mathcal{I}$.

depositors. This "parameter-free" property is a strong point of our contract compared with a deposit contract with deposit freeze policies.

This paper sheds light on the payment policy of banks in implementing a deposit contract. Under a sequential service constraint without retroactive levy, the obtained result shows that a refund cap is an important policy for run prevention and efficiency. Such a refund cap prohibits early depositors from fully withdrawing their deposits at one time and they are compelled to put up with relatively less consumption for the present, but ultimately, all early depositors can consume efficiently in equilibrium. Thus, our optimal payment policy renders a demand deposit contract similar to a time deposit contract in order to uniquely achieve an efficient outcome.

# 6 Appendix

## 6.1 Proof of Lemma 1

For notational convenience, we let $\hat{c}_q = \hat{c}_q(\hat{\theta})$, $q = E$ or $L$. The Lagrangian of the problem (6) is

$$L = \hat{\theta}u(c_E)+(1-\hat{\theta})u(c_L)+\lambda(\theta c_E^*-\hat{\theta}c_E)+\mu(R(1-\theta c_E^*)+\theta c_E^*-\hat{\theta}c_E-(1-\hat{\theta})c_L)$$

with some $\lambda \geq 0$ and $\mu \geq 0$. By using Kuhn-Tucker's theorem, we obtain

$$u'(\hat{c}_E) = u'(\hat{c}_L) + \lambda, \quad u'(\hat{c}_L) = \mu \tag{12}$$

with complementary slackness conditions $\lambda(\theta c_E^* - \hat{\theta}\hat{c}_E) = 0$ and $\mu(R(1 - \theta c_E^*) + \theta c_E^* - \hat{\theta}\hat{c}_E - (1 - \hat{\theta})\hat{c}_L) = 0$. In the optimum, we obtain $\hat{c}_E \leq \hat{c}_L$. If $\hat{c}_E = \hat{c}_L = \hat{c}$, that is, $\lambda = 0$, then the complementary slackness condition for $\mu$ implies $\hat{c} = R(1 - \theta c_E^*) + \theta c_E^*$. Since $R(1 - \theta c_E^*) = (1 - \theta)c_L^*$ and $(1 - \theta)c_L^* + \theta c_E^* > c_E^*$, we must have $\hat{c} > c_E^*$. Then, the first inequality constraint of (6) implies that $\hat{\theta} < \theta$. Hence, we obtain $\hat{c}_E < \hat{c}_L$ whenever $\hat{\theta} > \theta$ and the complementary slackness condition for $\lambda$ implies that $\theta c_E^* = \hat{\theta}\hat{c}_E$. Thus, we obtain $\hat{c}_E < c_E^* < c_L^* < \hat{c}_L$.

## 6.2 Proof of Proposition 1

Let $q_i(t)$ denote the amount of consumption that depositor $i$ consumes in $[0, t)$. Assumption 2 and our sequential service constraint imply that the bank knows $q_i(t)$ if depositor $i$ is served at time $t$. Let $\underline{c}_E \equiv \inf_{\hat{\theta}>\theta} \hat{c}_E(\hat{\theta})$. Then $\underline{c}_E > 0$ since $\lim_{c \to 0} u'(c) = \infty$. Taking into account the sequential service constraint and Assumptions 1 and 2, we define the provision function for a depositor $i$ as follows:

**Rule 1.** For all $t \in [0, K)$ with a fixed $K < T$, if $m_i(t) = 1$ and $q_i(t) < \bar{c}_E$, then

$$g_i(m_i, \cdot, t) = \bar{c}_E.$$

**Rule 2.** For all $t \in [K, T)$, if $m_i(t) = 1$ and $q_i(t) = \bar{c}_E$, then

$$g_i(m_i, \cdot, t) = c_E^{**}(\hat{\theta}) - \underline{c}_E.$$

**Rule 3.** For all $t \in [T, T']$, if $m_i(t) = 1$ and $q_i(t) < \bar{c}_E$, then

$$g_i(m_i, \cdot, t) = c_L^{**}(\hat{\theta}).$$

Lemma 1 ensures that $c_E^{**}(\hat{\theta}) - \underline{c}_E \geq 0$ for all $\hat{\theta} \in [0, 1]$. Any early withdrawal tender can be served $c_E^{**}(\hat{\theta})$ in $[0, T)$ since there is a bijection $\sigma : I \to [a, b)$ for arbitrary $a < b$. Hence, the provision function followed Rules 1–3 can provide $c_E^{**}(\hat{\theta})$ to $\hat{\theta}$ early depositors by time $T$. Since $c_E^{**}(\hat{\theta}) < c_L^{**}(\hat{\theta})$ for all $\hat{\theta} \in [0, 1]$, $m_i(t) = 1$ for all $t \in [0, T)$ ($m_i(t) = 0$ for all $t \in [0, T)$ and $m_i(t) = 1$ for all $t \in [T, T']$) is a strictly dominant behavior for early (late) depositor $i$. In equilibrium, $\hat{\theta} = \theta$ and the efficient outcome $(c_E^*, c_L^*)$ is uniquely achieved.

# References


1. Cooper, R. and T. Ross (1998), "Bank Runs: Liquidity Costs and Investment Distortions," *Journal of Monetary Economics*, 41: 27–38.

2. ——— (2002), "Bank Runs: Deposit Insurance and Capital Requirements," *International Economic Review*, 43: 55–72.

3. Diamond, D. and P. Dybvig (1983), "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91: 401–419.

4. Engineer, M (1989), "Bank Runs and the Suspension of Deposit Convertibility," *Journal of Monetary Economics*, 24: 443–454.

5. Ennis, H. and T. Keister (2009), "Bank Runs and Institutions: The Perils of Intervention," *American Economic Review*, 99: 1588–1607.

6. Green, E. and P. Lin (2003), "Implementing Efficient Allocations in a Model of Financial Intermediation," *Journal of Economic Theory*, 109: 1–23.

7. Martin, A (2006), "Liquidity Provision vs. Deposit Insurance: Preventing Bank Panics without Moral Hazard," *Economic Theory*, 28: 197–211.

8. Mas-Colell, A, M.D. Whinston, and J.R. Green (1995), *Microeconomic Theory*, Oxford University Press, New York