

# Incorporation of a Psychological Motive in the Implementation of Efficient Deposit Contracts\*

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## ABSTRACT

This paper provides a positive result for the unique implementation of the efficient deposit contract of the Diamond and Dybvig (J. Polit. Econ. 91 (1983) 401–419) model. The role of the bank here is to implement an efficient deposit contract. However, the realized outcome may be inefficient, either when the bank faces an aggregate risk on the preference state of depositors or when a self-fulfilling bank run occurs (or both). Regardless of the causes, a mild *psychological motive* held by a few depositors enables us to construct a provision scheme that fully reveals the true state and uniquely implements the efficient deposit contract concerning the state with iterative deletions of weakly dominated strategies.

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# 1 Introduction

This paper analyzes the Diamond–Dybvig (1983) model of demand deposit contracts and shows that almost all efficient deposit contracts are uniquely implementable with a few “socially conscious” depositors.

In the Diamond–Dybvig model, a risk-sharing contract between a bank and depositors is considered efficient only if the bank knows the *type state* of the depositors, that is, if the bank knows the number of depositors who need funds immediately. Moreover, if the bank knows the type state of depositors, Diamond and Dybvig (1983) show that an ex ante efficient risk-sharing contract can be uniquely implementable with a *deposit freeze policy*. Hence, the contract fails to ensure efficiency if and only if the bank does not know the type state. Indeed, if the bank faces an *aggregate risk* on a type state, that is, the bank does not know the exact type state, it may be optimal for all depositors to tender a full withdrawal before maturity, leading to a bank run. In such a case, Diamond and Dybvig (1983) propose deposit insurance provided by the government as a solution to prevent bank runs. However, it remains unclear whether such government intervention is required to prevent bank runs in an environment with aggregate risk.

This paper reexamines the implementation problem of efficient deposit contracts of the Diamond–Dybvig model in an environment with aggregate risk *without deposit insurance*. For this purpose, the paper introduces some *socially conscious* depositors. By using such depositors, I show that there is a *provision scheme* that uniquely implements an ex ante efficient deposit contract in an environment with aggregate risk and without government intervention such as deposit insurance.

A novel feature of this paper is the introduction of socially conscious depositors into the deposit contract design. There are two social categories in the economy. Depositors in the socially conscious category have a rational preference with a *psychological cost*, and depositors in the other category are assumed to have only a rational preference. A socially conscious depositor incurs an *arbitrarily small* psychological cost with an early withdrawal when he does not immediately require the funds, but more importantly, the cost

declines if *some other socially conscious depositors have already withdrawn their deposits* before his early withdrawal despite no immediate need for their funds.

Furthermore, I suggest another issue to be considered by the Diamond–Dybvig model: a *time-consistent budget constraint*. A time-consistent budget constraint refers to the relationship between the set of feasible contracts and the payment ability of the bank. We must consider whether the bank can offer a contract that provides a positive amount of consumption to zero-measured depositors after the bank repays all funds to their depositors. This issue is critical to the prevention of bank runs because the Diamond–Dybvig model represents the depositors as a continuum. However, previous studies have not seriously considered this issue. In this paper, we assume that the bank cannot offer any positive amount of consumption to depositors after the bank’s deposit is zero.

We construct a provision scheme for the deposit contract of the Diamond–Dybvig model. The provision scheme requires no government intervention such as deposit insurance. The main result of this paper is that the provision scheme under a time-consistent budget constraint uniquely implements almost all efficient contracts with iterative deletions of weakly dominated strategies if there are a few socially conscious depositors.<sup>1</sup> The contribution of this paper therefore lies in the exhibition of the successful incorporation of a slight psychological motive into the efficient deposit contract design problem.<sup>2</sup>

This paper is organized as follows. Section 2 describes the basic framework of the model. Section 3 introduces several definitions, including those of a time-consistent budget constraint and a psychological cost. Section 4 intuitively explains our main result. Section 5 presents the main result. Section 6 presents the concluding remarks.

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<sup>1</sup>Another approach toward the establishment of a unique outcome is the “global game” approach. See for example, Goldstein and Pauzner (2005).

<sup>2</sup>Psychological preferences and human interactions have been widely investigated in economics, e.g., Geanakoplos et al. (1989), Bernheim (1994), Dufwenberg and Lundholm (2001), Battigalli and Dufwenberg (2009), and Dufwenberg et al. (2011).

## 1.1 Related literature

This paper takes a mechanism-design approach to solve the unique implementation problem of efficient deposit contracts in the Diamond–Dybvig model. Green and Lin (2003) also take the same approach in a finite-trader version of the Diamond–Dybvig model without any psychological depositors (hereafter, the Green–Lin model). They show that the ex ante efficient contract is implementable in strictly dominant strategies. We observe that their affirmative result hinges on the finiteness of depositors. If depositors are finite, the behavior of a single depositor influences the outcome of a contract. As a result, the bank can incentivize a depositor to reveal his true type in the Green–Lin model *without* the time-consistent budget constraint problem. However, if depositors are represented by a continuum (i.e., as in the original Diamond–Dybvig model), the decision of a single depositor cannot influence the outcome of a contract, which fails to fully incentivize a depositor.

This paper assumes a psychological cost for some depositors. The most critical assumption concerning the psychological cost is that the cost of a depositor can be reduced depending on the other socially conscious depositors. This cost reduction property is consistent with the “broken windows theory” (Wilson and Kelling (1982)). According to the theory, people tend to become vandals once they observe small signs of social disorder. Keizer et al. (2008) empirically examine this theory using field experiments and find their results to support the theory. In our banking model, a socially conscious depositor becomes less conscious once some other socially conscious depositors have tendered a full withdrawal before maturity.

## 2 Model

### 2.1 Material environment $\mathcal{E}$

The model used in this paper is based on the Diamond–Dybvig (1983) model. Consider an economy with a single consumption good, a representative bank, and depositors. The bank implicitly faces competition with other identical banks, and its profit is assumed to be zero. The economy has three event

phases:  $E_0$ ,  $E_1$ , and  $E_2$ . There are two technologies available to the bank: storage and investment. Storage yields one unit of the consumption good in  $E_1$  per unit input in  $E_0$ . Investment yields  $R > 1$  units of the good at a maturity time in  $E_2$  per unit input in  $E_0$ . Premature liquidation of the investment yields one unit of the good in  $E_1$  per unit input in  $E_0$ ; that is, there is no liquidation cost for the investment.

Depositors are identical in  $E_0$  and are represented by the continuum  $I = [0, 1]$ . Each depositor has one unit consumption good as an endowment, which it deposits in the representative bank in  $E_0$ . At the beginning of  $E_1$ , a fraction of the depositors,  $\theta \in (0, 1)$ , which we call *early depositors*, face a liquidity shock. The remaining depositors,  $1 - \theta$ , which we call *late depositors*, do not face a liquidity shock. Early depositors must exit from the market by the end of  $E_1$ , but late depositors can still exist in  $E_2$ .

Early depositors value consumption in  $E_1$ , while late depositors value consumption in both  $E_1$  and  $E_2$ . Let  $c_n$  denote the consumption of depositors in  $E_n$  ( $n = 1, 2$ ). The payoffs of depositor  $i$  are given by

$$U_i(c_1, c_2) = \begin{cases} u(c_1) & \text{if } i \text{ is an early depositor} \\ u(c_1 + c_2) & \text{if } i \text{ is a late consumer,} \end{cases} \quad (1)$$

where  $u$  in (1) is a von Neumann–Morgenstern utility function  $u : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ , which is strictly increasing, strictly concave, and twice differentiable, and which satisfies  $u(0) = 0$ ,  $\lim_{c \rightarrow 0} u'(c) = \infty$ ,  $\lim_{c \rightarrow \infty} u'(c) = 0$ , and  $-u''(c)/u'(c) > 1$  for all  $c > 0$ .

An ex ante efficient allocation for *state*  $\theta$  is defined as the solution of the following optimization problem:

$$\begin{aligned} \max_{c_e, c_l} \quad & \theta u(c_e) + (1 - \theta)u(c_l) \\ \text{s.t.} \quad & (1 - \theta)c_l \leq R(1 - \theta c_e), \end{aligned} \quad (2)$$

where the subscript  $e$  ( $l$ ) denotes early (late). In the optimum, the solution  $(c_e^*(\theta), c_l^*(\theta))$  exists uniquely for each  $\theta \in (0, 1)$  and satisfies  $1 < c_e^*(\theta) <$

$c_l^*(\theta) < R$  for all  $\theta \in (0, 1)$ .<sup>3</sup> The constraint in (2) tells us that  $c_e^*(1) = 1$  and  $c_l^*(0) = R$ , while  $c_e^*(0)$  and  $c_l^*(1)$  are indeterminate. We define a *contract* as a pair of contingent consumptions on a realized state  $\theta$  and denote it by  $\mathbf{c} = (c_e(\cdot), c_l(\cdot))$ . A contract is said to be *efficient* if (i)  $\mathbf{c}$  is the solution of Problem (2) for all  $\theta \in (0, 1)$  and (ii)  $c_e(1) = 1$  and  $c_l(0) = R$ . We denote by  $C$  the set of efficient contracts. Throughout the paper, I assume that the bank does not know the realized state  $\theta$  in  $E_0$  and that whether a depositor is early or late is his private information. We refer to the environment defined above as the *material environment*  $\mathcal{E}$ .

### 3 Definitions

Because our interest lies in the prevention of bank runs, we focus solely on the behavior in  $E_1$  without loss of generality. For tractability, let us denote  $E_1 = [0, T)$  and  $E_2 = [T, T']$ , where  $t = 0$  refers to the time of liquidity shock and  $t = T$  refers to the time by when early depositors have completely exited from the market *and* the maturity time of the investment.

#### 3.1 Provision scheme

The bank implements a contract with a provision scheme. We denote the *message* of a depositor  $i$  by a measurable function  $m_i : [0, T'] \rightarrow \{0, 1\}$ , where *action*  $m_i(t) = 1$  ( $0$ ) implies that depositor  $i$  tenders (does not tender) a full withdrawal at time  $t$ . Let  $m_i = (m_i(t))_{t \in [0, T']} \in M_i$ , where  $M_i$  is the set of all possible messages of depositor  $i$ . We denote the profile of the messages of all depositors and that of depositors other than depositor  $i$  by  $m \in M \equiv \prod_{i \in I} M_i$  and  $m_{-i} \in M_{-i} \equiv \prod_{j \neq i} M_j$ , respectively. We write  $m = (m \setminus m_i)$  if we emphasize a message of depositor  $i$ . Let  $g_i(m, t)$  denote the provision of the good for depositor  $i$  at time  $t$ , where  $g_i : M \times [0, T'] \rightarrow \mathbb{R}_+$ . We refer to  $\mathcal{M} = (M_i, g_i(\cdot, \cdot))_{i \in I}$  as a *provision scheme*. A provision scheme describes when and how much depositors are served by the bank.

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<sup>3</sup>These derivations owe to Diamond and Dybvig (1983).

**Definition 1** A provision scheme achieves allocation  $(c_e, c_l)$  with a message profile  $m$  if for any early depositor  $i$ , there exists a finite set  $\{t_1^i, \dots, t_N^i\} \subset E_1$  such that  $\sum_{n=1}^N g_i(m, t_n^i) = c_e$  and for any late depositor  $j$ , there exists a finite set  $\{t_1^j, \dots, t_K^j\} \subset E_1 \cup E_2$  such that  $\sum_{k=1}^K g_j(m, t_k^j) = c_l$ .

### 3.2 Time-consistent budget constraint

I introduce a new constraint into our material environment  $\mathcal{E}$ . Let  $B_t$  denote the amount of the deposits that the bank holds at time  $t$  and  $\mathcal{L}(X)$  denote the Lebesgue measure of a set  $X$ .

**Assumption 1** (Time-consistent budget constraint) *The feasible contracts of the bank are subject to the following constraint: If  $\mathcal{L}(B_t) = 0$ , then  $g_j(\cdot, t') = 0$  for all  $t' \geq t$  and  $j \in I$ .*

Because  $c_e^*(1) = 1$  in Problem (2), this assumption implies that  $c_l^*(1) = 0$ .

Without this assumption, the Diamond–Dybvig model is free from the bank run problem. To see this point clearly, Suppose that  $u(c) = c^{(1-\gamma)}(1-\gamma)^{-1}$ , where  $\gamma > 1$  is the relative risk aversion parameter. In this case, the solution of Problem (2) is easily derived as

$$c_e^*(\theta) = \frac{1}{\theta + (1-\theta)R^{\frac{1-\gamma}{\gamma}}}, \quad c_l^*(\theta) = R^{\frac{1}{\gamma}}c_e^*(\theta).$$

Let  $\hat{\theta}$  denote the number of full withdrawal tenders in  $E_1$ . Suppose that  $\theta < 1$  but  $\hat{\theta} = 1$ . Because the bank does not know the true state, the bank provides  $c_e^*(\hat{\theta}) = 1$  to all depositors in  $E_1$  and the obtained result is inefficient. To prevent such full withdrawals, the bank must provide more favorable consumption to a depositor in  $E_2$ . In this case,  $c_l^*(\hat{\theta}) = R^{\frac{1}{\gamma}}$ . Here, we confront a problem: Can the bank provide  $R^{\frac{1}{\gamma}}$  to a depositor in  $E_2$ ? Because the bank fully liquidates the investment and pays them back to all depositors in  $E_1$ , the bank has no funds in  $E_2$ . Thus, we observe that the bank cannot provide  $R^{\frac{1}{\gamma}}$  to any depositor.

One may think, however, that the bank can offer any positive consumption in that case because there remain at most zero-measured depositors in

$E_2$ . If we allow the bank to offer positive consumption to zero-measured depositors, the bank run problem becomes “no problem” because we can incentivize late depositors by offering positive consumption. To clearly confirm this point, as in the original Diamond–Dybvig model, suppose that the bank commits to provide  $c_e = c_e^*(\theta) > 1$  to any withdrawal tender in  $E_1$ . In this case, Diamond and Dybvig (1983) point out that  $\hat{\theta} = 1$  is an undesired Nash equilibrium behavior because the bank goes bankrupt in  $E_1$  when providing the good to the amount of  $\tilde{\theta} = 1/c_e$  depositors. However, if the bank can provide any positive consumption to zero-measured depositors, it is feasible to offer  $c_l^*(\theta)$  in  $E_2$  up to zero-measured depositors even if the bank has provided  $c_e$  to  $\tilde{\theta}$  depositors in  $E_1$ . In this case, a late depositor finds it profitable to unilaterally tender a full withdrawal in  $E_2$ . Hence,  $\hat{\theta} = 1$  is no longer Nash equilibrium when  $\theta < 1$ , and we can solve the bank run problem. Thus, we should consider the time-consistent budget constraint in the Diamond–Dybvig model.

The time-consistent budget constraint does not matter in the Green–Lin model. The Green–Lin model assumes finite depositors. Let  $\xi \in \{0, 1, \dots, N\}$  denote the number of early depositors. Then, we can identify an efficient allocation  $(\tilde{c}_e^*(\xi), \tilde{c}_l^*(\xi))$  in the Green–Lin model with the allocation  $(c_e^*(\xi/N), c_l^*(\xi/N))$  in our model. Let  $\hat{\theta} = \hat{\xi}/N$ , where  $\hat{\xi}$  is the number of early full withdrawal tenders in the Green–Lin model. Then,  $\hat{\theta} = 1$  corresponds to  $\hat{\xi} = N$ . If  $\hat{\xi} = N$ , the efficient allocation is  $(\tilde{c}_e^*(N), \tilde{c}_l^*(N))$ , where  $\tilde{c}_e^*(N) = 1$ . Because there are finite depositors and  $c_l^*((N-1)/N) > 1$ , the allocation  $(\tilde{c}_e^*(N-1), \tilde{c}_l^*(N-1)) = (c_e^*((N-1)/N), c_l^*((N-1)/N))$  can be achieved by unilateral deviation from  $\hat{\xi} = N$  regardless the value of  $\tilde{c}_l^*(N)$ . However, if the set of depositors is represented by a continuum, we must consider the time-consistent budget constraint because the allocation  $(c_e^*(1), c_l^*(1))$  is unchanged by unilateral deviation.

### 3.3 Psychological cost

We introduce socially conscious depositors into the material environment  $\mathcal{E}$ . The socially conscious depositors want to achieve an efficient allocation



and hence are willing to avoid early withdrawal when they are “late.” To simplify the model, I assume that *the socially conscious depositors are all late depositors* and that each incurs a *psychological cost* with early withdrawal. Hereafter, we refer to the socially conscious depositors as  $\alpha$ -late depositors and the late depositors who do not have the socially conscious preference as  $\beta$ -late depositors.

More importantly, the psychological cost declines once some other  $\alpha$ -late depositors have already tendered an early withdrawal. This cost reduction implies that an  $\alpha$ -late depositor becomes less motivated to achieve the efficient outcome. For tractability, for a given  $\theta$ , we divide the set of depositors into three sets,  $I_\theta(e)$ ,  $I_\theta(\alpha)$ , and  $I_\theta(\beta)$ , such that  $I_\theta(l) = I_\theta(\alpha) \cup I_\theta(\beta)$  and  $I_\theta(\alpha) \cap I_\theta(\beta) = \emptyset$ . Let  $\eta_i$  denote a *type* of depositor  $i$ ,  $\eta_i \in \{\alpha, \beta, e\}$ , where  $\alpha/\beta/e$  denote  $\alpha$ -late/ $\beta$ -late/early depositors, respectively. Under a provision scheme, we introduce the number  $\tau_i$  such that

$$\tau_i \equiv \inf \{t \in [0, T'] \mid m_i(t) = 1\}.$$

If  $m_i(t) = 0$  for all  $t \in [0, T']$ , we let  $\tau_i = T'$ .

**Definition 2** A function  $D_i : M \rightarrow \mathbb{R}$  represents a psychological cost if, for all  $\theta \in [0, 1]$ ,  $i \in I_\theta(\alpha)$ , and  $m \in M$ ,

$$\tau_i \geq T \Rightarrow D_i(m) = 0, \quad (3)$$

$$\forall j \in I_\theta(\alpha), \tau_i \leq \tau_j < T \Rightarrow D_i(m) = -d_1 < 0, \quad (4)$$

and

$$\exists j \in I_\theta(\alpha) \setminus \{i\}, \tau_j < \tau_i < T \Rightarrow D_i(m) = -d_2 > -d_1 \quad (5)$$

with  $d_2 \geq 0$ .

Properties (3) and (4) imply that depositor  $i$  is better off with respect to  $D_i$  if he does not tender an early withdrawal. In addition, Properties (4) and (5) imply that depositor  $i$  is better off with respect to  $D_i$  if he waits to tender

his early withdrawal until some other  $\alpha$ -late depositors tender a withdrawal or when he waits to tender a withdrawal by the maturity time  $T$ .

The cost reduction in Property (5) indicates that the motivation for efficiency lessens once some other  $\alpha$ -late depositors have undertaken socially inefficient behavior, that is, early withdrawal. This cost reduction property is consistent with the empirical result of Keizer et al. (2008), who use field experiments to determine whether a norm violation by some people causes subsequent norm violations by other people.<sup>4</sup> Keizer et al. (2008) report that once people observe one norm violation, they are likely to violate other norms. In our model, the norm is the achievement of a socially efficient allocation. It is costly for an  $\alpha$ -late depositor to violate the norm, but this cost lessens if some other  $\alpha$ -late depositors have already violated the norm.

In this paper, I assume that an  $\alpha$ -late depositor  $i$  has the following additive separable payoff function:

$$u(g_i(m, t)) + D_i(m). \quad (6)$$

To avoid a trivial case, I further assume that the cost itself is sufficiently small compared to the consumption of the initial endowment.

**Assumption 2** (Small psychological cost) *The value  $d_1$  satisfies:*

$$u(1) - d_1 > u(0).$$

This assumption excludes the case in which  $\alpha$ -late depositors do not tender any early withdrawal before  $T$  because of a large psychological cost.

I refer to the environment in which there are socially conscious depositors in  $\mathcal{E}$  such that their preferences are defined with (6) subject to Assumption 2 as the *psychological environment*  $\mathcal{E}_p$ .

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<sup>4</sup>Some research attempts to incorporate findings from social psychology into economic theory. Matsushima (2009), for instance, incorporates a psychological preference for social conformity into implementation theory.

## 4 Implementation without sequential service constraints

The aim of this section is to provide an intuitive explanation of how we implement an efficient allocation in the psychological environment  $\mathcal{E}_p$ . Throughout this section, we assume the following for simplicity:

**Assumption 3** (S1:) *There are  $\theta$  early depositors,  $\delta(1 - \theta)$   $\alpha$ -late depositors, and  $(1 - \delta)(1 - \theta)$   $\beta$ -late depositors, where  $\theta, \delta \in (0, 1)$ . In addition, this information is common knowledge shared by the depositors. (S2:) *The provision of the consumption good occurs in  $\{0, t_1\} \subset E_1$  and  $\{t_2\} \subset E_2$  for some  $t_1 \in (0, T)$  and  $t_2 \in [T, T']$ . (S3:) *There is no sequential service constraint; that is, the bank can provide the good after counting the number of withdrawal tenders in each  $t \in \{0, t_1, t_2\}$ .***

We will define a sequential service constraint in Section 5.1. Under Assumption 3, we obtain the following proposition.

**Proposition 1** *In a psychological environment  $\mathcal{E}_p$  with Assumption 3, there exists a provision scheme  $\mathcal{M}$  that achieves the allocation  $(c_e^*(\theta), c_l^*(\theta))$  with a unique optimal message profile  $m^*$ .*

**Proof.** Consider the following payment scheme,  $\mathcal{M}$ :

1. Let  $\hat{\theta}_0$  denote the number of withdrawal tenders in  $t = 0$ . The bank provides  $c_e^*(\hat{\theta}_0)$  to  $p\hat{\theta}_0$  depositors and nothing to  $(1 - p)\hat{\theta}_0$  depositors, where  $p \in (0, 1)$ .
2. Let  $\hat{\theta}_1$  denote the number of withdrawal tenders in  $t = t_1$ . The bank provides  $c_e^*(\hat{\theta}_0)$  to depositors if and only if  $\hat{\theta}_1 \leq (1 - p)\hat{\theta}_0$ .
3. In  $t = t_2$ , the bank provides  $c_l^*(\hat{\theta}_0)$  to depositors if  $\hat{\theta}_0 < 1$ ; if  $\hat{\theta}_0 = 1$ , the time-consistent budget constraint implies that the bank provides 0.

We show that  $\mathcal{M}$  achieves the allocation  $(c_e^*(\theta), c_l^*(\theta))$  with a message profile  $m^*$  such that  $m_j^*(0) = m_j^*(t_1) = 1$  for any early depositor  $j$  and  $m_i^*(0) = m_i^*(t_1) = 0$  and  $m_i^*(t_2) = 1$  for any late depositor  $i$ .

For an early depositor  $j$ , if  $m_j^*(0) = m_j^*(t_1) = 1$ , then his payoff is  $pu(c_e^*(\hat{\theta}_0)) + (1-p)\Gamma(m \setminus m_j^*)u(c_e^*(\hat{\theta}_0))$ , where  $\Gamma(m \setminus m_j^*) = 1$  if  $\hat{\theta}_1 \leq (1-p)\hat{\theta}_0$ ; otherwise it equals 0. If  $m_j'(0) = 0$  or  $m_j'(t_1) = 1$ , then his payoff is  $\Gamma(m \setminus m_j')u(c_e^*(\hat{\theta}_0))$  or  $pu(c_e^*(\hat{\theta}_0))$ . Because of continuum depositors,  $\Gamma(m \setminus m_j^*) = \Gamma(m \setminus m_j')$ ; thus,  $m_j(0) = m_j(t_1) = 1$  are the optimal actions for all early depositors.

Following this reasoning, we consider the decision of an  $\alpha$ -late depositor. First, suppose that an  $\alpha$ -late depositor  $i$  expects that some other  $\alpha$ -late depositors tender a full withdrawal in  $t = 0$ . If  $m_i(0) = m_i(t_1) = 1$ , his payoff is  $pu(c_e^*(\hat{\theta}_0)) + (1-p)\Gamma(m)u(c_e^*(\hat{\theta}_0)) - d_1$ . If  $m_i'(0) = 0$  and  $m_i'(t_1) = 1$ , then  $\Gamma(m \setminus m_i')u(c_e^*(\hat{\theta}_0)) - d_2$ . Hence, for a sufficiently small  $p$ , we obtain

$$\begin{aligned} & pu(c_e^*(\hat{\theta}_0)) + (1-p)\Gamma(m)u(c_e^*(\hat{\theta}_0)) - d_1 - \left( \Gamma(m \setminus m_i')u(c_e^*(\hat{\theta}_0)) - d_2 \right) \\ &= p(1 - \Gamma(m))u(c_e^*(\hat{\theta}_0)) - (d_1 - d_2) \\ &\leq pu(c_e^*(\hat{\theta}_0)) - (d_1 - d_2) \\ &< 0. \end{aligned}$$

Hence,  $m(0) = 1$  is not optimal in this case. Next, suppose that an  $\alpha$ -late depositor  $i$  expects that no other  $\alpha$ -late depositors tender a full withdrawal in  $t = 0$ . Then, depositor  $i$  deduces that  $\hat{\theta}_0 < 1$  because of assumption S1. In  $t = t_2$ , depositor  $i$  can definitely be served  $c_i^*(\hat{\theta}_0)$ , which is greater than  $c_e^*(\hat{\theta}_0)$ , by taking the actions  $m_i(t_0) = m_i(t_1) = 0$  and  $m_i(t_2) = 1$ . In both the cases, depositor  $i$  finds it better not to tender a full withdrawal in  $t = 0$ . Thus, no  $\alpha$ -late depositor tenders a full withdrawal in  $t = 0$ .

Following this reasoning, a  $\beta$ -late depositor deduces that  $\hat{\theta}_0 < 1$  is certain because of assumption S1 and that he can definitely be served  $c_i^*(\hat{\theta}_0)$  in  $t = t_2$ . Hence, it is better for all  $\beta$ -late depositors not to tender a full withdrawal in  $t = 0$  and  $t_1$ ; thus, all late depositors deduce that  $\hat{\theta}_0 = \theta$ . Any late depositor therefore tenders a full withdrawal only in  $t = t_2$ , and the bank achieves efficient allocation such that all  $\theta$  early depositors receive  $c_e^*(\theta)$  in  $E_1$  and all  $1 - \theta$  late depositors receive  $c_i^*(\theta)$  in  $E_2$ . ■

It is important for this affirmative result to take  $p$  such that  $p \in (0, 1)$ . If  $p = 1$ , the second stage of  $\mathcal{M}$  has no meaning; the bank provides  $c_e^*(\hat{\theta}_0)$  in the first stage and  $c_l^*(\hat{\theta}_0)$  or 0 in the third stage. In this case, however,  $\hat{\theta}_0 = 1$  constitutes a Nash equilibrium because of the time-consistent budget constraint (Assumption 1) and the small psychological cost (Assumption 2). If  $p = 0$ , depositors are not served in the first stage of  $\mathcal{M}$ . In this case, the choice of withdrawal tender in  $t = 0$  or  $t = t_1$  is indifferent for early depositors; thus, the provision scheme with  $p = 0$  fails to ensure that  $\hat{\theta}_0 = \theta$ .

## 5 Implementation with a sequential service constraint

### 5.1 Sequential service constraint

A *sequential service* means that the provision at time  $t$  can depend only on information concerning the amount the bank has served by  $t$  and not on any future information. To formalize a sequential service in our continuous-time model, I first impose the following constraint on our environment.

**Assumption 4** *For each  $t \in [0, T']$ , if there exists a depositor  $i$  such that  $g_i(\cdot, t) > 0$ , then  $g_j(\cdot, t) = 0$  for all  $j \in I \setminus \{i\}$ .*

Assumption 4 states that the bank can provide the good to at most a single depositor at each time. This assumption breaks S2 in Assumption 3. I further assume that any “retroactive levy” of banks is prohibited; that is, banks cannot make depositors repay served consumption goods.

**Assumption 5** *The bank is subject to a no-retroactive-levies constraint: For all  $i \in I$  and  $t \in [0, T']$ , if  $g_i(\cdot, t) = c_t > 0$ , depositor  $i$  consumes  $c_t$  at time  $t$ .*

The set of periods in which a depositor  $i$  are served by a time  $t$  is

$$H_i(t) \equiv \{k \in [0, t] \mid g_i(\cdot, k) > 0\}.$$

Let  $m_i[t] \equiv (m_i(k))_{k \in H_i(t)}$  and we refer to  $m_i[t]$  as a *history* of depositor  $i$  by time  $t$ . The set of depositors who have already been served positive

consumption from the bank by time  $t$  is given by

$$\Psi(t) \equiv \{j \in I \mid H_j(t) \neq \emptyset\}.$$

The bank must provide the good contingent on the withdrawals that have occurred so far. The available information of the bank in providing the good to a depositor  $i$  at a time  $t$  therefore only contains the action of depositor  $i$  at time  $t$  and history  $m_j[t]$  for all  $j \in \Psi(t)$ ; this condition is our *sequential service constraint* of the bank.

**Definition 3** (Sequential service constraint) *For each  $i \in I$ ,  $m \in M$ , and  $t \in [0, T']$ ,*

$$g_i(m, t) = g_i(m_i(t), (m_j[t])_{j \in \Psi(t)}, t).$$

Note that the sequential service constraint implies that the bank cannot know the number of total withdrawal tenders at time 0. Suppose that two depositors tender a full withdrawal at time 0. The bank can provide the good to only one depositor at that time because of Assumption 4 and because all the information available to the bank in providing the good at time 0 is that there is one depositor who tenders a full withdrawal. Furthermore, Assumption 5 makes it impossible for the bank to re-obtain the consumption from served depositors.<sup>5</sup>

## 5.2 Provision scheme and solution concepts for implementation

Let  $q_i(t)$  denote the amount of consumption that the bank has provided to depositor  $i$  in  $[0, t)$ . Our sequential service constraint implies that the bank knows  $q_i(t)$  if the bank provides the good to depositor  $i$  at time  $t$ . We pick  $k_1$  and  $k_2$  in  $(0, T)$  such that  $0 < k_1 < k_2 < T$  and define the phases such that  $\Delta_1 \equiv [0, k_1)$ ,  $\Delta_2 \equiv [k_1, k_2)$ ,  $\Delta_3 \equiv [k_2, T)$ , and  $\Delta_4 \equiv [T, T']$ . We consider

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<sup>5</sup>Our sequential service constraint is less demanding than that used in the research of a finite-trader version of the Diamond–Dybvig model. See Green and Lin (2003), Peck and Shell (2003), Andolfatto et al. (2007), Andolfatto and Nosal (2008), and Ennis and Keister (2009).

a provision scheme  $\mathcal{M}^* = (M_i, g_i(\cdot, \cdot))_{i \in I}$  that comprises these phases.

**Phase 1:** For all  $t \in \Delta_1$  and  $m \in M$ ,

$$g_i(m \setminus m_i, t) = \begin{cases} 1 & \text{w.p. } p \\ 0 & \text{w.p. } 1 - p \end{cases}$$

if  $m_i(t) = 1$  and  $q_i(t) < 1$ ; otherwise,  $g_i(m, t) = 0$  with probability one, where  $p \in (0, 1)$  is such that  $0 < pR < 1$  and  $pu(R) < d_1 - d_2$ .

**Phase 2:** Let  $\hat{\theta}$  denote the number of full withdrawal tenders in  $\Delta_1$ . For all  $t \in \Delta_2$ ,  $g_i(\cdot, t) = c_e^*(\hat{\theta}) - 1$  if  $q_i(t) = 1$ ; otherwise,  $g_i(\cdot, t) = 0$ .

**Phase 3:** Let  $Q(t)$  denote the set of depositors who have been served  $c_e^*(\hat{\theta})$  by time  $t$ ; that is,

$$Q(t) \equiv \mathcal{L}\left(\{j \in \Psi(t) \mid \exists t' \leq t, q_j(t') = c_e^*(\hat{\theta})\}\right).$$

For all  $t \in \Delta_3$  and  $m \in M$ ,

$$g_i(m \setminus m_i, t) = \begin{cases} c_e^*(\hat{\theta}) & \text{if } Q(t) \leq \hat{\theta} \\ 0 & \text{otherwise} \end{cases}$$

if  $m_i(t) = 1$  and  $q_i(t) = 0$ ; otherwise,  $g_i(m, t) = 0$ .

**Phase 4:** For all  $t \in \Delta_4$  and  $m \in M$ , if  $\hat{\theta} < 1$ ,

$$g_i(m \setminus m_i, t) = \frac{R(1 - \hat{\theta}c_e^*(\hat{\theta}))}{1 - \hat{\theta}}$$

whenever  $m_i(t) = 1$  and  $q_i(t) = 0$ ; otherwise,  $g_i(m, t) = 0$ .

Under Assumption 4, the scheme  $\mathcal{M}^*$  can provide to at most one depositor at each time. To investigate the strategic behavior of depositors, it is important to know the order in which depositors are served. Let  $\omega_k(t) \in I$  denote a depositor who can be served at time  $t$  in Phase  $k$ . We assume that

the scheme  $\mathcal{M}^*$  is subject to the following law.

**Assumption 6** For any Phase  $k \in \{1, 2, 3, 4\}$  in  $\mathcal{M}^*$ , the order in which depositors are served is determined a priori by a bijection  $\omega_k : \Delta_k \rightarrow I$ .

Let  $\omega_k^{-1}$  denote an inverse of  $\omega_k$ . For any  $i$  and  $j$  in  $I$  such that  $i \neq j$ , if  $\omega_k^{-1}(i) < \omega_k^{-1}(j)$ , then depositor  $i$  can be served earlier than  $j$  in Phase  $k$ . Under Assumption 6, the provision function in  $\mathcal{M}^*$  depends on  $\omega$  rather than  $t$ ; hence, we describe the provision function for  $i$  as  $g_i(m, \omega_k)$ . Let  $\Omega_k$  denote the set of all possible bijections  $\omega_k$  in Phase  $k$  and  $\omega \in \Omega \equiv \Omega_1 \times \Omega_2 \times \Omega_3 \times \Omega_4$ . We assume that depositors do not know a realized  $\omega$ .

Let us denote the payoff function of depositor  $i$  by

$$U_i(g_i, m, \omega) = \begin{cases} u(g_i(m, \omega)) + D_i(m) & \text{if } i \text{ is } \alpha\text{-late} \\ u(g_i(m, \omega)) & \text{otherwise,} \end{cases}$$

where  $D_i(m)$  is the cost function defined in Definition 2. Given a type  $\eta_i$ , a message  $m'_i$  is *weakly dominated* against  $\hat{M}_{-i} \subset M_{-i}$  if, for all  $\omega \in \Omega$  and  $m_{-i} \in \hat{M}_{-i}$ , there exists a message  $m''_i$  that satisfies

$$U_i(g_i, m''_i, m_{-i}, \omega) \geq U_i(g_i, m'_i, m_{-i}, \omega) \quad (7)$$

and for some  $\omega \in \Omega$  and  $m_{-i} \in \hat{M}_{-i}$ , there exists a message  $m_i$  that satisfies  $U_i(g_i, m, \omega) > U_i(g_i, m'_i, m_{-i}, \omega)$ . If (7) holds strictly, then message  $m'_i$  is *strictly dominated* against  $\hat{M}_{-i}$ . If there exists a message  $m''_i$  such that for all  $\omega \in \Omega$  and  $m_{-i} \in \hat{M}_{-i}$ , (7) holds and for some  $\omega \in \Omega$  and  $m_{-i} \in \hat{M}_{-i}$ , (7) holds strictly, then message  $m''_i$  *weakly dominates* message  $m'_i$  against  $\hat{M}_{-i}$ . Let  $s_i$  denote a pure *strategy* of depositor  $i$ ,  $s_i : \{e, \alpha, \beta\} \rightarrow M_i$ . I denote by  $S_i$  the set of pure strategies of depositor  $i$ . A strategy  $s''_i \in S_i$  *weakly dominates*  $s'_i \in S_i$  against  $\hat{S}_{-i} \subset S_{-i}$  if for all  $\eta_i \in \{e, \alpha, \beta\}$ ,  $\omega \in \Omega$ , and  $s_{-i} \in \hat{S}_{-i}$ ,

$$U_i(g_i, s''_i(\eta_i), s_{-i}, \omega) \geq U_i(g_i, s'_i(\eta_i), s_{-i}, \omega) \quad (8)$$

and for all  $\eta_i \in \{e, \alpha, \beta\}$ , there exists some  $\omega \in \Omega$  and  $s_{-i} \in \hat{S}_{-i}$  such



that (8) holds strictly. Let  $K$  denote a bounded subset of  $\mathbb{R}_+$ . Consider a sequence of sets indexed by  $k \in K$ ,  $\{S_i(k)\}_{k \in K}$ , such that [1]  $S_i(a) = S_i$ , where  $a = \inf K$ , [2]  $S_i(k) \subseteq S_i(k')$  iff  $k \geq k'$ , and [3] any  $s'_i \in S_i(k') \setminus S_i(k)$ , if it exists, is weakly dominated against  $S_{-i}(k')$ . I refer to the sequence that satisfies properties [1] through [3] as a *deletion sequence*. A strategy  $s_i^*$  is *weakly iteratively undominated* if there exists a deletion sequence such that  $s_i^* \in \bigcap_{k \in K} S_i(k)$ .

**Definition 4** *The provision scheme  $\mathcal{M}^* = (M, g)$  implements an efficient contract  $\mathbf{c}^* = (c_e^*(\cdot), c_i^*(\cdot))$  in weakly iteratively undominated strategies if for any  $\theta \in [0, 1]$  and any weakly iteratively undominated strategy profile  $s^*$ , the provision scheme  $\mathcal{M}^*$  achieves the allocation  $(c_e^*(\theta), c_i^*(\theta))$  with the message profile  $m^* = s^*(\theta)$ .*

### 5.3 Main theorem

We make an assumption concerning the existence ratio of depositors.

**Assumption 7** (i) If  $\theta \in (0, 1)$ , then  $\mathcal{L}(I_\theta(\alpha)) > 0$ . (ii) For all  $\theta \in [0, 1]$  and  $\eta \in \{e, \alpha, \beta\}$ , if  $I_\theta(\eta) \neq \emptyset$ , then  $\mathcal{L}(I_\theta(\eta)) > 0$ .

Assumption 7(i) implies that if there are late depositors, some of them have the socially conscious preference. Assumption 7(ii) implies that if a depositor finds himself a type  $\eta$ , he is sure that there are many type- $\eta$  depositors. Note that Assumption 7 is used to generalize S1 in Assumption 3. We describe the psychological environment that satisfies Assumption 7 as  $\mathcal{E}_p^*$ .

**Theorem 1** *Suppose that any provision scheme is subject to the sequential service constraint with Assumptions 4, 5, and 6. In a psychological environment  $\mathcal{E}_p^*$ , the payment scheme  $\mathcal{M}^*$  implements any efficient contract  $c^* \in C^*$  in weakly iteratively undominated strategies, where*

$$C^* = \{c^* \in C \mid c_e^*(0) \in (0, R), c_i^*(1) = 0\}.$$

**Remark.** (1) The proof of this theorem is analogous to that of Proposition 1. (2) The result is valid even if the number of socially conscious depositors

is arbitrary low as far as it has a positive Lebesgue measure. **Proof.** For convenience, we describe an action of a message  $m_i$  for which  $m_i = s_i(\eta)$  as  $s_i(\eta, t)$ , i.e.,  $s_i(\eta, t) = m_i(t)$ , where  $m_i = s_i(\eta)$ . We define the earliest time of  $s_i(\eta, t) = 1$  for late depositor  $i$  as

$$\tau(s_i(\eta)) \equiv \inf\{t \in [0, T'] \mid s_i(\eta, t) = 1\}$$

for  $\eta \in \{\alpha, \beta\}$ .

**(1. Early depositors):** Consider a strategy such that  $s_i^*(e, t) = 1$  for all  $t \in [0, T)$ . For an arbitrary message profile  $m_{-i} \in M_{-i}$ , his payoff is

$$pu(c_e^*(\hat{\theta})) + (1-p)u(g_i(m \setminus s_i^*(e), \omega_3)).$$

We show that message  $s_i^*(e)$  weakly dominates any message  $s_i(e)$  such that  $s_i(e, t') = 0$  for some  $t' \in [0, T)$  and  $s_i(e, t) = s_i^*(e, t)$  for all  $t \in [0, T) \setminus \{t'\}$ . If  $i = \omega_1(t')$ , then Assumptions 4 and 6 imply that depositor  $i$  cannot be served, and hence, his payoff is  $u(g_i(m \setminus s_i(e), \omega_3))$  or  $pu(c_e^*(\hat{\theta}))$ . Because  $u(g_i(m \setminus s_i^*(e), \omega_3)) = u(g_i(m \setminus s_i(e), \omega_3))$ , the payoff difference is  $p\{u(c_e^*(\hat{\theta})) - u(g_i(m \setminus s_i(e), \omega_3))\}$  or  $(1-p)u(g_i(m \setminus s_i(e), \omega_3))$ . If  $Q(\omega_3^{-1}(i)) \leq (1-p)\hat{\theta}$ , then  $u(g_i(m \setminus s_i(e), \omega_3)) = u(c_e^*(\hat{\theta}))$ ; otherwise,  $u(g_i(m \setminus s_i(e), \omega_3)) = 0$ . Because  $c_e^*(0) > 0$ , the message  $s_i^*(e)$  weakly dominates  $s_i(e)$  against  $M_{-i}$ . We define  $S_i(-1)$  as  $S_i(-1) \equiv \{s_i \in S_i \mid s_i(e, t) = 1, \forall t \in [0, T)\}$  and

$$M_i(-1) \equiv \{m_i \mid \exists s_i \in S_i(-1), \exists \eta \in \{e, \alpha, \beta\}, m_i = s_i(\eta)\}.$$

**(2.  $\alpha$ -late depositors):** Suppose that  $S(-1)$  is given. When  $\eta_i = \alpha$ , Assumption 7(ii) ensures that  $\mathcal{L}(I_\theta(\alpha)) > 0$ . Pick any  $k \in [0, k_2]$  and define

$$S_i(k) \equiv \{s_i \in S_i(-1) \mid \tau(s_i(\alpha)) \geq k\}.$$

Consider a strategy of depositor  $i$ ,  $s'_i \in S_i(0)$ , such that  $\tau(s'_i(\alpha)) = 0$  and  $s'_i(\alpha, t) = 0$  for all  $t \in \Delta_4$ . Suppose that there exists a depositor  $j \in I_\theta(\alpha) \setminus \{i\}$

such that  $\tau(s_j(\alpha)) = 0$ . Then, the maximum payoff of depositor  $i$  is

$$pu(c_e^*(\hat{\theta})) + (1 - p)u(g_i(m \setminus s'_i(\alpha), \omega_3)) - d_1. \quad (9)$$

Let  $s''_i(\alpha)$  denote a message such that  $\tau(s''_i(\alpha)) = k_2$  and  $s''_i(\alpha, t) = s'_i(\alpha, t)$  for all  $t \geq k_2$ . Then,  $i$ 's payoff is

$$u(g_i(m \setminus s''_i(\alpha), \omega_3)) - d_2. \quad (10)$$

Because  $g_i(m \setminus s'_i(\alpha), \omega_3) = g_i(m \setminus s''_i(\alpha), \omega_3)$  and the maximum of  $g_i(m \setminus s'_i(\alpha), \omega_3)$  is  $c_e^*(\hat{\theta})$ , the difference of (9) and (10) is

$$\begin{aligned} & p\left(u(c_e^*(\hat{\theta})) - u(g_i(m \setminus s''_i(\alpha), \omega_3))\right) - (d_1 - d_2) \\ & \leq pu(c_e^*(\hat{\theta})) - (d_1 - d_2) \\ & < 0 \end{aligned}$$

because  $c_i^*(0) < R$  and the value of  $p$ . Next, consider a strategy  $s'_i \in S_i(0)$  such that  $\tau(s'_i(\alpha)) = 0$  and  $s'_i(\alpha, t) = 1$  for some  $t \in \Delta_4$ . In this case, for all  $m \in \prod_{i \in I} M_i(-1)$ , if  $g_i(m \setminus s'_i(\alpha), \omega_4) = 0$  for all  $\omega_4 \in \Omega_4$ , then the above reasoning applies and the message  $s''_i(\alpha)$  is a more profitable message for depositor  $i$ . If  $g_i(m \setminus s'_i(\alpha), \omega_4) > 0$  for some  $\omega_4 \in \Omega_4$ , then  $i$ 's maximum payoff is  $u(c_i^*(\hat{\theta})) - d_1$ . Hence, depositor  $i$  can be better-off by changing the message to  $s_i(\alpha)$  such that  $\tau(s_i(\alpha)) = T$  and  $s_i(\alpha, t) = s'_i(\alpha, t)$  for all  $t \geq T$ . We can easily check that the reasoning for these two cases is valid for the case in which  $\tau(s_j(\alpha)) \in (0, k_2)$  for all  $j \in I_\theta(\alpha) \setminus \{i\}$ . Suppose that for all  $j \in I_\theta(\alpha) \setminus \{i\}$ ,  $\tau(s_j(\alpha)) \in \Delta_3$ . In this case, Assumption 7 implies that it is strictly better-off for depositor  $i$  to send a message  $s_i(\alpha)$  such that  $\tau(s_i(\alpha)) = T$  and  $s_i(\alpha, t) = 1$  for all  $t \in \Delta_4$ . Therefore, any strategy  $s'_i \in S_i(0)$  such that  $\tau(s'_i(\alpha)) = 0$  is weakly dominated by a strategy  $s_i \in S_i(k')$  for some  $k' > 0$ . Applying this reasoning to any  $k \in (0, k_2)$ , any strategy  $s'_i \in S_i(k)$  such that  $\tau(s'_i(\alpha)) = k$  is shown to be weakly dominated by a strategy  $s_i \in S_i(k')$  for some  $k' > k$ . Hence, we obtain a deletion sequence  $\{S_i(k)\}_{k \in [0, k_2]}$  such that any  $s_i \in \bigcap_{k \in [0, k_2]} S_i(k)$  satisfies  $\tau(s_i(\alpha)) \geq k_2$ .

**(3.  $\beta$ -late depositors):** Suppose that  $S(k_2)$  is given. When  $\eta_i = \beta$ , depositor  $i$  deduces that  $\mathcal{L}(I_\theta(\alpha)) > 0$  because of Assumption 7 and that  $\hat{\theta} < 1$ . Hence, the  $\beta$ -late depositor  $i$  rationally expects to obtain  $c_i^*(\hat{\theta})$  in  $[T, T']$  for certain. Let

$$S_i(T) \equiv \{s_i \in S_i(k_2) \mid \tau(s_i(\beta)) = T, s_i(\beta, t) = 1, \forall t \in [T, T']\}.$$

**(4.  $\alpha$ -late depositors):** Suppose that  $S(T)$  is given. For any  $\alpha$ -late depositor  $i$ , the strategy  $s_i$  such that  $s_i(\alpha, t) = 1$  for all  $t \in [T, T']$  weakly dominates any other strategy in  $S_i(T)$  against  $S_{-i}(T)$ . Let

$$S_i(T') \equiv \{s_i \in S_i(T) \mid \tau(s_i(\alpha)) = T, s_i(\alpha, t) = 1, \forall t \in [T, T']\}.$$

The above reasoning shows that any strategy  $s_i \in S_i(T')$  is an iteratively weakly undominated strategy. Furthermore, for any  $\theta \in [0, 1]$ , the message profile  $s(\theta)$  for any  $s \in S(T')$  achieves the allocation  $(c_e^*(\theta), c_l^*(\theta))$ . Hence, the provision scheme  $\mathcal{M}^*$  implements any efficient contract  $c^* \in C^*$  in iteratively undominated strategies. ■

The provision scheme  $\mathcal{M}^*$  gathers the state information  $\theta$  in Phase 1. In this information gathering, the probability  $p$  plays a crucial role. We can interpret  $p$  as the threshold that the bank stops to provide the good before maturity. For instance, the bank may stop to provide if it observes that a full withdrawal tender continuously occurs in  $[0, \theta']$  for some  $\theta' < k_1$ . In this case, we can interpret the probability as  $p = \theta'/k_1$ ; the bank concludes that such a continuous tendering is a signal of self-fulfilling bank runs, and the early deposit freeze in an environment with aggregate risk implies the bank's preemptive move to bank runs.

I should note that our result crucially depends on the assumption of no liquidation cost, as in Diamond and Dybvig (1983).<sup>6</sup> If there is a liquidation cost, our provision scheme  $\mathcal{M}^*$  generally fails to implement an efficient con-

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<sup>6</sup>For a banking model like Diamond–Dybvig with a liquidation cost for investment, see Cooper and Ross (1998).

tract. To confirm this fact, suppose that there is a liquidation cost  $\kappa > 0$  per unit liquidation for the investment. Because the bank in our model invests all funds in  $E_0$ , we obtain  $1 - (1 - \kappa)^{-1} < 0$ , which implies that the scheme  $\mathcal{M}^*$  fails to achieve the allocation  $(c_e^*(\theta), c_l^*(\theta))$  if  $\theta$  is sufficiently close to 1. Furthermore, even if  $1 - \hat{\theta}c_e^*(\hat{\theta})(1 - \kappa)^{-1} > 0$  for some  $\hat{\theta}$ , we cannot offer  $c_l^*(\hat{\theta})$  for  $1 - \hat{\theta}$  late depositors because  $R(1 - \hat{\theta}c_e^*(\hat{\theta})(1 - \kappa)^{-1}) < c_l^*(\hat{\theta})(1 - \hat{\theta})$ . Therefore, even in a psychological environment  $\mathcal{E}_p^*$ , the provision scheme  $\mathcal{M}^*$  cannot achieve the allocation  $(c_e^*(\theta), c_l^*(\theta))$  for some  $\theta$ .

## 6 Concluding remarks

This paper revisits the Diamond–Dybvig (1983) model in an environment with an aggregate risk for states and establishes the positive result that bank runs can be prevented without government intervention such as deposit insurance if some late depositors have a socially conscious preference. Notice that our positive result is obtained even if the number of socially conscious depositors is low.

However, our positive result relies on the technological assumption that there is no liquidation cost for investment, as in Diamond and Dybvig (1983). Because of this assumption, a representative bank can be free from any distortion by liquidation, and hence, it can offer an efficient allocation for any realized state.

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