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A Real Options Game: Investment on the Project with Operational Options and Fixed Costs

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A Real Options Game: Investment on the Project with Operational Options and Fixed Costs[*]

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Abstract

In this paper, we analyze investment decision on the 'entry-exit' project, which can be active and suspended by paying some cost, in a duopoly setting. The model incorporates Dixit (1989) and Huisman and Kort (1999). That is, we propose a new extension of the model that captures competitive nature in the recent trend. Then we show it is optimal that the firm must start producing at the beginning of the project, and the leader is more encouraged to invest in a duopoly than monopoly while the follower is more discouraged.

Keywords: Entry-exit; duopoly; operational options
JEL classification: D81; C73

1 Introduction

Real options approach, in other words, investment under uncertainty is classically studied by Brennan and Schwartz (1985) and McDonald and Siegel (1986), and basically summarized by Dixit and Pindyck (1994). Dixit and Pindyck (1994) provide the most basic model in Ch.5. When a project investment is irreversible and the project value is uncertainty, they formulate a firm's project value maximizing problem as a optimal stopping problem and show the optimal investment timing, so that they derive the value of the option to invest. This analysis directly applies the firm's decision to entry in a new/existing market if the firm has not enter in the market yet. Dixit and Pindyck (1994) also analyze investment decision on the project that can be suspended without cost in Ch.6. Furthermore,

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Dixit and Pindyck (1994) analyze 'entry-exit' decision that the project can be active and suspended by paying some cost in Ch.7. Note that 'exit' means 'suspend' in this paper unlike Alvarez (1998, 1999) which study 'shut-down' decision with some cost. Dixit (1989) and Brekke and Øksendal (1991, 1994) also study entry and exit decisions in detail. Abel and Eberly (1996) study a firm's investment problem when investment is characterized by costly reversibility.

However, the literature above does not consider firms' competitive nature. In real business environment, firms face stiff competition and consider not only their own strategies but also competitors' strategies. There are the competitive interactions between competing firms. Then, real options approach is naturally extended to analyze the firms' competitive nature by using game theory. In case of preemption, there is the risk that the firm may earn less profit if its competitor invests earlier. Game theory results in that the firm must invest earlier than monopoly, in contradiction to real options approach. Dixit and Pindyck (1994) incorporate competitive nature into real options approach properly at the earliest date in Ch.9. While they investigate an oligopolistic industry, they do not derive the equilibrium by game theory. On the other hand, Grenadier (1996) applies their framework to the real-estate investment problem, and succeeds in deriving the equilibrium. His equilibrium remains the problem that simultaneous investment is eliminated, whereas Huisman and Kort (1999) extends the theoretical framework by resolving the problem.

While the above literature analyze only the competitive situation by assuming firms do not have operational options, Takashima et al. (2008) consider symmetric and asymmetric firms competition with operational options in the electricity market. Entry and exit decisions in a duopoly are investigated by Lambrecht (2001), in particular, market entry, company closure and capital structure. Ruiz-Aliseda (2003) and Amir and Lambson (2003) investigate it by game theoretic approaches.

However, these literature do not analyze the situation of Ch.7 in Dixit and Pindyck (1994). So, in this paper we analyze investment decision on the entry-exit for a project with transition costs in a duopoly setting. We assume that two firms are identical and their roles are endogenously determined. One firm can be leader by investing the project before the other. The firms can start (enter) the project by investing the initial cost. However, just investing the project does not enable the firm to do business. We assume that it incurs the cost to make the project active, that is, sell a product. Furthermore, the firms can suspend (exit) the project by paying the cost and make the project reactive (reenter) by paying the cost. The firm 's aim is to maximize their profit derived from the project in duopoly. To solve the firms' problems, we first solve the single firm's profit maximizing problem as in Dixit (1989) and Dixit and Pindyck (1994, Ch.7). Next, we solve each firm's problem in duopoly. Consequently, we provide the equilibria in the firms'

investment game. To derive the equilibria, we use the strategy space and equilibrium concept defined by Huisman and Kort (1999). Then, we show three types of equilibria. Furthermore, we show numerical examples and comparative static results of the thresholds which determine the investment, suspend, and reactive timing. From the results we find that the leader is more encouraged to invest and the follower is more discouraged.

The rest of the paper is organized as follows. Section 2 describes the firm's problem. Section 3 provides the equilibria, i.e., the firm's optimal investment strategy in our duopoly setting. Next, we present numerical examples in section 4. Then, we discuss some important results in section 5. Lastly, section 6 concludes the paper.

2 The Model

Suppose that two identical firms consider entering a new market, so that the market is a duopoly. The two firms are labeled 1 and 2. By index i we refer to an arbitrary firm and by j to the 'other firm.' In order to enter the market, they invest a project with the initial cost I. We assume that they can sell a product in the market at the moment of investing the initial cost.

The project can become active by paying some cost K, then the firm can produce a unit flow of output at the variable cost C. Moreover, the project can be suspended by paying an exit cost E, and the firm can reenter by paying K again at some future time. In this context, the term 'reenter' means that the firm resumes the product salses. The product will yield the sales according to a downward-sloping inverse demand function $D(Q_t)$. Q_t denotes the number of firms which have invested the project. $D(\cdot)$ is a differential function with $D'(\cdot) < 0$, which ensures the first mover's advantage. This market is characterized by evolving uncertainty in the state of demand, so that the demand is subject to random shocks derived by the shock variable X_t. Then, the demand function is of the following form:

$$P_t = D(Q_t)X_t, \tag{1}$$

where P_t denotes the output price at time t. We assume that X_t follows a geometric Brownian motion:

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}W_t, \quad X_0 = x, \tag{2}$$

where μ is the instantaneous expected growth rate of X_t, σ (> 0) is the associated volatility, and W_t is a standard Brownian motion.

The project has three state variables, the demand shock X_t, the number of firms Q_t and discrete variable that indicates whether the project is active (1) or not (0). Let a right-continuous function with left limits Z_t denote the state 0 or 1. The profit function

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of both firms is given by

$$\pi(X_t, Q_t) = \big(D(Q_t)X_t - C\big)Z_t. \tag{3}$$

We first consider the situation that the firms already have invested. Then, the firm's problem is to choose the timing of making the project active or suspending the project in order to maximize the expected discounted profit. Let θ_m be the m-th timing of making the project active or suspend the project. Suppose $Q_t = q$, the single firm's value function is given by

$$J(x, q) = \sup_{w \in \mathcal{W}} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\pi(X_t, q)\mathrm{d}t - \sum_{m=1}^\infty e^{-\rho\theta_m} H(Z_{\theta_m^-}, Z_{\theta_m})\mathbf{1}_{\{\theta_m < \infty\}}\right], \tag{4}$$

where $\rho\ (> \mu)$ denotes a discount rate,

$$w = (\theta_1, \theta_2, \dots, \theta_m, \dots; \zeta_1, \zeta_2, \dots, \zeta_m, \dots) \tag{5}$$

denotes the collection of the stopping time θ_m and the control of the state $\zeta_m = Z_{\theta_m}$, $\mathcal{W}$ denotes the collection of the admissible controls and

$$H(0, 0) = H(1, 1) = 0, \tag{6}$$
$$H(0, 1) = K, \tag{7}$$
$$H(1, 0) = E. \tag{8}$$

Equation (4) can be decomposed into the value function of inactive state $J_0(x, q)$ and active state $J_1(x, q)$. From Dixit (1989), they are given by

$$J_0(x, q) = G_0(q)x^{\beta_1}, \tag{9}$$
$$J_1(x, q) = G_1(q)x^{\beta_2} + \frac{D(q)x}{\delta} - \frac{C}{\rho}, \tag{10}$$

respectively, where $\delta = \rho - \mu$. $\beta_1 > 1$ and $\beta_2 < 0$ are respectively the solution to the following characteristic equation: $1/2\sigma^2\beta(\beta - 1) + \mu\beta - \rho = 0$. The unknown parameters $G_0(q)$ and $G_1(q)$ are found numerically from the value-matching conditions:

$$J_0(\overline{X}(q), q) = J_1(\overline{X}(q), q) - K, \tag{11}$$
$$J_1(\underline{X}(q), q) = J_0(\underline{X}(q), q) - E, \tag{12}$$

and smooth-pasting conditions:

$$J_0'(\overline{X}(q), q) = J_1'(\overline{X}(q), q), \tag{13}$$
$$J_1'(\underline{X}(q), q) = J_0'(\underline{X}(q), q), \tag{14}$$

where $\overline{X}(q)$ and $\underline{X}(q)$ are the optimal threshold to reenter and suspend respectively, which are found from the above conditions.

Next, we consider the situation that the firms have not invested yet. The firms problems are to choose the timing of investment in order to maximize the expected discounted profit. In a duopoly setting, the firm i's decision problem is given by

$$
\begin{aligned}
V^i(x) = \sup_{\tau^i \in \mathcal{T}} \mathbb{E} \Bigg[& \sup_{w \in \mathcal{W}} \mathbb{E} \Bigg[\int_{\tau^i \wedge \tau^j}^{\tau^i \vee \tau^j} \mathrm{e}^{-\rho t} \pi(X_t, 1) \mathrm{d}t \\
& - \sum_{m=1}^{\infty} \mathrm{e}^{-\rho \theta_m} H(Z_{\theta_m^-}, Z_{\theta_m}) \mathbf{1}_{\{\tau^i \wedge \tau^j \le \theta_m < \tau^i \vee \tau^j\}} \Bigg] \mathbf{1}_{\{\tau^i < \tau^j\}} \\
& + \sup_{w \in \mathcal{W}} \mathbb{E} \Bigg[\int_{\tau^i \vee \tau^j}^{\infty} \mathrm{e}^{-\rho t} \pi(X_t, 2) \mathrm{d}t - \sum_{m=1}^{\infty} \mathrm{e}^{-\rho \theta_m} H(Z_{\theta_m^-}, Z_{\theta_m}) \mathbf{1}_{\{\tau^i \vee \tau^j \le \theta_m < \infty\}} \Bigg] \\
& \hspace{10cm} - \mathrm{e}^{-\rho \tau^i} I \Bigg], \\
= \sup_{\tau^i \in \mathcal{T}} \mathbb{E} \Bigg[& \mathrm{e}^{-\rho \tau^i \wedge \tau^j} J(X_{\tau^i \wedge \tau^j}, 1) \mathbf{1}_{\{\tau^i < \tau^j\}} \\
& + \mathrm{e}^{-\rho \tau^i \vee \tau^j} \left(J(X_{\tau^i \vee \tau^j}, 2) - J(X_{\tau^i \vee \tau^j}, 1) \mathbf{1}_{\{\tau^i < \tau^j\}} \right) - \mathrm{e}^{-\rho \tau^i} I \Bigg], \quad (15)
\end{aligned}
$$

where τ^i denotes the stopping time for firm i to invest and $\mathcal{T}$ denotes the collection of admissible stopping times. The second equality holds by strong Markov property of X_t. Since $J(x, q)$ can be decomposed into two states, equation (15) can be rewritten into

$$
\begin{aligned}
V_k^i(x) = \sup_{\tau^i \in \mathcal{T}} \mathbb{E} \Bigg[& \mathrm{e}^{-\rho \tau^i \wedge \tau^j} J_k(X_{\tau^i \wedge \tau^j}, 1) \mathbf{1}_{\{\tau^i < \tau^j\}} \\
& + \mathrm{e}^{-\rho \tau^i \vee \tau^j} \left(J_k(X_{\tau^i \vee \tau^j}, 2) - J_k(X_{\tau^i \vee \tau^j}, 1) \mathbf{1}_{\{\tau^i < \tau^j\}} \right) - \mathrm{e}^{-\rho \tau^i} I \Bigg], \quad (16)
\end{aligned}
$$

$$
V^i(x) = \max\{V_0^i(x), V_1^i(x)\}. \quad (17)
$$

Equation (17) claims that both firms must choose initial state k as well as investment time τ^i.

There are three patterns of investment. If $\tau^i < \tau^j$, firm i can earn higher profit until firm j enters into the market at τ^j. In this case, firm i is called *the leader*, and its value function is denoted by $L^i(x)$. If $\tau^i > \tau^j$, firm i waits to enter and can earn no profit until τ^i. In this case, firm i is called *the follower*, and its value function is denoted by $F^i(x)$. If $\tau^i = \tau^j$, both firms earn lower profit since they enter into the market simultaneously. This case is called *simultaneous investment*, and the associated value function is denoted by $M^i(x)$.

3 Equilibria

In this section, we solve the firm i's problem (17) and derive the equilibrium. Since dynamic games are usually solved backwards, we solve the maximum problem (17) at the

moment the leader has invested, i.e., $\tau^i \wedge \tau^j = 0$. In what follows, we omit the index i, j because two firms are identical.

First, we derive the value function of simultaneous investment. In this case, both firms invest simultaneously such that $Q_t = 2$ for all time, so we have

$$M(x) = \max\{J_0(x, 2), J_1(x, 2)\} - I. \tag{18}$$

Since $J_0(x, 2)$ crosses $J_1(x, 2)$ at the point $x \in (\underline{X}(2), \overline{X}(2))$, let $\tilde{X}(2)$ denote the corresponding demand shock value. Then, we have

$$M(x) = \begin{cases} J_0(x, 2) - I = G_0(2)x^{\beta_1} - I, & \text{for } x < \tilde{X}(2), \\ J_1(x, 2) - I = G_1(2)x^{\beta_2} + \dfrac{D(2)x}{\delta} - \dfrac{C}{\rho} - I, & \text{for } x \geq \tilde{X}(2). \end{cases} \tag{19}$$

Because the simultaneous investment is not optimal, for $x < \tilde{X}(2)$, both firms do not produce from the beginning of the project until $X_t \geq \overline{X}(2)$.

Next, since the leader has already invested the project, the value function of the follower is

$$F(x) = \sup_{\tau \in \mathcal{T}} \mathbb{E}\left[\sup_{w \in \mathcal{W}} \mathbb{E}\left[\int_\tau^\infty e^{-\rho t}\pi(X_t, 2)\mathrm{d}t - \sum_{m=1}^\infty e^{-\rho\theta_m}H(Z_{\theta_m^-}, Z_{\theta_m})\mathbf{1}_{\{\tau \leq \theta_m < \infty\}}\right] - e^{-\rho\tau}I\right],$$

$$= \sup_{\tau \in \mathcal{T}} \mathbb{E}\left[e^{-\rho\tau}\left(J(X_\tau, 2) - I\right)\right],$$

$$= \sup_{\tau \in \mathcal{T}} \mathbb{E}\left[e^{-\rho\tau}\left(\max\{J_0(X_\tau, 2), J_1(X_\tau, 2)\} - I\right)\right], \tag{20}$$

where τ denotes the stopping time for the follower to invest. Given the constant threshold of the follower X^{F}, τ is the form of

$$\tau = \inf\{t > 0 : X_t \geq X^{\mathrm{F}}\}. \tag{21}$$

Equation (20) satisfies the following ODE:

$$\frac{1}{2}\sigma^2 x^2 F''(x) + \mu x F'(x) - \rho F(x) = 0, \tag{22}$$

with boundary conditions:

$$F(0) = 0, \tag{23}$$
$$F(X^{\mathrm{F}}) = \max\{J_0(X^{\mathrm{F}}, 2), J_1(X^{\mathrm{F}}, 2)\} - I, \tag{24}$$
$$F'(X^{\mathrm{F}}) = \max\{J_0'(X^{\mathrm{F}}), J_1'(X^{\mathrm{F}})\}. \tag{25}$$

The second condition is called the value-matching condition, and the third is called the smooth-pasting condition. By solving equation (22) with equation (23)–(25), we have

$$F(x) = \begin{cases} Ax^{\beta_1}, & \text{for } x < X^{\mathrm{F}}, \\ J_1(x, 2) - I, & \text{for } x \geq X^{\mathrm{F}}, \end{cases} \tag{26}$$

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where A and X^{F} are the solutions of nonlinear simultaneous equation (24) and (25) corresponding to $J_1(x, 2)$. Note that the optimal threshold for $J_0(x, 2)$ does not exist.

Next, we consider the leader's problem. Suppose that the follower plays the optimal policy τ, the value function of the leader is

$$
\begin{aligned}
L(x) &= \mathbb{E}\left[\sup_{w \in \mathcal{W}} \mathbb{E}\left[\int_0^\tau \mathrm{e}^{-\rho t}\pi(X_t, 1)\mathrm{d}t - \sum_{m=1}^\infty \mathrm{e}^{-\rho\theta_m}H(Z_{\theta_m^-}, Z_{\theta_m})\mathbf{1}_{\{\theta_m < \tau\}}\right] - I \right. \\
&\quad \left. + \sup_{w \in \mathcal{W}} \mathbb{E}\left[\int_\tau^\infty \mathrm{e}^{-\rho t}\pi(X_t, 2)\mathrm{d}t - \sum_{m=1}^\infty \mathrm{e}^{-\rho\theta_m}H(Z_{\theta_m^-}, Z_{\theta_m})\mathbf{1}_{\{\tau \le \theta_m < \infty\}}\right]\right], \\
&= \mathbb{E}\left[J(x, 1) - I + \mathrm{e}^{-\rho\tau}\big(J(X_\tau, 2) - J(X_\tau, 1)\big)\right], \\
&= \max\{J_0(x, 1), J_1(x, 1)\} - I + \max_{k \in \{0,1\}} \mathbb{E}\left[\mathrm{e}^{-\rho\tau}\big(J_k(X_\tau, 2) - J_k(X_\tau, 1)\big)\right]. \quad (27)
\end{aligned}
$$

Let $\tilde{L}(x)$ be the last term of equation (27), it satisfies the following ODE:

$$
\frac{1}{2}\sigma^2 x^2 \tilde{L}''(x) + \mu x \tilde{L}'(x) - \rho\tilde{L}(x) = 0, \quad (28)
$$

with boundary conditions

$$
\tilde{L}(0) = 0, \quad (29)
$$

$$
L(X^{\mathrm{F}}) = F(X^{\mathrm{F}}). \quad (30)
$$

Notice that since equation (27) is not maximum problem, the smooth-pasting condition is not necessary, and X^{F} is known. Since $J_0(x, 1)$ crosses $J_1(x, 1)$ at the point $x \in (\underline{X}(1), \overline{X}(1))$, we define the corresponding demand shock value as $\tilde{X}(1)$. By solving equation (28) with equations (29) and (30), we have

$$
L(x) = \begin{cases} J_0(x, 1) + Bx^{\beta_1} - I, & \text{for } x < \tilde{X}(1), \\ J_1(x, 1) + Bx^{\beta_1} - I, & \text{for } \tilde{X}(1) \le x < X^{\mathrm{F}}, \\ J_1(x, 2) - I, & \text{for } x \ge X^{\mathrm{F}}, \end{cases} \quad (31)
$$

where

$$
B = (G_1(2) - G_1(1))(X^{\mathrm{F}})^{\beta_2 - \beta_1} + \frac{D(2) - D(1)}{\delta}(X^{\mathrm{F}})^{1-\beta_1}. \quad (32)
$$

Now we are in a position to derive the equilibrium firm's problem. To this end, we need the following proposition.

Proposition 1 *There exists a unique value for x, which we denote by X^{L}, such that*

$$
L(X^{\mathrm{L}}) = F(X^{\mathrm{L}}), \text{ and } 0 < X^{\mathrm{L}} < X^{\mathrm{F}}. \quad (33)
$$

The proof is almost same as Takashima et al. (2008) and omitted. We define the stopping time

$$
\lambda = \inf\{t > 0 : X_t \ge X^{\mathrm{L}}\}. \quad (34)
$$

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Unfortunately, X^{L} must be found numerically. Due to proposition 1, we can use the strategy space and equilibrium concept defined by Huisman and Kort (1999). This concept can be traced back to Fudenberg and Tirole (1985).

Proposition 2 *There are three types of equilibria depending on the value of x.*

1. *If $x \in (0, X^{\mathrm{L}}]$, there are two possible outcomes. In the first, firm 1 is the leader and invests the project at time λ, and firm 2 is the follower and invests at time τ with probability $1/2$. The second is the symmetric counterpart, and the probability that both firms invest simultaneously is zero.*

2. *If $x \in (X^{\mathrm{L}}, X^{\mathrm{F}}]$, there are three possible outcomes. In the first, firm 1 is the leader and invests at time 0, and firm 2 is the follower and invests at time τ with probability $\frac{F(x)-M(x)}{L(x)+F(x)-2M(x)}$. The second is the symmetric counterpart. In the third, both firms invests simultaneously at time 0 with probability $\frac{L(x)-F(x)}{L(x)+F(x)-2M(x)}$.*

3. *If $x \in (X^{\mathrm{F}}, \infty)$, then both firms invests at time 0 with probability 1.*

The proof is almost same as Takashima et al. (2008) again and omitted. Proposition 2 claims that there is no simultaneous investment where both firms can earn less profit, if the game starts with low demand shock.

4   Numerical Examples

In this section we numerically calculate the optimal thresholds: $\underline{X}(1)$, $\overline{X}(1)$, $\underline{X}(2)$, $\overline{X}(2)$, X^{L} and X^{F}. Furthermore, we present a comparative statics analysis of the thresholds by changing parameters: volatility σ and the entry cost K. Since volatility σ represents the degree of uncertainty, it is the most important parameter in a real options model. The entry cost K is needed only in the entry-exit model. Therefore, we focus these parameters in this section. We assume that the hypothetical value of the parameters are as follows: $\mu = 0.02$, $\sigma = 0.20$, $\rho = 0.04$, $D(1) = 2$, $D(2) = 1$, $C = 5$, $K = 10$, $E = 5$ and $I = 50$. Then, we have the optimal thresholds: $\underline{X}(1) = 1.53$, $\overline{X}(1) = 3.93$, $\underline{X}(2) = 3.06$, $\overline{X}(2) = 7.87$, $X^{\mathrm{L}} = 3.34$ and $X^{\mathrm{F}} = 11.12$.

Figure 1 displays the value functions of the leader, the follower and simultaneous investment. Their shapes are almost same as Takashima et al. (2008).

Figure 2 displays the comparative statics of the thresholds with respect to σ. Although the leader's threshold X^{L} is less than the monopoly restart threshold $\overline{X}(1)$, the follower's threshold X^{F} is much greater than the duopoly restart threshold $\overline{X}(2)$. This implies that the leader's investment is more encouraged to invest in a duopoly than monopoly and the follower's is more discouraged.
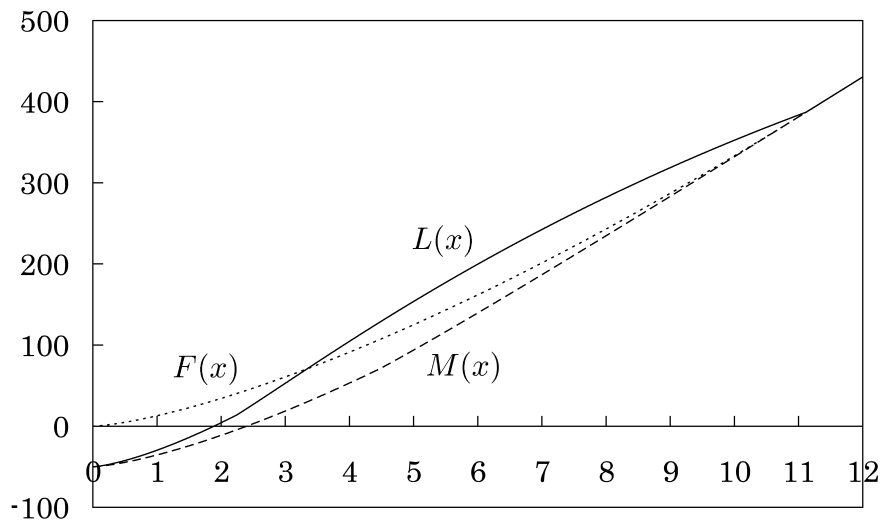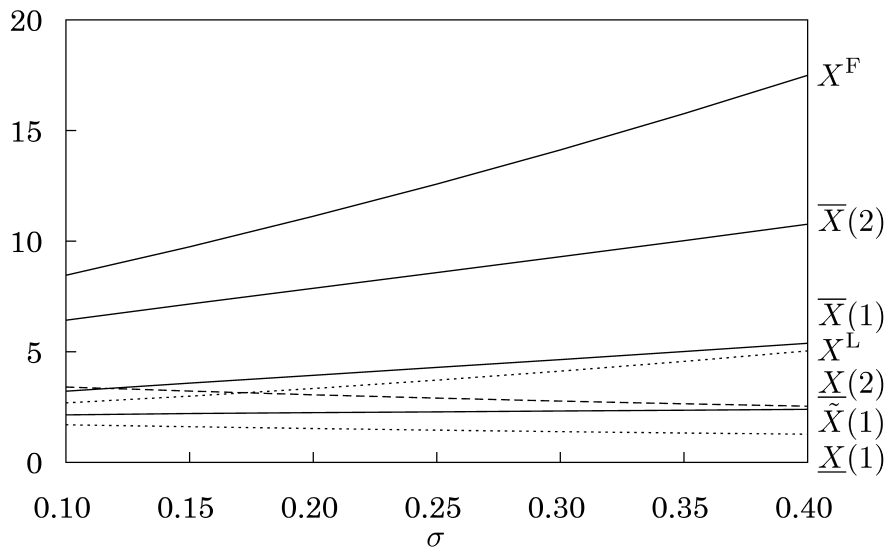
Figure 1: The value functions

Figure 2: The comparative statics of the thresholds with respect to σ
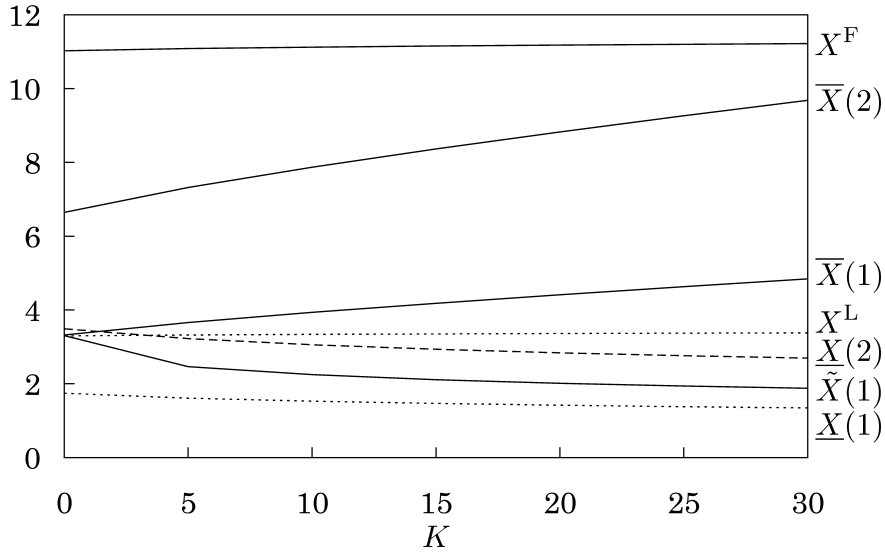
9

Figure 3: The comparative statics of the thresholds with respect to K

Figure 3 displays the comparative statics of the thresholds with respect to K. Compared with σ, entry cost K has less impact on the thresholds, with the exception of the entry thresholds $\overline{X}(1)$ and $\overline{X}(2)$. In particular, leader's and follower's investment thresholds are almost free of the influence of entry cost, which is similar to exit cost E. This shows that entry and exit costs have impact on the strategies after investment, however, no impact before investment.

Figure 4 and 5 display the comparative statics of the follower's and the leaders thresholds with respect to σ, for three cases. The first case (Case 1) is the project without operational options (Ch.5 in Dixit and Pindyck (1994)), the second case (Case 2) is the operational option without costs (Ch.6), and the third case (Case 3) is the operational option with fixed costs (Ch.7 and our model).

In figure 4, Case 3 intermediates between Case 1 and Case 2. In Case 3, $K = E = 0$ means Case 2, and $K = E = \infty$ means Case 1 since the option is no longer exercised. Since the difference between Case 1 and Case 2 is the operational option value, Case 1 coincides Case 2 when σ is near 0. To the contrary, the differences among each option value become larger when σ is large.

In figure 5, since the leader's threshold is not optimized, we can not discuss similarly to the follower's in figure 4. Although there are no longer orderly relations between Case 1 and Case 2, Case 3 has the smallest thresholds for all σ. This implies the leader's investment on the entry-exit project is most encouraged, while the follower ' s is more discouraged than the project of the operational option without costs.
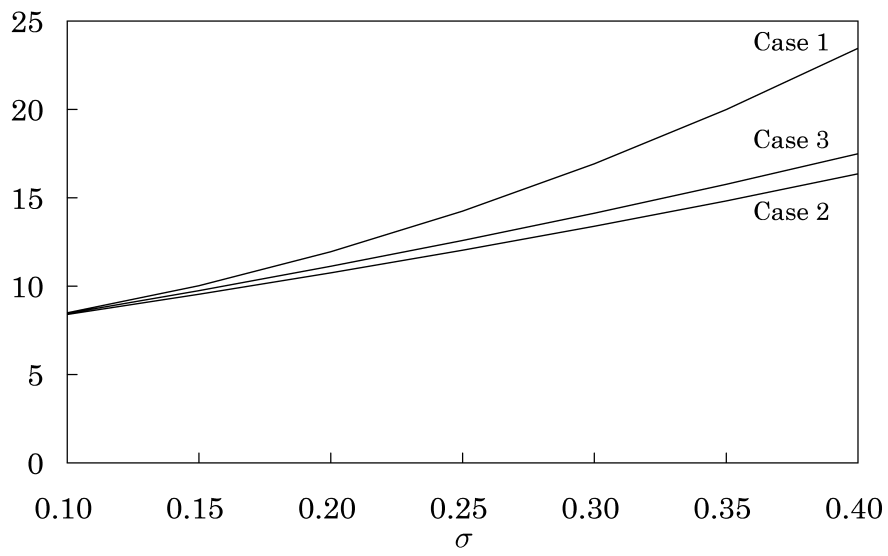
10

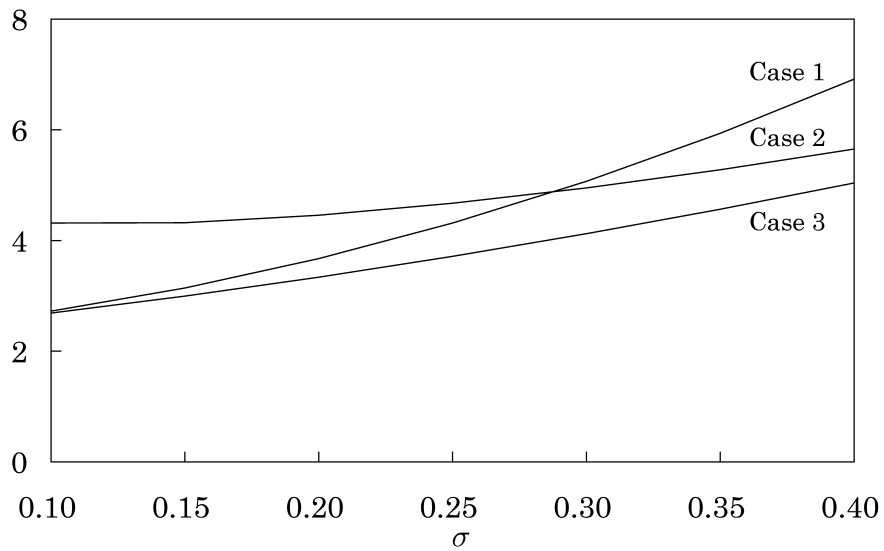Figure 4: The comparative statics of the follower's thresholds X^{F} with respect to σ for three cases

Figure 5: The comparative statics of the leader's thresholds X^{L} with respect to σ for three cases

5 Discussion

Numerical results in the previous section give some important results below.

First, there is the question that the leader suspends the project when the follower invests. If it is ture, then equation (26) does not hold because $Q_\tau = 1$. However, numerical results show X^{F} is greater than $\overline{X}(1)$. The leader is sure to activate the project when $x \geq \overline{X}(1)$, so that $Q_\tau = 2$. Therefore, we can claim the fact that $\overline{X}(1) < X^{\mathrm{F}}$ ensures the validity of equation (26).

Next, there is the question that X^{L} is low enough to damage the leader's value. The leader is sure to idle the project when $x \leq \underline{X}(1)$. If the leader activates the project in this region, the threat of preemption makes the leader's value much lower. However, numerical results show X^{L} is greater than $\underline{X}(1)$. Therefore, we can claim the optimality of leader's investment is ensured by the fact that $\underline{X}(1) < X^{\mathrm{L}}$.

In addition, since $\tilde{X}(1) < X^{\mathrm{L}}$ and $(\tilde{X}(2) <) \overline{X}(2) < X^{\mathrm{F}}$, the follower and the leader must invest the project with the state 1. This describes that there is no reason to incur the investment cost I only to keep the project idle for some time. We can find the same property as Dixit and Pindyck (1994, Ch.6).

Due to the above claims, we use proposition 2 in relief. Finally, we explains the firm's optimal actions in the equilibrium. We assume that the initial value x is sufficiently low. Then, the equilibrium is as follows:

1. At the first moment that X_t exceeds X^{L}, firm i becomes the leader with state 1 with probability 1/2,

2. when X_t falls below $\underline{X}(1)$, firm i suspends the project,

3. when X_t exceeds $\overline{X}(1)$, firm i reenters the project,

4. at the first moment that X_t exceeds X^{F} $(> \overline{X}(1))$, firm j invests with state 1,

5. when X_t falls below $\underline{X}(2)$, both firms suspend the project,

6. when X_t exceeds $\overline{X}(2)$, both firms reenter the project.

6 Conclusion

In this paper, we have analyzed investment decision on the entry-exit project in a duopoly setting. Then we have shown it is optimal that the firm must start producing at the beginning of the project, and there are no simultaneous investment if the initial demand shock is sufficiently low. Furthermore, the comparative statics of thresholds imply the leader is more encouraged to invest and the follower is more discouraged. Entry-exit

costs, which are the characteristic of this project, have impact on the entry-exit strategies after investment, however, no impact investment strategies.

For future research, we will analyze the case in which Q_t means the supply of products in the market. It is natural that the supply determines the price, however, Q_t could change after the firm invests the project. Consequently, there is possibility that equation (15) no longer holds. Furthermore, we will try an abandonment decision in duopoly, similar to Murto (2004) or Goto and Ohno (2006).

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