No. 2026-008

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August 22, 2026

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Research Institute of Business Administration (Sanken), Waseda University
Working Paper Series, No. 2026-008

Working paper note. An unpublished Japanese-language manuscript formed the basis for this expanded English-language working paper. The baseline model and the numerical results contained in that manuscript are unchanged; the present version adds international literature positioning and a supplementary post-withdrawal numerical appendix. The first two papers in the research program are forthcoming in this series as RIBA Working Paper No. 2026-006 (*Two Dynamics toward Mental Quieting: A Comparative Model of Faith-Mediated and Self-Observation Pathways*) and No. 2026-007 (*When Memory Governs Memory: An Endogenous-Operator Model of Recursive Psychological Activity*). The Japanese originals of those two papers are scheduled to appear in *Waseda Shogaku (The Waseda Commercial Review)*, No. 472 (September 15, 2026) and No. 473 (December 15, 2026), respectively. A reproduction script (`reproduce_g1_numerics.py`) accompanies this paper.

Abstract

This paper models the conditions under which externally initiated observation becomes object-sustained, thereby raising the intensity of candidate openings toward functional non-closure and, conditional on object sensitivity, the intensity of model-level attention entry. Building on a research program that formalizes self-referential psychological memory and functional closure, we endogenize, in reduced form, the intensity of *candidate openings* toward functional non-closure: external tokens initiate task contact; contact feeds object-centered curiosity; curiosity renders the memory's own dynamics visible; curiosity and visibility jointly build object sensitivity; and tokens—or the direct evaluation of attention—feed an acquisition–achievement pressure that works against all of these. Candidate openings form a counting process whose first-opening time we analyze; model-level attention entries are the object-sensitivity-thinned subprocess, and measured entry is an operational proxy containing misclassification. Analytically, early curiosity and the initial opening probability are inverted-U in token intensity; mutual reinforcement of curiosity and visibility creates a watershed beyond which observation persists after tokens are withdrawn, rationalizing tapering designs; rewarding attention directly backfires under an explicit comparative condition; and no finite design guarantees a first opening within

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a finite horizon—a finite-horizon boundary distinct from cure-fraction structure. Numerically, with a single parameter set, a permanently high reward performs worse than no reward while in force and collapses after withdrawal, whereas a moderate starter followed by tapering preserves early openings and survives withdrawal at roughly a third of the cumulative cost; under equal budgets, front-loaded bursts fail where spread allocations succeed. A post-withdrawal appendix tabulates these trajectories. Total spending alone does not determine the outcome; its temporal allocation can reverse it.

Keywords: attention; functional non-closure; candidate opening; intrinsic motivation; token economy; motivational crowding; interest development; counting process; bistability; handoff.
JEL classification: C61; D83; D91; M41.

1 Introduction

People often begin an activity for reasons external to the activity itself: to escape discomfort, to earn money or grades, to win approval. Incentive schemes can exploit this fact and often increase initial contact with a target activity; token economies are a canonical example (Kazdin and Bootzin, 1972). Yet a well-documented complication follows the initiation. A meta-analysis of 128 experiments found that tangible rewards contingent on engagement, completion, or performance significantly undermined subsequently measured intrinsic motivation ($d = -0.40$, -0.36, and -0.28, respectively; Deci et al., 1999), and field evidence reviewed by Gneezy et al. (2011) indicates that modest incentives can perform worse than none at all. In functional communication training, reviews reach a parallel conclusion from the applied side: behavior established under dense reinforcement often requires planned schedule thinning for practical maintenance (Hagopian et al., 2011; updated by Kranak and Brown, 2023). The design problem, then, is not whether external rewards move behavior—they do—but whether, and under what dynamic conditions, externally initiated engagement can hand itself over to a drive that survives the reward's withdrawal.

This paper studies that handoff problem for a particular, and, we will argue, particularly demanding target: attention. Throughout, attention denotes the first-person correlate of *low functional closure* combined with *preserved sensitivity to the present object*—an operator-level construct inherited from the research program summarized below and formalized in Section 3. Object-centered curiosity, which plays a central role in our model, is a mediating state that may sustain observation after external rewards are withdrawn; it is not itself attention. The construct is likewise distinct from selective or executive attention, from flow, and from decentering, though it has partial affinities with each; Section 2.5 locates it against these neighbors. Two features make attention, so understood, a hard case for incentive design. First, attempts to command it tend to backfire: the instruction "pay attention" installs a new achievement task and a new self-monitoring operator, and self-monitoring pressure is, in the model developed below, the principal antagonist to candidate openings: it tends to raise closure and lower the opening intensity (cf. ironic process theory; Wegner, 1994). Second, evaluating attention directly—turning it into a key performance indicator—risks converting observation itself into one more arena of acquisition, a dynamic familiar from the literatures on measure corruption and multitask distortion (Campbell, 1979; Holmström and Milgrom, 1991).

The paper is the third in a research program whose two preceding papers are written in Japanese (Suzuki, 2026a,b). The first compared faith-mediated and self-observation pathways toward mental quieting. The second modeled self-referential psychological memory as a system that endogenously generates operators of concealment, correction, punishment, justification, and control, and re-enters the outputs of those operators as present facts. The degree to which operator outputs are re-entered was formalized as functional closure, written c_t; the first-person experience corresponding to low closure combined with preserved object sensitivity was identified with attention. What those papers left exogenous is the question taken up here: under what conditions does closure open at all?

Our answer deliberately does not model attention itself. We model *candidate openings*. External tokens $u(t)$ initiate task contact $\mathcal{E}(u)$; contact feeds object-centered curiosity q; curiosity renders the memory's own dynamics visible (v); curiosity and visibility jointly build object sensitivity κ; and tokens—or the direct evaluation of attention—feed an acquisition–

achievement pressure state g that works against all of the above. These mediating states determine a candidate-opening intensity $\lambda(t) = \lambda_0 \exp(\theta_q q + \theta_v v + \theta_\kappa \kappa - \theta_g g)$, the intensity of a counting process $N(t)$ of instantaneous opening episodes toward functional non-closure, with first-opening time $\tau_O := \inf\{t \geq 0 : N(t) \geq 1\}$. A candidate opening is not yet attention. At the model level, attention-entry events are the thinned process N_A whose episodes additionally satisfy the object-sensitivity condition (intensity $\lambda_A = \lambda\, \pi(\kappa)$, first attention-entry time $\tau_A := \inf\{t \geq 0 : N_A(t) \geq 1\}$); at the measurement level, entry is certified by an operational proxy $\hat{\tau}_A$ built from estimated closure and sensitivity, and that proxy contains misclassification. This three-layer ontology—candidate openings, model-level attention entries, measured entries—is maintained throughout the paper.

Five results organize what follows. First, early-stage curiosity and the initial opening probability are non-monotonic in token intensity: too little fails to initiate contact, while too much inflates pressure, yielding an interior maximum (an inverted U). Second, the mutual reinforcement of curiosity and visibility creates bistability: beyond a saddle watershed (separatrix), observation persists after tokens are withdrawn. This formalizes *handoff* and rationalizes tapering designs—a moderate starter, then gradual withdrawal once interest has formed. Third, directly rewarding attention is counterproductive under an explicit comparative condition: unless its contact-promoting effect outweighs the self-monitoring pressure it installs, an attention KPI lowers the candidate-opening intensity. Fourth, under the model's finite-intensity assumption, no finite design guarantees a first opening within a finite horizon: over any finite horizon the integrated intensity is finite, so the probability of a first candidate opening remains strictly below one. This is a finite-horizon boundary of the present model, not a cure-fraction result (Section 2.4 states the contrast); we read it as the formal expression of attention's non-coercive character. Fifth, numerical illustrations with a single fixed parameter set show that a permanently high reward delays first openings below even the no-reward baseline while it is in force (cumulative opening probability 0.50 versus 0.70 over the horizon); that a moderate starter followed by tapering preserves early openings while sustaining a high post-withdrawal opening intensity at roughly a third of the cumulative cost; that, holding the budget fixed, front-loaded burst allocations collapse where spread allocations succeed; and that when policies are withdrawn at the end of the horizon, the high fixed schedule collapses to the origin while the moderate schedule persists (Appendix table). Total spending alone does not determine the outcome; its temporal allocation can reverse it.

Against the literatures reviewed in Section 2, the paper makes five contributions. (i) It decomposes one large, black-box wager—"token seeking may turn into interest in observation itself"—into designable, observable, and testable transitions, while stating exactly the non-coercive residual that no design removes. (ii) It endogenizes, in reduced form, the conditional arrival intensity of candidate openings, rather than positing entry exogenously. (iii) It recasts motivational crowding as state dynamics: where meta-analytic evidence establishes static undermining, the model derives withdrawal-contingent, allocation-contingent, and baseline-relative consequences. (iv) It formalizes the transition from situational to individual interest (Hidi and Renninger, 2006) as a basin handoff in a bistable curiosity–visibility system. (v) It connects the psychology of attention to the statistics of counting processes—thinning, frailty, and hazard-based non-guarantee—so that measurement, individual differences, and the finite-horizon boundary sit within one probabilistic frame.

Two boundary statements are worth making at the outset. The nearest conceptual neighbor is the reward-learning framework of Murayama (2022), in which knowledge acquisition self-boosts over long horizons and is vulnerable to extrinsic incentives; our model can be read as equipping that account with explicit state dynamics, an entry point process, and a budget-design problem. The sharpest empirical discriminator is behavioral momentum theory (Nevin and Grace, 2000), which predicts that richer reinforcement histories produce more persistent behavior; for high fixed rewards our model predicts the opposite sign after withdrawal, and Hypothesis H4 is constructed to separate the two accounts (the post-withdrawal trajectories supporting this claim are tabulated in the Appendix).

Finally, why attention? Not because we privilege it metaphysically. Within the program's model of self-referential memory, the functional non-closure corresponding to attention is the one bottleneck left un-endogenized: the point at which the coupling of re-entry and rule formation can change without stacking yet another controller on top of the system. The paper does not claim to manufacture attention, and its central proposition shows that no finite design can guarantee it within a finite horizon. In practical terms, the design problem is to lower what the first paper of the program called the "luxury level" of attention—its de facto entry cost—without converting attention into one more object of management. The aim is thus narrower and, we believe, more useful: to shrink the designable part of the uncertainty surrounding entry into attention, and to state precisely the non-coercive part that remains. Section 2 positions the model; Sections 3–4 present the construction; Section 5 gives analytical results; Section 6 reports numerical policy comparisons; Section 7 develops measurement and hypotheses; Sections 8–9 discuss design and interpretive questions; Section 10 states limitations; Section 11 concludes.

2 Related Work

2.1 Incentives and intrinsic motivation

The undermining of intrinsic motivation by tangible rewards has been documented, contested, and meta-analyzed for five decades. Early experiments documented overjustification and contingency effects (Deci, 1971; Lepper et al., 1973; Ryan et al., 1983), later integrated into self-determination theory (Ryan and Deci, 2000). The meta-analysis of Deci et al. (1999) established significant negative effects of engagement-, completion-, and performance-contingent tangible rewards on free-choice intrinsic motivation; the opposing meta-analysis of Cameron and Pierce (1994) and the ensuing exchange (Cameron et al., 2001; Deci et al., 2001) define the controversy's contours, and a later synthesis finds intrinsic motivation and incentives predicting different performance components (Cerasoli et al., 2014). Economics formalized *motivation crowding* (Frey and Jegen, 2001), most influentially as a static signaling mechanism in which rewards convey information about the task or the principal's beliefs and thereby crowd out engagement (Bénabou and Tirole, 2003); field studies summarize the design lesson as "pay enough or don't pay at all" (Gneezy et al., 2011), and neuroimaging locates the undermining effect in reward circuitry (Murayama et al., 2010). What this literature does not provide is a state-space account of *when* crowding accumulates and *what happens after withdrawal*: its comparisons are static or one-shot, and the time path of incentives is not a choice variable. The present model supplies exactly that layer—crowding as the dynamics of a pressure state—and derives consequences invisible to static designs: a maintained high reward falling below the no-reward baseline, and same-budget allocations succeeding or failing depending on their concentration in time.

2.2 Interest development and curiosity

Research on interest and curiosity offers the phenomena our mediating states are built to capture. Classical accounts treat curiosity as arousal under collative variables (Berlyne, 1960) or as an information gap (Loewenstein, 1994), with interest as a distinct knowledge emotion (Silvia, 2008). The standard developmental account is the four-phase model of Hidi and Renninger (2006), in which triggered situational interest may become maintained, emerge as individual interest, and finally stabilize. Neuroscience links curiosity states to dopaminergic circuits and memory benefits (Gruber et al., 2014; Kidd and Hayden, 2015; Gottlieb et al., 2013), and computational work formalizes intrinsic motivation as learning progress or compression progress for artificial agents (Oudeyer and Kaplan, 2007; Schmidhuber, 2010). Closest to our aims, the reward-learning framework of Murayama (2022) integrates curiosity, interest, and trait interest as one long-run knowledge-acquisition process that is self-boosting yet vulnerable to extrinsic reward (see also Murayama et al., 2019). The gap is formal: the four-phase model describes stages without stating transition conditions; computational curiosity designs internal rewards but does not model their competition with external tokens or the withdrawal problem; and the reward-learning framework, while conceptually adjacent, contains no explicit state dynamics, no entry point process, and no budget-allocation problem. Our bistable curiosity–visibility system, with its saddle watershed, is offered as a dynamical formalization of the situational-to-individual transition, and our tapering and burst results as its design consequences.

2.3 Persistence, schedule thinning, and behavioral momentum

Applied behavior analysis has long faced the practical version of our question: how to maintain behavior established under dense reinforcement once reinforcement becomes sparse. Token economies (Kazdin and Bootzin, 1972) and functional communication training generated a mature practice of *reinforcement schedule thinning*, with reviews and updated recommendations cataloguing procedures, outcomes, and the resurgence of problem behavior during thinning (Tiger et al., 2008; Hagopian et al., 2011; Kranak and Brown, 2023). The dominant quantitative theory of persistence is behavioral momentum theory (Nevin and Grace, 2000; Nevin and Shahan, 2011): reinforcers accumulated in a stimulus context confer "mass," so richer histories predict greater resistance to disruption. Two gaps matter here. First, thinning practice lacks a state-level account of *why* and *when* gradual withdrawal succeeds; our handoff proposition and the watershed hypothesis (H6) provide one, and H4 states the resulting tapering prediction. Second, and more sharply, behavioral momentum theory and the present model make opposite predictions for the withdrawal of high fixed rewards: momentum predicts persistence increasing with reinforcement rate, whereas in our model a high fixed reward inflates pressure and suppresses curiosity and sensitivity, so that when rewards are withdrawn at the end of the evaluation horizon the high-reward system collapses to the origin while a moderate-reward system persists (post-withdrawal trajectories are tabulated in the Appendix). Hypothesis H4 is constructed as the discriminating test between the two accounts, with the dependent variable, withdrawal timing, and disruption operation to be matched across paradigms in implementation.

2.4 Stochastic formulation: counting processes and asymptotically autonomous dynamics

The probabilistic layer of the model draws on the standard counting-process toolkit—intensity-based processes in the lineage of Cox (1972), Aalen (1978), and Andersen and Gill (1982); thinning of an inhomogeneous Poisson process (Lewis and Shedler, 1979), which relates candidate openings to model-level attention entries; and frailty mixtures (Vaupel et al., 1979) for population heterogeneity. These are introduced where used, in the model and measurement sections. One boundary deserves emphasis. The model's non-guarantee is a *finite-horizon* result: over any finite horizon the integrated intensity is finite, so the probability of a first candidate opening is strictly below one. Under the bounded-input policies considered here, the mediating states remain bounded and the candidate-opening intensity is bounded away from zero; hence the cumulative intensity diverges and a first opening is almost sure over an infinite horizon. It is therefore not a cure-fraction result. Cure models in survival analysis (Boag, 1949; Berkson and Gage, 1952; Chen et al., 1999; for a review, Amico and Van Keilegom, 2018) describe populations in which a fraction never experiences the event because the cumulative hazard remains bounded on the infinite horizon; connecting the present framework to that structure would require extensions—an integrable baseline intensity, a frailty atom at zero susceptibility, or explicit susceptibility classes—which we leave to future work. Finally, the handoff proof uses convergence theory for asymptotically autonomous systems: the planar Poincaré–Bendixson trichotomy of Thieme (1992), stated explicitly as Theorem 2.1 in Castillo-Chavez and Thieme (1995) and extending Markus (1956, Theorem 7); details are confined to the Appendix. These tools are devices; the claimed contribution lies in the mechanism.

2.5 Attention constructs: selective attention, flow, decentering, and functional non-closure

Because the paper's title term is heavily overloaded, we state how our construct relates to its neighbors. Selective and executive attention concern orienting and control capacities within information processing (Posner and Petersen, 1990); functional non-closure is not a capacity but an operator-level property—a reduced tendency for the outputs of concealment, correction, punishment, and justification operators to be re-entered as present facts. Flow describes absorption under a skill–challenge balance (Csikszentmihalyi, 1990). Unlike flow, functional non-closure requires neither absorption nor a skill–challenge balance: memory dynamics may remain visible while sensitivity to the present object is preserved, and the negative state in the model is not goal structure itself but the acquisition–achievement pressure it often carries. Decentering and cognitive defusion (Bernstein et al., 2015; Bolderston et al., 2019), with measurement instruments such as the CFQ (Gillanders et al., 2014), are the closest measurement-level relatives and inform the operational proxy $\hat{c}_t$; the regulation styles of contemplative practice—focused attention and open monitoring (Lutz et al., 2008)—describe how practitioners work with attention rather than what entry into it is. Functional non-closure with object sensitivity is offered as a construct at the level of memory operations: complementary to, and not reducible to, these neighbors.

Positioning

Computational accounts of meditative attention—from the regulation-style taxonomy (Lutz et al., 2008) to active-inference models of meditative states (Laukkonen and Slagter, 2021, and subsequent formal implementations)—model the dynamics of attention *during* practice. Our scoping search, reported separately, identified no model that jointly treats external incentive design, mediating-state dynamics, and the arrival of candidate openings into attention; pending complementary database searches, we advance this as the paper's location rather than as a priority claim. Within that location, the model lets the central findings of four literatures—the inverted U of incentives, the fragility documented by crowding research, the self-boosting of interest, and the practical wisdom of tapering—emerge from one parameter set, together with one discriminating prediction: that under equal budgets and a common evaluation horizon, the temporal allocation of incentives can reverse the outcome of withdrawal.

3 Concepts, research question, and scope

3.1 The design problem and its scope

This paper studies the following design problem: use external tokens or other external occasions to initiate contact with an observation task, and then design the conditions under which the driving source of engagement shifts from reward acquisition to the object itself. The program is not a technique for manufacturing attention. It does not assume that external input directly produces attention; it concerns the conditions that change the intensity with which transitions toward attention can occur.

The entry envisaged here is not restricted to dedicated practitioners or advanced trainees. Ordinary workers, students, patients, and members of organizations first contact a task for external reasons; the structure of the object then becomes interesting; and observation may continue for its own sake. In this sense the design problem is a deliberately ambitious attempt to lower the de facto "luxury level" of attention—its entry cost.

3.2 Why focus on attention?

The focus on attention does not rest on the assumption that attention is an ultimately good state. The theoretical reason is that the preceding models showed that a decline in functional closure weakens the self-referential loop, but left exogenous *why* closure declines. The practical reason is that commanding attention directly can generate a new operator that monitors "am I attending?" and "am I observing correctly?"

The question of this paper is therefore not

> how to produce attention,

but

> how far one can arrange the conditions under which a transition toward functional non-closure can occur, without commanding, evaluating, or coercing attention.

3.3 Decomposing one large wager

Treated as a single black box, the wager

$$\text{token seeking} \longrightarrow \text{interest in observation itself} \tag{1}$$

gives no information, upon failure, about where the process stopped. We decompose it into the following transitions.

1. External tokens initiate contact with the task (in the model, contact is reduced to an instantaneous variable $\mathcal{E}(u)$ rather than a state variable; see Section 7.4).
2. Contact activates object-centered curiosity.
3. Curiosity renders visible the repetitions, contradictions, and low-rank stereotypy of the memory's own dynamics.
4. Visibility raises sensitivity to the object itself.
5. Curiosity, visibility, and object sensitivity jointly raise the possibility of an opening toward functional non-closure.
6. At the same time, tokens strengthen acquisition, achievement, and self-evaluation, and act in the opposite direction.

The decomposition distinguishes failure points: participation never occurred; engagement never moved beyond the reward motive; curiosity arose but was reabsorbed into self-evaluation; or inward looking succeeded at the price of losing the external object.

3.4 Attention, non-instrumental observation, and object sensitivity

Definition 1 (Non-instrumental observation)**.** *Observation is called* non-instrumental *when it persists even as the expectation of external reward or future gain declines or is removed, and when its persistence does not have the maintenance, expansion, or correction of the self-image as its primary purpose. The experience of "observing despite oneself, because it is interesting" is a typical symptom of this state, not its definition.*

The preceding papers named *attention* the first-person experience corresponding to low functional closure. To separate it clearly from dissociation and withdrawal, this paper characterizes the entry into attention by two conditions. First, the outputs of self-referential operators are only weakly re-entered as present facts—closure is low. Second, sensitivity to new information arriving from the external object is preserved. Conceptually,

$$\text{entry into attention} \;\approx\; \{c_t \downarrow,\; \kappa_t \uparrow\}, \tag{2}$$

where c_t is the degree of functional closure and κ_t is object sensitivity. A state in which fear is visible, the impulse to manipulate the fear is visible, and the object in front of one is still not lost, is the intuitive example of these two axes.

Two terminological cautions are in order. First, "attention" here does not refer to selective or executive attention in cognitive psychology—the mechanism that allocates limited processing resources to particular inputs or tasks (Posner and Petersen, 1990). It is the name of a first-person experience corresponding to lowered functional closure in the program's framework, characterized not by the efficiency of resource allocation but by the joint occurrence of weakened self-referential re-entry and preserved object sensitivity. Second, it is operationally distinct from flow (Csikszentmihalyi, 1990). Flow is often described as absorption in the object accompanied by a loss of self-awareness; the attention of this paper is a state in which the visibility v of memory dynamics is preserved while object sensitivity κ is high—inner movement remains visible while the outward opening persists.

We also delimit the variables' scope of application. The central application of the model is to tasks whose object is the movement of self-referential memory itself. There, q is curiosity about memory dynamics as an object, v is their visibility, and κ is the residual opening to external reality beyond them. When the object is a general external task—as in an introductory mathematics example—reading v as "visibility of the object's structure" is an analogical extension. The requirement in equation (7) of Section 4.2 that object sensitivity be built through the product qv—that is, that object sensitivity not rise without inner visibility—is a formalization suited to the central application; routes such as absorbed perception, in which high external sensitivity arises without introspective visibility, lie outside the κ dynamics of this paper (Section 10).

3.5 Three kinds of uncertainty

The paper divides the uncertainty that remains in the design problem into three kinds.

1. **Epistemic uncertainty:** not yet knowing which mediating variables matter and what the parameters are. It can be reduced by theory, observation, and experiment.

2. **Design uncertainty:** not knowing which token amount, timing, task difficulty, tapering rule, or evaluation method is best. It can be reduced by comparative experiments and optimization.

3. **Non-coercive uncertainty:** even with the conditions arranged, the actual occurrence of object-centered curiosity and functional non-closure cannot be guaranteed from outside. The more one tries to guarantee it, the more attention can flip into one more object of control.

The role of this third paper of the program is to shrink the first two uncertainties as far as possible and to state the third explicitly within the theory.

4 The basic model

4.1 Variables and external inputs

Time is continuous, $t \geq 0$, and the external token intensity is $u(t) \geq 0$. Tokens are not restricted to money; they include points, eligibility, course credit, praise, and access to the task—any external occasion that activates entry behavior. The degree to which tokens activate contact with the task is represented by the saturating function

$$\mathcal{E}(u) = 1 - e^{-\alpha u}, \qquad \alpha > 0, \tag{3}$$

with $\mathcal{E}(0) = 0$, $\mathcal{E}'(u) > 0$, and $\mathcal{E}''(u) < 0$.

There are four endogenous states.

- $q(t) \in [0,1]$: object-centered curiosity—the degree of wanting to see more of the object itself, not of the reward.
- $v(t) \in [0,1]$: visibility of memory dynamics—the degree to which stereotyped responses, self-contradictions, and the repetition of operators are visible as structure.
- $g(t) \geq 0$: acquisition–achievement pressure—the pressure to attain rewards, success, self-evaluation, and "correct observation."
- $\kappa(t) \in [0,1]$: object sensitivity—the degree to which new information from the external object enters the response.

In addition, $z(t) \geq 0$ denotes the intensity of institutions that evaluate, grade, or reward attention itself. z is distinguished from the ordinary entry token u: u is paid for contact with the task, whereas z is tied directly to inner outcomes—attention, quiet, correct observation.

4.2 Dynamics of the mediating states

Object-centered curiosity is driven by externally occasioned contact, sustained by the visibility of the object, and inhibited by acquisition–achievement pressure:

$$\dot{q} = a_q \mathcal{E}(u)(1-q) + \rho v q(1-q) - \{d_q + b_q g\} q, \tag{4}$$

with $a_q, \rho, d_q, b_q > 0$. The first term is the starter effect of tokens initiating contact; the second is the object-intrinsic effect by which structure, once visible, invites further curiosity; the third combines natural decay with inhibition by achievement pressure.

Visibility rises with curiosity and decays naturally:

$$\dot{v} = a_v q(1-v) - d_v v, \qquad a_v, d_v > 0. \tag{5}$$

The interpretation is that v rises as repetitions become visible as objects—"the same excuse appeared again," "the punishing memory has begun to punish the very self that punishes."

Acquisition–achievement pressure is raised by ordinary tokens u and by the attention KPI z, and decays naturally:

$$\dot{g} = a_g u + a_z z - d_g g, \qquad a_g, a_z, d_g > 0. \tag{6}$$

Tokens open the entrance while also strengthening the external reason for acquisition. The attention KPI, because it directly evaluates whether one attended or observed correctly, is positioned as an input that raises self-monitoring pressure in a purer form.

Object sensitivity rises when curiosity and visibility are jointly high, and is lowered by achievement pressure:

$$\dot{\kappa} = a_\kappa q v(1-\kappa) - \{d_\kappa + b_\kappa g\}\kappa, \qquad a_\kappa, d_\kappa, b_\kappa > 0. \tag{7}$$

This equation expresses that merely looking at inner content is not yet attention: fear, the manipulation of fear, and the external object must share the same present, with the response channel to the object open. The route by which achievement pressure lowers object sensitivity (the $b_\kappa g$ term) is consistent with the finding that pressure heightens attention to the self and thereby impairs skilled performance (Baumeister, 1984).

If the initial values satisfy $q(0), v(0), \kappa(0) \in [0,1]$ and $g(0) \geq 0$, the solutions of (4)–(7) remain in the same region, so q, v, κ can be read as proportions and g as a nonnegative pressure.

4.3 The point process of candidate openings

The continuous closure degree c_t of the preceding papers is coarse-grained here into opening events. The occurrence of openings that can lead to an entry into functional non-closure is modeled as a counting process $N(t)$ whose intensity depends on the mediating states, and the time of its first event is written $\tau_O := \inf\{t \geq 0 : N(t) \geq 1\}$ (the first-opening time). τ_O is neither "attention itself" nor the entry into attention itself; it is the first time of a *candidate* opening that can lead to entry (the measurement-level qualification of attention entry is given in Section 7.3). The candidate-opening intensity while in the closed state is

$$\lambda(t) = \lambda_0 \exp\{\theta_q q(t) + \theta_v v(t) + \theta_\kappa \kappa(t) - \theta_g g(t)\}, \tag{8}$$

with all coefficients positive. Curiosity, visibility, and object sensitivity raise the ease of an opening; acquisition–achievement pressure lowers it. Given the parameters and inputs, $\lambda(t)$ is determined by the dynamics of the mediating states; it is the candidate-opening intensity at time t. The baseline intensity $\lambda_0 > 0$ represents candidate openings arising spontaneously from routes other than those treated here (tokens and materials). The probability of a first event in a small interval $\Delta > 0$ is

$$\mathbb{P}\{\tau_O \leq t + \Delta \mid \tau_O > t, \mathcal{F}_t\} = 1 - \exp\Big\{-\int_t^{t+\Delta} \lambda(s)\, ds\Big\} \simeq 1 - e^{-\lambda(t)\Delta}, \tag{9}$$

where $\mathcal{F}_t$ is the information up to time t and the rightmost expression is the local approximation that treats λ as constant over the interval. We fix the terminology as follows. Each event of N is a candidate-opening episode (henceforth, opening episode), and the one-period opening probability at time t is $p_\Delta(t) := 1 - \exp\{-\int_t^{t+\Delta} \lambda(s)\, ds\}$ with $\Delta = 1$ (equal to the exact form in (9); at $t = 40$ and near steady states the difference from the local approximation does not appear at the displayed precision, while during fast transients Section 6 reports the local approximation and the accompanying reproduction files provide both the local and the exact interval probabilities). At the individual level, the mediating-state dynamics (4)–(7) are deterministic given the parameters ϕ and the inputs, so N is an inhomogeneous Poisson process; by its independent-increments property, the number of events in $(t, t+\Delta]$ is independent of the past episode history. Hence $p_\Delta(t)$ is an opening probability conditioned only on the state path, the inputs, and the individual parameters—not on the past event history—and it coincides with (9) conditioned on $\tau_O > t$. At the population level, integrating over the distribution of individual differences, event counts in different intervals can be dependent through the common ϕ (a frailty mixture; Section 7.4). This formulation separates arranging conditions from manufacturing attention deterministically from outside. Because the paper does not treat re-closure after attention (Section 10), τ_O is defined for the first opening only, but opening episodes themselves can recur: $N(t)$ counts the instantaneous occurrence of opening opportunities, not the occupation time of an attention state, so repeated openings are well defined without positing re-closure dynamics. An opening episode is a candidate opening on which the measurement-level $\bar{\kappa}$ condition is not imposed; at the measurement level, only episodes with $\hat{\kappa} \geq \bar{\kappa}$ are qualified as attention entries (Section 7.3). Section 6 reports the cumulative probability of a first opening, $\mathbb{P}(\tau_O \leq T)$, separately from the pointwise one-period opening probability $p_\Delta(t)$.

Model-level attention entries. For completeness at the model level, define the attention-entry process N_A as the subprocess of N whose episodes additionally satisfy the object-sensitivity condition. Concretely, an episode at time t is retained with probability $\pi(\kappa(t))$, where $\pi : [0, 1] \to [0, 1]$ is nondecreasing (for example, $\pi(\kappa) = \mathbf{1}\{\kappa \geq \bar{\kappa}\}$ or a smoothed version). By the thinning construction for inhomogeneous Poisson processes (Lewis and Shedler, 1979), N_A is itself an inhomogeneous Poisson process with intensity

$$\lambda_A(t) = \lambda(t)\,\pi(\kappa(t)) \leq \lambda(t). \tag{10}$$

Its first event defines the *model-level attention-entry time*

$$\tau_A := \inf\{t \geq 0 : \, N_A(t) \geq 1\}, \tag{11}$$

the model-level counterpart of τ_O; the measurement proxy $\hat{\tau}_A$ of Section 7.3 estimates τ_A subject to misclassification. The three-layer ontology of the paper is thus: candidate openings N (this subsection), model-level attention entries N_A (this paragraph), and measured entries $\hat{\tau}_A$ (Section 7.3), which add measurement error and misclassification on top of N_A.

Three supplementary remarks. First, no explicit dynamics are given for the closure degree c_t in this paper. Equation (8) is a reduced-form expression of the candidate-opening intensity as a function of the mediating states, in place of modeling the internal dynamics of closure decline; deriving λ from a stochastic-process model of c_t is left to future work (Section 10). Second, at the measurement level an entry event is operationalized by the criterion "closure indicator at most $\bar{c}$ and object-sensitivity indicator at least $\bar{\kappa}$" (Section 7.3, equation (30)); κ is thus used at two levels—as a covariate of the opening intensity in the model, and as a qualification condition against dissociation and withdrawal in measurement. Third, if one wishes to exclude the reading that visibility v raises the opening intensity in the absence of object sensitivity, a robustness check introducing the interaction $\theta_{v\kappa} v\kappa$ into (8) is available; the paper retains the additive form (the numerical examples of Section 6 are based on it).

4.4 The model as a whole

Figure 1 summarizes the two competing pathways of the model.

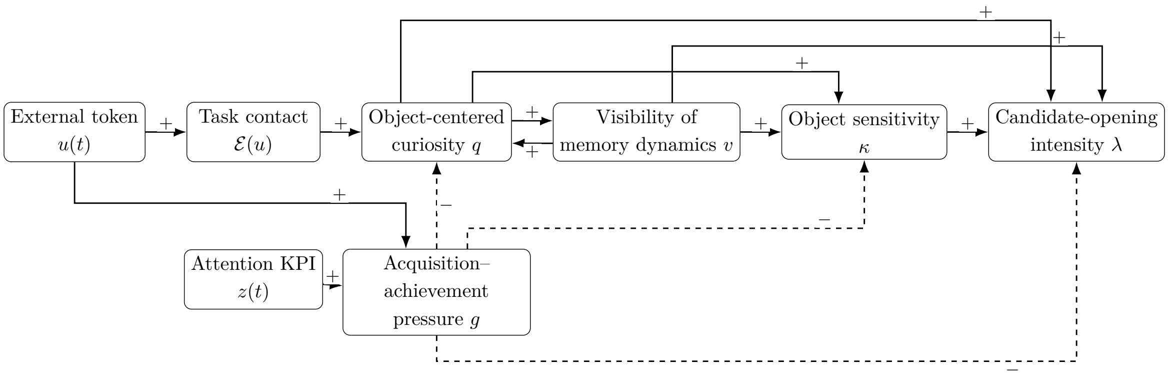

Figure 1: Two competing pathways of the model. Tokens can promote openings toward non-closure through contact, curiosity, and the visibility of memory dynamics, while also acting in the opposite direction through acquisition–achievement pressure. Solid lines denote positive effects; dashed lines denote negative effects.

5 Analytical results

5.1 The early-stage inverted U

Immediately after observation begins, visibility v can be taken as still low. Approximating g as equilibrating quickly relative to the tokens, write

$$\bar{g}(u, z) = \frac{a_g u + a_z z}{d_g}. \tag{12}$$

The quasi-steady value of curiosity in the early stage with $v \simeq 0$ is

$$\bar{q}_E(u, z) = \frac{a_q \mathcal{E}(u)}{a_q \mathcal{E}(u) + d_q + b_q \bar{g}(u, z)}. \tag{13}$$

Proposition 1 (Interior maximum of early curiosity)**.** *Let $z = 0$. Then $\bar{q}_E(u, 0)$ is continuous on $u \geq 0$, $\bar{q}_E(0, 0) = 0$, and $\bar{q}_E(u, 0) \to 0$ as $u \to \infty$. Hence it attains at least one interior maximum $u_q^* > 0$. An interior solution satisfies*

$$\frac{\mathcal{E}'(u)}{\mathcal{E}(u)} = \frac{b_q a_g / d_g}{d_q + (b_q a_g / d_g) u}. \tag{14}$$

The proposition expresses that too small a token fails to initiate contact, while too large a token lets acquisition pressure overwhelm curiosity. The relation between reward and curiosity is neither monotonically increasing nor monotonically decreasing.

In the early stage, set $v \simeq \kappa \simeq 0$ and define the opening index

$$I_E(u) := \theta_q \bar{q}_E(u, 0) - \theta_g \bar{g}(u, 0). \tag{15}$$

Corollary 1 (Interior maximum of the early opening probability)**.** *If*

$$\theta_q \frac{a_q \alpha}{d_q} > \theta_g \frac{a_g}{d_g} \tag{16}$$

holds, then $I_E'(0) > 0$. Since $I_E(u) \to -\infty$ as $u \to \infty$, the early opening probability $1 -$

$\exp[-\lambda_0 e^{I_E(u)}\Delta]$ *(local-constancy approximation) attains at least one interior maximum.*

Condition (16) means that, in the early stage, the starter effect of a small increase in tokens outweighs the accompanying increase in achievement pressure. Note that $\bar{q}_E(0,0) = 0$ implies $I_E(0) = 0$, so the early one-period opening probability at $u = 0$ is $p_\Delta = 1 - e^{-\lambda_0 \Delta} > 0$: opening episodes occur with positive probability even without external tokens (the no-reward row of Section 6). The design studied here is not the only route into attention; as stated in Section 3.1, it is an auxiliary route that lowers attention's "luxury level" (the intercept in the right panel of Figure 2 corresponds to this baseline probability).

5.2 The steady state of object sensitivity

Holding q, v, g fixed, the steady state of (7) is

$$\bar{\kappa}(q, v, g) = \frac{a_\kappa q v}{a_\kappa q v + d_\kappa + b_\kappa g}. \tag{17}$$

Proposition 2 (Comparative statics of object sensitivity)**.** *For $q > 0$ and $v > 0$,*

$$\frac{\partial \bar{\kappa}}{\partial q} > 0, \qquad \frac{\partial \bar{\kappa}}{\partial v} > 0, \qquad \frac{\partial \bar{\kappa}}{\partial g} < 0. \tag{18}$$

Curiosity and visibility maintain contact with the object; achievement pressure narrows it. A state in which entanglement with inner content is merely weak, while object sensitivity is also low, is therefore a candidate for withdrawal or dissociation, not for attention.

5.3 Handoff after starter withdrawal and the two basins

The core of the design is not that tokens become a permanent driving source, but that the driving source is handed over to curiosity about, and visibility of, the object. To examine this possibility, withdraw the tokens and the attention KPI, $u = z = 0$, and consider the limit in which achievement pressure has sufficiently decayed, $g = 0$. Equations (4)–(5) become

$$\dot{q} = q\{\rho v(1-q) - d_q\}, \tag{19}$$

$$\dot{v} = a_v q(1-v) - d_v v. \tag{20}$$

The origin $(0, 0)$ is the state in which observation fails to start or fades out.

Proposition 3 (Two basins of the curiosity–visibility system)**.** *Assume*

$$\rho > d_q, \qquad \Delta_h := (\rho - d_q)^2 - \frac{4\rho d_q d_v}{a_v} > 0. \tag{21}$$

Then, in addition to the origin, (19)–(20) *possess two positive equilibria*

$$q_\pm = \frac{(\rho - d_q) \pm \sqrt{\Delta_h}}{2\rho}, \qquad v_\pm = \frac{a_v q_\pm}{d_v + a_v q_\pm}. \tag{22}$$

The origin is locally asymptotically stable, (q_-, v_-) is a saddle, and (q_+, v_+) is locally asymptotically stable.

The proposition shows that the consequences of token withdrawal are not uniform. If the starter carries the state beyond the saddle watershed, curiosity and visibility support each other, and observation can persist without external reward; if the watershed is not crossed, the state returns to the origin. The wager of the design is thus decomposed: not a single leap from tokens to attention, but first an observable intermediate question—can the curiosity–visibility system cross its basin boundary?

Condition (21) also reads as a design statement. Raising the efficiency of visibility formation a_v and lowering its decay d_v enlarge Δ_h and lower the saddle q_-. The design of observation materials that make stereotypy, self-contradiction, and operator repetition easy to find as structure therefore governs both the feasibility of handoff itself ($\Delta_h > 0$) and the height of the watershed. The role of materials is taken up again in Sections 7.2 and 8.1.

Corollary 2 (Handoff of the driving source through reward tapering)**.** *Suppose condition* (21) *of Proposition 3 holds, and let $\mathcal{B}$ denote the basin of attraction of* (q_+, v_+) *for the limiting system* (19)–(20)*. For every compact set K contained in the interior of $\mathcal{B}$ there exists $\varepsilon_K > 0$ such that, if $(q(t_0), v(t_0)) \in K$ and $0 \leq g(t_0) < \varepsilon_K$ at withdrawal ($u = z = 0$ for $t > t_0$), the trajectory converges to* (q_+, v_+)*. Hence, once sufficient contact has brought* (q, v) *deep into the basin and the residual pressure has decayed sufficiently, observation persists as* $u(t)$ *is tapered and withdrawn.*

The corollary establishes persistence conditional on the withdrawal state; the numerical examples of Section 6 illustrate one tapering policy that reaches such a state.

The result shows that "more reward for longer" is not necessarily better. Rewards are useful as starters, but once the object-intrinsic cycle is established they leave achievement pressure behind, so tapering can be advantageous. Note that the basin argument of Corollary 2 concerns the autonomous system (19)–(20) with $u = z = 0$ and $g = 0$, whereas a tapering path passes transiently through $g > 0$; strictly, the taper must be sufficiently gradual, or the state at withdrawal sufficiently deep inside the basin (Appendix A.5).

5.4 The attention-KPI paradox

When attention or quiet itself is evaluated and its attainment is tied to rewards, punishments, or comparisons, $z > 0$. The asymmetry between u and z is grounded in the operator framework of the preceding papers. The entry token u is contingent on external behavior—touching the material, trying for a set time. The attention KPI z, by contrast, demands a judgment of an inner outcome: did one attend, did one observe correctly? That judgment re-enters its own output (succeeded/failed) as a present fact and registers it in the formation history of the next self-monitoring rule—precisely the generation of an endogenous operator as formalized by Suzuki (2026b). Although z is reduced in the model to an input into g, its substance is the installation of a new monitoring–punishing operator: an institutional version of the ironic process by which direct control of mental states backfires through the monitoring it requires (Wegner, 1994). From (12) and (8), the direct effect with q, v, κ fixed is as follows.

Proposition 4 (Direct counter-effect of the attention KPI)**.** *Fix q, v, κ, u and let g be at its quasi-steady value* (12)*. Then*

$$\frac{\partial \lambda}{\partial z} = -\theta_g \frac{a_z}{d_g} \lambda < 0. \tag{23}$$

Hence the stronger the direct evaluation of attention itself, the lower the candidate-opening intensity.

Proposition 4, however, is a direct effect with the mediating states fixed, and it grants z no positive channel. In that form the paradox is closer to a restatement of the modeling assumption that z enters only g than to a derived result. In practice, evaluating attention can increase contact with the practice: a meditation app's streak record, for instance, is a visualization of an inner outcome and yet can raise contact time. We therefore extend the contact function to $\mathcal{E}(u + \psi z)$, where $\psi \geq 0$ is the degree to which attention evaluation also promotes entry behavior—a coefficient converting the KPI intensity z into token-equivalent contact promotion, so that u and ψz add on the same contact scale. Define early-stage curiosity and the opening index ($v \simeq 0$, g quasi-steady) as

$$\bar{q}_E(u, z; \psi) = \frac{A}{A + B}, \qquad A := a_q \mathcal{E}(u + \psi z), \quad B := d_q + \frac{b_q}{d_g}(a_g u + a_z z), \tag{24}$$

$$I_E(u, z; \psi) := \theta_q \bar{q}_E(u, z; \psi) - \theta_g \frac{a_g u + a_z z}{d_g}. \tag{25}$$

Proposition 5 (Attention-KPI effect with a positive contact channel)**.** *The partial derivative of the early opening index with respect to z is*

$$\frac{\partial I_E}{\partial z} = \theta_q \frac{\psi a_q \mathcal{E}'(u + \psi z)\, B - A\, b_q a_z / d_g}{(A + B)^2} - \theta_g \frac{a_z}{d_g}. \tag{26}$$

Consequently:

1. *If $\psi = 0$ (pure inner-outcome evaluation), then $\partial I_E/\partial z < 0$ always. The attention KPI then lowers the early opening probability through both the pressure channel (the θ_g term) and the curiosity channel (the fall of $\bar{q}_E$).*

2. *If $\psi > 0$, then $\partial I_E/\partial z < 0$ is equivalent to*

$$\theta_q \frac{\psi a_q \mathcal{E}'(u + \psi z)\, B}{(A + B)^2} < \frac{a_z}{d_g} \left\{ \theta_g + \theta_q \frac{A\, b_q}{(A + B)^2} \right\}. \tag{27}$$

 The left side is the contact-promoting effect of attention evaluation; the right side is the installation cost of self-monitoring pressure.

By Proposition 5, the counter-effect of the attention KPI is not an assumption but a falsifiable dominance condition—whether contact promotion outweighs self-monitoring pressure (the early opening probability is a monotone increasing function of I_E, so the sign conclusions carry over directly). The numerical examples of Section 6 use $\psi = 0$; the values of Table 2 are unaffected by the extension. Empirically, one should distinguish contact-promoting visualization (institutions with large ψ) from attainment-grading assessment (small ψ); this is Hypothesis H5. In either case, the distinction stands between rewarding entry behavior and rewarding the inner outcome called attention: tokens may attach to touching the material or trying for a set time, whereas turning "became quiet" or "attended correctly" into a KPI risks generating a new monitor.

5.5 The non-coercive uncertainty that remains to the end

Proposition 6 (Finite-horizon non-guarantee)**.** *For any* $0 < T < \infty$*, if* $0 < \int_0^T \lambda(s)\,ds < \infty$*, then*

$$0 < \mathbb{P}(\tau_O \le T) = 1 - \exp\left\{-\int_0^T \lambda(s)\,ds\right\} < 1. \tag{28}$$

Under the model's finite-intensity assumption, therefore, no finite tokens, no finite design of materials, and no finite time can guarantee a first opening with probability one. The model-level attention-entry process is the candidate-opening process thinned by object sensitivity (the process N_A *of Section 4.3); its probability of a first event within a finite horizon is therefore also below one, and the entry into attention likewise cannot be guaranteed. Since the measurement proxy* $\hat{\tau}_A$ *may contain misclassification, this containment is a model-level relation.*

The proposition is not a weakness of the design but a structural boundary. External design can raise the candidate-opening intensity, but the attempt to guarantee attention turns attention back into an object of control. The third paper does not eliminate the wager; it removes what recognition and design can remove, and states explicitly the genuine wager that remains. Two clarifications. First, the result follows almost directly from the paper's model choice—a finite-intensity stochastic hazard—and is positioned as an explicit statement of the design's structural boundary, not as a deep analytical theorem. Second, we separate the formal result from its interpretation. As a mathematical fact, within the absolutely continuous, finite-intensity hazard family adopted here, achieving probability one within finite time by intensity manipulation alone requires divergence of the cumulative intensity; the paper interprets that singular limit as lying outside the range of non-coercive design. Under this interpretation, the non-guarantee is the formal expression of attention's non-coercive character.

Contrast with cure-fraction structure. The non-guarantee above is a *finite-horizon* boundary and should not be read as a cure-fraction result. Under the bounded-input policies considered in this paper, the mediating states remain bounded and $\lambda(t)$ is bounded away from zero, so the cumulative intensity diverges and a first opening is almost sure over an infinite horizon: the first-opening distribution is proper in every scenario studied here. Cure models in survival analysis (Boag, 1949; Berkson and Gage, 1952; Chen et al., 1999; Amico and Van Keilegom, 2018) describe the different situation in which a fraction of the population never experiences the event because the cumulative hazard remains bounded on the infinite horizon. Connecting the present framework to that structure would require extensions—an integrable baseline intensity $\lambda_0(t)$, a frailty atom at zero susceptibility, or explicit susceptibility classes—which we leave to future work.

6 Numerical illustrations

6.1 Parameter setting

This section is not a fit to empirical data but an illustration that makes the model's structure visible. The values of Table 1 are used. The contact function is $\mathcal{E}(u) = 1 - e^{-2u}$, and the evaluation interval for opening probabilities is $\Delta = 1$. Initial values are $q(0) = v(0) = g(0) = \kappa(0) = 0$ for all schedules. The cumulative opening probability and the median opening time are computed along the same baseline trajectory by numerically integrating the cumulative intensity

$\Lambda(t) = \int_0^t \lambda(s)\, ds$, as $\mathbb{P}(\tau_O \le T) = 1 - e^{-\Lambda(T)}$ and as the first time $\Lambda(t) = \log 2$, respectively (Section 6.4). Time is in uncalibrated theoretical units, and the no-reward cumulative level is governed by the product of the baseline intensity and the horizon, $\lambda_0 T$ ($= 1.2$ here); sensitivity to λ_0 and T is a caveat in interpreting the examples.

Table 1: Parameters for the numerical examples

Symbol	Meaning	Value	Symbol	Meaning	Value
α	Contact ramp-up	2.0	a_q	Contact to curiosity	1.2
ρ	Visibility-driven curiosity	1.5	d_q	Natural decay of curiosity	0.4
b_q	Curiosity inhibition by pressure	0.8	a_v	Formation of visibility	1.0
d_v	Decay of visibility	0.3	a_g	Tokens to pressure	1.0
a_z	Attention KPI to pressure	0.8	d_g	Decay of pressure	0.8
a_κ	Object-sensitivity formation	1.2	d_κ	Object-sensitivity decay	0.2
b_κ	Sensitivity loss under pressure	0.8	λ_0	Baseline opening intensity	0.03
θ_q, θ_v	Curiosity, visibility coefficients	3.0, 1.0	θ_κ, θ_g	Sensitivity, pressure coefficients	1.5, 1.0

Condition (21) of Proposition 3 is satisfied, with

$$q_- = 0.1333, \quad v_- = 0.3077, \qquad q_+ = 0.6000, \quad v_+ = 0.6667. \tag{29}$$

The lower equilibrium is the watershed; the upper is the local attractor of object-intrinsic observation that can persist after token withdrawal.

6.2 Token intensity and early curiosity

The left panel of Figure 2 shows contact $\mathcal{E}(u)$, early curiosity $\bar{q}_E(u, 0)$, and normalized achievement pressure $\bar{g}/(1+\bar{g})$ against token intensity u. Contact grows with saturation while pressure keeps growing, so curiosity peaks at an interior point. The right panel approximates the early opening probability, which peaks near $u \approx 0.25$.

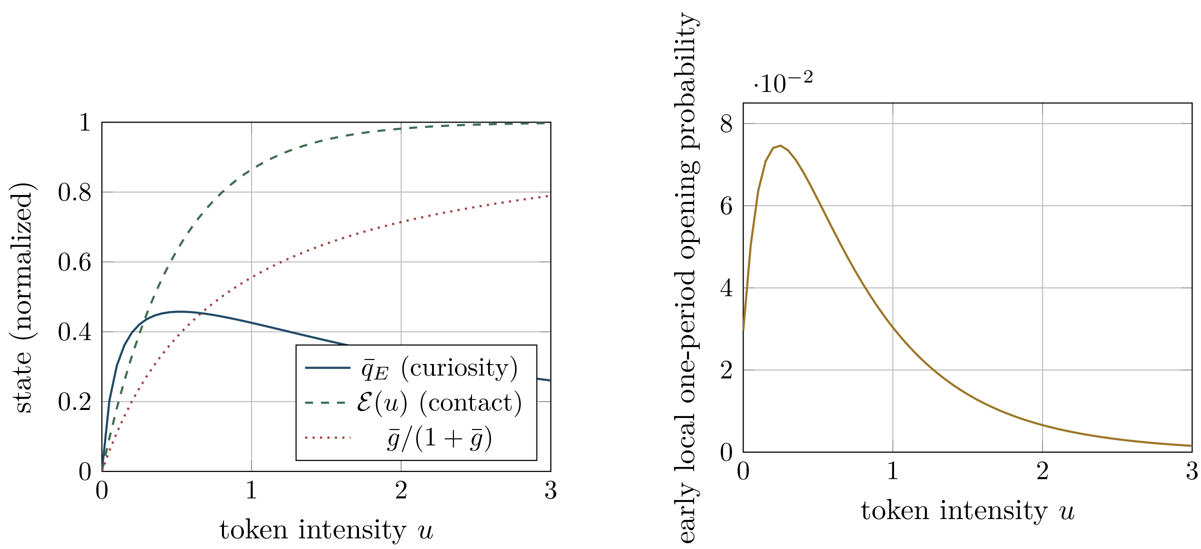

Figure 2: The dual effect of tokens. If tokens are too small, contact does not begin; if too large, achievement pressure suppresses curiosity and the opening probability.

6.3 Handoff of the driving source

Figure 3 shows the nullclines of the q–v system after token withdrawal. The origin and the upper equilibrium are stable; the lower equilibrium is the saddle that separates them. The role of the starter is not to produce attention directly but to carry the state to the far side of the lower watershed.

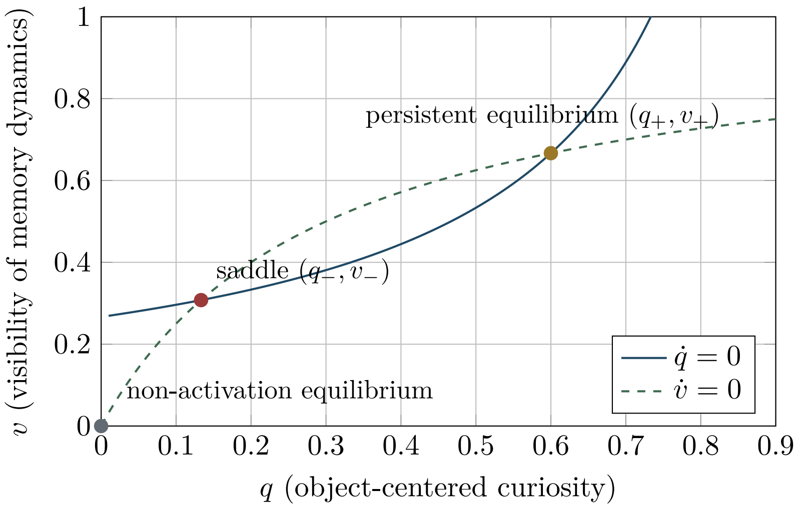

Figure 3: Two basins after token withdrawal. Beyond the lower saddle, curiosity and visibility can sustain each other.

6.4 Comparing four institutional designs

We compare four schedules.

1. **Moderate fixed:** $u(t) = 0.8$.
2. **High fixed:** $u(t) = 2.0$.
3. **Tapered:** $u = 0.8$ for $t < 8$, decreasing linearly to zero over $8 \le t < 16$, and zero thereafter.
4. **Attention KPI:** $u = 0.8$ plus direct evaluation of attention at $z = 0.8$.

Figure 4 shows the trajectories of the moderate fixed, high fixed, and tapered schedules. The opening-probability panel reports the local approximation $1 - e^{-\lambda(t)}$; at $t = 40$ and near steady states it agrees with the exact interval probability to the displayed digits, while during fast transients it understates the exact one-period probability. Under tapering, the initial tokens carry q and v past the watershed, and g then decays, so q and κ stay high and the opening probability rises sharply. Under the high fixed schedule, contact is ample but g is large, suppressing object sensitivity and the opening probability.

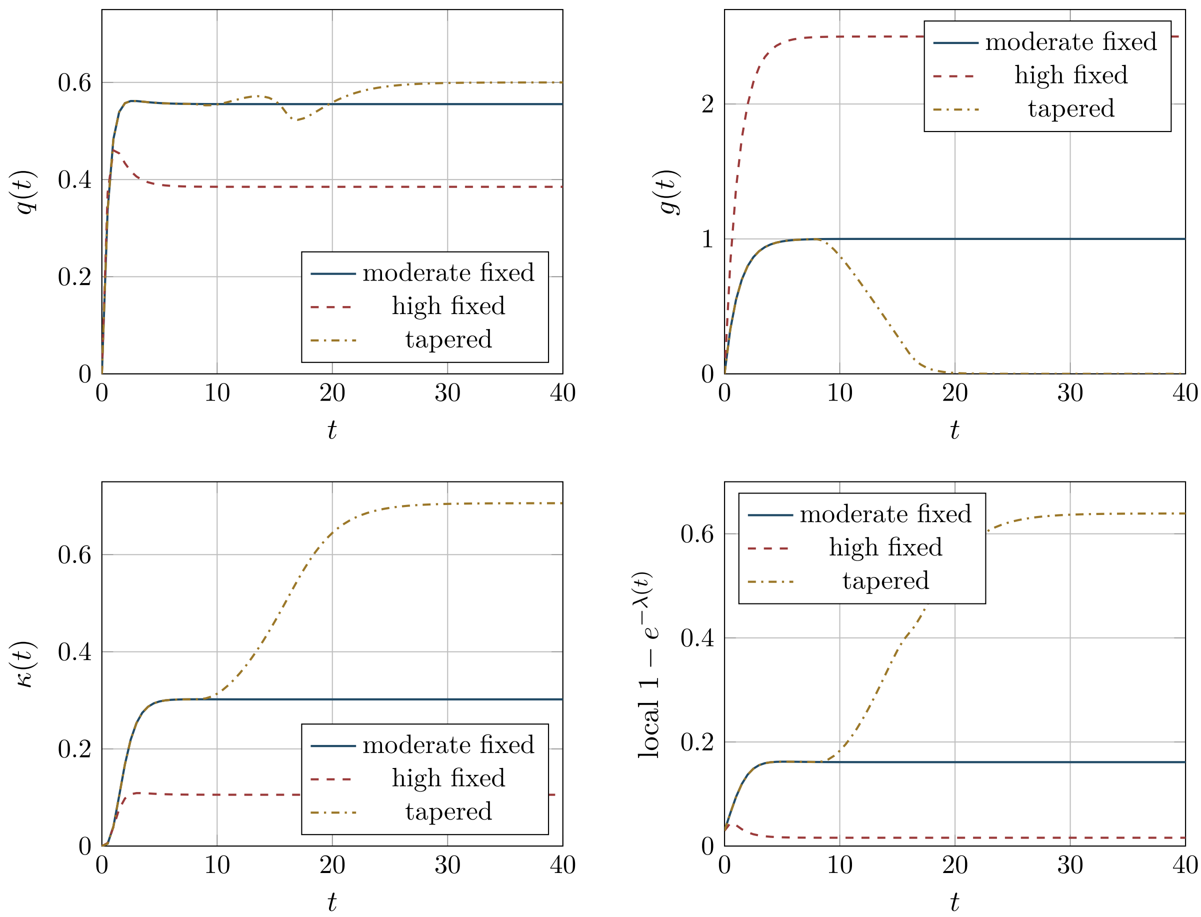

Figure 4: Dynamic comparison of fixed and tapered rewards. The tapering policy hands the driving source over to the curiosity–visibility system and lowers achievement pressure.

Table 2 reports the states at $t = 40$ and three opening indicators—the one-period opening probability $p_\Delta(40)$, the cumulative opening probability $\mathbb{P}(\tau_O \leq 40)$, and the median opening time—together with the cumulative token expenditure U, and alongside the no-reward schedule driven by the baseline intensity alone. The attention KPI, although it pays the same entry reward as the moderate fixed schedule, raises g and worsens q, κ, and all three opening indicators.

Table 2: States and opening indicators by schedule at $t = 40$. The one-period opening probability $p_\Delta(40)$ is computed with the locally constant approximation $1 - e^{-\lambda(40)\Delta}$ ($\Delta = 1$), which agrees with the exact interval integral to the displayed digits; $\mathbb{P}(\tau_O \leq 40) = 1 - e^{-\Lambda(40)}$; the median τ_O is the first time at which $\Lambda(t) = \log 2$; and $U = \int_0^{40} u\, dt$ is cumulative token spending. The corresponding opening intensities are $\lambda(40) = 0.030$ (no reward), 0.176 (moderate fixed), 0.016 (high fixed), 1.019 (tapered), and 0.039 (attention KPI).

Schedule	q	v	g	κ	$p_\Delta(40)$	$\mathbb{P}(\tau_O \leq 40)$	Median τ_O	Spend U
no reward	0.000	0.000	0.000	0.000	0.030	0.6988	23.1	0.0
moderate fixed	0.555	0.649	1.000	0.302	0.161	0.9989	5.0	32.0
high fixed	0.385	0.562	2.500	0.106	0.016	0.5023	39.7	80.0
tapered	0.600	0.667	0.000	0.706	0.639	>0.9999	5.0	9.6
attention KPI	0.418	0.582	1.800	0.151	0.038	0.7980	16.8	32.0

Four structural readings follow from the numbers. First, doing nothing has its own baseline: even without rewards, roughly seventy percent experience a candidate opening within forty periods through λ_0 (first row of Table 2). Against that baseline, the high fixed schedule falls *below* the spontaneous level (cumulative probability 0.50 versus 0.70; median 39.7 versus 23.1)—the sharpest expression of crowding out, in which excessive reward not only fails to promote openings but suppresses them. Note, however, that the openings under no reward and high fixed include many candidate openings with low κ, which pass the measurement-level $\bar{\kappa}$ filter less readily (Sections 4.3 and 7.3). Second, the first-opening distributions of the moderate fixed and tapered schedules coincide up to $t = 8$, and the median of 5.0 is determined on this common segment. Their distributions differ afterwards, but the cumulative probabilities to $t = 40$ are both essentially one. The advantage of tapering therefore lies not in whether a first opening occurs, but in the fact that a high post-withdrawal one-period opening probability (0.639 versus 0.161; in intensities, 1.02 versus 0.18, about 5.8-fold) is sustained at less than a third of the cumulative expenditure (9.6 versus 32.0)—the handoff of the driving source is achieved without sacrificing the first opening. Third, the schedule that evaluates attention directly delays the first opening (median 16.8) and lowers both the cumulative and the one-period opening probabilities under the same entry reward. Fourth, under no schedule does the cumulative probability reach one in finite time; the non-coercive uncertainty remains (Proposition 6).

As a further reference, four alternative allocations with the same cumulative expenditure as the tapered schedule, $U = 9.6$, were computed under the same baseline dynamics. A low constant schedule spreading the budget over the forty periods ($u = 0.24$) achieves handoff ($q \approx 0.63$ at $t = 40$, one-period opening probability 0.48, cumulative > 0.9999, median 3.9). By contrast, front-loaded burst allocations concentrating the same budget—$u = 9.6$ for one period, $u = 4.8$ for two, or $u = 2.4$ for four—all collapse to the origin in the curiosity–visibility system after withdrawal, and handoff fails. Moreover, the cumulative opening probabilities of the first two (0.664, 0.667) fall even below the no-reward level (0.699): the concentrated injection sends achievement pressure g soaring and suppresses even the spontaneous openings during the injection period. Under the present parameters, therefore, equal budgets divide qualitatively by their temporal allocation (H7a), and the watershed binds against concentrated allocations. Distinguishing between diffuse allocations such as tapering and the low constant, on the other hand, requires setting the budget or the parameters near the watershed threshold (Section 7.1). Table 2 illustrates that tapering *can* be advantageous; it does not establish a general dominance of tapering under equal budgets.

Finally, the taper of this section is a time-based rule (linear decrease from $t = 8$), but as Figure 4 shows, the state under $u = 0.8$ has essentially reached quasi-steady values (well above the watershed) by $t = 8$, so the rule effectively approximates a state-dependent rule—taper after curiosity reaches a threshold. As a policy matter, adaptive tapering keyed to the individual's q (or an observable proxy of it) is the primary prescription (Corollary 2, Section 7.2), and the formulation as an optimal control problem—maximizing $\mathbb{P}(\tau_O \leq T)$ under the budget constraint $\int_0^T u\,dt \leq U$—is left to future work (Section 11).

7 Falsifiable predictions and empirical design

7.1 Principal hypotheses

The model implies at least the following hypotheses.

H1 (Entry effect) As token intensity rises from near zero, the task contact rate and early observation time increase.

H2 (Inverted U) The relations between token intensity and object-centered curiosity, and between token intensity and non-closure indicators, are conditionally inverted-U shaped.

H3 (Mediation) The positive effects of tokens run through task contact and visibility; the negative effects run through acquisition–achievement pressure.

H4 (Tapering advantage) Conditions that taper tokens after initial contact yield higher post-withdrawal voluntary re-engagement, object sensitivity, and low-closure indicators than fixed high-reward conditions.

H5 (Attention-KPI paradox) Conditions that evaluate and reward attention, quiet, or correct observation itself raise self-monitoring and achievement pressure and lower non-closure indicators relative to conditions that reward entry behavior only. The effect is strongest for attainment-grading assessment without contact promotion (institutions with small ψ in Proposition 5).

H6 (Watershed) The probability that observation persists after token withdrawal does not increase smoothly in the pre-withdrawal pair (q, v); it exhibits a steep threshold near the saddle watershed. If the distribution of initial states puts sufficient mass on both sides of the watershed and noise and measurement error are not excessive, long-run outcomes can be bimodal.

H7a (Path dependence through concentration) Holding cumulative token expenditure and the evaluation horizon fixed, allocations that concentrate the input into a short period can fail at handoff by sending achievement pressure soaring, while spread allocations can succeed (supported by the numerical example of Section 6.4).

H7b (Shape effects among diffuse allocations) Among diffuse allocations under equal budgets and a common evaluation horizon, the path "start moderately, taper after interest has formed" yields higher post-withdrawal persistence and non-closure indicators than "keep a low constant level." Identifying this difference requires controlling the actual injection period and setting the budget and task near the watershed threshold (a hypothesis for future theory and testing).

H8 (Non-guarantee) Even under optimized designs, individual and trial variation in the first opening within finite time remains, and its cumulative probability does not reach one.

7.2 Basic experimental design

A first-stage experiment could randomly assign participants to the following conditions:

1. a no-reward condition;
2. low, moderate, and high fixed-token conditions;
3. a condition in which a moderate token is tapered from partway;
4. an attention-KPI condition in which "did you attend?" rather than entry behavior is evaluated.

The observation object should be material in which the stereotyped responses of self-referential memory are comparatively easy to find—for example, a short task in which participants can record a primary reaction to a stimulus, the secondary operations that conceal, suppress, or justify it, and the simultaneously present external task. Designs that force disclosure of clinical distress or secrets must be avoided; one should begin with low-intrusiveness experimental tasks.

Tokens attach to entry behaviors—contacting the material for a set time, keeping an optional observation log—and are not tied to the attainment of attention, quiet, or correct answers. In the tapering condition, tokens are reduced stepwise after the individual's curiosity or voluntary re-engagement reaches a set level.

7.3 Measurement

The main measurement candidates are as follows.

- **Contact and participation:** task start rates, dwell time, voluntary revisit rates.
- **Object-centered curiosity:** state items such as "I want to see what happens next" and "I would continue without a reward"; spontaneous information search.
- **Visibility:** the degree to which the same stereotyped responses, self-contradictions, and operation sequences can be identified as structure.
- **Acquisition–achievement pressure:** attention to rewards, self-evaluation of success and failure, felt pressure to "observe correctly."
- **Object sensitivity:** change detection in the external task, response times, context-appropriate response revision.
- **Closure and decentering:** state scales such as the State Cognitive Fusion Questionnaire, decentering, and nonreactive awareness (Bolderston et al., 2019; Bernstein et al., 2015).
- **Non-instrumentality:** persistence of observation after token withdrawal, freedom to stop, reduced self-evaluation.

The closure degree c_t and object sensitivity κ_t can be estimated as latent variables with dynamic structural equation models, state-space models, or hidden Markov models. At the measurement level, the timing of an entry event is operationalized through the estimated latent states $\hat{c}_t, \hat{\kappa}_t$ as

$$\hat{\tau}_A := \inf\{t \geq 0 : \hat{c}_t \leq \bar{c},\ \hat{\kappa}_t \geq \bar{\kappa}\}. \tag{30}$$

At the model level, the attention-entry process N_A and its intensity $\lambda_A(t) = \lambda(t)\pi(\kappa_t)$ were defined in Section 4.3 by thinning the candidate-opening process with the object-sensitivity qualification π (principal example $\pi(\kappa) = \mathbf{1}\{\kappa \geq \bar{\kappa}\}$). The thinning depends on κ alone because the candidate-opening event N itself already represents the low-closure side of the opening (a transient fall of c); the only condition to be added is object sensitivity. The proxy $\hat{\tau}_A$ of (30) is the measurement-level counterpart of the first event of the thinned process N_A, and it contains misclassification—misses and false alarms—originating in latent-state estimation error. As to correspondence, $\hat{c}_t \leq \bar{c}$ corresponds to the *detection* of an opening episode's occurrence, and $\hat{\kappa}_t \geq \bar{\kappa}$ to the *qualification* condition. The exact correspondence between the threshold rule and the thinning (an observation model including the detection process for openings), and the derivation of λ from a stochastic-process model of c_t, are left to future work (Section 11). In particular, by not expressing attention through c_t alone but distinguishing the state with low c_t and high κ_t, confusion with dissociation, indifference, and withdrawal is avoided.

7.4 How far can the wager be reduced?

Experiments can estimate the links of (1) separately. Writing the individual and task parameters jointly as ϕ, the mediating-state path is deterministic given ϕ and the design $u(\cdot)$, and the finite-horizon cumulative opening probability is

$$\mathbb{P}(\tau_O \leq T \mid u(\cdot), \phi) = 1 - \exp\left\{ - \int_0^T \lambda(q_t, v_t, \kappa_t, g_t; \phi)\, dt \right\}. \tag{31}$$

Individual differences enter through the parameter distribution Π:

$$\mathbb{P}(\tau_O \leq T \mid u(\cdot)) = \int \mathbb{P}(\tau_O \leq T \mid u(\cdot), \phi)\, d\Pi(\phi), \tag{32}$$

a schematic frailty-mixture representation that omits measurement error and latent-state estimation uncertainty. Diagnosis of failure points corresponds to stagewise parameter estimation: (i) contact—the early response to $\mathcal{E}(u)$ (a_q; experimentally, task start rates and dwell time); (ii) curiosity to visibility—a_v (formation of structure identification); (iii) object sensitivity—a_κ, b_κ (maintenance of responding to the external task); (iv) pressure—a_g, a_z, b_q; (v) opening—the θ coefficients and λ_0. Note that in the decomposition of Section 3.3, "contact" is reduced to the instantaneous variable $\mathcal{E}(u)$ rather than a state. An extension that makes contact explicit as a stock $x(t)$ (with $\dot{x} = a_x \mathcal{E}(u)(1-x) - d_x x$ and the first term of (4) replaced by $a_q x(1-q)$) would separate entry failure from curiosity failure still more directly, but requires a full recomputation of the numerical examples and is left to the next paper. Even if all parameters were known, however, the stochastic uncertainty of Proposition 6 would remain. That residual is the wager that remains to the end.

8 Implications for education, clinical practice, organizations, and AI design

8.1 Educational design

In education, rewards, credits, and grades often open the entrance. But if the same external pressure is maintained after interest in the content itself has formed, the task can revert to a means of producing "the evaluated me." The model suggests moderate support for entry behavior, materials that raise the visibility of the object, tapering of rewards after interest has formed, and abstention from evaluating attention itself.

The difference between "reluctant language class" and "doing mathematics because it is interesting" lies not in the amount of activity but in the driving source. In the former, the self-image, evaluation, and the desire to finish exploit the task; in the latter, task-relevant memory is recruited by the demands of the object. The design problem of this paper is the handoff from the former to the latter.

8.2 Clinical implications

In clinical settings, instructions to "eliminate anxiety," "observe correctly," or "be mindful" can create new self-monitoring. This paper does not reject treatment in general or functional control: securing safety, medication, sleep regulation, and impulse control can be indispensable. What is at issue is self-referential control that keeps processing inner experience toward an ideal state.

An intervention in the spirit of this design does not make attention an attainment target; it offers a low-stakes entry point from which the stereotyped dynamics of memory can become an object of curiosity. Clinical application, however, requires research ethics, safety, attention to individual differences, and monitoring for symptom exacerbation; treatment guidelines cannot be derived directly from the theoretical model of this paper.

8.3 Organizations and management accounting

Management accounting has a control structure of targets, measurement, variances, and correction. Indicators open entrances to behavior; yet the indicators themselves can internalize self-image, comparison, and defense, generating a "manager" inside the person. High-frequency AI-based evaluation simultaneously carries the potential to promote contact, learning, and improvement, and to amplify achievement pressure and self-monitoring.

The model suggests the following design principles. First, indicators should support entry behavior and learning opportunities, and should not grade inner attention or personal worth. Second, once interest in the object has formed, the salience of external incentives should be lowered. Third, object sensitivity, flexible revision, and learning from errors should be measured, rather than self-protective KPI attainment alone. Fourth, AI should be used not as a monitor that commands "attend correctly" but as an aid that makes the repetition of memory dynamics visible.

These principles are consistent with the contract theory of multitask agency. As Holmström and Milgrom (1991) showed, attaching strong incentives to measurable tasks diverts effort from hard-to-measure but important tasks, so that when the latter matter, deliberately weakening incentives on the measurable tasks can be optimal. In the present frame, entry behavior (contact) is measurable while attention is hard to measure. Attempts to measure and reward attention itself ($z > 0$) generate an inner-state version of Campbell (1979)'s observation that indicators corrupt as they are used for decisions (Campbell's law) and of a closely related measurement-gaming principle (Propositions 4 and 5). "Moderate rewards restricted to measurable entry behavior, with reduced salience after interest forms" can thus be understood as a species of the low-powered-incentive solution to the multitask problem.

8.4 Low-rank stereotyped responses as observation material

When self-preservation is strong, high-dimensional reality is sometimes compressed into a small number of stereotyped operators: denial, minimization, blame shifting, justification, sympathy solicitation, sanction bargaining. As an informal suggestion in the notation of Suzuki (2026b), with reality Y_t (high-dimensional), memory history H_t, and the response-generating map Γ,

$$\Gamma(H_t) = \sum_{k=1}^{K} w_k(H_t) a_k b_k^\top, \qquad K \ll \dim Y_t, \tag{33}$$

where $a_k b_k^\top$ are rank-one stereotyped response templates, $w_k(H_t)$ are history-dependent weights, and $K \ll \dim Y_t$ expresses that the effective dimension of the response is far below that of reality. Reality Y_t is high-dimensional, but the response is projected onto a low-rank self-preserving subspace.

This low-rank property is not the protagonist of the third paper, but it is a strong candidate for observation material. It can begin as a game of finding others' stereotyped responses, until one sees that the same operators exist in oneself. It must not become a verdict table for condemning others: what matters is not the wording itself but whether the utterance opened or closed contact with reality.

9 Discussion

9.1 Avoiding the mystification of attention

The paper does not posit attention as a mysterious entity, an independent self, or a supreme control operator. The entry into attention is operationalized as a stochastic state transition in which lowered closure and raised object sensitivity occur together. This allows one to ask which external occasions and mediating states change the opening intensity, without presupposing the metaphysical question "from where does attention descend?"

The operationalization does not explain the whole of attention. τ_O is an indicator of candidate openings that can lead to entry; it proves nothing about the phenomenology, duration, or depth of attention, or the relations among attention, freedom, stillness, and love. Those are left to subsequent research.

9.2 What is it for observation to cease being a means of acquisition?

"Observation ceases to be a means of acquisition" largely overlaps with "observing despite oneself, because it is interesting," but not completely: interestingness itself can become a new object of acquisition. The paper identifies non-instrumental observation by the combination of persistence after reward withdrawal, reduced self-evaluation pressure, and maintained object sensitivity.

Nor is the transition a simple substitution of internal for external reward. If it becomes "I must maintain the interesting state," the self-referential operator rises again. Object-centered curiosity is therefore not attention itself but the hinge from reward purpose to non-instrumental observation.

9.3 Simultaneous visibility of fear, manipulation, and the object

Attention may not be a two-stage process of first erasing fear and then returning to the object. There is a state in which fear exists, the operations that hide, suppress, or correct fear also exist, and the object in front of one is seen at the same time. In that state, fear and manipulation have not vanished, but they are not the sovereign that chooses the response to the object.

κ in this paper expresses this opening to the external object. Lowering closure alone cannot be distinguished from dissociation and withdrawal. High q and v, low g, and high κ are the operational signature of the attention this design aims at.

9.4 Naturalness, aliveness, efficiency

Memory-driven activity spends large internal administrative costs on maintaining, monitoring, punishing, and correcting the self-image; throughput is high but effective response to reality is low. When self-referential closure is weak, functional memory works actively while excess self-monitoring declines. In the first person, this can feel like moving from "finitely shallow and opaque" to "deep and transparent," from "hollow and artificial" to "simply natural," from "wastefully inefficient" to "matter-of-factly efficient."

These words are, at this stage, phenomenological candidates accompanying the transition to functional non-closure, not theoretical variables. Future experience sampling and qualitative research should examine their correspondence with closure and object sensitivity.

9.5 Shrinking the wager versus eliminating it

The paper does not eliminate the wager. Optimizing tokens, materials, visibility, and tapering policies cannot generate attention with probability one. Decomposing the large black box, however, makes the following possible:

- separating the problem of contact not occurring from the problem of not moving to curiosity;
- identifying the case in which curiosity arose but achievement pressure was too strong;
- separating absorption in the interior from attention that preserves object sensitivity;
- comparing fixed and tapered rewards;
- testing the counter-effect of attention KPIs.

What remains at the end is condensed into a single non-coercive point: when the conditions are arranged, does observation actually cease to be a means of acquisition?

10 Limitations

The paper has at least nine limitations. First, the model is a theoretical scaffold; its parameters are not estimated from data. Second, curiosity, visibility, achievement pressure, and object sensitivity are each reduced to a single scalar. Third, the kinds of tokens, their contingencies, announcement, fairness, and social meaning are collapsed into a single u. Fourth, candidate openings toward the entry into attention are coarse-grained into a counting process with state-dependent intensity, without directly modeling the continuous internal dynamics of the closure degree c_t. Fifth, transitions from attention back to re-closure are not treated. Sixth, the curiosity–visibility bistability of Proposition 3 is one possible mechanism, not a necessary psychological law. Seventh, existing findings on the effects of external rewards are condition-dependent, and the paper's inverted U must be tested as a theoretical prediction. Eighth, applications to clinical, educational, and organizational settings must weigh ethics, safety, cultural differences, and individual variation. Ninth, acquisition–achievement pressure g is treated as a purely exogenous variable driven by external inputs and decaying exponentially after they stop, abstracting from the internalization of pressure (self-reinforcement and history dependence). In the framework of Suzuki (2026b), control operators can self-propagate, so this abstraction is not conservative for the tapering advantage (Corollary 2, H4), while it is conservative for the attention-KPI paradox (Proposition 4): if internalization is allowed, the monitor installed by a KPI persists after withdrawal, and the counter-effect becomes more durable. Internalized control can be interpreted as belonging to the closure degree c_t within this framework, but the formalization of history dependence in g is left to future work.

The paper also does not prove why attention, freedom, stillness, or love should be ultimately desirable. What it shows is only that the functional non-closure corresponding to attention is one candidate state that maintains object sensitivity to reality without multiplying the infinite regress of self-referential control.

11 Conclusion

This paper endogenized, in reduced form and within an externally assisted entry model, the state-dependent arrival intensity of *candidate openings* toward functional non-closure, which the two preceding papers had left exogenous; model-level attention entries are represented separately by the thinned intensity $\lambda_A(t) = \lambda(t)\pi(\kappa_t)$. External tokens can initiate contact with the task and raise object-centered curiosity and the visibility of memory dynamics; at the same time, they can foreground acquisition, achievement, and self-evaluation, lowering object sensitivity and the opening intensity.

The model distinguishes external tokens, object-centered curiosity, visibility, acquisition–achievement pressure, and object sensitivity; represents the candidate openings from the closed state toward functional non-closure as a counting process with state-dependent intensity; and analyzes its first time. Analytically, early curiosity and the early opening probability can attain interior maxima in token intensity; the interaction of curiosity and visibility can create two basins that persist after token withdrawal; turning attention into a KPI can lower the opening intensity; and, under the finite-intensity assumption, the transition cannot be guaranteed from outside within finite time. Numerically, a fixed high reward delays the first opening itself and falls below the no-reward baseline in cumulative opening probability; equal budgets can fail at handoff when allocated in concentrated bursts; and the tapering policy sustains a high post-withdrawal opening intensity at low cumulative expenditure without sacrificing the first opening.

The role of this third paper is therefore not to justify the large wager "give tokens and attention will arise." It is to decompose that wager into the smaller transitions of contact, curiosity, visibility, object sensitivity, and non-closure; to reduce the uncertainty that design and testing can reduce; and to state explicitly the non-coercive uncertainty that remains. The design does not manufacture attention. It removes conditions that prevent attention from arising, arranges conditions under which it can arise, and raises the intensity of candidate openings and, conditional on sufficient object sensitivity, the intensity of model-level attention entries.

The next tasks are to operationalize the mediating variables and to run experiments comparing token amounts, reward targets, and tapering rules—while evaluating neither attention nor quiet itself, and distinguishing support for entry behavior from the formation of interest in the object. Further questions include whether low-rank stereotyped memory responses can serve as observation material, how object sensitivity κ relates to appropriate and timely response to reality, and how the positive experiential modes after attention appear. Model extensions include: (i) making the entry stage explicit through a contact stock $x(t)$ (Section 7.4); (ii) formulating the optimal control problem of maximizing the cumulative opening probability under a budget constraint and deriving front-loading–tapering structure (Section 6.4); (iii) generalizing $\dot{\kappa}$ to admit absorbed-perception routes and checking robustness to the interaction $\theta_{v\kappa}v\kappa$ in the opening intensity (Sections 3.4 and 4.3); (iv) formalizing the internalization and history dependence of the pressure g (Section 10); and (v) deriving the opening intensity λ from a stochastic-process model of the closure degree c_t.

A Proofs

A.1 Proof of Proposition 1

Set $z = 0$ and write $A(u) := a_q\mathcal{E}(u)$ and $B(u) := d_q + (b_q a_g/d_g)u$. Then

$$\bar{q}_E(u, 0) = \frac{A(u)}{A(u) + B(u)}.$$

Since $A(0) = 0$ and $B(0) = d_q > 0$, we have $\bar{q}_E(0, 0) = 0$. As $u \to \infty$, $A(u) \to a_q$ remains bounded while $B(u) \to \infty$, so $\bar{q}_E(u, 0) \to 0$. Moreover $\bar{q}_E'(0) = a_q\alpha/d_q > 0$, so the function increases near zero. By continuity and the vanishing limit at infinity, it attains at least one interior maximum. Setting the derivative to zero at an interior point gives

$$A'(u)B(u) - A(u)B'(u) = 0,$$

that is, $A'/A = B'/B$, which is Eq. (14). □

A.2 Proof of Corollary 1

The early opening index is $I_E(u) = \theta_q\bar{q}_E(u, 0) - \theta_g(a_g/d_g)u$. Hence

$$I_E'(0) = \theta_q\frac{a_q\alpha}{d_q} - \theta_g\frac{a_g}{d_g},$$

which is positive under condition (16). On the other hand, $\bar{q}_E$ is bounded and the second term decreases linearly in u, so $I_E(u) \to -\infty$. The opening probability is a monotone increasing function of I_E, so it attains at least one interior maximum. □

A.3 Proof of Proposition 2

Write $s := a_\kappa qv$. Then $\bar{\kappa} = s/(s + d_\kappa + b_\kappa g)$ is increasing in s and decreasing in g, and s is increasing in each of q and v whenever $q > 0$ and $v > 0$. Partial differentiation of Eq. (17) in each argument gives the claim immediately. □

A.4 Proof of Proposition 3

At a positive equilibrium $q > 0$, Eq. (19) gives

$$\rho v(1 - q) = d_q,$$

and Eq. (20) gives

$$v = \frac{a_v q}{d_v + a_v q}.$$

Substituting the latter into the former yields

$$\rho q^2 - (\rho - d_q)q + \frac{d_q d_v}{a_v} = 0.$$

Its discriminant is Δ_h, and under condition (21) the quadratic has two positive roots $q_- < q_+$; the corresponding $v_\pm$ are given by Eq. (22).

The Jacobian at the origin is

$$J(0,0) = \begin{pmatrix} -d_q & 0 \\ a_v & -d_v \end{pmatrix},$$

with eigenvalues $-d_q$ and $-d_v$, so the origin is locally asymptotically stable. At a positive equilibrium the trace of the Jacobian is negative. Using the equilibrium conditions, the determinant can be written as

$$\det J(q,v) = a_v q\{d_q + 2\rho q - \rho\}.$$

Since $q_- < (\rho - d_q)/(2\rho)$, the determinant is negative there and (q_-, v_-) is a saddle; since $q_+ > (\rho - d_q)/(2\rho)$, the determinant is positive, which together with the negative trace makes (q_+, v_+) locally asymptotically stable. □

A.5 Proof of Corollary 2

We first record an auxiliary fact.

Lemma 1 (No closed orbits in the limiting system)**.** *The autonomous system* (19)–(20) *has no closed orbit in the open region* $D := \{(q,v) : q > 0,\ v > 0\}$.

Proof of the lemma. Take the Dulac function $B(q,v) = 1/(qv)$. With f_1 and f_2 the right-hand sides of (19)–(20),

$$Bf_1 = \rho(1-q) - \frac{d_q}{v}, \qquad Bf_2 = \frac{a_v(1-v)}{v} - \frac{d_v}{q},$$

and

$$\frac{\partial(Bf_1)}{\partial q} + \frac{\partial(Bf_2)}{\partial v} = -\rho - \frac{a_v}{v^2} < 0$$

throughout D. Since D is simply connected, Dulac's criterion rules out closed orbits in D. □

Proof of Corollary 2. By Proposition 3, (q_+, v_+) is a locally asymptotically stable equilibrium of the autonomous system (19)–(20), and its basin of attraction $\mathcal{B}$ is an open set containing (q_+, v_+). Along post-withdrawal paths, Eq. (6) with $u = z = 0$ gives $g(t) = g(t_0)e^{-d_g(t-t_0)}$, which decays exponentially, so the (q,v) dynamics form an asymptotically autonomous system whose limiting system is (19)–(20). By Lemma 1 the limiting system has no closed orbits; combined with the Poincaré–Bendixson theorem, every orbit of the limiting system in the invariant box $[0,1]^2$ converges to an equilibrium. All three equilibria are hyperbolic, and no cyclic chain of equilibria exists (the unstable manifold of the saddle runs only toward the two asymptotically stable equilibria). Hence, by the convergence theorem for asymptotically autonomous systems (Thieme, 1992; see also Markus, 1956; Castillo-Chavez and Thieme, 1995), every forward-bounded solution of the perturbed system converges to one of the equilibria of the limiting system. That the limit is (q_+, v_+) for withdrawal states in the basin is checked in four steps. (i) By the local asymptotic stability of (q_+, v_+), there exists inside $\mathcal{B}$ a positively invariant neighborhood U, obtainable as a sublevel set of a local Lyapunov function. The strict negativity of the Lyapunov derivative is robust to bounded perturbation terms, so U remains positively invariant for the perturbed system as long as the post-withdrawal residual-pressure terms (such as $-b_q g q$) are sufficiently small. (ii) Every compact set K in the interior of the basin enters U within some finite time T_K under the flow of the limiting system, by compactness of K and continuity of the flow. (iii) On the finite interval $[t_0, t_0 + T_K]$, if $g(t_0)$ at withdrawal is sufficiently small, Gronwall's inequality keeps the perturbed solution uniformly close to the limiting solution, so perturbed solutions started in K also enter U. (iv) After entering U, the perturbation invariance in (i), the decay $g(t) \to 0$, and the fact that the chain-recurrent set of the limiting system inside U is $\{(q_+, v_+)\}$ alone imply convergence to (q_+, v_+). $\square$

A.6 Proof of Proposition 4

Substitute the quasi-steady value $g = (a_g u + a_z z)/d_g$ into Eq. (8) and differentiate in z with q, v, κ, and u held fixed; this yields Eq. (23). $\square$

A.7 Proof of Proposition 5

Differentiating $\bar{q}_E = A/(A + B)$ in z gives

$$\frac{\partial \bar{q}_E}{\partial z} = \frac{A_z B - A B_z}{(A + B)^2}, \qquad A_z = \psi a_q \mathcal{E}'(u + \psi z), \quad B_z = \frac{b_q a_z}{d_g},$$

and the z-derivative of the second term of Eq. (25) is $-\theta_g a_z/d_g$, which yields Eq. (26). If $\psi = 0$ then $A_z = 0$; since $A \geq 0$ and $B > 0$, the first term is nonpositive (negative when $u > 0$) and the second is negative, so $\partial I_E/\partial z < 0$. For $\psi > 0$, the equivalent condition (27) follows by setting Eq. (26) below zero and rearranging. Since the early opening probability $1 - \exp[-\lambda_0 e^{I_E} \Delta]$ is monotone increasing in I_E, the sign conclusions carry over to the opening probability. $\square$

A.8 Proof of Proposition 6

For a counting process with time-varying intensity $\lambda(t)$, the finite-horizon cumulative probability of the first event is given by Eq. (28). If $0 < \int_0^T \lambda(s)\, ds < \infty$, the exponential term lies strictly between 0 and 1, and so does the cumulative probability. Moreover, since $0 \leq \pi(\kappa) \leq 1$, the model-level attention-entry intensity of Section 4.3 satisfies $\lambda_A(t) = \lambda(t)\pi(\kappa_t) \leq \lambda(t)$, hence $\Lambda_A(T) \leq \Lambda(T) < \infty$ and

$$0 \leq \mathbb{P}(\tau_A \leq T) = 1 - e^{-\Lambda_A(T)} \leq \mathbb{P}(\tau_O \leq T) < 1,$$

with the left inequality strict if and only if $\Lambda_A(T) > 0$. In particular, the first attention entry can never be guaranteed within a finite horizon. $\square$

B Variables and parameters

Symbol	Meaning
$u(t)$	External token intensity; the external trigger that initiates entry behavior.

Symbol	Meaning
$z(t)$	Intensity of institutions that directly evaluate attention, quiet, or correct observation itself.
$\mathcal{E}(u)$	Task contact induced by tokens; a saturating function.
$q(t)$	Object-centered curiosity; the degree of wanting to see more of the object.
$v(t)$	Visibility of memory dynamics; the degree to which stereotyped responses, self-contradictions, and operator repetitions are visible.
$g(t)$	Acquisition–achievement pressure; pressure toward rewards, success, self-evaluation, and correct observation.
$\kappa(t)$	Sensitivity to the external object; the degree to which new information from reality enters the response.
c_t	Degree of functional closure from the preceding papers; no explicit dynamics are given here, and it enters only the measurement criterion (Eq. (30)).
τ_O	First-opening time: the time of the first opening episode of the counting process with state-dependent opening intensity $\lambda(t)$ (episodes may recur, but τ_O is restricted to the first; re-closure is not treated).
$\lambda(t)$	Candidate-opening intensity (the arrival intensity of opening episodes); the baseline value λ_0 is the spontaneous rate along routes outside the present design.
$p_\Delta(t)$	One-period opening probability at time t, $1-\exp\{-\int_t^{t+\Delta}\lambda\,ds\}$ ($\Delta=1$; the locally constant approximation is $1-e^{-\lambda(t)\Delta}$).
$\lambda_A(t)$	Model-level attention-entry intensity $\lambda(t)\pi(\kappa_t)$: the intensity of the thinned process N_A (Section 4.3).
τ_A	First model-level attention-entry time: the time of the first event of the thinned process N_A (Eq. (11); Section 4.3).
$\hat{\tau}_A$	Measured qualification time of the attention entry (Eq. (30)); a measurement proxy for the first event of the thinned process N_A (Sections 4.3 and 7.3).
a_q, ρ	Coefficients by which contact and visibility raise curiosity.
d_q, b_q	Natural decay of curiosity; inhibition by achievement pressure.
a_v, d_v	Formation and decay of visibility.
a_g, a_z, d_g	Conversion of tokens and the attention KPI into pressure; decay of pressure.
$a_\kappa, d_\kappa, b_\kappa$	Formation of object sensitivity, its natural decay, and its reduction by achievement pressure.
$\theta_q, \theta_v, \theta_\kappa, \theta_g$	Coefficients by which each state enters the opening intensity.
ψ	Degree to which attention evaluation also promotes entry behavior (contact); the extension of Section 5.4.
$\bar{c}, \bar{\kappa}$	Closure and object-sensitivity thresholds used in the measured entry qualification (Eq. (30)).

C Post-withdrawal trajectories under the baseline parameter set

All five policies of Section 6 are truncated at the end of the evaluation horizon ($u = z = 0$ for $t > 40$) and the mediating-state system is integrated to $t = 300$ (RK4, step 0.005; parameters and initial conditions as in Table 1). The last column reports $\Pr\{N(80)-N(40) \geq 1\}$, the probability of *at least one candidate-opening episode* during the first forty post-withdrawal periods, computed as $1-\exp\{-(\Lambda(80)-\Lambda(40))\}$ along each trajectory (valid by the independent-increments property of the conditional counting process).

Schedule	State at $t = 40$ (q, v, g, κ)	(q, v) at $t = 300$	$\lambda(300)$	Handoff	$\Pr\{N(80) - N(40) \geq 1\}$
No reward	$(0.000, 0.000, 0.000, 0.000)$	$(0.000, 0.000)$	0.030	—	0.699
Moderate fixed	$(0.555, 0.649, 1.000, 0.302)$	$(0.600, 0.667)$	1.019	yes	≈ 1.000
High fixed	$(0.385, 0.562, 2.500, 0.106)$	$(0.000, 0.000)$	0.030	**no**	0.749
Attention KPI	$(0.418, 0.582, 1.800, 0.151)$	$(0.600, 0.667)$	1.019	yes	≈ 1.000
Tapered	$(0.600, 0.667, 0.000, 0.706)$	$(0.600, 0.667)$	1.019	yes	≈ 1.000

A controlled sensitivity analysis fixes (q, v, κ) at each policy's withdrawal state and varies only the residual pressure, locating the critical value g^* by bisection (survival criterion: $q > 0.3$ after 300 reward-free periods from the tabulated withdrawal state):

Withdrawal state (q, v, κ) held fixed	Critical g^*	Actual g	Outcome
High fixed $(0.385, 0.562, 0.106)$	1.853	2.500	collapse
Attention KPI $(0.418, 0.582, 0.151)$	1.988	1.800	survival (narrow)
Moderate fixed $(0.555, 0.649, 0.302)$	2.435	1.000	survival

Notes. (i) Among the policy-generated withdrawal states examined here, the KPI state remains in the upper basin whereas the high-fixed state collapses. Because q and v also differ across states, that comparison does not by itself identify a scalar threshold in g; the controlled analysis above shows that the critical residual pressure g^* varies with the formed withdrawal state—in the present handoff calculation it is determined by the initial (q, v), and is higher for the more developed (q, v) states examined here—rather than being a scalar constant. (κ matters for the opening intensity but does not enter the q–v–g handoff dynamics; recomputing g^* with κ varied leaves the value unchanged.) The values are reported to three decimals; the fourth decimal varies with the classification horizon and with the rounding of the tabulated state (e.g., 1.9878–1.9894 for the KPI state across the variants examined), leaving the ordering and all conclusions unchanged. (ii) These post-withdrawal outcomes are the quantity relevant to the comparison with behavioral momentum theory: the richer reinforcement history (high fixed) yields *less* post-withdrawal persistence, the opposite sign to the momentum prediction. The within-horizon cumulative probabilities of Table 2 (e.g., 0.50 versus 0.70) concern maintained schedules and are a different quantity. (iii) The attention-KPI schedule illustrates that within-horizon harm and post-withdrawal persistence can dissociate: while in force it suppresses openings (Table 2), but by withdrawal the accumulated (q, v) has crossed the watershed, and its residual pressure $g = 1.8$ falls just below the critical value $g^* = 1.988$ for its state—a narrow survival. (iv) The transient rise in λ while pressure decays explains why even the collapsing high-fixed schedule records 0.749 over $(40, 80]$, slightly above the no-reward 0.699, before settling at the baseline intensity.

The accompanying script `reproduce_g1_numerics.py` regenerates all values in this appendix and in Section 6 (RK4 integration, trapezoidal cumulative intensity, withdrawal runs, and the bisection for g^*).

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