No. 2026-004

Verification as Disturbance in Accounting Measurement: Coherence Costs of Audit-Evidence Projection

May 7, 2026

Takanori Suzuki

Verification as Disturbance in Accounting Measurement: Coherence Costs of Audit-Evidence Projection

Takanori Suzuki*

May 7, 2026

Research Institute of Business Administration (Sanken), Waseda University
Working Paper Series, No.2026-004

Working paper note. This version is prepared for circulation in the Working Paper Series of the Research Institute of Business Administration (Sanken), Waseda University. A revised version may be submitted to a journal.

Abstract

Accounting verifiability is usually treated as a desirable attribute of a reported number. This paper develops a different, complementary view: imposing verification is an operation on the measurement map itself. When a measurement map W is made verifiable by projection onto an audit-evidence supported subspace, the map is transformed into $E_V W$, or under partial verification into $E_{V,\tau} W = (1-\tau)W + \tau E_V W$. This operation increases evidentiary support, but it can disturb dynamic coherence whenever the verification projection fails to commute with accounting dynamics. For a coherent candidate $T : R \to H_Y$ on the recirculation subspace, the paper proves the exact identities

$$G_{\mathrm{ver}}(T;\tau) = \tau \|(I - E_V)T\|_{\mathrm{HS}}, \qquad D_{\mathrm{coh}}(T;\tau) = \tau \|[\delta_Y, E_V]T\|_{\mathrm{HS}}.$$

It then defines the coherence price of verification, derives the verification–disturbance frontier, and solves the minimum-disturbance problem for a target verification gain g. The resulting formula,

$$D_E^*(g) = g \min_{T \in \mathcal{K}_g(E)} \frac{\|[\delta_Y, E_V]T\|_{\mathrm{HS}}}{\|(I - E_V)T\|_{\mathrm{HS}}},$$

shows that the cost of verification depends on the target gain, the audit-evidence partition, the wealth-side dynamics, and the recirculation channels exposed to verification. The paper applies the framework to fair value measurement, OCI recycling, impairment recognition, sustainability assurance, audit quality, and standards-setting. The central message is that verification is not free: making accounting numbers verifiable can change what is measured.

Keywords: accounting measurement; verifiability; audit evidence; coherence; disturbance; conditional expectation; fair value; OCI; impairment; sustainability assurance.
JEL classification: M40, M41, M42, M48.

*Faculty of Commerce, Waseda University. Email: suzuki@waseda.jp. ORCID: 0009-0008-8955-7003.

1 Introduction

1.1 The ordinary view of verifiability

Accounting verifiability is usually introduced as a desirable attribute of a reported number. A measurement supported by independent evidence appears safer, more auditable, and more reliable than a measurement that cannot be traced to contracts, confirmations, market prices, physical counts, or other institutional sources of evidence. This view is correct. Financial reporting would lose much of its disciplining force if reported amounts were not constrained by evidence that another party could inspect, reproduce, or challenge.

But the view is incomplete. In many accounting settings, making a measurement verifiable does not simply confirm a number that has already been produced. It changes the measurement that can be reported. Restricting a fair-value estimate to observable inputs, recognizing impairments only after evidentiary thresholds are met, recycling OCI items only upon realization, or redesigning sustainability metrics so that they become assurance-ready all improve evidentiary support by reshaping the measurement itself. The number becomes more verifiable, but not because a neutral verification label has been attached to an otherwise unchanged measurement. The evidence requirement has altered the admissible measurement map.

This paper studies that reshaping operation. We treat verifiability not only as a static attribute of an accounting measurement, but also as an operation on the measurement map. Formally, if W denotes a measurement map and E_V denotes the conditional expectation onto the audit-evidence supported subspace, then imposing full verification transforms

$$W \quad \longmapsto \quad E_V W. \tag{1}$$

Partial verification transforms W into

$$W \quad \longmapsto \quad E_{V,\tau} W, \qquad E_{V,\tau} = (1-\tau)I + \tau E_V, \qquad \tau \in [0,1]. \tag{2}$$

The question is therefore not only whether a measurement is verifiable. It is also what happens to the measurement when it is made verifiable.

The central claim of this paper is that verification has a coherence price. Verification increases evidentiary support, but when the verification projection fails to commute with accounting dynamics, it disturbs dynamic coherence. The disturbance is governed by the commutator

$$[\delta_Y, E_V] = \delta_Y E_V - E_V \delta_Y, \tag{3}$$

where δ_Y is the wealth-side differencing operator. In words, the commutator measures whether audit-evidence partitions are preserved by the dynamics of accounting measurement. If they are preserved, verification can be imposed without disturbing coherence. If they are not preserved, making a measurement more verifiable changes its dynamic content.

The argument is deliberately not introduced as a claim that verifiability has been misunderstood. Verifiability is a desirable attribute. The point is that the institutional act of making a measurement verifiable is also an operation. Once this operation is recognized, a new set of accounting questions becomes visible. How much verification gain is produced by imposing E_V? How much coherence disturbance is produced at the same time? Which measurement maps deliver verification cheaply in coherence terms? Which evidence partitions make verification expensive? And how should audit quality, standards-setting, and assurance be evaluated once verification is understood

as a transformation of measurement rather than merely as an attribute of the reported amount?

1.2 Examples that motivate the operation view

The operation view is easiest to see in familiar accounting settings.

First, fair value measurement is often described as relevant but difficult to verify. A common institutional response is to restrict measurement to observable inputs or to classify estimates by the quality of supporting evidence. Such restrictions increase evidentiary support, but they also change the measurement. A price process that is dynamically coherent from an economic perspective may be replaced by a measurement supported by a narrower evidence partition. The resulting number is more verifiable, but it may no longer track the same dynamics.

Second, impairment recognition is organized around evidentiary thresholds. Losses are recognized after particular conditions are satisfied; reversals may be restricted; timing is shaped by the verification structure. The threshold does not merely audit a previously defined loss. It determines which loss can be recognized and when. Making the write-down supportable may change the period in which economic deterioration is represented.

Third, OCI recycling routes items between stock-domain and flow-domain representations. Realization-based evidence may support recognition in profit or loss, but the act of waiting for realization changes how the accounting system represents the path of the item through time. P/L becomes more verifiable, yet wealth-side coherence may be rerouted through accumulated OCI.

Fourth, sustainability assurance illustrates the same logic in a contemporary setting. Metrics made assurance-ready are often easier to verify because they are standardized, observable, countable, or documentable. But assurance-readiness may also make them less dynamically informative. A metric designed around evidence availability may not be the metric that best tracks the underlying performance trajectory.

These examples are not applications added after the theory. They are the reason for the theory. They suggest that verification is an institutional operation that acts on measurement maps. The paper's formal apparatus is designed to make this operation visible.

1.3 The coherence price of verification

Let T be the restriction of a coherent measurement map to the recirculation subspace R. The two quantities that drive the paper are

$$v_E(T) := \|(I - E_V)T\|_{\mathrm{HS}}, \qquad d_E(T) := \|[\delta_Y, E_V]T\|_{\mathrm{HS}}. \tag{4}$$

The first quantity, $v_E(T)$, is the unverifiable component of the coherent measurement. It is the amount of verifiability defect that can be removed by imposing the projection E_V. The second quantity, $d_E(T)$, is the coherence disturbance generated by imposing verification. It is the size of the commutator exposure of the coherent measurement.

Under the soft verification operation $T \mapsto E_{V,\tau}T$, the paper shows that

$$G_{\mathrm{ver}}(T;\tau) = \tau v_E(T), \qquad D_{\mathrm{coh}}(T;\tau) = \tau d_E(T), \tag{5}$$

where G_{ver} is verification gain and D_{coh} is coherence disturbance. Thus each coherent candidate generates a line segment in the gain–disturbance plane. The ratio

$$\kappa_E(T) := \frac{d_E(T)}{v_E(T)} = \frac{\|[\delta_Y, E_V]T\|_{\mathrm{HS}}}{\|(I - E_V)T\|_{\mathrm{HS}}}, \tag{6}$$

whenever $v_E(T) > 0$, is the *coherence price of verification*. It is the coherence disturbance paid per unit of verification gain.

This price is the paper's central conceptual object. It changes the way verifiability is discussed. Instead of asking whether a measurement is verifiable, we ask how costly it is to make it verifiable. Instead of treating audit evidence as an unqualified improvement, we ask whether the evidence partition is dynamically aligned with accounting measurement. Instead of treating verification and coherence as two static desiderata, we study the institutional operation that increases one and disturbs the other.

1.4 Contributions

The paper makes four contributions.

First, it introduces verification-induced coherence disturbance as a concept in accounting measurement theory. The paper distinguishes the attribute view of verifiability from the operation view of verification. Verifiability can be assessed as a defect of a given measurement map. Verification, by contrast, can be imposed as an operation that transforms the map. This distinction is the basis for the entire analysis.

Second, it derives an exact identity for the disturbance generated by soft verification. If W_{coh} is coherent on the recirculation subspace and $T = W_{\mathrm{coh}}|_R$, then imposing partial verification yields

$$D_{\mathrm{coh}}(T;\tau) = \tau\|[\delta_Y, E_V]T\|_{\mathrm{HS}}. \tag{7}$$

Together with the verification-gain identity $G_{\mathrm{ver}}(T;\tau) = \tau\|(I - E_V)T\|_{\mathrm{HS}}$, this gives the exact gain–disturbance path generated by a coherent candidate. The commutator is not a metaphor. It is the algebraic object that measures how the verification operation interferes with accounting dynamics.

Third, it develops a measurement–disturbance frontier. For a compact set $\mathcal{K}$ of coherent candidate measurements, the attainable gain–disturbance set is

$$\mathcal{F}_E = \bigcup_{T\in\mathcal{K}} \{\tau(v_E(T), d_E(T)) : \tau \in [0,1]\}. \tag{8}$$

The set is generally star-shaped rather than convex. If a target verification gain g is imposed, the minimum disturbance is

$$D_E^*(g) = g \min_{T\in\mathcal{K}_g(E)} \frac{d_E(T)}{v_E(T)}, \qquad \mathcal{K}_g(E) := \{T \in \mathcal{K} : v_E(T) \geq g\}. \tag{9}$$

This formula turns the abstract statement that "verification is costly" into a comparative-statical object: the cost depends on the target gain, the candidate set, the verification regime, and the commutator exposure of the measurement.

Fourth, it translates the frontier into accounting and auditing implications. Fair value measurement, OCI recycling, impairment recognition, sustainability assurance, audit quality, and standards-setting can all be read as choices on, or movements of, the verification–disturbance frontier. The paper does not use these applications to motivate a single institutional claim. It uses them to show that the operation view makes several otherwise separate accounting phenomena visible through the same conceptual lens.

1.5 Scope and positioning

The paper is self-contained. The commutator identity developed below is derived within the present model rather than imported as an external theorem. The argument does,

however, belong to a broader research program on coherence, verifiability, and measurement design. The present paper focuses on one branch of that program: structural disturbance caused by imposing verification on a coherent measurement. It does not analyze strategic disturbance, in which firms change the accounting information they produce in anticipation of monitoring, and it does not endogenize the standards-setter's choice of the verification regime. Those issues are natural extensions, but they are not required for the claims made here.

The paper is also not a quantum-accounting paper. The formal structure is reminiscent of measurement-disturbance ideas in noncommutative measurement systems, but the justification is accounting-theoretic: audit-evidence partitions, admissible measurement maps, positivity and constant preservation, and the dynamic coherence of recirculating accounting readings. The point is not to borrow a physical metaphor. The point is to show that, in accounting, making a measurement verifiable can be an operation that changes the measurement map.

1.6 Plan of the paper

Section 2 situates the paper in the accounting measurement, verifiability, audit-evidence, and information-structure literatures. Section 3 develops the model. Section 4 proves the exact disturbance identity for soft verification. Section 5 derives the measurement–disturbance frontier and the minimum-disturbance formula for a target verification gain. Section 6 studies comparative statics of the coherence price of verification. Section 7 translates the theory into accounting and audit implications. Section 8 concludes.

2 Background and Related Literature

This section positions the paper relative to four conversations. The first is the accounting literature on verifiability, reliability, and audit evidence. The second is the measurement-theoretic literature on coherence. The third is the information-structure literature in which verification is represented by partitions or σ-algebras. The fourth is the emerging operation view of verification developed in this paper: verification is not only a property of an output, but an institutional transformation of a measurement map.

2.1 From reliability to verifiability

Verifiability occupies an unusual position in accounting theory. It is not identical to truth, relevance, reliability, faithful representation, or auditability, but it is connected to all of them. A verifiable measure is one for which independent parties can obtain sufficiently similar results using the same measurement basis, or for which a reported amount can be supported by evidence external to the preparer. In practice, this includes market prices, transaction documents, contractual terms, confirmations, physical counts, valuation inputs, and audit procedures.

In traditional debates, verifiability is often treated as a desirable constraint on measurement. Historical cost is more verifiable but may be less timely or less relevant. Fair value may be more relevant but less verifiable when inputs are not directly observable. Conservative recognition rules may increase evidentiary support but delay recognition. The recurring theme is a tradeoff between an information objective and an evidence constraint.

The present paper does not reject that framing. It changes the level at which the tradeoff is analyzed. The traditional framing asks whether a measurement outcome is

verifiable. This paper asks what operation is performed on the measurement map when verification is imposed. In many accounting settings, evidence requirements do not merely screen already-defined measurements; they define the subset of measurements that can be reported. When a fair-value estimate is restricted to observable inputs, the reported value is not simply the old estimate plus an audit label. It is a different measurement. When impairment recognition waits for evidentiary thresholds, the timing of the measurement changes. When sustainability metrics are redesigned for assurance, the measurement object changes.

This operation view gives a sharper account of the cost of verifiability. The cost is not simply lower relevance, lower timeliness, or lower informativeness in the abstract. It is a coherence disturbance: a failure of the verified measurement to commute with the relevant accounting dynamics.

2.2 Coherence in accounting measurement

The coherence concept used in this paper has roots in the axiomatic measurement tradition. Mattessich (1957, 1964, 1995) treated accounting as a measurement system with algebraic structure. Chambers (1966) emphasized contemporary equivalence and the consistency of measurement across the accounting cycle. In this tradition, coherence is a structural requirement: the measurement system should preserve the relationships that define the accounting object.

The present paper uses a dynamic version of coherence. A measurement map W is coherent on the recirculation subspace R if measuring and then differencing gives the same result as differencing and then measuring:

$$(\delta_Y W - W\delta_Z)P_R = 0. \tag{10}$$

Equivalently, for the restriction $T = W|_R$,

$$\delta_Y T = T\delta_R. \tag{11}$$

Thus coherence is an intertwining relation between the state-side dynamics and the wealth-side dynamics.

The departure from the classical measurement-theoretic tradition is that this paper does not treat coherence as an all-or-nothing admissibility criterion. Coherence is a structural target that can be disturbed by institutional verification. Its failure is not simply a defect to be ruled out. It is a quantity to be measured, priced, and compared. This is what allows the paper to define the coherence price of verification.

2.3 Verification as information structure

The information-structure literature provides the natural formal language for verification. Demski (1973, 1980), Christensen and Demski (2003), and Christensen and Feltham (2003) represent accounting information through partitions, information sets, or σ-algebras. In that framework, verifiable information is information measurable with respect to the available evidence structure.

This paper follows that tradition but uses its functional-analytic counterpart. Let $\mathcal{G}$ be an audit-evidence σ-algebra on the wealth-side reading space. The verification-supported subspace is

$$M_V = L^2(Y, \mathcal{G}, \nu|_{\mathcal{G}}) \subseteq H_Y. \tag{12}$$

The canonical projection onto M_V is the conditional expectation

$$E_V = \mathbb{E}[\,\cdot \mid \mathcal{G}\,]. \tag{13}$$

This modeling choice is substantive. An arbitrary orthogonal projection onto a closed subspace of H_Y need not preserve positivity or constants. Conditional expectation does. Since admissible accounting measurement maps must preserve nonnegativity and the unit constant, E_V is not merely a convenient projection; it is the accounting-admissible way to represent verification as information coarsening.

The point will matter again in the main results. If verification is imposed by $W \mapsto E_V W$, then $E_V W$ remains admissible when W is admissible. This would not be true for a generic projection. Thus the model cannot be cleanly separated into an abstract projection theorem plus a later accounting application. The projection used by the theory is already an accounting object: an audit-evidence conditional expectation.

2.4 From verifiability as attribute to verification as operation

The three literatures just reviewed largely treat verifiability as an attribute of a measurement or as a constraint imposed on admissible measurements. This paper shifts the focus to the operation by which the constraint is imposed. Given a measurement map W, one may first ask how far W is from being verifiable. That is the attribute view. But once an institution requires verification, the map itself is transformed into $E_V W$, or under partial verification into $E_{V,\tau} W$. That is the operation view.

This shift matters because the verification operation need not commute with accounting dynamics. If W is dynamically coherent before verification is imposed, $E_V W$ need not remain coherent afterwards. The coherence disturbance is governed by the commutator $[\delta_Y, E_V]$. The paper's contribution is to turn this observation into a gain–disturbance theory: how much verifiability is gained, how much coherence is disturbed, and how the resulting price depends on evidence partitions, dynamics, and recirculation channels.

The construction is accounting-specific. The projection E_V is not an arbitrary orthogonal projection onto a closed subspace of H_Y. It is the conditional expectation induced by an audit-evidence σ-algebra, and therefore preserves positivity and constants, as required for admissible accounting measurement. The domain on which disturbance is evaluated is not arbitrary either; it is the recirculation subspace R, where accrual reversals, deferred items, OCI-type recyclables, and other cycling items move between stock-domain and flow-domain representation. Thus the commutator is not merely a formal operator expression. It measures the failure of audit-evidence partitions to be preserved by wealth-side accounting dynamics.

2.5 Why the paper is not application-first

The paper is not organized around a single accounting application. It does not begin with fair value, OCI recycling, impairment, or sustainability assurance as the main phenomenon to be explained. Those applications are important, but they are not the primitive. The primitive is the operation view of verification.

This is a concept-first paper. Its purpose is to show that making a measurement verifiable is itself a measurement operation and that this operation can disturb dynamic coherence. Once that concept is in place, many accounting phenomena can be seen in a common language. Fair value restrictions, impairment thresholds, OCI realization rules, and assurance-ready metrics all become instances of the same general structure: evidence support is increased by transforming the measurement.

This does not make the paper abstract in a way that is detached from accounting. On the contrary, the concept is motivated by accounting practice and formalized using accounting-specific restrictions: audit-evidence σ-algebras, conditional expectations, positivity, constant preservation, recirculation subspaces, and accounting dynamics. The applications in Section 7 therefore function as translations rather than as afterthoughts. They show what becomes visible once the concept has been introduced.

3 Model

This section develops the formal framework. We define the spaces, measurement maps, verification projections, soft verification operators, and the two quantities whose relationship drives the paper: verification gain and coherence disturbance. The model is designed to be self-contained; the only objects used in the main results are introduced below.

3.1 Accounting state space and wealth-side reading space

Let $(Z, \mathcal{F}_Z, \mu)$ be a probability space. A point $z \in Z$ represents a complete configuration of accounting-relevant positions at an instant in the firm's measurement cycle. Let

$$H_Z := L^2(Z, \mathcal{F}_Z, \mu) \tag{14}$$

be the Hilbert space of square-integrable state readings. A state reading is an element $f \in H_Z$, identified up to null sets.

Let $(Y, \mathcal{F}_Y, \nu)$ be a probability space for wealth-side or reported-reading quantities, and let

$$H_Y := L^2(Y, \mathcal{F}_Y, \nu). \tag{15}$$

We assume probability normalization so that $1_Z \in H_Z$ and $1_Y \in H_Y$.

Let

$$\delta_Z : H_Z \to H_Z, \qquad \delta_Y : H_Y \to H_Y \tag{16}$$

be bounded linear differencing operators. The state-side operator δ_Z represents period-to-period change in state readings. The wealth-side operator δ_Y represents period-to-period change in reported or wealth-side readings. We keep the two operators notationally distinct because they act on different spaces; all commutators below are defined only after the relevant compositions have matching domains and codomains.

3.2 The recirculation subspace

The recirculation subspace

$$R \subseteq H_Z \tag{17}$$

contains those accounting readings whose values cycle between stock-domain and flow-domain representation. Typical directions include accrual reversals, deferred revenues and expenses, inventory cost flows, pension and hedge-related accumulations, and OCI items that may later be recycled into profit or loss, in the spirit of the recirculation logic introduced by Ijiri (1986).

Assumption 3.1 (Finite-dimensional recirculation)**.** *R is a closed finite-dimensional subspace of H_Z, with $\dim R = r < \infty$.*

Assumption 3.2 (Invariance)**.** *R is invariant under δ_Z: $\delta_Z(R) \subseteq R$.*

Assumption 3.2 allows us to define

$$\delta_R := \delta_Z|_R : R \to R. \tag{18}$$

Let $P_R : H_Z \to R$ denote the orthogonal projection onto R.

3.3 Measurement maps and admissibility

A measurement map is a bounded linear operator

$$W : H_Z \to H_Y. \tag{19}$$

The paper focuses on maps that satisfy the basic admissibility requirements for accounting measurement.

Definition 3.3 (Admissible measurement map)**.** A measurement map $W : H_Z \to H_Y$ is admissible if it satisfies:

(W1) linearity;

(W2) boundedness;

(W3) positivity preservation: $f \geq 0$ implies $Wf \geq 0$;

(W4) constant preservation: $W1_Z = 1_Y$.

The class of admissible measurement maps is denoted by $\mathcal{W}$.

The positivity and constant-preservation conditions matter because they prevent the formal measurement map from ceasing to be an accounting measurement. A map that sends nonnegative claims to negative reported quantities, or a map that fails to preserve the unit reading, may be mathematically legitimate but is not admissible for the accounting interpretation used here.

Trivial maps must be excluded when optimization over measurement maps is considered. For the present paper, the main candidate set will be defined directly on restrictions $T : R \to H_Y$, but it is useful to retain the full-space normalization.

Definition 3.4 (Non-degenerate admissible class)**.** For fixed $c_0 > 0$, define

$$\mathcal{W}_0 := \{W \in \mathcal{W} : \|WP_R\|_{\mathrm{HS}} = 1, \sigma_{\min}(W|_R) \geq c_0\}. \tag{20}$$

We assume $0 < c_0 \leq r^{-1/2}$ and $\mathcal{W}_0 \neq \varnothing$.

The Hilbert–Schmidt normalization fixes the scale of measurement on R. The singular-value condition prevents W from suppressing any recirculation direction. Without such a condition, one could reduce both verification and coherence defects by making the measurement map nearly zero on the relevant subspace.

3.4 Verification as an audit-evidence projection

Let $\mathcal{G} \subseteq \mathcal{F}_Y$ be an audit-evidence σ-algebra. It represents the distinctions that the institutional environment can support through independent evidence. The verification-supported subspace is

$$M_V := L^2(Y, \mathcal{G}, \nu|_{\mathcal{G}}) \subseteq H_Y. \tag{21}$$

The verification projection is the conditional expectation

$$E_V := \mathbb{E}[\,\cdot \mid \mathcal{G}\,] : H_Y \to M_V. \tag{22}$$

It is an orthogonal projection in L^2, satisfies $E_V^2 = E_V$, preserves positivity, and preserves constants.

The use of conditional expectation is central. If $M \subseteq H_Y$ is an arbitrary closed subspace, the orthogonal projection onto M need not preserve positivity. It may therefore transform an admissible measurement map into a non-admissible one. By contrast, if $W \in \mathcal{W}$, then $E_V W$ remains positive and constant-preserving. Thus E_V represents not just mathematical projection but accounting-admissible verification.

3.5 Coherence and verifiability as defects

For any bounded measurement map $W : H_Z \to H_Y$, define the coherence defect on R by

$$\varepsilon_{\mathrm{coh}}(W) := \|(\delta_Y W - W\delta_Z)P_R\|_{\mathrm{HS}}. \tag{23}$$

The map is coherent on R if this defect is zero. Equivalently, if $T = W|_R$, then

$$\delta_Y T = T\delta_R. \tag{24}$$

Define the verifiability defect by

$$\varepsilon_{\mathrm{ver}}(W) := \|(I - E_V)WP_R\|_{\mathrm{HS}}. \tag{25}$$

This is the part of the measured recirculation reading lying outside the verification-supported subspace.

Because R is finite-dimensional, both operators inside the Hilbert–Schmidt norms are finite-rank whenever W is bounded. The defects are therefore well-defined for every bounded W, not only for $W \in \mathcal{W}_0$.

3.6 Verification as an operation

The previous subsection treats verifiability as a defect of a given W. We now turn to verification as an operation on W.

For $\tau \in [0, 1]$, define the soft verification operator

$$E_{V,\tau} := (1 - \tau)I + \tau E_V. \tag{26}$$

Given a measurement map W, its partially verified version is

$$W_\tau := E_{V,\tau} W. \tag{27}$$

The endpoints are $W_0 = W$ and $W_1 = E_V W$. The parameter τ measures the intensity with which the institution imposes the verification-supported component.

This operation is the formal counterpart of making a measurement verifiable. It does not merely evaluate W; it changes W. The main results of Sections 4 and 5 ask how the two defects in (23) and (25) change under (27).

3.7 Coherent candidate class

The gain–disturbance analysis focuses on coherent candidates and asks what verification does to them. Let

$$\mathcal{H}_{\mathrm{op}} := \mathcal{L}(R, H_Y) \tag{28}$$

be the Hilbert space of bounded linear maps from R to H_Y, endowed with the Hilbert–Schmidt inner product. Since R is finite-dimensional, every such map is Hilbert–Schmidt.

Define the coherent subspace

$$\mathcal{C} := \{T \in \mathcal{H}_{\mathrm{op}} : \delta_Y T = T\delta_R\}. \tag{29}$$

The paper takes as given a compact set of coherent candidates

$$\mathcal{K} \subseteq \mathcal{C} \tag{30}$$

that are Hilbert–Schmidt normalized and non-degenerate. One may think of $\mathcal{K}$ as the set of feasible coherent restrictions $T = W|_R$ generated by admissible full-space maps, after scale and rank degeneracy have been ruled out. The compactness assumption is a technical device that ensures existence of minimizers in the target-gain problem.

Assumption 3.5 (Compact coherent candidate set)**.** *$\mathcal{K} \subseteq \mathcal{C}$ is nonempty and compact in the Hilbert–Schmidt topology. For every $T \in \mathcal{K}$, $\|T\|_{\mathrm{HS}} = 1$ and T satisfies the same non-degeneracy threshold imposed on admissible restrictions.*

This assumption is intentionally stated on restrictions $T : R \to H_Y$. The paper's disturbance frontier is an operator-space object. Full positive unital extensions matter for accounting admissibility, but the main frontier analysis depends only on the behavior of measurement maps on the recirculation subspace.

3.8 Verification gain and coherence disturbance

For $T \in \mathcal{K}$, define

$$v_E(T) := \|(I - E_V)T\|_{\mathrm{HS}}, \qquad d_E(T) := \|[\delta_Y, E_V]T\|_{\mathrm{HS}}. \tag{31}$$

These quantities are the building blocks of the theory. The first is the initial verifiability defect of the coherent candidate. The second is the commutator exposure of that candidate.

For a soft verification intensity τ, define the verification gain by

$$G_{\mathrm{ver}}(T;\tau) := \varepsilon_{\mathrm{ver}}(T) - \varepsilon_{\mathrm{ver}}(E_{V,\tau}T), \tag{32}$$

where T is identified with a map $R \to H_Y$. Define the coherence disturbance by

$$D_{\mathrm{coh}}(T;\tau) := \varepsilon_{\mathrm{coh}}(E_{V,\tau}T) - \varepsilon_{\mathrm{coh}}(T). \tag{33}$$

Since $T \in \mathcal{K} \subseteq \mathcal{C}$, $\varepsilon_{\mathrm{coh}}(T) = 0$. The exact identity proved in Section 4 will show that

$$G_{\mathrm{ver}}(T;\tau) = \tau v_E(T), \qquad D_{\mathrm{coh}}(T;\tau) = \tau d_E(T). \tag{34}$$

Thus the model reduces the verification operation to a two-dimensional path in gain–disturbance space.

The ratio

$$\kappa_E(T) := \frac{d_E(T)}{v_E(T)} \tag{35}$$

when $v_E(T) > 0$ is the coherence price of verification. It is the central object for the comparative statics and applications that follow.

3.9 Roadmap to the main results

The next two sections establish the technical core of the paper. Section 4 proves that soft verification removes unverifiable components linearly in τ and that, for coherent candidates, it creates coherence disturbance equal to $\tau\|[\delta_Y, E_V]T\|_{\mathrm{HS}}$. Section 5 uses these identities to define the measurement–disturbance frontier and to solve the minimum-disturbance problem for a target verification gain.

The remaining sections then ask what the frontier means for accounting. Section 6 studies how the coherence price of verification changes with the target gain, the verification intensity, the evidence partition, the wealth-side dynamics, and the recirculation domain. Section 7 translates the results into fair value measurement, OCI recycling, impairment recognition, sustainability assurance, audit quality, and standards-setting.

4 Verification-Induced Coherence Disturbance

This section establishes the paper's first main identity. The preceding section introduced verification as an operation on measurement maps rather than merely as a static property of a reported number. If E_V denotes the conditional expectation onto the audit-evidence σ-algebra, then imposing verification on a measurement map amounts to replacing W by $E_V W$. This operation improves verifiability by construction, but it need not preserve coherence. The purpose of this section is to measure exactly how much coherence is lost when a coherent measurement is partially or fully projected onto the verification-supported subspace.

The result is a soft-verification commutator identity. We do not focus only on the single extreme $\tau = 1$. Instead, we introduce a continuum of verification intensities $\tau \in [0, 1]$ and trace the gain–disturbance path generated by moving from an unconstrained coherent measurement to a fully verifiable one.

4.1 Soft verification

Let H_Z and H_Y be Hilbert spaces, let $R \subset H_Z$ be a finite-dimensional recirculation subspace, and let $P_R : H_Z \to R$ be the orthogonal projection. Let

$$\delta_Z : H_Z \to H_Z, \qquad \delta_Y : H_Y \to H_Y$$

be bounded differencing operators, and assume that R is δ_Z-invariant. We write

$$\delta_R := \delta_Z|_R : R \to R.$$

Let $E_V : H_Y \to M_V \subseteq H_Y$ be the verification projection. Substantively, E_V is not an arbitrary orthogonal projection onto a closed subspace of H_Y; it is the conditional expectation induced by an audit-evidence σ-algebra. This distinction is essential for the accounting interpretation: conditional expectation preserves positivity and constants, whereas an arbitrary orthogonal projection need not.

For $\tau \in [0, 1]$, define the *soft verification operator*

$$E_{V,\tau} := (1 - \tau)I_{H_Y} + \tau E_V = I_{H_Y} - \tau(I_{H_Y} - E_V). \tag{36}$$

The endpoints have direct interpretations. When $\tau = 0$, no verification is imposed and $E_{V,\tau} = I_{H_Y}$. When $\tau = 1$, verification is imposed fully and $E_{V,\tau} = E_V$. Intermediate values represent partial institutional enforcement of the verification-supported component.

For any bounded measurement map $W : H_Z \to H_Y$, define

$$\varepsilon_{\mathrm{coh}}(W) := \|(\delta_Y W - W\delta_Z)P_R\|_{\mathrm{HS}}, \tag{37}$$

$$\varepsilon_{\mathrm{ver}}(W) := \|(I_{H_Y} - E_V)WP_R\|_{\mathrm{HS}}. \tag{38}$$

Because R is finite-dimensional, the compositions in (37) and (38) are finite-rank whenever W is bounded, so the Hilbert–Schmidt norms are well defined.

Definition 4.1 (Coherent candidate)**.** A bounded measurement map $W_{\mathrm{coh}} : H_Z \to H_Y$ is called *coherent on* R if

$$(\delta_Y W_{\mathrm{coh}} - W_{\mathrm{coh}}\delta_Z)P_R = 0. \tag{39}$$

Equivalently, writing $T = W_{\mathrm{coh}}|_R$, coherence on R means

$$\delta_Y T = T\delta_R.$$

For a coherent candidate W_{coh}, the partially verified measurement is

$$W_\tau := E_{V,\tau}W_{\mathrm{coh}}. \tag{40}$$

The central question is: how much coherence defect is generated by replacing W_{coh} with W_τ?

4.2 Verification gain

We first record the effect of soft verification on the verifiability defect. This identity holds for any bounded measurement map, not only for coherent ones.

Lemma 4.2 (Verification gain identity)**.** *For every bounded measurement map $W : H_Z \to H_Y$ and every $\tau \in [0, 1]$,*

$$\varepsilon_{\mathrm{ver}}(E_{V,\tau}W) = (1 - \tau)\varepsilon_{\mathrm{ver}}(W). \tag{41}$$

Consequently, if verification gain is defined by

$$G_{\mathrm{ver}}(W; \tau) := \varepsilon_{\mathrm{ver}}(W) - \varepsilon_{\mathrm{ver}}(E_{V,\tau}W), \tag{42}$$

then

$$G_{\mathrm{ver}}(W; \tau) = \tau\varepsilon_{\mathrm{ver}}(W). \tag{43}$$

Proof. Using $E_V^2 = E_V$,

$$(I_{H_Y} - E_V)E_{V,\tau} = (I_{H_Y} - E_V)((1-\tau)I_{H_Y} + \tau E_V) = (1-\tau)(I_{H_Y} - E_V) + \tau(I_{H_Y} - E_V)E_V.$$

The final term vanishes because $(I_{H_Y} - E_V)E_V = E_V - E_V^2 = 0$. Hence

$$(I_{H_Y} - E_V)E_{V,\tau} = (1 - \tau)(I_{H_Y} - E_V).$$

Multiplying on the right by WP_R and taking Hilbert–Schmidt norms gives

$$\varepsilon_{\mathrm{ver}}(E_{V,\tau}W) = \|(I_{H_Y} - E_V)E_{V,\tau}WP_R\|_{\mathrm{HS}} = (1 - \tau)\|(I_{H_Y} - E_V)WP_R\|_{\mathrm{HS}},$$

which is (41). Equation (43) follows immediately from the definition of G_{ver}. □

The lemma says that soft verification removes a fraction τ of the unverifiable component. This is a useful normalization: the parameter τ is not merely a formal interpolation but has a direct interpretation as the proportion of the original verifiability defect eliminated by the institutional projection.

4.3 A general decomposition of coherence defect

Before turning to coherent candidates, it is useful to state the exact decomposition for an arbitrary W. The decomposition separates the pre-existing coherence defect of W from the additional defect created by the verification operation.

Lemma 4.3 (Coherence-defect decomposition)**.** *For every bounded measurement map $W : H_Z \to H_Y$ and every $\tau \in [0,1]$,*

$$(\delta_Y E_{V,\tau} W - E_{V,\tau} W \delta_Z) P_R = E_{V,\tau}(\delta_Y W - W \delta_Z) P_R + \tau [\delta_Y, E_V] W P_R. \tag{44}$$

Proof. Add and subtract $E_{V,\tau}\delta_Y W P_R$:

$$\begin{aligned}(\delta_Y E_{V,\tau} W - E_{V,\tau} W \delta_Z) P_R &= \delta_Y E_{V,\tau} W P_R - E_{V,\tau} W \delta_Z P_R \\ &= E_{V,\tau}(\delta_Y W - W \delta_Z) P_R + (\delta_Y E_{V,\tau} - E_{V,\tau}\delta_Y) W P_R.\end{aligned}$$

Since $E_{V,\tau} = (1-\tau) I_{H_Y} + \tau E_V$,

$$[\delta_Y, E_{V,\tau}] = \delta_Y E_{V,\tau} - E_{V,\tau}\delta_Y = \tau(\delta_Y E_V - E_V \delta_Y) = \tau [\delta_Y, E_V].$$

Substitution gives (44). □

For a general W, the two terms in (44) may reinforce, partially offset, or interact through the norm. Thus there is no universal exact identity for the norm of $\varepsilon_{\mathrm{coh}}(E_{V,\tau} W)$ in terms of the two separate norms. Exactness emerges when the initial measurement is coherent.

4.4 The exact disturbance identity

Theorem 4.4 (Exact coherence disturbance under soft verification)**.** *Let $W_{\mathrm{coh}} : H_Z \to H_Y$ be coherent on R. For $\tau \in [0,1]$, let $W_\tau = E_{V,\tau} W_{\mathrm{coh}}$. Then*

$$\varepsilon_{\mathrm{coh}}(W_\tau) = \tau \| [\delta_Y, E_V] W_{\mathrm{coh}} P_R \|_{\mathrm{HS}}. \tag{45}$$

Equivalently, if coherence disturbance is defined by

$$D_{\mathrm{coh}}(W_{\mathrm{coh}}; \tau) := \varepsilon_{\mathrm{coh}}(E_{V,\tau} W_{\mathrm{coh}}) - \varepsilon_{\mathrm{coh}}(W_{\mathrm{coh}}), \tag{46}$$

then, because $\varepsilon_{\mathrm{coh}}(W_{\mathrm{coh}}) = 0$,

$$D_{\mathrm{coh}}(W_{\mathrm{coh}}; \tau) = \tau \| [\delta_Y, E_V] W_{\mathrm{coh}} P_R \|_{\mathrm{HS}}. \tag{47}$$

In the squared-loss version,

$$\varepsilon_{\mathrm{coh}}(W_\tau)^2 = \tau^2 \| [\delta_Y, E_V] W_{\mathrm{coh}} P_R \|_{\mathrm{HS}}^2. \tag{48}$$

Proof. Apply Lemma 4.3 with $W = W_{\mathrm{coh}}$. By coherence on R,

$$(\delta_Y W_{\mathrm{coh}} - W_{\mathrm{coh}} \delta_Z) P_R = 0.$$

Therefore the first term on the right-hand side of (44) vanishes, and

$$(\delta_Y E_{V,\tau} W_{\mathrm{coh}} - E_{V,\tau} W_{\mathrm{coh}} \delta_Z) P_R = \tau [\delta_Y, E_V] W_{\mathrm{coh}} P_R.$$

Taking Hilbert–Schmidt norms gives (45). The disturbance formula (47) follows because the original map is coherent. Squaring (45) gives (48). □

Theorem 4.4 is the structural core of the verification-as-disturbance view. Verification does not merely remove unsupported components of the measurement; it actively changes the relationship between measurement and accounting dynamics. The amount of induced coherence defect is governed by the commutator between the wealth-side dynamics and the verification projection, evaluated on the coherent measurement's recirculation range.

4.5 Immediate consequences

Corollary 4.5 (Zero-disturbance condition)**.** *For a coherent candidate* W_{coh} *and* $\tau \in [0,1]$*,*

$$D_{\mathrm{coh}}(W_{\mathrm{coh}};\tau) = 0$$

if and only if either $\tau = 0$ *or*

$$[\delta_Y, E_V]W_{\mathrm{coh}}P_R = 0. \tag{49}$$

In particular, $[\delta_Y, E_V] \neq 0$ *as a global operator does not imply positive disturbance for every coherent candidate; the commutator must act nontrivially on the range of* $W_{\mathrm{coh}}P_R$*.*

Proof. This is immediate from (47) and non-negativity of the Hilbert–Schmidt norm. □

Corollary 4.6 (Disturbance price of verification for a fixed coherent map)**.** *Suppose* W_{coh} *is coherent on* R *and* $\varepsilon_{\mathrm{ver}}(W_{\mathrm{coh}}) > 0$*. Then, for every* $\tau \in (0,1]$*,*

$$\frac{D_{\mathrm{coh}}(W_{\mathrm{coh}};\tau)}{G_{\mathrm{ver}}(W_{\mathrm{coh}};\tau)} = \frac{\|[\delta_Y, E_V]W_{\mathrm{coh}}P_R\|_{\mathrm{HS}}}{\varepsilon_{\mathrm{ver}}(W_{\mathrm{coh}})}. \tag{50}$$

The ratio is independent of the verification intensity τ*.*

Proof. By Lemma 4.2,

$$G_{\mathrm{ver}}(W_{\mathrm{coh}};\tau) = \tau\varepsilon_{\mathrm{ver}}(W_{\mathrm{coh}}).$$

By Theorem 4.4,

$$D_{\mathrm{coh}}(W_{\mathrm{coh}};\tau) = \tau\|[\delta_Y, E_V]W_{\mathrm{coh}}P_R\|_{\mathrm{HS}}.$$

Dividing the second expression by the first gives (50). □

The ratio in (50) is the *disturbance price of verification* for the candidate W_{coh}: it measures how much coherence disturbance must be paid for one unit of verification gain along the soft-verification path generated by that candidate. The next section studies the lower envelope of these prices across admissible coherent candidates.

4.6 Accounting interpretation

The force of Theorem 4.4 is accounting-specific. The projection E_V is not an arbitrary orthogonal projection onto a closed subspace of H_Y. It is the conditional expectation induced by an audit-evidence σ-algebra and therefore preserves positivity and constants, as required for admissible accounting measurement. Likewise, the defect is not evaluated on an arbitrary domain but on the recirculation subspace R, where accrual reversals, deferred items, and recyclable components cycle between stock-domain and flow-domain representation. Thus the commutator $[\delta_Y, E_V]$ is interpreted not merely as a formal operator expression but as the failure of audit-evidence partitions to be preserved by wealth-side accounting dynamics.

Three accounting implications follow. First, verification has a coherence cost whenever audit evidence partitions are not stable under accounting dynamics. Second, the relevant cost is not only a property of the verification regime in isolation; it is the commutator of the regime with wealth-side differencing, evaluated on the recirculation directions actually measured by W_{coh}. Third, soft verification makes the tradeoff continuous: an institution can impose partial verification, thereby reducing unverifiable components while accepting a proportional coherence disturbance.

5 The Measurement–Disturbance Frontier

The previous section showed that, for each coherent candidate, soft verification generates a linear path in verification gain and coherence disturbance. This section aggregates those paths across a class of coherent candidates and characterizes the minimum coherence disturbance required to achieve a target level of verification gain.

The key message is deliberately modest. Without additional institutional assumptions, the attainable gain–disturbance set need not be convex. What follows directly from the exact identity is a star-shaped representation: each coherent candidate generates a ray from the origin, and the frontier is the lower envelope of those rays subject to the candidate's maximum verification capacity. Convexity emerges only if the institution permits randomized or convexified measurement designs and evaluates outcomes ex ante.

5.1 Candidate set and gain–disturbance coordinates

The frontier analysis is conducted on a class of coherent candidates. Since the disturbance identities depend only on restrictions to R, it is convenient to formulate the candidate set in the finite-dimensional operator space.

Assumption 5.1 (Compact coherent candidate class)**.** *Let $\mathcal{H}_{\mathrm{op}} = \mathcal{L}(R, M_f)$ be a finite-dimensional Hilbert–Schmidt operator space, where $M_f \subseteq H_Y$ is a finite-dimensional subspace containing the relevant verification-supported subspace. Let $\mathcal{K} \subseteq \mathcal{H}_{\mathrm{op}}$ be a nonempty compact set of coherent candidates such that*

$$\mathcal{K} \subseteq \{T \in \mathcal{H}_{\mathrm{op}} : \delta_Y T - T\delta_R = 0,\ \|T\|_{\mathrm{HS}} = 1,\ \sigma_{\min}(T) \geq c_0\}\,. \tag{51}$$

Each $T \in \mathcal{K}$ is assumed to admit at least one admissible positive unital extension $W_T : H_Z \to H_Y$ with $W_T|_R = T$.

The compactness assumption is not conceptually restrictive in the finite-dimensional setting: it records the normalization and non-degeneracy restrictions that prevent the trivial measurement from eliminating both gain and disturbance. The positive-unital-extension clause keeps the operator-space candidate set tied to admissible accounting measurement, rather than treating arbitrary Hilbert-space operators as accounting maps.

For $T \in \mathcal{K}$, define the candidate's *verification capacity*

$$v(T) := \|(I_{H_Y} - E_V)T\|_{\mathrm{HS}}, \tag{52}$$

and its *full-verification coherence disturbance*

$$d(T) := \|[\delta_Y, E_V]T\|_{\mathrm{HS}}. \tag{53}$$

For the soft verification intensity $\tau \in [0,1]$, the gain–disturbance pair generated by T is

$$p(T,\tau) := (G(T,\tau), D(T,\tau)) := (\tau v(T), \tau d(T)). \tag{54}$$

Here $G(T,\tau)$ is verification gain and $D(T,\tau)$ is coherence disturbance. Formula (54) is just Lemma 4.2 and Theorem 4.4 rewritten in the coordinates of the candidate set.

Definition 5.2 (Attainable gain–disturbance set)**.** The attainable gain–disturbance set is

$$\mathcal{F} := \left\{(g,d) \in \mathbb{R}^2_+ : (g,d) = p(T,\tau) \text{ for some } T \in \mathcal{K},\ \tau \in [0,1]\right\}. \tag{55}$$

5.2 Frontier representation and star-shapedness

Proposition 5.3 (Frontier representation)**.** *The attainable set has the representation*

$$\mathcal{F} = \bigcup_{T \in \mathcal{K}} \{\tau(v(T), d(T)) : \tau \in [0,1]\}. \tag{56}$$

Consequently, $\mathcal{F}$ is star-shaped with respect to the origin. If $\mathcal{K}$ is compact, then $\mathcal{F}$ is compact.

Proof. The representation (56) is the definition of $\mathcal{F}$ together with (54). To prove star-shapedness, let $(g,d) \in \mathcal{F}$. Then $(g,d) = \tau(v(T), d(T))$ for some $T \in \mathcal{K}$ and $\tau \in [0,1]$. For any $s \in [0,1]$,

$$s(g,d) = s\tau(v(T), d(T)) = (s\tau)(v(T), d(T)),$$

with $s\tau \in [0,1]$, hence $s(g,d) \in \mathcal{F}$. Thus $\mathcal{F}$ is star-shaped with respect to the origin. If $\mathcal{K}$ is compact, the map $(T,\tau) \mapsto \tau(v(T), d(T))$ is continuous on the compact set $\mathcal{K} \times [0,1]$. Its image $\mathcal{F}$ is therefore compact. □

The proposition deliberately does not claim convexity. Each coherent candidate generates a line segment from the origin to its full-verification point $(v(T), d(T))$. The attainable set is the union of these segments. A union of line segments from the origin is star-shaped, but it is not generally convex. This distinction matters for accounting interpretation: absent randomization or institutional convexification, the frontier is the lower envelope of available measurement designs, not automatically a smooth convex tradeoff curve.

Definition 5.4 (Disturbance frontier)**.** For $g \geq 0$, define the minimum disturbance needed to attain verification gain at least g by

$$D^*(g) := \inf\{d \geq 0 : (g', d) \in \mathcal{F} \text{ for some } g' \geq g\}. \tag{57}$$

Equivalently,

$$D^*(g) = \inf_{T \in \mathcal{K},\ \tau \in [0,1]} \{\tau d(T) : \tau v(T) \geq g\}. \tag{58}$$

The inequality $g' \geq g$ in (57) is intentional. An institution that requires at least a target gain g may choose a design that delivers more than the minimum target if doing so reduces disturbance through a different candidate. The next theorem shows that, despite this apparent flexibility, the optimum binds the target for the chosen candidate.

5.3 Minimum disturbance for a target verification gain

Let

$$v_{\max} := \max_{T \in \mathcal{K}} v(T), \tag{59}$$

which exists by compactness of $\mathcal{K}$ and continuity of v. For $g > 0$, define

$$\mathcal{K}_g := \{T \in \mathcal{K} : v(T) \geq g\}. \tag{60}$$

Theorem 5.5 (Minimum disturbance for a target verification gain)**.** *Under Assumption 3.5, the target-gain disturbance function satisfies:*

(i) $D^*(0) = 0$.

(ii) If $g > v_{\max}$, the target is infeasible and $D^*(g) = +\infty$.

(iii) If $0 < g \le v_{\max}$, then

$$D^*(g) = g \min_{T \in \mathcal{K}_g} \frac{d(T)}{v(T)}. \tag{61}$$

Moreover, the minimum is attained: there exists $T_g^* \in \mathcal{K}_g$ such that

$$D^*(g) = g \frac{d(T_g^*)}{v(T_g^*)}. \tag{62}$$

An optimal verification intensity is

$$\tau_g^* := \frac{g}{v(T_g^*)}. \tag{63}$$

Proof. Part (i) follows because $\tau = 0$ gives $G(T, 0) = 0$ and $D(T, 0) = 0$ for every $T \in \mathcal{K}$.

For part (ii), if $g > v_{\max}$, then for every $T \in \mathcal{K}$ and every $\tau \in [0, 1]$,

$$G(T, \tau) = \tau v(T) \le v(T) \le v_{\max} < g.$$

Thus no pair is feasible.

Now suppose $0 < g \le v_{\max}$. Fix $T \in \mathcal{K}$. If $v(T) < g$, then $G(T, \tau) = \tau v(T) < g$ for every $\tau \le 1$, so T cannot support the target. Hence any feasible T must lie in $\mathcal{K}_g$.

For a fixed $T \in \mathcal{K}_g$, the constraint $\tau v(T) \ge g$ is equivalent to

$$\tau \ge \frac{g}{v(T)}.$$

Because $D(T, \tau) = \tau d(T)$ and $d(T) \ge 0$, the smallest disturbance for this fixed T is obtained by making the constraint bind:

$$\tau = \frac{g}{v(T)}.$$

The resulting disturbance is

$$D\left(T, \frac{g}{v(T)}\right) = \frac{g}{v(T)} d(T) = g \frac{d(T)}{v(T)}.$$

Taking the infimum over $T \in \mathcal{K}_g$ gives

$$D^*(g) = g \inf_{T \in \mathcal{K}_g} \frac{d(T)}{v(T)}.$$

The set $\mathcal{K}_g$ is closed in $\mathcal{K}$ because v is continuous, and it is nonempty because $g \le v_{\max}$. Since $\mathcal{K}$ is compact, $\mathcal{K}_g$ is compact. On $\mathcal{K}_g$, $v(T) \ge g > 0$, so the ratio $d(T)/v(T)$ is continuous. By the Weierstrass theorem, the infimum is attained, yielding (61)–(63). □

Theorem 5.5 converts the soft-verification identity into an optimization result. The target-gain problem does not require solving an abstract variational problem from scratch. It reduces to choosing the coherent candidate with the lowest disturbance price among those with sufficient verification capacity.

5.4 Disturbance price and frontier shape

Definition 5.6 (Disturbance price of verification)**.** For $T \in \mathcal{K}$ with $v(T) > 0$, define

$$\kappa(T; E_V) := \frac{d(T)}{v(T)} = \frac{\|[\delta_Y, E_V]T\|_{\mathrm{HS}}}{\|(I_{H_Y} - E_V)T\|_{\mathrm{HS}}}. \tag{64}$$

The quantity $\kappa(T; E_V)$ is the coherence cost of one unit of verification gain along the soft-verification path generated by T. In these terms,

$$D^*(g) = g \min_{T \in \mathcal{K}_g} \kappa(T; E_V). \tag{65}$$

Corollary 5.7 (Monotonicity and average disturbance price)**.** *The function $D^*(g)$ is nondecreasing on $[0, v_{\max}]$. Moreover, for $0 < g \le v_{\max}$,*

$$\frac{D^*(g)}{g} = \min_{T \in \mathcal{K}_g} \kappa(T; E_V), \tag{66}$$

and $D^*(g)/g$ is nondecreasing in g.

Proof. If $0 \le g_1 < g_2 \le v_{\max}$, every pair feasible for target g_2 is feasible for target g_1. Therefore the infimum for g_1 cannot exceed the infimum for g_2, so $D^*(g_1) \le D^*(g_2)$. For the average price, if $g_1 < g_2$, then $\mathcal{K}_{g_2} \subseteq \mathcal{K}_{g_1}$. Taking the minimum of κ over the smaller set cannot decrease the value. Equation (66) then implies that $D^*(g)/g$ is nondecreasing. □

The monotonicity of the average disturbance price has a simple accounting interpretation. Low verification-gain targets can be met by choosing designs with low disturbance price, even if their total verification capacity is limited. High verification-gain targets exclude low-capacity designs, forcing the institution to select candidates with greater verification reach, which may come at a higher disturbance price.

Corollary 5.8 (Linear regions of the frontier)**.** *Suppose $T^\dagger \in \mathcal{K}$ minimizes $\kappa(T; E_V)$ over all $T \in \mathcal{K}$ with $v(T) > 0$. Then, for all $g \in (0, v(T^\dagger)]$ such that no lower-price candidate is capacity-feasible only on a smaller interval,*

$$D^*(g) = g\kappa(T^\dagger; E_V). \tag{67}$$

When g exceeds $v(T^\dagger)$, the candidate $T^\dagger$ can no longer support the target and the frontier switches to another candidate. Thus the lower envelope is composed of capacity-limited rays.

Proof. This is a direct reading of (65). On any interval of target gains for which the same candidate $T^\dagger$ minimizes κ among $\mathcal{K}_g$, the expression is linear in g with slope $\kappa(T^\dagger; E_V)$. At gains above $v(T^\dagger)$, $T^\dagger \notin \mathcal{K}_g$ and cannot be chosen. □

5.5 Convexified frontiers and randomized institutional designs

The preceding results do not require convexity. In some institutional settings, however, a regulator or auditor may randomize across measurement designs or combine procedures ex ante. If such randomized designs are evaluated by expected gain and expected disturbance, the relevant attainable set is the convex hull of $\mathcal{F}$.

Definition 5.9 (Randomized gain–disturbance designs)**.** A randomized design is a probability distribution π over $\mathcal{K} \times [0,1]$. Its ex ante gain and disturbance are

$$\bar{G}(\pi) := \int_{\mathcal{K}\times[0,1]} \tau v(T)\, d\pi(T,\tau), \tag{68}$$

$$\bar{D}(\pi) := \int_{\mathcal{K}\times[0,1]} \tau d(T)\, d\pi(T,\tau). \tag{69}$$

Proposition 5.10 (Convexified frontier under randomization)**.** *If randomized designs are admissible and evaluated by ex ante gain and disturbance, then the attainable ex ante set is the convex hull* $\mathrm{co}(\mathcal{F})$*. The corresponding minimum ex ante disturbance for target gain* g *is the lower convex envelope of the deterministic frontier* $D^*(g)$*.*

Proof. Every degenerate distribution on a point (T,τ) yields the deterministic point $p(T,\tau) \in \mathcal{F}$. Every finite randomization over points of $\mathcal{F}$ yields a convex combination of those points. Conversely, every convex combination of finitely many points in $\mathcal{F}$ is generated by the corresponding finite-support distribution. In finite-dimensional $\mathbb{R}^2$, closure of these finite combinations gives $\mathrm{co}(\mathcal{F})$ for general probability distributions. The lower boundary of this convex set at target gain g is precisely the lower convex envelope of the deterministic frontier. □

This proposition should be read carefully. It is not a claim that the deterministic measurement map obtained by averaging operators has gain and disturbance equal to the averaged pair; norms need not behave linearly under such averaging. It is an ex ante statement about randomized institutional designs. The proposition is useful when audit or verification procedures are randomized, rotated, sampled, or otherwise mixed in a way that makes expected gain and expected coherence cost the relevant institutional criterion.

5.6 Accounting interpretation and implications

The frontier analysis yields a disciplined way to speak about the price of verification. A verification regime is not evaluated only by the amount of unverifiable information it removes, but also by the coherence disturbance it induces. The key object is not merely the commutator $[\delta_Y, E_V]$ in isolation, but the ratio

$$\kappa(T; E_V) = \frac{\|[\delta_Y, E_V]T\|_{\mathrm{HS}}}{\|(I_{H_Y} - E_V)T\|_{\mathrm{HS}}},$$

which measures the institutional cost of converting unverifiable recirculation readings into verification-supported readings for a given coherent candidate.

Several implications follow.

First, a verification regime can be powerful but expensive. A candidate with large $v(T)$ can support high verification-gain targets, but if its disturbance price $\kappa(T; E_V)$ is large, using it may substantially disrupt dynamic coherence. Such regimes correspond to measurement systems that are audit-friendly but dynamically misaligned.

Second, low-disturbance candidates may have limited capacity. A candidate with small $\kappa(T; E_V)$ may be attractive for low target gains, but if $v(T)$ is small, it cannot support a high-verification regime. This captures a common accounting problem: a narrowly tailored measurement procedure may be coherent and cheap in disturbance terms, but insufficient when institutions demand broad verification.

Third, verification reform changes the frontier through two channels. Refining the audit-evidence σ-algebra may reduce the unverifiable component of some candidates,

but it also changes the commutator $[\delta_Y, E_V]$ and therefore the disturbance price. More evidence is not automatically better in every dimension; what matters is whether the evidence partition is aligned with accounting dynamics on the recirculation subspace.

Fourth, randomized verification can convexify the institutional frontier. This provides an accounting rationale for sampling, rotation, and randomized audit procedures that is distinct from traditional error-detection arguments. Randomization may allow the institution to attain ex ante gain–disturbance combinations that no single deterministic procedure can deliver.

The section therefore reframes verification as a design choice with a measurable coherence cost. The next sections build on this frontier by deriving comparative statics and translating them into accounting, audit, and standards-setting implications.

6 Comparative Statics of the Disturbance Price

Sections 4 and 5 establish the technical core of the argument. Imposing verification through the soft operator

$$E_{V,\tau} = (1-\tau)I + \tau E_V$$

improves evidentiary support in a mechanically transparent way, but it also creates a coherence disturbance whenever the verification projection fails to commute with the wealth-side dynamics. For a coherent candidate $T : R \to H_Y$, the gain–disturbance path is

$$G_{\mathrm{ver}}(T;\tau) = \tau v_E(T), \qquad D_{\mathrm{coh}}(T;\tau) = \tau d_E(T), \tag{70}$$

where

$$v_E(T) := \|(I - E_V)T\|_{\mathrm{HS}}, \qquad d_E(T) := \|[\delta_Y, E_V]T\|_{\mathrm{HS}}. \tag{71}$$

The notation emphasizes the dependence on the verification regime $E = E_V$. The ratio

$$\kappa_E(T) := \frac{d_E(T)}{v_E(T)} = \frac{\|[\delta_Y, E_V]T\|_{\mathrm{HS}}}{\|(I - E_V)T\|_{\mathrm{HS}}} \tag{72}$$

whenever $v_E(T) > 0$, is the central comparative-static object of the paper. It is the *coherence price of verification*: the coherence disturbance paid per unit of verification gain along the path generated by the candidate T.

This section asks how that price changes when the institutional target, the verification regime, the wealth-side dynamics, and the recirculation domain change. The point is not to claim that verification is undesirable. The point is sharper: verification is not free. A verification regime has a price schedule, and that price schedule is determined by how audit-evidence partitions interact with accounting dynamics.

Throughout this section let $\mathcal{K}$ denote the compact set of coherent candidate restrictions considered in Section 5. Elements of $\mathcal{K}$ are Hilbert–Schmidt normalized operators $T : R \to H_Y$ satisfying

$$\delta_Y T = T\delta_R,$$

together with the non-degeneracy restrictions imposed in the preceding sections. For a target verification gain $g \geq 0$, define

$$\mathcal{K}_g(E) := \{T \in \mathcal{K} : v_E(T) \geq g\}, \qquad v_{\max}(E) := \max_{T\in\mathcal{K}} v_E(T), \tag{73}$$

and recall from Section 5 that the minimum coherence disturbance required to obtain target gain g is

$$D_E^*(g) = g \min_{T\in\mathcal{K}_g(E)} \kappa_E(T), \qquad 0 < g \leq v_{\max}(E). \tag{74}$$

When $g > v_{\max}(E)$, the target is infeasible; when $g = 0$, the minimum disturbance is zero.

6.1 The target verification gain

The first comparative-static parameter is the target gain g itself. Equation (74) shows that increasing g has two effects. It scales the required disturbance directly, and it also reduces the set of candidate measurements available to achieve the target. The second effect is the important one: stringent verification targets force the institution to abandon low-price candidates if those candidates do not have enough unverified component to deliver the required gain.

Proposition 6.1 (Monotonicity of the average disturbance price)**.** *Suppose* $0 < g_1 \leq g_2 \leq v_{\max}(E)$*. Then*

$$\mathcal{K}_{g_2}(E) \subseteq \mathcal{K}_{g_1}(E),$$

and consequently

$$\frac{D_E^*(g_1)}{g_1} \leq \frac{D_E^*(g_2)}{g_2}. \tag{75}$$

Thus the minimum disturbance per unit of verification gain is weakly increasing in the target gain.

Proof. The inclusion is immediate from the definition: if $v_E(T) \geq g_2$ and $g_2 \geq g_1$, then $v_E(T) \geq g_1$. Since

$$\frac{D_E^*(g)}{g} = \min_{T \in \mathcal{K}_g(E)} \kappa_E(T),$$

taking the minimum over the smaller set $\mathcal{K}_{g_2}(E)$ cannot produce a smaller value than taking the minimum over $\mathcal{K}_{g_1}(E)$. This proves (75). □

The proposition does not say that $D_E^*(g)$ must be convex or differentiable. In general it need not be either. The frontier is an envelope of line segments generated by different coherent candidates. At low verification targets, a low-price candidate may dominate. Once the target exceeds that candidate's verification capacity $v_E(T)$, the institution must switch to another candidate. The frontier can therefore have kinks. Those kinks have accounting meaning: they mark points at which further verification can no longer be obtained by intensifying the same measurement design and requires a change in the underlying design.

Corollary 6.2 (Linear regions of the frontier)**.** *Suppose there exists* $T^\dagger \in \mathcal{K}$ *such that*

$$\kappa_E(T^\dagger) = \min_{T \in \mathcal{K}} \kappa_E(T), \qquad v_E(T^\dagger) > 0.$$

Then for every $g \in (0, v_E(T^\dagger)]$*,*

$$D_E^*(g) = g\,\kappa_E(T^\dagger). \tag{76}$$

Proof. For $g \leq v_E(T^\dagger)$, $T^\dagger \in \mathcal{K}_g(E)$. Since $T^\dagger$ minimizes κ_E over the whole of $\mathcal{K}$, it also minimizes κ_E over the subset $\mathcal{K}_g(E)$. Equation (74) gives (76). □

The corollary clarifies why the target-gain formulation is useful. Small verification improvements may be obtained at a stable marginal price. Large verification improvements can require institutional redesign. In accounting terms, low levels of additional audit

support may be obtained by applying a verification procedure more intensely. High levels of audit support may require changing what is measured, how the evidence partition is drawn, or which components of the accounting state are allowed to remain dynamically coherent.

6.2 Verification intensity

For a fixed coherent candidate T, the verification intensity τ has no hidden comparative statics. From (70), both gain and disturbance are linear in τ:

$$\frac{\partial G_{\mathrm{ver}}(T;\tau)}{\partial \tau} = v_E(T), \qquad \frac{\partial D_{\mathrm{coh}}(T;\tau)}{\partial \tau} = d_E(T). \tag{77}$$

Thus τ controls the quantity of verification imposed, while $\kappa_E(T)$ controls the institutional price of imposing it. This separation is useful. Two regimes can impose the same intensity τ but have very different consequences because their commutators differ. Conversely, two regimes can have the same commutator exposure but different verification gain capacities because their unverified components differ.

The practical implication is simple but important. A gentle verification policy is not automatically low-cost. If $\kappa_E(T)$ is large, even a small increase in verification intensity creates a large coherence disturbance per unit of verification gain. Conversely, if $\kappa_E(T)$ is small, the institution can increase verification intensity with little loss of dynamic coherence. The audit question is therefore not only how much verification to impose; it is whether the evidence partition makes verification cheap or expensive in coherence terms.

6.3 Refinement of the verification regime

Next consider changes in the verification regime. Let E_1 and E_2 be two verification projections with nested verification-supported subspaces,

$$M_1 \subseteq M_2,$$

so that E_2 is a refinement of E_1. In the Hilbert-space setting this nesting implies that

$$E_1E_2 = E_2E_1 = E_1.$$

Define

$$Q := E_2 - E_1.$$

Then Q is the orthogonal projection onto the incremental evidence subspace $M_2 \cap M_1^{\perp}$.

Proposition 6.3 (Two effects of verification refinement)**.** *For every $T \in \mathcal{K}$,*

$$v_{E_2}(T)^2 = v_{E_1}(T)^2 - \|QT\|_{\mathrm{HS}}^2. \tag{78}$$

Moreover,

$$[\delta_Y, E_2]T = [\delta_Y, E_1]T + [\delta_Y, Q]T, \tag{79}$$

so that

$$\left|d_{E_2}(T) - d_{E_1}(T)\right| \le \|[\delta_Y, Q]T\|_{\mathrm{HS}}. \tag{80}$$

Proof. Because $E_2 = E_1 + Q$ and Q is orthogonal to E_1, we have

$$I - E_1 = (I - E_2) + Q,$$

with orthogonal ranges for $I - E_2$ and Q. Hence

$$\|(I - E_1)T\|_{\mathrm{HS}}^2 = \|(I - E_2)T\|_{\mathrm{HS}}^2 + \|QT\|_{\mathrm{HS}}^2,$$

which gives (78). Equation (79) follows from linearity of the commutator:

$$[\delta_Y, E_2] = [\delta_Y, E_1 + Q] = [\delta_Y, E_1] + [\delta_Y, Q].$$

The bound (80) is the reverse triangle inequality applied to (79). □

The proposition separates two effects that are often conflated. Refining the verification regime mechanically reduces the unverified component of a fixed measurement, as (78) shows. But the coherence disturbance need not fall. The new evidence subspace Q may be dynamically aligned with δ_Y, in which case $[\delta_Y, Q]T$ is small, or it may cut across the dynamics, in which case $[\delta_Y, Q]T$ is large. Refinement reduces residual unverifiability; it does not necessarily reduce the coherence price of verification.

This is the main comparative-static warning of the section. More evidence categories, more observable inputs, or a finer audit-evidence partition are not automatically better from the standpoint of dynamic coherence. They are better when the refined partition tracks the movement of accounting states. They are costly when the refined partition slices the state space in a way that is unstable under the accounting dynamics.

6.4 Wealth-side dynamics and dynamic alignment

The term $d_E(T)$ depends on the wealth-side dynamics only through the commutator $[\delta_Y, E]$. The decomposition

$$[\delta_Y, E] = (I - E)\delta_Y E - E\delta_Y(I - E) \tag{81}$$

reveals the two channels of disturbance. The first term,

$$(I - E)\delta_Y E,$$

is leakage: a currently verifiable reading becomes unverifiable after differencing. The second term,

$$E\delta_Y(I - E),$$

is mixing: an unverifiable reading is moved by the dynamics into the verifiable subspace. Both matter for the norm of the full commutator.

Proposition 6.4 (Lipschitz comparative statics in wealth-side dynamics)**.** *Let δ'_Y be another bounded wealth-side differencing operator and set $\Delta := \delta'_Y - \delta_Y$. Then for every $T \in \mathcal{K}$,*

$$|d_E^{\delta'_Y}(T) - d_E^{\delta_Y}(T)| \le \|[\Delta, E]T\|_{\mathrm{HS}} \le 2\|\Delta\|_{\mathrm{op}}\|T\|_{\mathrm{HS}}. \tag{82}$$

Proof. By definition,

$$[\delta'_Y, E]T = [\delta_Y, E]T + [\Delta, E]T.$$

The first inequality follows from the reverse triangle inequality. For the second,

$$\|[\Delta, E]T\|_{\mathrm{HS}} \le \|\Delta ET\|_{\mathrm{HS}} + \|E\Delta T\|_{\mathrm{HS}} \le \|\Delta\|_{\mathrm{op}}\|ET\|_{\mathrm{HS}} + \|E\|_{\mathrm{op}}\|\Delta T\|_{\mathrm{HS}}.$$

Since E is an orthogonal projection, $\|E\|_{\mathrm{op}} = 1$ and $\|ET\|_{\mathrm{HS}} \le \|T\|_{\mathrm{HS}}$. This gives the bound. □

The proposition formalizes the idea of dynamic alignment. If two accounting environments differ only slightly in their wealth-side dynamics, their disturbance prices differ only slightly. If the dynamics are changed in a way that cuts across the evidence partition, the commutator changes. Thus the important issue is not the complexity of the dynamics in isolation; it is whether the dynamics preserve the evidence partition used for verification.

In the limiting case, if E commutes with δ_Y on the range of all candidates in $\mathcal{K}$, then $d_E(T) = 0$ for every $T \in \mathcal{K}$. Verification is then free in coherence terms. Conversely, if the candidate range lies mostly in directions on which $[\delta_Y, E]$ is large, verification is expensive. This is the formal version of the paper's conceptual claim: verification is not merely a property of evidence; it is a relation between evidence and dynamics.

6.5 Recirculation channels

The recirculation subspace R determines which directions of the accounting state are exposed to the verification operation. Because R is finite-dimensional, the exposure can be decomposed direction by direction. Let $\{e_1, \dots, e_r\}$ be an orthonormal basis of R. Then

$$d_E(T)^2 = \sum_{i=1}^{r} \|[\delta_Y, E]Te_i\|_{H_Y}^2, \qquad v_E(T)^2 = \sum_{i=1}^{r} \|(I-E)Te_i\|_{H_Y}^2. \tag{83}$$

This identity is elementary, but its accounting interpretation is powerful. The recirculation subspace does not add disturbance by dimension alone. It adds channels through which disturbance can occur. A new recirculation direction that is mapped into the kernel of $[\delta_Y, E]$ adds little or no structural disturbance. A new direction that is strongly exposed to the commutator can substantially increase the disturbance price.

This observation is important for applications. Accrual reversals, deferred items, OCI recyclables, hedge reserves, and sustainability-performance accumulations may all be recirculation channels, but they need not have the same disturbance profile. The relevant question is whether the audit-evidence partition tracks the dynamics of each channel. A broad reporting regime with many recirculation channels can be low-cost if the evidence partitions are dynamically stable. A narrow regime can be high-cost if its few recirculation channels are poorly aligned with verification.

6.6 Randomized verification procedures

The frontier derived in Section 5 is deterministic. Each pair (T, τ) generates one point in the gain–disturbance plane,

$$(G_{\mathrm{ver}}(T;\tau), D_{\mathrm{coh}}(T;\tau)).$$

If the institution permits ex ante randomization across verification procedures, the relevant object becomes the convex hull of the deterministic attainable set. This is not a claim about averaging accounting numbers after the fact. It is a claim about randomized institutional procedures: audit sampling, rotating evidence tests, randomized verification thresholds, or randomized inspections of recirculation channels.

Proposition 6.5 (Convexification by ex ante randomization)**.** *Let $\mathcal{F}_E$ denote the deterministic attainable set in the gain–disturbance plane. If ex ante randomization across finitely many deterministic procedures is institutionally admissible and performance is evaluated in expected gain and expected disturbance, then the randomized attainable set is*

$$\mathrm{co}(\mathcal{F}_E), \tag{84}$$

the convex hull of $\mathcal{F}_E$. In particular, the randomized minimum disturbance for target gain g is weakly no larger than the deterministic minimum disturbance $D_E^*(g)$.

Proof. A randomized procedure chooses deterministic designs (T_j, τ_j) with probabilities $p_j \geq 0$, $\sum_j p_j = 1$. Expected gain and expected disturbance are

$$\sum_j p_j G_{\mathrm{ver}}(T_j; \tau_j), \qquad \sum_j p_j D_{\mathrm{coh}}(T_j; \tau_j).$$

Thus every randomized procedure generates a convex combination of deterministic attainable points. Conversely, every finite convex combination can be implemented by the corresponding randomization. The randomized attainable set is therefore $\mathrm{co}(\mathcal{F}_E)$. Since the deterministic set is contained in its convex hull, minimizing over the convexified set cannot yield a higher disturbance for the same target gain. □

The result should be interpreted with care. Randomization may reduce the structural disturbance frontier in expectation, but it can introduce implementation costs, communicability problems, and strategic anticipation by firms. Those issues belong to a broader theory of accounting measurement disturbance. The present paper uses randomization only to clarify the geometry of the verification-induced structural frontier.

6.7 Summary of the comparative statics

The comparative statics of the model can be summarized in one sentence: the cost of verification depends less on how much evidence exists than on how that evidence is positioned relative to accounting dynamics. Increasing the target gain g raises the minimum average disturbance price by shrinking the feasible candidate set. Increasing the verification intensity τ moves linearly along a candidate's ray. Refining the evidence partition reduces residual unverifiability but can raise or lower the coherence price depending on dynamic alignment. Changing the wealth-side dynamics matters only through the commutator with the verification projection. Enlarging the recirculation domain adds channels, not automatic cost. Randomization convexifies the attainable set but raises institutional questions beyond the deterministic model.

These results are not yet accounting applications. They are the comparative-static grammar of the concept. The next section translates that grammar into accounting measurement, audit, and standards-setting implications.

7 Accounting and Audit Implications

The preceding sections introduce a way of seeing verification that differs from the usual one. Verification is not merely a desirable attribute attached to a measurement after the fact. It is an operation that changes the measurement map. It may improve evidentiary support, but when the evidence partition is not preserved by accounting dynamics, it also disturbs coherence. The question is therefore not simply whether verification is good. The question is what its coherence price is.

This section translates that idea into accounting and audit settings. The purpose is not to offer full case studies. The purpose is to show that once verification is viewed as a disturbance-generating operation, several familiar accounting problems become instances of the same structure. Fair value measurement, OCI recycling, impairment recognition, sustainability assurance, audit quality, and standards-setting can all be read as choices on a verification–disturbance frontier.

7.1 The coherence price of verification

The paper's central accounting object is

$$\kappa_E(T) = \frac{\|[\delta_Y, E]T\|_{\mathrm{HS}}}{\|(I-E)T\|_{\mathrm{HS}}}. \tag{85}$$

The numerator is the coherence disturbance created when verification is imposed on the coherent candidate T. The denominator is the verifiability defect available to be reduced by imposing verification. Their ratio is the coherence price of verification.

This ratio changes the way verifiability is discussed. A measurement is not merely more or less verifiable. It has a verification capacity $v_E(T)$, a disturbance exposure $d_E(T)$, and a price $\kappa_E(T)$. A regime with a low price makes verification cheap: evidentiary support can be improved without materially disturbing dynamic coherence. A regime with a high price makes verification expensive: evidentiary support is obtained only by distorting the dynamic relationship between stock-domain and flow-domain measurement.

The target-gain function

$$D_E^*(g) = g \min_{T \in \mathcal{K}_g(E)} \kappa_E(T) \tag{86}$$

then gives the accounting interpretation of the frontier. A standards-setter, auditor, or reporting system that demands a target level g of verification gain must pay at least $D_E^*(g)$ in coherence disturbance. The object $D_E^*(g)$ is not a welfare function; it is a structural price schedule. It tells us how the accounting system must move if it insists on making a measurement more audit-supported.

7.2 Fair value measurement and observable-input restrictions

Fair value measurement illustrates the distinction between evidentiary support and dynamic coherence. Observable inputs increase audit support because they tie measurement to independently observable prices, transactions, or market data. In the model, the observable-input restriction is a verification regime E that projects measurements onto the subspace supported by market evidence. Imposing the restriction reduces the unverified component of a measurement:

$$v_E(T) = \|(I-E)T\|_{\mathrm{HS}}.$$

But the restriction can also disturb coherence if the evidence partition is not stable under the wealth-side dynamics:

$$d_E(T) = \|[\delta_Y, E]T\|_{\mathrm{HS}}.$$

The framework therefore distinguishes two cases that are often conflated. If the observable-input partition tracks the relevant economic dynamics, then $[\delta_Y, E]T$ is small and fair-value verification is cheap. In that regime, market evidence supports the measurement without forcing the report away from the dynamic path of the asset or liability. If, however, the evidence partition is discontinuous relative to the underlying dynamics–for example, if an item moves across evidence categories, if market observability is episodic, or if the relevant price dynamics occur in dimensions not captured by the audit-supported inputs–then $[\delta_Y, E]T$ can be large. The measurement becomes more verifiable, but less dynamically coherent.

This reframes the familiar debate over observable inputs and fair value measurement (Whittington, 2008; Linsmeier, 2016). The issue is not whether observable inputs are

reliable and unobservable inputs are unreliable. The issue is whether the evidence partition induced by observability is dynamically aligned with the accounting object being measured. A Level-1-style evidence partition can be low-price for actively traded instruments whose value dynamics are themselves market-price dynamics. The same kind of partition can be high-price for instruments whose reported value depends on contract-specific, managerial, or long-horizon state variables not preserved by the observable-input partition.

Thus fair value measurement is not placed on a one-dimensional relevance–reliability scale. It is placed on a frontier. Observable-input restrictions produce verification gain. Their coherence price depends on the commutator between the wealth dynamics and the evidence partition.

7.3 OCI recycling and recirculation

OCI recycling is the natural accounting setting for the recirculation language of the model. Items enter equity through other comprehensive income, remain in an accumulated stock-domain account, and may later be recycled into profit or loss. The reporting system thereby moves items between stock-domain and flow-domain representations. This is precisely the kind of cyclic movement represented by the recirculation subspace R.

In a recycling regime, verification is often event-dependent. A fair-value movement may be recognized outside profit or loss while unrealized, and a realization event may later support recognition in profit or loss. In the notation of the model, the realization event changes the relevant evidence partition. A measurement that was dynamically coherent in the wealth domain may become projected into a profit-or-loss-supported evidence subspace. The disturbance is governed not by recycling as a label, but by

$$[\delta_Y, E]T.$$

If the evidence supporting recycling is aligned with the way the item evolves over time, the coherence price is low. If realization is only weakly related to the economic dynamics of the item, the price is high.

This interpretation is useful because it avoids two oversimplifications. First, it avoids saying that recycling is always desirable because it enhances realized-performance verification. Second, it avoids saying that recycling is always undesirable because it disrupts comprehensive-income coherence. The model asks a different question: what is the disturbance price of using realization-based evidence to make a recirculating item verifiable in profit or loss?

The directional decomposition in (83) gives the relevant diagnostic. For each recirculation component $e_i \in R$, the term

$$\|[\delta_Y, E]Te_i\|_{H_Y}$$

measures how strongly that component is exposed to the verification-induced disturbance. Accrual reversals, deferred items, cash-flow hedge reserves, and OCI recyclables can therefore have different disturbance prices even within the same reporting system. A single recycling rule can be cheap for one channel and expensive for another.

The implication is that recycling design should not be evaluated only by whether it improves profit-or-loss verifiability. It should also be evaluated by the coherence price it imposes on the recirculation channels to which it applies.

7.4 Impairment recognition and asymmetric thresholds

Impairment recognition provides a second illustration, closely related to conservatism and asymmetric timeliness debates (Basu, 1997; Watts, 2003a,b). An impairment standard typically requires evidentiary support before a write-down is recognized. The evidentiary threshold protects against unverifiable write-downs, but it also introduces asymmetry: losses may be recognized under specified evidence conditions while gains are not recognized symmetrically. In the present framework, the threshold is a verification device. It pushes the measurement toward an evidence-supported subspace.

The target-gain result gives a way to describe the cost of this device. Raising the evidentiary target corresponds to increasing g in

$$D_E^*(g) = g \min_{T \in \mathcal{K}_g(E)} \kappa_E(T).$$

The higher the target, the smaller the feasible candidate set $\mathcal{K}_g(E)$. At low targets, the reporting system may obtain additional evidentiary support by intensifying an existing impairment test. At high targets, it may have to switch to a measurement design with a higher disturbance price. This is the formal counterpart of a familiar institutional observation: stringent impairment verification can make reported losses more audit-supported while making the measurement less symmetric over time.

The model does not imply that impairment thresholds are bad. It says that they occupy a point on the verification–disturbance frontier. A conservative recognition rule may be justified because it reduces unverifiable measurement components. But it should not be described as costless. Its cost is the coherence disturbance created by using an evidence threshold that is not dynamically symmetric.

This also explains why impairment rules are difficult to evaluate by a single attribute such as reliability. A rule can be reliable in the sense of being evidence-supported and yet costly in the sense of distorting dynamic coherence. The relevant design question is not whether impairment recognition is verifiable, but whether the verification gain is worth its coherence price.

7.5 Sustainability assurance and assurance-ready metrics

Sustainability reporting and assurance make the paper's conceptual point especially visible. Assurance-ready metrics are often designed to be independently supportable: they are tied to documents, third-party data, sensor readings, contractual records, or standardized calculation protocols. This improves verifiability. But it can also change the performance object being measured.

In the model, an assurance-ready metric corresponds to a measurement map projected toward an evidence-supported subspace. The unverified component falls, but the metric may become less dynamically coherent with the underlying sustainability performance process. For example, a carbon metric that is easy to verify because it is based on invoice-level energy purchases may fail to track operational emissions dynamics in real time. A human-capital metric that is easily assured because it is based on headcount categories may fail to track training intensity, turnover quality, or organizational learning. A supply-chain metric that is verifiable through supplier documentation may not preserve the dynamics of actual supply-chain risk.

The model's implication is not anti-assurance. It is more precise. Assurance-ready design creates a potential gain–disturbance tradeoff. The verification gain is valuable, but the metric's coherence with dynamic performance should be evaluated separately. A

sustainability metric is not improved simply because it becomes easier to assure. It is improved when assurance can be added at a low coherence price.

This point may be especially important in emerging reporting domains. When the measurement object is not yet institutionally settled, the evidence partition can shape the object itself. If assurance requirements are imposed too early or too narrowly, they may induce metrics that are highly verifiable but dynamically thin. In the language of the model, the reporting system may choose a low-v metric by construction, but only by moving to a high-d design.

7.6 Audit quality as gain and disturbance

The audit implication of the paper is straightforward: audit quality has at least two dimensions. The first is verification gain. An audit procedure reduces the part of a measurement not supported by independent evidence. The second is coherence disturbance. The same procedure may alter the measurement map in a way that weakens its relationship to accounting dynamics. A high-quality audit procedure is therefore not merely one that produces more evidence. It is one that produces evidence aligned with the dynamics of the accounting object.

This perspective changes the interpretation of audit evidence. In a static view, better audit evidence is finer, more independent, or more objective evidence. In the present view, better audit evidence is evidence whose partition is stable under the relevant accounting dynamics. If the evidence partition drifts relative to the object, the audit may obtain verification by changing what is being measured. If the partition tracks the object, the audit can obtain verification without large coherence disturbance.

The ratio $\kappa_E(T)$ is therefore an audit-quality diagnostic. Low $\kappa_E(T)$ indicates that the audit procedure can increase evidentiary support at low coherence cost. High $\kappa_E(T)$ indicates that the procedure creates a large dynamic distortion per unit of verification gain. This ratio is not a substitute for conventional audit-quality concepts such as independence, competence, or detection probability. It adds a measurement-theoretic dimension: the dynamic cost of making a number auditable.

The framework also clarifies why audit procedures can have different effects across accounts. The same procedure may be low-price for cash, receivables, or exchange-traded securities, and high-price for internally generated intangibles, long-term provisions, or sustainability metrics. The difference is not merely that some accounts are hard to audit. The difference is that their evidence partitions interact differently with their dynamics.

7.7 Standards-setting as choosing a frontier point

For standards-setting, the paper's message is that verifiability requirements choose points on a frontier. A standard that demands stronger evidence support is not simply moving the system upward along a quality scale. It selects a target verification gain g, a verification regime E, and an admissible class of measurement designs $\mathcal{K}$. Those choices imply a minimum disturbance

$$D_E^*(g) = g \min_{T \in \mathcal{K}_g(E)} \kappa_E(T).$$

This makes the standards-setting problem more disciplined. A standard setter can ask three questions. First, what verification gain is required? Second, which evidence partition supplies that gain? Third, what coherence price does that partition impose on

the relevant recirculation channels? The third question is often implicit. The present framework makes it explicit.

The result also explains why standards-setting reforms may have non-monotone consequences. Refining the evidence partition reduces the unverified component of a measurement, but it can increase the commutator if the refinement is dynamically misaligned. Thus more granular verification is not necessarily lower-cost verification. A better standard is not always one with more evidence categories. It is one in which the evidence categories are drawn along dynamically meaningful lines.

This point is especially relevant for mixed-measurement systems. Such systems do not choose between historical cost and fair value once and for all. They assign different measurement bases to different items, often depending on business model, realization status, market observability, or risk-management designation. In the present framework, each assignment defines part of an evidence partition. The quality of the system depends on how those partitions interact with the dynamics of the items they classify.

7.8 What the applications show

The applications are intentionally diverse. Fair value, OCI recycling, impairment, sustainability assurance, audit quality, and standards-setting are not the same problem. But the same conceptual move helps to analyze them. Verification is an operation. It reduces the unverified component of a measurement. It can also disturb coherence. The size of that disturbance depends on the commutator between accounting dynamics and the verification regime.

The value of the theory is therefore not that it solves one application. It is that it makes a class of questions visible. Instead of asking whether a measurement is verifiable, we can ask what it costs to make it verifiable. Instead of asking whether audit evidence is strong, we can ask whether the evidence partition is dynamically aligned. Instead of asking whether a standard improves reliability, we can ask where it moves the reporting system on the verification–disturbance frontier.

That is the paper's conceptual contribution. The familiar question is: how can accounting numbers be made more verifiable? The question opened by this paper is: what does making them verifiable do to the measurement itself?

8 Conclusion

This paper began with an ordinary idea and then changed its direction. The ordinary idea is that verifiability is a desirable attribute of accounting numbers. A number supported by independent evidence is easier to audit, easier to defend, and easier to accept institutionally. The paper does not deny that view. It argues that the view is incomplete. In many accounting settings, making a number verifiable is not merely an after-the-fact evaluation of a number already produced. It is an operation that acts on the measurement map itself.

The formal expression of that operation is simple. If W is a measurement map and E_V is the conditional expectation onto the audit-evidence supported subspace, full verification transforms W into $E_V W$. Partial verification transforms W into $E_{V,\tau} W$, where

$$E_{V,\tau} = (1-\tau)I + \tau E_V, \qquad \tau \in [0,1].$$

Once verification is written this way, its cost becomes visible. For a coherent candidate

$T : R \to H_Y$, verification gain and coherence disturbance are

$$G_{\mathrm{ver}}(T;\tau) = \tau v_E(T), \qquad D_{\mathrm{coh}}(T;\tau) = \tau d_E(T),$$

where

$$v_E(T) = \|(I - E_V)T\|_{\mathrm{HS}}, \qquad d_E(T) = \|[\delta_Y, E_V]T\|_{\mathrm{HS}}.$$

The coherence price of verification is therefore

$$\kappa_E(T) = \frac{d_E(T)}{v_E(T)}.$$

This ratio is the paper's central concept. It measures how much dynamic coherence is disturbed per unit of verification gained.

The main theoretical results follow from this operation view. Section 4 showed that soft verification removes unverifiable components linearly in τ and, for coherent candidates, creates coherence disturbance linearly in τ. Section 5 aggregated these linear paths into a measurement–disturbance frontier and solved the minimum-disturbance problem for a target verification gain:

$$D_E^*(g) = g \min_{T \in \mathcal{K}_g(E)} \frac{d_E(T)}{v_E(T)}.$$

Section 6 showed how this price changes with the target gain, the verification intensity, the audit-evidence partition, the wealth-side dynamics, and the recirculation channels. Section 7 then translated the formal results into accounting and audit settings: fair value measurement, OCI recycling, impairment recognition, sustainability assurance, audit quality, and standards-setting.

The paper's conceptual contribution can be put plainly. Verification is not free. It is not merely an attribute that attaches to a reported number after measurement has been completed. It is an institutional operation that can improve evidentiary support by changing the measurement map. When the audit-evidence partition is dynamically aligned with accounting measurement, the coherence price of verification can be low. When it is not aligned, verification may produce a more supportable number by disturbing the dynamic structure of the measured object.

The paper also clarifies the role of audit evidence. Stronger evidence is not automatically better in every dimension. A refined evidence partition reduces the unverified component of a measurement, but it can increase commutator exposure if the refinement cuts across accounting dynamics. The relevant institutional question is therefore not only how much evidence is available. It is whether the evidence partition tracks the movement of accounting states.

Several limitations define the next steps. First, the paper studies structural disturbance. It does not model strategic disturbance, in which firms change the accounting information they produce in anticipation of monitoring. Second, the verification regime E is treated as given. A standards-setting model would endogenize the choice of E, the target gain g, and the candidate set $\mathcal{K}$. Third, the paper uses Hilbert–Schmidt norms. Entropic or information-theoretic measures of verification loss and coherence disturbance may capture other features of audit evidence and disclosure. Fourth, the paper works on a finite-dimensional recirculation subspace. Infinite-dimensional extensions are mathematically natural but institutionally more demanding.

These limitations are not defects of the concept. They indicate where the concept can travel. If verification is an operation, then audit quality is not only the strength of

evidence; it is also the coherence price at which evidence is obtained. If standards-setting chooses evidence partitions, then standards-setting chooses a point on a verification–disturbance frontier. If assurance-ready metrics are designed around what can be verified, then assurance may improve support while changing the measured object. The familiar question is how accounting numbers can be made more verifiable. The question opened by this paper is what making them verifiable does to the measurement itself.

A Additional Technical Details

The main proofs are included in the text because the paper's central claims are short algebraic and geometric arguments. This appendix records several auxiliary facts that are useful for checking the scope of the results and for extending the framework.

A.1 Restriction to the recirculation subspace

Let $W : H_Z \to H_Y$ be bounded and let $T = W|_R$. Since P_R has range R, the finite-rank operator WP_R factors as

$$WP_R = TP_R.$$

For Hilbert–Schmidt norms on the finite-dimensional domain R, $\|T\|_{\mathrm{HS}}$ is computed by choosing any orthonormal basis $\{e_i\}_{i=1}^r$ of R:

$$\|T\|_{\mathrm{HS}}^2 = \sum_{i=1}^{r} \|Te_i\|_{H_Y}^2.$$

All gain and disturbance quantities used in the paper depend only on T, not on the behavior of W outside R. This is why the frontier analysis can be conducted on the compact coherent candidate set $\mathcal{K} \subseteq \mathcal{L}(R, H_Y)$.

A.2 Why conditional expectation is used

If $M \subseteq H_Y$ is an arbitrary closed subspace, the orthogonal projection P_M need not preserve positivity. Therefore, even if W is positive, $P_M W$ may fail to be positive. In accounting terms, an arbitrary projection can transform a nonnegative accounting reading into a negative reported quantity. Conditional expectation avoids this problem. If $E_V = \mathbb{E}[\cdot \mid \mathcal{G}]$, then $f \geq 0$ implies $E_V f \geq 0$, and $E_V 1_Y = 1_Y$. Hence $W \in \mathcal{W}$ implies $E_V W \in \mathcal{W}$. This is why the verification operation in the paper is modeled as conditional expectation onto an audit-evidence σ-algebra rather than as an arbitrary projection.

A.3 A two-dimensional illustration

Consider the simplest case in which $R = \mathrm{span}\{e\}$ is one-dimensional and $H_Y = \mathbb{R}^2$ with orthonormal basis (y_1, y_2). Let

$$Ey_1 = y_1, \qquad Ey_2 = 0,$$

so that $M = \mathrm{span}\{y_1\}$ is the verification-supported subspace. Let the wealth-side dynamics be

$$\delta_Y = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

Let $Te = u_1 y_1 + u_2 y_2$ with $\|T\| = 1$. Then

$$v_E(T) = \|(I - E)T\| = |u_2|.$$

The commutator is

$$[\delta_Y, E] = \delta_Y E - E\delta_Y = \begin{pmatrix} 0 & -b \\ c & 0 \end{pmatrix}.$$

Thus

$$d_E(T)^2 = b^2 u_2^2 + c^2 u_1^2.$$

The coherence price is

$$\kappa_E(T) = \frac{\sqrt{b^2 u_2^2 + c^2 u_1^2}}{|u_2|},$$

whenever $u_2 \neq 0$. The formula shows the two sources of disturbance. The coefficient b measures how unverifiable directions move into the verified coordinate; the coefficient c measures how verified directions leak into the unverified coordinate. If c is large and u_1 is large, then a measurement that is already mostly verifiable can be very expensive to make fully verifiable, because the verified component is dynamically unstable.

A.4 Alternative loss functions

The paper uses Hilbert–Schmidt norms because they provide a transparent geometry on $\mathcal{L}(R, H_Y)$ and yield exact linear gain–disturbance paths under the soft verification operator. Other loss functions are possible. If squared disturbance is used, then for coherent T,

$$D^{(2)}_{\mathrm{coh}}(T; \tau) := \|\delta_Y E_{V,\tau} T - E_{V,\tau} T \delta_R\|^2_{\mathrm{HS}} = \tau^2 d_E(T)^2.$$

The corresponding target-gain problem becomes quadratic in g:

$$D^*_{E,2}(g) = g^2 \min_{T \in \mathcal{K}_g(E)} \frac{d_E(T)^2}{v_E(T)^2}.$$

The substantive interpretation is unchanged, but the marginal price of verification is no longer constant along a fixed ray. This distinction may matter for empirical or regulatory applications in which large coherence disturbances are penalized more than proportionally.

B Notation Summary

Symbol	Meaning
H_Z, H_Y	accounting state space and wealth-side reading space
R	finite-dimensional recirculation subspace
P_R	orthogonal projection onto R
$\delta_Z, \delta_Y, \delta_R$	state-side, wealth-side, and restricted differencing operators
W	measurement map $H_Z \to H_Y$
T	restriction $W\|_R : R \to H_Y$
$\mathcal{G}$	audit-evidence σ-algebra
M_V	verification-supported subspace
E_V	conditional expectation onto M_V
$E_{V,\tau}$	soft verification operator $(1-\tau)I + \tau E_V$
$\mathcal{K}$	compact set of coherent candidate restrictions
$v_E(T)$	initial unverifiable component $\|(I - E_V)T\|_{\mathrm{HS}}$
$d_E(T)$	commutator exposure $\|[\delta_Y, E_V]T\|_{\mathrm{HS}}$
$\kappa_E(T)$	coherence price of verification $d_E(T)/v_E(T)$
$D_E^*(g)$	minimum disturbance for target verification gain g

References

Basu, S. 1997. The conservatism principle and the asymmetric timeliness of earnings. *Journal of Accounting and Economics* 24(1): 3–37.

Chambers, R. J. 1966. *Accounting, Evaluation and Economic Behavior*. Englewood Cliffs, NJ: Prentice-Hall.

Christensen, J. A., and J. S. Demski. 2003. *Accounting Theory: An Information Content Perspective*. New York: McGraw-Hill/Irwin.

Christensen, P. O., and G. A. Feltham. 2003. *Economics of Accounting, Volume I: Information in Markets*. Boston, MA: Kluwer Academic Publishers.

Demski, J. S. 1973. The general impossibility of normative accounting standards. *The Accounting Review* 48(4): 718–723.

Demski, J. S. 1980. *Information Analysis*. Reading, MA: Addison-Wesley.

Ijiri, Y. 1986. A framework for triple-entry bookkeeping. *The Accounting Review* 61(4): 745–759.

Linsmeier, T. J. 2016. Revised model for presentation in statement(s) of financial performance: Potential implications for measurement in the conceptual framework. *Accounting and Business Research* 46(5): 485–498.

Mattessich, R. 1957. Towards a general and axiomatic foundation of accountancy. *Accounting Research* 8(4): 328–355.

Mattessich, R. 1964. *Accounting and Analytical Methods*. Homewood, IL: Richard D. Irwin.

Mattessich, R. 1995. *Critique of Accounting: Examination of the Foundations and Normative Structure of an Applied Discipline*. Westport, CT: Quorum Books.

Watts, R. L. 2003a. Conservatism in accounting part I: Explanations and implications. *Accounting Horizons* 17(3): 207–221.

Watts, R. L. 2003b. Conservatism in accounting part II: Evidence and research opportunities. *Accounting Horizons* 17(4): 287–301.

Whittington, G. 2008. Fair value and the IASB/FASB conceptual framework project: An alternative view. *Abacus* 44(2): 139–168.