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Money Confidence in the New Keynesian Model

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Abstract

We present a micro-founded model which extends the standard closed-economy New Keynesian model to incorporate money confidence. Our main finding is that the more varying the money confidence, the bigger the effects of a monetary policy shock on output and inflation.

Keywords: Money confidence, New Keynesian model, Monetary policy shocks

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1 Introduction

What happens after an exogenous shock to monetary policy is one of the main topics of monetary economics. However, at least as far as I know, none of the literature on this topic has taken public confidence on money into consideration.

Money confidence is important since changes in the degree of it may affect the effects of monetary policy on the economy. For example, when monetary authorities increase the money supply, people may lose confidence on money and reduce money demand, thinking that inflation, which decreases purchasing power and lowers the value of money, is likely to happen. This could result in the different dynamic responses of macroeconomics variables from those without money confidence.

Thus, this paper is designed as a first step to construct a model which introduces money confidence and to explore its implications. Specifically, we include it into a New Keynesian DSGE framework and investigate the effect of a monetary policy shock on output and inflation. In addition, the model we present here can be interpreted as the one which makes a distinction between convertible and fiat money. If money has intrinsic value, the degree of confidence on it should be unchanged. Therefore, money represents convertible money in that case. Money is fiat money if the degree of confidence varies over time. Our main finding is that the more varying the money confidence, the bigger the effects of a monetary policy shock on output and inflation.

The rest of this paper is organized as follows. Section 2 describes the model. Section 3 describes the calibration of the model and solves it through simulation. Section 4 concludes.

2 The Elements of the Model

We extend the standard closed-economy New Keynesian model of, for example, Galí (2008) in two ways. Firstly and the most importantly, money confidence as well as real balances is included as an argument of the utility function. As a result of that, money demand depends on money confidence directly. Secondly, government purchases are introduced.

2.1 Consumers

The world is inhabited by a continuum of infinitely-lived consumers, who seek to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \Phi_t \frac{m_t^{1-\nu}}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

subject to the budget constraint

$$\int_0^1 P_t(i)\ C_t(i)\ di + Q_t\ B_t + M_t \leq B_{t-1} + M_{t-1} + W_t\ N_t - T_t + D_t \qquad (2)$$

for $t = 0, 1, 2, \cdots$. In (1), E_0 is the mathematical expectation operator based on the information at period 0, and $\beta \in (0, 1)$ is the discount factor. C_t is a Dixit-Stiglitz aggregator given by

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$$

where $C_t(i)$ is the quantity of the consumption of good i in period t, and $\epsilon > 1$ is the elasticity of substitution between goods. m_t denotes real money balances, and Φ_t denotes the money confidence, which is a key feature of this model. If $\Phi_t = 1$ for any time period, which leads to the standard money-in-utility function, money can be interpreted as convertible money. The reason is that the degree of confidence on convertible money remains unchanged since it has intrinsic value. On the other hand, if Φ_t varies over time, money represents fiat money. N_t rdenotes hours of work satisfying $0 \leq N_t \leq 1$. The parameters, σ, ν, and φ, are positive constants.

In (2), $P_t(i)$ is the price of $C_t(i)$, B_t is the quantity of one period nominally riskless bonds purchased in period t which pay one unit of money at maturation, and Q_t is its price. W_t is nominal wage, T_t is the nominal tax collections by the government, and D_t denotes dividends from ownership of firms.

The consumer minimizes the cost function $\int_0^1 P_t(i)\ C_t(i)di$ with respect to $C_t(i)$ given C_t. At the optimum,

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t. \qquad (3)$$

Therefore, ϵ can also be interpreted as the price elasticity of demand for good i. P_t is the aggregate price index and defined as

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}} \qquad (4)$$

which leads to $\int_0^1 P_t(i)C_t(i)di = P_t\ C_t$. Using this relationship and defining $\mathcal{A}_t \equiv B_{t-1} + M_{t-1}$ as the total financial wealth at the beginning of period t, the budget constraint can be rewritten as

$$P_t\ C_t + Q_t\ \mathcal{A}_{t+1} + (1 - Q_t)M_t \leq \mathcal{A}_t + W_t\ N_t - T_t + D_t.$$

The first-order conditions with respect to C_t, N_t, $\mathcal{A}_{t+1}$ and m_t (which can be denoted byM_t/P_t) with the utility maximization problem can be written as

$$[C_t]: \quad C_t^{-\sigma} - \lambda_t = 0 \tag{5}$$

$$[N_t]: \quad -N_t^{\varphi} + \lambda_t \frac{W_t}{P_t} = 0 \tag{6}$$

$$[A_{t+1}]: \quad Q_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \right\} \tag{7}$$

$$[m_t]: \quad \Phi_t (M_t/P_t)^{-\nu} - \lambda_t (1 - Q_t) = 0 \tag{8}$$

where λ_t is a Lagrange multiplier associated with the period t budget constraint. Combining (5) with (6) and taking logs of both sides, we have the log-linear labor supply schedule:

$$w_t - p_t = \sigma \; c_t + \varphi \; n_t. \tag{9}$$

Substituting $\lambda_t = C_t^{-\sigma}$ coming from (5) into (7) and taking a first-order Taylor expansion of the resulting expression around a steady state with constants rate of inflation and consumption growth leads to a standard Euler equation

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} \left(i_t - E_t\{\pi_{t+1}\} - \rho \right) \tag{10}$$

where $c_t \equiv \log C_t$ and we define the inflation rate between $t-1$ and t as $\pi_t = P_t/P_{t-1} - 1 \simeq p_t - p_{t-1}$ (having defined $p_t \equiv \log P_t$). We can interpret $i_t \equiv -\log Q_t$ as the nominal interest rate and $\rho \equiv -\log \beta$ as the consumer's discount rate.

The optimality condition (8) can be rewritten as

$$\frac{M_t}{P_t} = \Phi_t^{1/\nu} C_t^{\sigma/\nu} (1 - \exp\{-i_t\})^{-1/\nu} \tag{11}$$

which can be interpreted as a demand for real balances. As is clear from this expression, money demand depends on money confidence, Φ_t. Equation (11) can be rewritten in approximate log-linear form as[1]

$$m_t - p_t = \frac{1}{\nu} \; \phi_t + \frac{\sigma}{\nu} \; c_t - \eta \; i_t \tag{12}$$

where $\phi_t \equiv \log \Phi_t$. Here $1/\nu$ is now interpreted as the confidence elasticity, and $\eta \equiv \frac{1}{\nu(\exp\{i\}-1)} \simeq \frac{1}{\nu i}$ is the interest semi-elasticity of money demand. This money demand function dictates that consumers will demand more money to hold if money confidence rises, and vise versa.

[1]We use the fact that $\log(1 - \exp\{-i_t\}) \simeq const. + \frac{1}{\exp\{i\}-1} i_t$ and drop constants.

2.2 Firms

We assume a continuum of firms indexed by $i \in [0,1]$, producing a differentiated good with an identical technology. All firms are owned by consumers. The production function takes the form:

$$Y_t(i) = A_t\ N_t(i)^{1-\alpha}, \qquad \alpha \in (0,1) \tag{13}$$

where A_t is a time-varying exogenous productivity. Labor is the only factor of production and there is no capital stock or accumulation. Here we introduce nominal rigidity as the modal in Calvo (1983). In each period, a fraction $\theta \in (0,1)$ of firms is unable to adjust its price, while the remaining fraction $(1-\theta)$ of firms can set its price optimally. Therefore, θ can be interpreted as the degree of price stickiness, and the average price duration is given by $1/(1-\theta)$.[2]

Under the Calvo-type assumption, the aggregate price (4) can be expressed in the log-linear form as

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}). \tag{14}$$

where $p_t^* = \log P_t^*$.

2.3 Government

We assume that the government purchases quantity $G_t(i)$ of good i. As in the case of private sector's, let $G_t = [\int_0^1 G_t(i)^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}}$ denote an index of public consumption. The government seeks to maximize this consumption index G_t for any level of expenditures $\int_0^1 P_t(i)G_t(i)di$. Then, their demand functions for individual goods will have the same form as the private sector's:

$$G_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} G_t.$$

For simplicity, we also assume that the government runs a balanced budget each period and that the government expenditure are financed by means of lump-sum taxes, issuing bonds, and seignorage.

$$P_t G_t = T_t + Q_t B_t + M_t - M_{t-1}.$$

2.4 Optimal Price Setting by Firms

A firm that changes its price in period t chooses P_t^* to maximize

$$\sum_{k=0}^{\infty} \theta^k\ E_t\left\{Q_{t,t+k}(P_t^*\ Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}))\right\}$$

[2]This follows the fact that j periods of price duration occur with the probability of $\theta^{j-1}(1-\theta)$. Therefore, the average price duration is given by $1(1-\theta) + 2\theta(1-\theta) + 3\theta^2(1-\theta) + \cdots = (1-\theta)(1+2\theta+3\theta^2+\cdots) = 1/(1-\theta)$.

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} (C_{t+k} + G_{t+k}) \quad \text{for } k = 0, 1, 2, \cdots.$$

where $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right)$ is the stochastic discount factor, $\Psi_t(\cdot)$ is the nominal cost function in period t, and $Y_{t+k|t}$ is output in $t+k$ for a firm last reset its price in t.

2.5 Equilibrium

Goods market clearing condition dictates that each output good is allocated among consumers and the government:

$$Y_t(i) = C_t(i) + G_t(i) \quad \text{for all } i \text{ and all } t.$$

This implies

$$Y_t = C_t + G_t.$$

In steady state we assume $G = S_G\ Y$ where S_G is the share of public consumption in output. Log-linearizing the goods market equilibrium condition:

$$y_t = (1 - S_G)\ c_t + S_G\ g_t \tag{15}$$

where $y_t = \log Y_t$ and $g_t = \log G_t$. Using the above condition, the Euler equation for output is given by

$$y_t = E_t\{y_{t+1}\} - \frac{1 - S_G}{\sigma} (i_t - E_t\{\pi_{t+1}\} - \rho) - S_G\ E_t\{\Delta g_{t+1}\} \tag{16}$$

Labor market clearing condition can be written by $N_t = \int_0^1 N_t(i)\ di$. Then, by using the production function (13) and the demand function (3), the log-linearized first-order condition associated with this profit maximization problem and Eq. (14) imply

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \lambda\ \widehat{mc}_t \tag{17}$$

where $\widehat{mc}_t$ denotes the log deviation of real marginal cost from its steady state value and $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$.[3]

Manipulation of Eqs. (9), (13), and (15) leads to the relationship between mc_t and y_t.

$$\begin{aligned} mc_t &\equiv (w_t - p_t) - mpn_t \\ &= (\sigma\ c_t + \varphi\ n_t) - (y_t - n_t) - \log(1-\alpha) \\ &= \left(\frac{\sigma(1 - S_G) + (1 - S_G)(\varphi + \alpha)}{(1 - S_G)(1 - \alpha)}\right) y_t - \frac{1+\varphi}{1-\alpha}\ a_t - \frac{S_G}{1 - S_G} - \log(1-\alpha) \end{aligned} \tag{18}$$

[3]See Galí (2008, Ch.3) for a derivation.

where mpn_t denotes the log of the marginal product of labor. Now, we define the natural level of output y_t^n as the equilibrium level of output under flexible prices:

$$mc = \left(\frac{\sigma(1-S_G)+(1-S_G)(\varphi+\alpha)}{(1-S_G)(1-\alpha)}\right) y_t^n - \frac{1+\varphi}{1-\alpha}\, a_t - \frac{S_G}{1-S_G} - \log(1-\alpha) \tag{19}$$

Subtracting (19) from (18) yields

$$\widehat{mc}_t = \left(\frac{\sigma(1-S_G)+(1-S_G)(\varphi+\alpha)}{(1-S_G)(1-\alpha)}\right) \widetilde{y}_t \tag{20}$$

where $\widetilde{y}_t \equiv y_t - y_t^n$ is the output gap.

Combining (20) with (17) yields the New Keynesian Phillips curve:

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t \tag{21}$$

where $\kappa \equiv \lambda\ \left(\frac{\sigma(1-S_G)+(1-S_G)(\varphi+\alpha)}{(1-S_G)(1-\alpha)}\right)$.
The second key equation make use of the Euler equation for output.

$$y_t = E_t\{y_{t+1}\} - \frac{1-S_G}{\sigma}\ (i_t - E_t\{\pi_{t+1}\} - \rho) - S_G\ E_t\{\Delta g_{t+1}\} \tag{22}$$

Rewriting (22) in terms of the output gap yields the dynamic IS equation.

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1-S_g}{\sigma}\ (i_t - E_t\{\pi_{t+1}\} - r_t^n) \tag{23}$$

where $r_t^n \equiv \rho + \sigma E_t\{\Delta y_{t+1}^n\} - \frac{\sigma S_G}{1-S_G}\ E_t\{\Delta g_{t+1}\}$.
Combining the goods market equilibrium condition (15) and the money demand equation (12) gives;

$$m_t - p_t = \frac{1}{\sigma}\ \phi_t + \frac{1}{1-S_G}\ y_t - \frac{S_G}{1-S_G}\ g_t - \eta\ i_t \tag{24}$$

2.6 The System

The economy is described by the following equilibrium conditions.

$$\begin{aligned}
\widetilde{y}_t &= E_t\{\widetilde{y}_{t+1}\} - \frac{1-S_G}{\sigma}\ (i_t - E_t\{\pi_{t+1}\} - r_t^n) \\
\pi_t &= \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t \\
m_t - p_t &= \frac{1}{\sigma}\ \phi_t + \frac{1}{1-S_G}\ y_t - \frac{S_G}{1-S_G}\ g_t - \eta\ i_t.
\end{aligned}$$

We assume that money confidence follows the following process:

$$\phi_t = \rho_\phi\ \phi_{t-1} - \rho_{\phi m}\ \Delta m_t + \epsilon_t^\phi$$

where $\rho_\phi \in [0,1)$, $\rho_{\phi m} > 0$ and $\{\epsilon_t^\phi\}$ is white noise. The second term in the RHS captures the intuitive idea that money confidence is undermined when money supply increases since people expect the purchasing power of money to fall because of inflation. We assume also that Δm_t follows the AR(1) process:

$$\Delta m_t = \rho_m \, \Delta m_{t-1} + \epsilon_t^m$$

where $\rho_m \in [0,1)$ and $\{\epsilon_t^m\}$ is white noise.

3 Calibration and Simulation

In this section we analyze dynamic responses to a monetary policy shock. Here we also solve the model where money confidence remains unchanged (i.e., $\phi_t = 0$) to compare the results.

3.1 Parameter Values

In the baseline calibration, we take the period in the model to correspond to a quarter and use the common parameter values in the literature. It is assumed $\beta = 0.99$, $\sigma = 1$, $\varphi = 1$, $\epsilon = 6$ as in Blanchard and Galí (2010). The parameter θ, which measures the degree of price stickiness, is assumed to be 0.75 (implying an average price duration of four quarters) as in Mankiw and Reis (2002). We set $\alpha = 1/3$, $\rho_m = 0.5$ as in Galí (2008), and the steady-state share of public consumption in output to $S_G = 0.25$ as in Kirsanova and Wren-Lewis (2007). Since we have no evidence on ν, ρ_ϕ, and $\rho_{\phi m}$, we set $\nu = 1$, $\rho_\phi = 0.9$ and $\rho_{\phi m} = 1$ respectively. The benchmark parameters are displayed in Table 1.

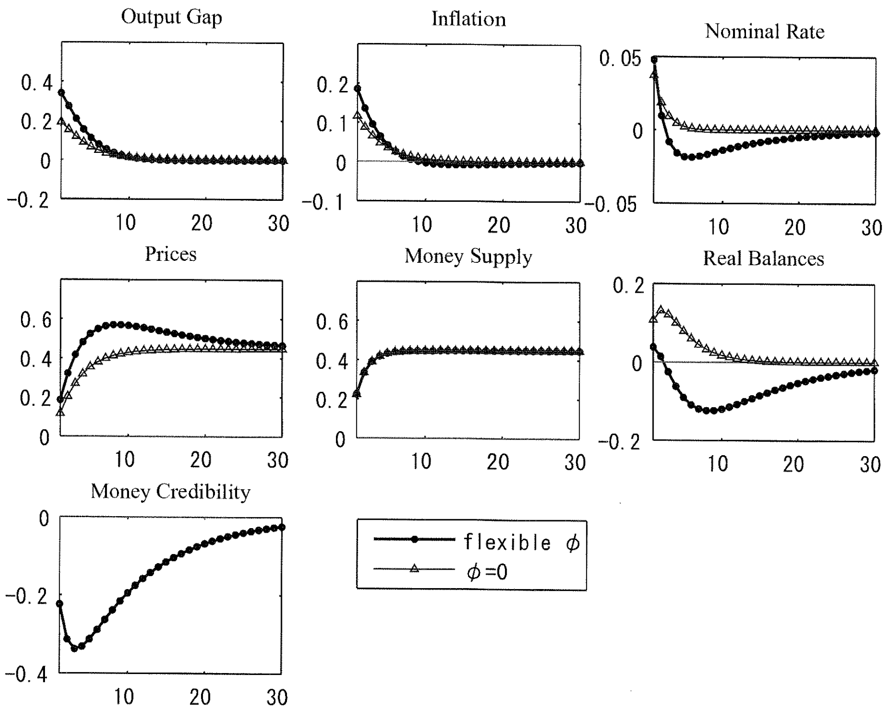
Table 1: Common Parameter Values

Parameter	Description	Value
β	Time discount factor	0.99
σ	Coefficient of relative risk aversion	1
φ	Disutility of labor	1
ϵ	Elasticity of substitution	6
α	Coefficient of returns to scale	1/3
θ	Degree of price stickiness	0.75
S_G	Public consumption share	0.25
ν	Reciprocal of confidence elasticity of money demand	1
ρ_m	AR(1) coefficient on Δm_{t-1}	0.5

3.2 The Dynamic Effects of a Monetary Policy Shock

Figure 1 shows the dynamic effects of an increase of 0.05% in ϵ_t^m. It is clear

Figure 1: Dynamic Response to a Monetary Policy Shock



that directions of changes in variables are the same but that fluctuations are larger in the case of the time-varying money confidence than in that of unchanged one. The expansionary monetary policy shock leads to an instantaneous decline in money confidence when it varies. The maximum impact of the rise in money supply on money confidence occurs at three quarters. By this gradual fall in money confidence, consumers' demand for real balances also fall gradually. Since the real balances (M_t/P_t) fall in spite of the rise in money supply (M_t), prices (P_t) must go up. As a result, inflation responds instantly, which is responsible for the rise in output. However, the model fails to generate an inertial response in inflation. This is a well-known key shortcoming of NKPC, which lacks the backward-looking components.

4 Conclusion

This paper has modified the standard New Keynesian model to include money confidence in order to investigate how changes in it affect the effects of monetary policy shocks on aggregate dynamics such as output and inflation. We find that the dynamic responses of them get larger as money confidence varies more greatly.

The model we presented here is, however, still parsimonious since it eliminates some important features. For example, it abstracts from capital and investment. We should also incorporate rule-of-thumb price setters as in Steinsson (2003) and staggered wage contracts as in Christiano, Eichenbaum, and Evans (2005) to generate inflation inertia. By abandoning the assumption of a balanced budget by the government, linking the budget deficit to money confidence could be a significant extension. Moreover, to judge the responses predicted by the model describe well the real economy would require an assessment against empirical evidence obtained, for example, from a VAR-based analysis. This would be an interesting subject for future research, and one which might have implications for the design of the models used by policy-makers.

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