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Hideaki Goto

Waseda Institute for Advanced Study, Waseda University, Tokyo, Japan

WIAS 早稲田大学高等研究所
Waseda Institute for Advanced Study

1-6-1 Nishiwaseda, Shinjuku-ku, Tokyo 169-8050, Japan
Tel: +81-3-5286-2460 ; Fax: +81-3-5286-2470

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Hideaki Goto†

Abstract

This paper presents a model of the endogenous formation of altruism during individuals' lifetimes and attempts to find determinants of the distribution of altruism within a society. Young individuals are randomly and repeatedly matched to play a game that represents a socio-economic environment. Individuals are assumed to become more or less altruistic when they encounter opponents whom they infer to be more or less altruistic, respectively, than they are. We show that if the game is supermodular, individuals tend to become more homogeneous when spillover effects increase.

Key Words: preference formation; distribution of altruism; supermodularity; spillovers.

JEL Classification: C72, D64, D62.

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†Waseda Institute for Advanced Study (WIAS), Waseda University, 1-6-1 Nishi-Waseda, Shinjuku-ku, Tokyo 169-8050, Japan. Tel.: +81-3-5286-2120; Fax: +81-3-5286-2470; E-mail: h-goto@aoni.waseda.jp

1 Introduction

This paper presents a model of the endogenous formation of altruism and attempts to find socio-economic determinants of the distribution of altruism within a society. It has been found that individual altruism is formed primarily by the combined effects of genes, parents, peers and society, and the socio-economic environment or culture (Cavalli-Sforza and Feldman [10]; Boyd and Richerson [8], [23]; Henrich et al. [14]; Levy-Garboua [18]; Bowles and Gintis [7]). This paper represents a society's socio-economic environment using a game and, treating the effects of genes and parents as exogenous, studies how individual altruism is formed through interactions with other people via this game. We aim to determine the characteristics of this game that affect the formation and distribution of altruism within a society.

More precisely, this paper studies the following situation. Individuals care not only about their own material payoffs but also about other individuals' payoffs. The extent to which they care about others' payoffs, or the degree of altruism, varies from person to person. Young individuals are randomly and repeatedly matched to play a game that represents a socio-economic environment. Their initial degrees of altruism, which could be interpreted as the effects of genes and parents on altruism, are exogenously given. Their opponents' altruism cannot be observed directly, but it can be inferred by the opponents' actions. In other words, we restrict attention to a class of games in which greater actions reflect greater degrees of altruism. We then suppose that: (*i*) when an individual meets with others who are inferred to be more or less altruistic than he is, he becomes more or less altruistic, respectively; (*ii*) the greater the difference between his own action and his opponents' actions, the more his degree of altruism changes; and (*iii*) individual altruism is determined by the equilibrium actions of all opponents, that is, their average equilibrium action. Given these assumptions, we ask what characteristics of the game affect altruism and its distribution, and we find that supermodularity and the extent of positive spillover effects are important.[1] To be more precise, we show that if the game played in the society is supermodular, individuals tend to become more homogeneous in terms of altruism as spillover effects increase. In particular, if the equilibrium actions are linear with respect to the degree of altruism, the distribution of altruism when spillover effects are small is a mean-preserving spread of that under larger spillover effects, given the same initial distribution of altruism. The basic intuition underlying the results is as follows. In a supermodular game, the equilibrium actions of altruistic individuals, which are monotonically increasing in the degree of altruism, are greater under greater spillovers because the marginal benefits to others of an increase in their own actions are

[1] A game is said to be *supermodular* if players' actions are strategic complements, that is, the best response of each player is increasing in his opponents' actions (see Definition 2 in subsection 2.5). A game is said to have *positive spillovers* if an increase in a player's action raises his opponents' payoffs (see Definition 1 in subsection 2.5).

greater. More importantly, the equilibrium actions increase with an increase in the degree of altruism in accordance with the extent of spillovers. Thus, the difference between the actions of individuals with different levels of altruism is greater under larger spillovers. As a result, for most individuals, the differences between the average action and their own actions are also greater and, therefore, their altruism is driven closer to the level of altruism of individuals who chose the average action. If the equilibrium actions are linear with respect to the degree of altruism, the action of an individual with the average initial degree of altruism is the average action under both smaller and larger spillover effects. Thus, individuals' altruism tends to move to the same level, but it does so to a greater extent under larger spillover effects. Therefore, the resulting distribution under larger spillover effects second-order stochastically dominates that under smaller spillover effects. This implies, for example, that people who live close to one another and engage in joint production, such as irrigation agriculture in a rural village, are more likely to exhibit similar levels of altruism than those who live apart and engage in more or less independent work, as is often the case in cities.

In the last two decades, much experimental evidence has indicated that people exhibit altruism in various situations (Camerer [9]) and, in parallel, economists have begun to recognize that altruism is a non-negligible factor in important decisions on public good provision, intergenerational transfers, and charitable donations, to name a few. These findings have directed attention to the process through which altruism is formed, creating two strands of economics literature. One strand either explicitly employs evolutionary game theory as an analytical tool or implicitly uses the concept of evolutionary selection, and this strand tries to determine the conditions under which altruism is evolutionarily stable or, more generally, has a payoff advantage (see Bester and Guth [3]; Bolle [6]; Possajennikov [21]; Dekel et al. [11]; Heifetz et al. [13]; Alger [1]).[2] The idea that altruistic individuals could have a payoff advantage is insightful. However, evolutionary approaches to altruism formation seem to have a serious limitation because they are based on the assumption that the number of individuals with certain preferences is increasing in the material or monetary payoff, which does not apply to altruism. First, this assumption may rest on another assumption that a parent's number of children is increasing in his material or monetary payoff, which has not been found in reality. In fact, an *inverse* association between income per adult and fertility has long been observed both across countries and across households within a country (Becker et al. [2]; Kuznets [16]). In addition, children are not exact reproductions of their parents. Thus, even if the number of children were increasing in parental income, it does not follow that the level of altruism

[2]In a similar context, Kockesen et al. [15] systematically explore the conditions under which individuals who are concerned with both their absolute and relative payoffs have a payoff advantage. They find that those individuals could have higher payoffs under supermodularity as well as under submodularity with certain additional conditions. Sethi and Somanathan [25] investigate the conditions under which reciprocity is stable.

of materially successful individuals would eventually prevail in a society. Second, this strand of literature may instead be based on another assumption that materially successful preferences are imitated by a larger number of individuals than less successful ones, which also does not apply to altruism. Individuals do not become altruistic by imitating altruistic agents precisely because those agents are materially successful. In this paper, therefore, we do not assume that the number of individuals with a certain level of altruism is increasing in the associated material payoff. Rather, we assume that an individual's altruism is based on how other people behave toward him as well as on the characteristics of his interactions with other people. In contrast to most results based on evolutionary selection mechanisms, heterogeneity in a society remains unless the initial distribution of altruism is degenerate, which is more realistic in light of the observed variety in altruism across individuals (Camerer [9]; Henrich et al. [14]).

Another strand of the literature analyzes how individual preferences are formed and transmitted to offspring, taking into account the effect of peers (Bisin and Verdier [4], [5]; Saez-Marti and Sjogren [24]; Pichler [20]). In one of their contributions [4], Bisin and Verdier, who first rigorously investigated cultural transmission in economics, present a model in which children acquire the preferences of either their parents or a role model in the population with some probability that can be controlled by parents. They then establish conditions under which, for example, heterogeneous preferences are globally stable. Saez-Marti and Sjogren [24] and Pichler [20] extend and elaborate the basic idea of Bisin and Verdier [4] so as to incorporate, respectively, the possibility of biased cultural transmission from peers and the formation of continuous preferences. These papers are closely related to this paper in that they appropriately consider the effect of peers and society on preference formation. On the other hand, this paper differs from theirs in that we emphasize the characteristics of interactions between individuals that affect the formation and distribution of altruism. To our knowledge, this has not been investigated rigorously in the existing literature.

The remainder of the paper is organized as follows. The next section describes the model. Section 3 studies the effect of larger spillover effects on altruism and its distribution. In section 4, we make sure that the results obtained for the model with two-player games continue to hold for a model with N-player games. Section 5 concludes the paper and discusses issues that remain unaddressed.

2 The Model

2.1 Environment

We consider an economy populated by a continuum of individuals, the set of whom is denoted by I. Individuals have different levels of altruism and live for two periods, youth and old age. More precisely, individuals are young in period 1 and old in period 2. Although the specific length of each period does not affect our analysis, period 1 is taken to be sufficiently long for individuals to establish the extent of their altruism. Time t is measured continuously. At every t in period 1, all individuals are randomly matched to play a two-player game $G = (X, \{\pi^1, \pi^2\})$. X is the action space of each player, which is assumed to be a compact subset of the real line $[\underline{a}, \overline{a}] \in \boldsymbol{R}$. $\pi^i : X \times X \to \boldsymbol{R}$ $(i = 1, 2)$ is the *material* payoff function of player i. We do not assume any *a priori* heterogeneity among individuals that changes the primitives of the game. Thus, we focus on symmetric games, which implies that there exists a function $\pi : X \times X \to \boldsymbol{R}$ such that $\pi^i = \pi(x_i, x_j)$, where x_i is player i's action and x_j is the opponent's action. The function π is assumed to be bounded and twice continuously differentiable.[3] It follows from Young's theorem that $\pi_{12}(x_i, x_j) = \pi_{21}(x_i, x_j)$, where $\pi_{12} = \partial^2\pi/\partial x_j \partial x_i$ and $\pi_{21} = \partial^2\pi/\partial x_i \partial x_j$. Let π_{11} and π_{22} be the second-order derivatives with respect to the first and the second argument, respectively. We assume $\pi_{11}(x_i, x_j) < 0$ and $\pi_{22}(x_i, x_j) < 0$ throughout this paper. The first inequality shows that π exhibits diminishing marginal returns to the player's own actions. The second inequality means that the marginal payoff of player i with respect to his opponent's action is decreasing in the opponent's action. The game played by the whole population, in which randomly paired individuals play the game G, will be referred to as the *population game with* G. An equilibrium of the population game will be called a *population equilibrium*.

2.2 Utility function

We next introduce the utility function of an altruistic individual. An individual is regarded as more altruistic the more she cares about other individuals' material payoffs. Let y_i and y_j, respectively, denote the material payoffs to individuals i and j, who happen to be paired. The utility function of individual i with extent of altruism λ_i is given by

$$U_i(y_i, y_j, \lambda_i) = y_i + \lambda_i y_j. \tag{1}$$

Throughout our analysis, we will focus on the case where λ_i is weakly positive. If λ_i is strictly positive, the individual cares not only about his own payoff but also about his opponent's payoff. As λ_i rises, his utility increases more with an increase in his opponent's payoff. Thus, a strictly positive λ_i can be interpreted as representing the degree of altruism. Instead, if $\lambda_i = 0$, the individual is exclusively concerned with his own payoff, or equivalently, is selfish. This case is most often assumed in the economics literature. Alger [1] and Levine [17], for example, have used (1) or a similar utility

[3] One-sided derivatives are assumed to exist at $\underline{a}$ and $\overline{a}$.

function for different analytical goals from this paper's.[4] In the game G, player i's utility can be written as a function of his and his opponent's (= player j's) actions:

$$U_i(x_i, x_j, \lambda_i) = \pi(x_i, x_j) + \lambda_i \pi(x_j, x_i), \tag{2}$$

where $\pi(x_j, x_i)$ in the second term on the right-hand side is the opponent's material payoff. We assume that given any expectations of his opponent's actions, the optimal action of an individual lies in his action space X.[5]

2.3 Distribution and formation of altruism

We suppose that the initial levels of young children's altruism at the beginning of period 1 are mutually independent and distributed according to a density function f, which is assumed to be continuous and to have compact and convex support $\Lambda = [\lambda_{min}, \lambda_{max}]$, where $0 \leq \lambda_{min} < \lambda_{max} < \infty$. The key feature of our model is that the degrees of individuals' altruism change over time depending on their experiences. There could be a variety of possible ways to formalize the endogenous formation of altruism. In particular, we want to consider both *what* causes altruism formation and *when* this formation occurs. First, if an individual could observe the extent of altruism of other people with whom he interacts, his altruism might change directly because of their altruism. In reality, however, it is almost impossible, or at least quite difficult, to tell how altruistic other people are.[6] Consequently, individuals most often infer others' altruism, and their personality traits in general, from what they *do*. Furthermore, most people tend to be deeply affected by others' *behavior*. Therefore, we assume that an individual cannot observe his opponents' altruism and that his altruism changes depending on his opponents' actions. To be more precise, we restrict attention to games in which greater actions imply that an opponent is more altruistic, and we assume that an individual's altruism changes according to the difference between his opponent's action and his own action. For example, suppose that two individuals were matched to play such a game and that one individual took a greater action than the other individual. Then, we postulate that the former's altruism falls and the latter's altruism rises. It is true that, in general, an opponent's action reflects both his altruism *and* his expectations. As noted below, however, our focus will be on equilibrium situations in which all individuals share the same correct expectations

[4]To be more precise, Alger [1] considers both altruism and negative altruism, or spite, and investigates the evolutionarily stable degrees of altruism when each generation plays a public goods game once. The utility function in Levine [17] incorporates not only altruism and spite but also reciprocity. His paper aims to pin down the values of the parameters in his utility function so as to explain experimental results successfully. The exploration of an evolutionary process through which those values are realized is suggested as future research.

[5]Precise conditions for this assumption will be given wherever appropriate.

[6]This is particularly true if individuals are randomly matched with strangers, which is the case in this paper.

about opponents' actions. Furthermore, there are many cases in which people attribute others' behavior to their personal traits, implicitly assuming that other people should have more or less similar expectations as themselves. Second, we suppose that experiences in childhood and young adulthood have a significantly larger impact on altruism than those in old age. Thus, for simplicity, we assume that individuals' altruism is formed based on their experiences during period 1. It is also assumed that each individual's altruism is determined based on his aggregate experience during that period.

In sum, an individual's altruism changes from its initial level λ_i to its level in old age λ_i^{old} according to

$$\lambda_i^{old}(\lambda_i, \Delta_i) = \lambda_i + h(\Delta_i), \tag{3}$$

where Δ_i is the difference between the average action of his opponents when he was young, m, and his own action, x_i. The function $h : \boldsymbol{R} \to \boldsymbol{R}$, which is assumed to be strictly increasing and satisfies $h(0) = 0$, transforms the difference Δ_i into units of altruism. As will become clear in what follows, in games with positive spillovers, an opponent is inferred to be more altruistic as he chooses greater actions. Hence, an individual becomes more or less altruistic when his opponents' actions are greater or lower, respectively, than his own action. Thus, the difference Δ_i is measured as $\Delta_i = m - x_i$. For our purposes, h could be any strictly increasing function with $h(0) = 0$, as long as it retains the order of λ_i (i.e., if $\lambda_i < \lambda_i'$, then $\lambda_i^{old} < \lambda_i^{old\prime}$).

2.4 Information structure and equilibrium

The information structure of the population game is as follows. When playing G, an individual knows neither his opponent's altruism nor the population distribution of altruism. He chooses his actions based on his own altruism and his expectations of others' actions. In equilibrium, all individuals share common expectations that are fulfilled by their actions. We assume that individuals play their equilibrium actions repeatedly over period 1. This could be interpreted, as in Dekel et al. [11], as individuals learning to form correct expectations and to play equilibrium actions from their experiences much faster than their preferences change. An equilibrium of the population game is given by equilibrium actions and their distribution, $\{x^*(\lambda_i), \varphi^*\}$, where $x^*(\lambda_i)$ is the optimal action of an individual with λ_i given the self-fulfilling expectations about opponents' actions, $\varphi^*(x)$.

2.5 Definitions and assumptions

As mentioned above, we focus on games in which greater actions reflect greater altruism. We therefore restrict ourselves to a certain class of games. The concept below is used to characterize such games.

Definition 1. (Kockesen et al. [15])
A two-player game G is said to have *positive spillovers* if

$$\left(\frac{\partial \pi(x_i, x_j)}{\partial x_j} \equiv\right) \pi_2(x_i, x_j) > 0 \quad \text{for } \forall (x_i, x_j) \in X^2.$$

G has positive spillovers if an increase in the opponent's action raises the individual's material payoff. Because the game is symmetric, the opponent's payoff also increases as the individual takes greater actions, which is more important from the point of view of altruistic individuals. This implies that in games with positive spillovers, more altruistic individuals are inclined to choose greater actions.

We also use the following concept.

Definition 2. (Topkis [26])
A two-player game G is said to be *supermodular* if $\pi_{12}(x_i, x_j)(= \pi_{21}(x_i, x_j)) \geq 0$ for $\forall (x_i, x_j) \in X^2$.

This concept is familiar in the economics literature. In supermodular games, the actions of two players are strategic complements, that is, a greater opponent's action induces a player to choose a greater action. From (2), it can be easily checked that if G is supermodular, the second-order cross derivatives of the utility function are weakly positive, $\partial^2 U_i / \partial x_i \partial x_j = \partial^2 U_i / \partial x_j \partial x_i \geq 0$. The supermodularity of G will be used to prove Proposition 2 and Lemma 1 in subsection 3.2.

To study the effect of the characteristics of a game played in a society on the distribution of altruism, we will compare the distributions of old individuals' altruism in two societies in which different games, G and $\hat{G}$, are played. In general, however, these games have multiple equilibria, and thus, the results may change as we compare different pairs of equilibria across societies. To avoid this problem, we restrict attention to games that have a unique equilibrium. The following assumptions, which Mason and Valentinyi [19] show are sufficient conditions for Bayesian games to have a unique monotone pure-strategy equilibrium, turn out to ensure the existence of a unique population equilibrium:[7]

Assumption 1. There exists an $\omega \in (0, \infty)$ such that for all $x_i, x_j \in X$ and $\lambda_i \in \Lambda$,

$$|\pi_1(x_i, x_j) + \lambda_i \pi_2(x_j, x_i)| \leq \omega.$$

Assumption 2. There exists a $\nu \in [0, \infty)$ such that $f(\lambda_i) \leq \nu$ for all $\lambda_i \in \Lambda$.

Assumption 3. There exists a $\delta > 2\nu\omega$ such that $\pi_2(x_j, x_i) \geq \delta$ for all $(x_j, x_i) \in X^2$.

In this paper, we focus on supermodular games with, from Assumption 3, positive spillovers. Examples of games that fall into this class include the standard Bertrand

[7] A *monotone pure strategy* is a pure strategy that is increasing in type, which is, in this case, the degree of altruism.

game with differentiated products (Fudenberg and Tirole [12]; Kockesen et al. [15]) and a public good game with a linear or an increasing-return production function. Furthermore, as long as there are weakly increasing returns, many real-life situations in which, for example, people collaborate in the production of food, commercial products, or security could be represented by a supermodular game with spillovers.

3 Spillovers and the Distribution of Altruism

As described above, we focus on supermodular games with positive spillovers. In this section, we study how the distribution of old individuals' altruism varies as the game played in the society changes. In particular, we investigate the effect of the extent of spillover effects on the distribution of altruism by comparing two distributions of altruism that result from two games with different extents of spillover effects. Before doing so, in the next subsection, we confirm that the population game under consideration has a unique monotone pure-strategy equilibrium.

3.1 Equilibrium

Suppose that an individual with altruism λ_i expects that his opponents will choose their actions according to a continuous density function, $\varphi(x)$. Then, the individual solves the following maximization problem:

$$\max_{x_i \in X} \quad \int_{x\in X} \left[\pi(x_i, x) + \lambda_i \pi(x, x_i)\right] \varphi(x) dx.$$

Because π is continuously differentiable and φ is continuous, the integrand in the above expression is continuous with respect to x_i and x and continuously differentiable with respect to x_i. It follows that the order of differentiation and integration is interchangeable. Thus, the first-order condition is given by[8]

$$\int_{x\in X} [\pi_1(x_i, x) + \lambda_i \pi_2(x, x_i)]\varphi(x) dx = 0.$$

Let $x_i(\lambda_i)$ denote the optimal action of the individual. Because the derivative of the left-hand side of the above expression with respect to x_i exists and is negative (by the assumptions that $\pi_{11} < 0$ and $\pi_{22} < 0$), $x_i(\lambda_i)$ is continuously differentiable for $\lambda_i > 0$ and

$$\frac{dx_i(\lambda_i)}{d\lambda_i} = -\frac{\int_{x\in X} \pi_2(x, x_i)\varphi(x)dx}{\int_{x\in X}[\pi_{11}(x_i, x) + \lambda_i \pi_{22}(x, x_i)]\varphi(x)dx}. \tag{4}$$

[8]The condition under which the optimal actions of all individuals lie in X is $\int_{x\in X}[\pi_1(\overline{a}, x) + \lambda_{max}\pi_2(x, \overline{a})]\varphi(x)dx \leq 0 \leq \int_{x\in X}[\pi_1(\underline{a}, x) + \lambda_{min}\pi_2(x, \underline{a})]\varphi(x)dx$.

Because there are spillovers ($\pi_2 > 0$), the numerator of the fraction on the right-hand side is positive, and we know that the denominator is negative. Hence, $dx_i/d\lambda_i > 0$, that is, the optimal actions are monotonically increasing in altruism given *any* expectations, including the correct ones. Therefore, any equilibrium in which individuals share correct expectations consists of monotone strategies as long as an equilibrium exists. Intuitively, this is because a more altruistic individual chooses a greater action by partly sacrificing his own material payoff to benefit his opponents through spillovers.

Under the assumptions stated in the previous section, it can be shown that the population game in which the two-player game G is played has a unique pure-strategy equilibrium. As the above argument suggests, the equilibrium is in monotone strategies.

Proposition 1. *Consider a population game in which altruistic individuals are randomly paired to play a supermodular game with positive spillovers, G. Then, under Assumptions 1–3, the population game has a unique monotone pure-strategy equilibrium.*

Proof. Let $G' = \{X, \Lambda, f, U_i\}$ be a two-player symmetric Bayesian game in which players' types are independent. The utility function U_i is given by (2), and a density function f gives players' common beliefs about their opponents' types. Because G' is symmetric, if there exists a unique equilibrium, it must be symmetric (Theorem 4.5 in Reny [22]). Thus, we restrict attention to a symmetric equilibrium. First, it will be shown that the Bayesian game G' has a unique monotone pure-strategy equilibrium. Next, we show that there exists a one-to-one correspondence between a symmetric Bayesian equilibrium of G' and an equilibrium of the population game with G.

Existence and uniqueness of a monotone pure-strategy equilibrium of G':

This part of the proof is based on Mason and Valentinyi [19]. Assumptions 3, 1, and 2 correspond to assumptions U1, U2, and D2 in their paper. Because players' types are independent in our model, their assumption D1 always holds as well. They establish that if assumptions U1, U2, D1, and D2 hold, Bayesian games have a unique monotone pure-strategy equilibrium. Therefore, the Bayesian game G' has a unique monotone pure-strategy equilibrium.

One-to-one correspondence between an equilibrium of G' and an equilibrium of the population game:

Next, we show that there is a one-to-one correspondence between a Bayesian equilibrium of G', $s^*(\lambda_i)$, and the equilibrium actions of individuals in the population game, $x^*(\lambda_i)$.
(1) $s^*(\lambda_i) \Rightarrow x^*(\lambda_i)$: In a Bayesian equilibrium $s^*(\lambda_i)$, given the expectations about the opponents' types $f(\lambda)$ and strategy $s^*(\lambda)$, the optimal strategy for both players is $s^*(\lambda_i)$ for each possible $\lambda_i \in \Lambda$. In other words, $s^*(\lambda_i)$ for each $\lambda_i \in \Lambda$ is the optimal action

for a player given the expectation that the other player will choose $s^*(\lambda)$ with density $f(\lambda)$. Let $\varphi^*(x) \equiv \varphi(s^*(\lambda)) = f(\lambda)$ be the probability density function of opponents' actions in the population game. φ^* means that the density of $x = s^*(\lambda)$ is given by $f(\lambda)$. Thus, the optimal action of an individual with altruism λ_i in the population game is $s^*(\lambda_i)$. If every individual in the population game chooses an action according to $x^*(\lambda_i) = s^*(\lambda_i)$, the probability density with which an opponent takes action $x = s^*(\lambda)$ is $\varphi^*(x) = f(\lambda)$. Thus, the expectation φ^* is self-fulfilling. This means that $x^*(\lambda_i) = s^*(\lambda_i)$ and $\varphi^*(x)$ $(\equiv \varphi(x^*(\lambda)) = f(\lambda))$ constitute a population equilibrium.
$\underline{(2)\ x^*(\lambda_i) \Rightarrow s^*(\lambda_i)}$: Conversely, let $\{x^*(\lambda_i), \varphi^*(x)\}$ be a population equilibrium. That is, $x^*(\lambda_i)$ is the optimal action of an individual with λ_i given the probability density of his opponents' actions, $\varphi^*(x)$. The support of $\varphi^*(x)$ is given by the set of x such that $x = x^*(\lambda)$ for some $\lambda \in \Lambda$. Because altruism is distributed according to f and because the optimal actions are strictly increasing in altruism, it must be the case that $\varphi^*(x) = \varphi(x^*(\lambda)) = f(\lambda)$. This means that if an opponent chooses $x^*(\lambda)$ with probability density $f(\lambda)$, the optimal action for an individual with altruism λ_i is $x^*(\lambda_i)$. Thus, $s^*(\lambda_i) = x^*(\lambda_i)$ for both players under the expectations of types $f(\lambda)$ is a Bayesian equilibrium of G'.

From (1) and (2), we have $x^*(\lambda_i) = s^*(\lambda_i)$ for $\forall \lambda_i \in \Lambda$. Therefore, if G' has a unique monotone pure-strategy equilibrium, the population equilibrium must also be a unique monotone pure-strategy equilibrium. This completes the proof of Proposition 1. □

Because the equilibrium actions in the population game and the equilibrium strategy in the corresponding Bayesian game are equivalent, we will use whichever is convenient for our purposes in the following analysis.

3.2 Extent of spillover effects and altruism

Given that the population game has a unique monotone pure-strategy equilibrium, we next investigate how the ultimate distribution of altruism changes as the extent of spillover effects varies. In particular, we compare two supermodular games with positive spillovers, $\hat{G} = (X, \hat{\pi})$ and $G = (X, \pi)$, where the spillovers and their overall marginal effects are greater in the former than in the latter.

To be more precise, we additionally make the following assumptions. First, we assume that $\hat{\pi}_2(x_j, x_i)$ is greater than $\pi_2(x_j, x_i)$ such that $\pi_2(x_j, x^*(\lambda_i)) < \hat{\pi}_2(x_j, \hat{x}^*(\lambda_i))$ holds for $\forall x_j \in X$ and $\forall \lambda_i \in \Lambda$, where $x^*(\lambda_i)$ and $\hat{x}^*(\lambda_i)$ are the equilibrium actions in G and $\hat{G}$, respectively, and that $\pi_{22}(x_j, x_i) < \hat{\pi}_{22}(x_j, x_i)$ for $\forall (x_j, x_i) \in X^2$. That is, both the spillovers and their marginal changes are greater in $\hat{G}$ than in G. Second, under positive spillovers, the marginal payoff to an individual's own action is assumed to decrease at a slower rate as his opponent's action increases: $\pi_{112}(x_i, x_j) \geq 0$. It is also assumed that the marginal spillover benefits to the opponent are non-decreasing as the opponent's action increases: $\pi_{221}(x_j, x_i) \geq 0$. Let us call $\hat{G}$ a game with *larger spillover effects* than G. In order to focus on the effect of spillovers, the effect of an individual's action on his own payoff is the same in the two games: $\pi_1 = \hat{\pi}_1$ and $\pi_{11} = \hat{\pi}_{11}$. Also, we focus on the case in which the marginal payoffs to the individual and his opponent with respect to the individual's action decrease at a weakly slower rate as the individual's action increases: $\pi_{111}(x_i, x_j) \geq 0$ and $\pi_{222}(x_j, x_i) \geq 0$.

As already noted, altruistic individuals choose greater actions as the spillover benefits to their opponents increase. Hence, the following result is quite intuitive (though the proof is somewhat complicated):

Proposition 2. *Consider two population games in which the supermodular games G and $\hat{G}$ are played, where $\hat{G}$ has larger spillover effects than G. Then, the equilibrium action for an individual with any level of strictly positive altruism is strictly greater in the population game with $\hat{G}$ than in the population game with G.*

Proof. Let $x^*(\lambda_i)$ and $\hat{x}^*(\lambda_i)$ be the unique population equilibria with G and $\hat{G}$, respectively. Also, let $s^*(\lambda_i)$ and $\hat{s}^*(\lambda_i)$ be the unique Bayesian equilibria of G' and $\hat{G}' = \{X, \Lambda, f, \hat{U}_i\}$, respectively, where $\hat{U}_i(x_i, x_j, \lambda_i) = \hat{\pi}(x_i, x_j) + \lambda_i \hat{\pi}(x_j, x_i)$.

To prove the proposition, we show that the Bayesian equilibrium of $\hat{G}'$ is greater than that of G' for all positive λ_i, that is, $s^*(\lambda_i) < \hat{s}^*(\lambda_i)$ for all $\lambda_i \in \Lambda'$, where $\Lambda' \equiv \Lambda \backslash \{0\}$. If so, we can conclude that $x^*(\lambda_i) < \hat{x}^*(\lambda_i)$ because, as is clear from the proof of Proposition 1, the equilibrium actions in the two population games are given by $x^*(\lambda_i) = s^*(\lambda_i)$ and $\hat{x}^*(\lambda_i) = \hat{s}^*(\lambda_i)$.

Let $s(\lambda)$ be an arbitrary monotone pure strategy. Given $s(\lambda)$, a player solves the following maximization problems for each λ_i in the Bayesian games G' and $\hat{G}'$, respectively:

$$\max_{x_i} \quad L(x_i, \lambda_i; s, f) \equiv \int_{\lambda \in \Lambda} [\pi(x_i, s(\lambda)) + \lambda_i \pi(s(\lambda), x_i)] f(\lambda) d\lambda$$

and

$$\max_{x_i} \quad \hat{L}(x_i, \lambda_i; s, f) \equiv \int_{\lambda \in \Lambda} [\hat{\pi}(x_i, s(\lambda)) + \lambda_i \hat{\pi}(s(\lambda), x_i)] f(\lambda) d\lambda.$$

Because the integrands in the above expressions are continuous with respect to λ and continuously differentiable with respect to x_i, the order of differentiation and integration is, again, interchangeable. Thus, the first-order conditions are given by

$$L_1(x_i, \lambda_i; s, f) \equiv \int_{\lambda \in \Lambda} [\pi_1(x_i, s(\lambda)) + \lambda_i \pi_2(s(\lambda), x_i)] f(\lambda) d\lambda = 0 \tag{5}$$

and

$$\hat{L}_1(x_i, \lambda_i; s, f) \equiv \int_{\lambda \in \Lambda} [\hat{\pi}_1(x_i, s(\lambda)) + \lambda_i \hat{\pi}_2(s(\lambda), x_i)] f(\lambda) d\lambda = 0. \tag{6}$$

From $\pi_1 = \hat{\pi}_1$ and $\pi_2 < \hat{\pi}_2$, we have $L_1 < \hat{L}_1$ for all $x_i \in X$ and $\lambda_i \in \Lambda'$. In addition, from $\pi_{11} = \hat{\pi}_{11}(< 0)$ and $\pi_{22} < \hat{\pi}_{22}(< 0)$, the second-order derivatives of L and $\hat{L}$ satisfy $L_{11} < \hat{L}_{11} < 0$ for $\forall \lambda_i \in \Lambda'$. Therefore, the optimal action for any positive λ_i is greater in $\hat{G}'$ than in G'.[9] (The optimal action for $\lambda_i = 0$ is the same for G' and $\hat{G}'$.)

Let Σ be the set of monotone pure strategies. We equip Σ with the following metric, which has been introduced by Mason and Valentinyi [19]:

$$d(s, s') \equiv \sup_{\varrho \in X}\{\lambda'_i - \lambda_i | s(\tau_i) < \varrho < s'(\tau_i) \text{ or } s'(\tau_i) < \varrho < s(\tau_i), \forall \tau_i \ s.t. \ \lambda_i \leq \tau_i \leq \lambda'_i\}. \quad (7)$$

Intuitively speaking, this metric is the supremum of the horizontal distance between two strategies. As Mason and Valentinyi point out, (Σ, d) is a complete metric space. Next, let $T : \Sigma \to \Sigma$ and $\hat{T} : \Sigma \to \Sigma$ denote the best-response functions in games G' and $\hat{G}'$, respectively. From the above argument, $Ts(\lambda) < \hat{T}s(\lambda)$ for $\lambda_i \in \Lambda'$. Because $\hat{G}'$ is supermodular, $\hat{L}_1$ is weakly increasing in $s(\lambda)$. Combining this fact with $Ts(\lambda) < \hat{T}s(\lambda)$ yields $T^2s(\lambda) < \hat{T}^2s(\lambda)$ for all $\lambda \in \Lambda'$, where $T^2s(\lambda) = T(Ts(\lambda))$ and $\hat{T}^2s(\lambda) = \hat{T}(\hat{T}s(\lambda))$. By induction, we have $T^n s(\lambda) < \hat{T}^n s(\lambda)$ for any $n \in \boldsymbol{N}$, where $\boldsymbol{N}$ denotes the set of natural numbers. Hence, $\lim_{n\to\infty} T^n s(\lambda) \leq \lim_{n\to\infty} \hat{T}^n s(\lambda)$. Under Assumptions 1–3, T and $\hat{T}$ are contractions (Theorem 4 in Mason and Valentinyi [19]). Thus, by the contraction mapping theorem, we have: $s^*(\lambda) = \lim_{n\to\infty} T^n s(\lambda) \leq \lim_{n\to\infty} \hat{T}^n s(\lambda) = \hat{s}^*(\lambda)$. We next show that the strict inequality $s^*(\lambda) < \hat{s}^*(\lambda)$ holds for all $\lambda \in \Lambda'$. From $s^*(\lambda) \leq \hat{s}^*(\lambda)$, we have $L_1(x_i, \lambda_i; s^*, f) < \hat{L}_1(x_i, \lambda_i; \hat{s}^*, f)$ for $\forall x_i \in X$ and $\forall \lambda_i \in \Lambda'$. Thus, from $L_{11}, \hat{L}_{11} < 0$, $s^*(\lambda_i) < \hat{s}^*(\lambda_i)$ holds for $\forall \lambda_i \in \Lambda'$. Therefore, we can conclude that the equilibrium actions in the population game with $\hat{G}$ are strictly greater than those in the population game with G for all altruistic individuals: $(s^*(\lambda_i) =)\ x^*(\lambda_i) < \hat{x}^*(\lambda_i)\ (= \hat{s}^*(\lambda_i))$ for $\forall \lambda_i \in \Lambda'$. □

Furthermore, we obtain the following result, which leads to Proposition 3 below.

Lemma 1. *A marginal increase in the equilibrium action with an increase in altruism is greater in the population game with $\hat{G}$ than in the population game with G. Therefore, the difference between the equilibrium actions for any two individuals with different levels of altruism is greater in the population game with $\hat{G}$ than in the population game with G.*

Proof. Differentiating s^* and $\hat{s}^*$ with respect to λ_i yields:

$$\frac{ds^*(\lambda_i)}{d\lambda_i} = -\frac{\int_{\lambda\in\Lambda} \pi_2(s^*(\lambda), s^*(\lambda_i)) f(\lambda) d\lambda}{\int_{\lambda\in\Lambda} [\pi_{11}(s^*(\lambda_i), s^*(\lambda)) + \lambda_i \pi_{22}(s^*(\lambda), s^*(\lambda_i))] f(\lambda) d\lambda} \ (> 0) \quad (8)$$

and

$$\frac{d\hat{s}^*(\lambda_i)}{d\lambda_i} = -\frac{\int_{\lambda\in\Lambda} \hat{\pi}_2(\hat{s}^*(\lambda), \hat{s}^*(\lambda_i)) f(\lambda) d\lambda}{\int_{\lambda\in\Lambda} [\hat{\pi}_{11}(\hat{s}^*(\lambda_i), \hat{s}^*(\lambda)) + \lambda_i \hat{\pi}_{22}(\hat{s}^*(\lambda), \hat{s}^*(\lambda_i))] f(\lambda) d\lambda} \ (> 0). \quad (9)$$

[9] The optimal actions for all $\lambda_i \in \Lambda$ lie in X if $\hat{L}_1(\overline{a}, \lambda_{max}; s, f) \leq 0 \leq L_1(\underline{a}, \lambda_{min}; s, f)$ holds.

From the assumption stated at the beginning of this subsection, the supermodularity of $\hat{\pi}$, and Proposition 2, $\pi_2(s^*(\lambda), s^*(\lambda_i)) < \hat{\pi}_2(s^*(\lambda), \hat{s}^*(\lambda_i)) \leq \hat{\pi}_2(\hat{s}^*(\lambda), \hat{s}^*(\lambda_i))$ for all $\lambda_i, \lambda \in \Lambda$. Hence, the numerator of the fraction on the right-hand side of (9) is greater than that on the right-hand side of (8). Then, from $\pi_{111}, \pi_{112} \geq 0$ and Proposition 2, $\pi_{11}(s^*(\lambda_i), s^*(\lambda)) \leq \pi_{11}(\hat{s}^*(\lambda_i), s^*(\lambda)) \leq \pi_{11}(\hat{s}^*(\lambda_i), \hat{s}^*(\lambda)) = \hat{\pi}_{11}(\hat{s}^*(\lambda_i), \hat{s}^*(\lambda))(< 0)$ holds for all $\lambda_i, \lambda \in \Lambda$. In addition, from $\pi_{222}, \pi_{221} \geq 0$, $\pi_{22} < \hat{\pi}_{22}$, and Proposition 2, we have $\pi_{22}(s^*(\lambda), s^*(\lambda_i)) \leq \pi_{22}(\hat{s}^*(\lambda), \hat{s}^*(\lambda_i)) < \hat{\pi}_{22}(\hat{s}^*(\lambda), \hat{s}^*(\lambda_i))(< 0)$. Thus, the denominator of the fraction on the right-hand side of (9) is negative but greater than that on the right-hand side of (8). Therefore, $(0 <)\ ds^*(\lambda_i)/d\lambda_i < d\hat{s}^*(\lambda_i)/d\lambda_i$ holds for $\forall \lambda_i \in \Lambda'$. From $x^*(\lambda_i) = s^*(\lambda_i)$ and $\hat{x}^*(\lambda_i) = \hat{s}^*(\lambda_i)$ for $\forall \lambda_i \in \Lambda$, we have $dx^*(\lambda_i)/d\lambda_i < d\hat{x}^*(\lambda_i)/d\lambda_i$ for $\forall \lambda_i \in \Lambda'$.

It follows that for any λ_i and $\lambda'_i(> \lambda_i)$,

$$x^*(\lambda'_i) - x^*(\lambda_i) = \int_{\lambda_i}^{\lambda'_i} \frac{dx^*(\lambda)}{d\lambda} d\lambda < \int_{\lambda_i}^{\lambda'_i} \frac{d\hat{x}^*(\lambda)}{d\lambda} d\lambda = \hat{x}^*(\lambda'_i) - \hat{x}^*(\lambda_i).$$

Thus, the difference between the optimal actions of any two individuals with different levels of altruism is greater in the population game with $\hat{G}$ than in the population game with G. □

As an individual's action increases, his marginal utility decreases. However, under positive spillovers, the marginal payoffs to the individual and his opponent with respect to his action decrease at a weakly slower rate as the opponent's action increases ($\pi_{112}(x_i, x_j) \geq 0$ and $\pi_{221}(x_j, x_i) \geq 0$). Because all individuals choose greater actions in $\hat{G}$, the marginal utility of each individual with respect to his own action decreases more slowly with an increase in his action in $\hat{G}$ than in G. In addition, because of greater spillovers, a marginal increase in the degree of altruism raises the marginal utility with respect to the individual's own action more in $\hat{G}$ than in G. The above proposition results from these two factors.

Each young individual's altruism changes according to the difference between the average action of his opponents and his own action (see Eq. (3)). From Lemma 1, we obtain the following result:

Proposition 3. *Suppose that there are two societies, society G and society $\hat{G}$, with the same initial distribution of altruism among young children. In societies G and $\hat{G}$, individuals are randomly and repeatedly paired to play the supermodular games G and $\hat{G}$, respectively, where $\hat{G}$ has larger spillover effects than G. Then, either (i) the minimum altruism among old individuals in society $\hat{G}$ is higher than that in society G, (ii) the maximum altruism among old individuals in society $\hat{G}$ is lower than that in society G, or both. As the equilibrium actions in these societies become closer to linear with respect to the degree of altruism, it becomes more likely that both (i) and (ii) occur.*

Proof. Let m and $\hat{m}$ be the average equilibrium actions in societies G and $\hat{G}$, respectively. Let x^{*-1} and $\hat{x}^{*-1}$ denote the inverse function of the equilibrium actions x^* and $\hat{x}^*$, respectively. Then, either (i) $x^{*-1}(m) < \hat{x}^{*-1}(\hat{m})$, (ii) $x^{*-1}(m) > \hat{x}^{*-1}(\hat{m})$, or (iii) $x^{*-1}(m) = \hat{x}^{*-1}(\hat{m})$ holds. In the first case, from $dx^*(\lambda_i)/d\lambda_i < d\hat{x}^*(\lambda_i)/d\lambda_i$, the difference between the average action and an individual's equilibrium action in society $\hat{G}$, $\hat{\Delta}_i = \hat{m} - \hat{x}^*(\lambda_i)$, is greater than that in society G, $\Delta_i = m - x^*(\lambda_i)$, for all λ_i weakly less than $\hat{x}^{*-1}(\hat{m})$. Consequently, the levels of altruism of old individuals whose initial altruism levels were included in the range $[\lambda_{min}, \hat{x}^{*-1}(\hat{m})]$ are higher in society $\hat{G}$ than in society G. Therefore, the minimum altruism in society $\hat{G}$ is higher than that in society G: $\min\{\lambda_i^{old}\}_{i\in I} < \min\{\hat{\lambda}_i^{old}\}_{i\in I}$. On the other hand, in case (ii), Δ_i and $\hat{\Delta}_i$ satisfy $\hat{\Delta}_i < \Delta_i$ for all $\lambda_i \in [\hat{x}^{*-1}(\hat{m}), \lambda_{max}]$. This means that the altruism levels of old individuals whose initial altruism levels were included in the range $[\hat{x}^{*-1}(\hat{m}), \lambda_{max}]$ are lower in society $\hat{G}$ than in society G. Hence, the maximum altruism in society $\hat{G}$ is lower than that in society G: $\max\{\hat{\lambda}_i^{old}\}_{i\in I} < \max\{\lambda_i^{old}\}_{i\in I}$. In case (iii), $\Delta_i < \hat{\Delta}_i$ for $\lambda_i \in [\lambda_{min}, x^{*-1}(m))$, and $\hat{\Delta}_i < \Delta_i$ for $\lambda_i \in (x^{*-1}(m), \lambda_{max}]$. It follows that $\lambda_i^{old}(\lambda_i, \Delta_i) < \hat{\lambda}_i^{old}(\lambda_i, \hat{\Delta}_i)$ for $\lambda_i \in [\lambda_{min}, x^{*-1}(m))$ and $\hat{\lambda}_i^{old}(\lambda_i, \hat{\Delta}_i) < \lambda_i^{old}(\lambda_i, \Delta_i)$ for $\lambda_i \in (x^{*-1}(m), \lambda_{max}]$. Therefore, the distribution of old individuals' altruism in society $\hat{G}$ second-order stochastically dominates that of society G, or equivalently, the latter is a mean-preserving spread of the former. As x^* and $\hat{x}^*$ become closer to linear, $x^{*-1}(m)$ and $\hat{x}^{*-1}(\hat{m})$ approach the average initial altruism $\bar{\lambda}(\equiv \int_{\lambda\in\Lambda} \lambda f(\lambda) d\lambda)$ and, as a result, it becomes more likely that both $\min\{\lambda_i^{old}\}_{i\in I} < \min\{\hat{\lambda}_i^{old}\}_{i\in I}$ and $\max\{\hat{\lambda}_i^{old}\}_{i\in I} < \max\{\lambda_i^{old}\}_{i\in I}$ hold, that is, the range of old individuals' altruism in society $\hat{G}$ is included in that of society G. □

In the special case of linear equilibrium actions, we have a more clear-cut result as follows:

Corollary 1. *If the equilibrium actions in both societies are linear with respect to altruism, the distribution of old individuals' altruism in society G is a mean-preserving spread of that of society $\hat{G}$.*

Proof. This directly follows from the proof of Proposition 3 because if x^* and $\hat{x}^*$ are linear, we have $x^{*-1}(m) = \hat{x}^{*-1}(\hat{m}) = \bar{\lambda}$. □

The above results state that if two societies start from the same initial distribution of young children's altruism and if the interactions between individuals in these societies can be represented by a supermodular game, then the range of old individuals' altruism in the society with smaller spillover effects is never included in that of the society with larger spillover effects. Furthermore, when the equilibrium actions of individuals are proportional to the degree of altruism, individuals in the society with larger spillover effects become more homogeneous than those in the society with smaller spillover effects.

We do not obtain the above stated results from increases in productivity (i.e., increases in $\pi_1(x_i, x_j)$ and $\pi_{11}(x_i, x_j)$). This is because, although an increase in an opponent's action raises spillover benefits to the opponent (by supermodularity), an increase in an individual's action reduces spillover benefits ($\pi_{22}(x_j, x_i) < 0$). It follows that the marginal increase in the equilibrium actions with respect to the degree of altruism under higher productivity may or may not be greater than that under lower productivity (see (8) and (9), in particular the numerators). Thus, we do not obtain Proposition 3 and Corollary 1. The results are also ambiguous in the cases of submodular games with positive or negative spillovers. As seen in the proof of Proposition 2, the best response given an opponent's strategy $s(\lambda)$, $Ts(\lambda)$, is greater the greater the spillovers, whether positive or negative.[10] If the game is submodular, however, the best response is lower when the opponent's strategy is greater. Thus, it cannot be determined which of $T^2s(\lambda)$ and $\hat{T}^2s(\lambda)$ is greater, where $\hat{T}$ is the best response function for the game with greater spillovers. Furthermore, in the case of supermodular games with negative spillovers, the equilibrium actions are inversely monotone in the degree of altruism but are still greater in a game with smaller negative spillovers. Then, the marginal increase in the equilibrium actions with respect to altruism for a game with smaller negative spillovers may or may not be greater than that for a game with larger negative spillovers, because the absolute values of the numerator and the denominator of the fraction showing the former are both smaller than those of the fraction showing the latter (see, again, (8) and (9)). Therefore, both supermodularity and positive spillovers constitute sufficient conditions for Proposition 3 and Corollary 1.

4 Extension: N-player Games

The two-player games considered so far can be extended to N-player games. Suppose that individuals are matched to play an N-player symmetric game, $G_N = (\{1, 2, ..., N\}, X, \pi)$, where the payoff function is now defined on the set of vectors of N players' actions, $\pi : X^N \to \boldsymbol{R}$. Let $\boldsymbol{x}_{-i}$ denote the actions of the other players, $\boldsymbol{x}_{-i} = (x_1, ..., x_{i-1}, x_{i+1}, ..., x_N) \in X^{N-1}$. The payoff function satisfies $\pi^i = \pi(x_i, \boldsymbol{x}_{-i}) = \pi(x_i, \boldsymbol{x}_{\sigma(-i)})$, where σ is a permutation of $1, ..., i-1, i+1, ..., N$. Suppose also that an individual cares about the material payoffs of all the other individuals playing the game in addition to his own payoff. The

[10] We regard smaller negative spillovers as greater than larger negative spillovers.

utility function of an individual with λ_i can be written as[11]

$$U_i^N(x_i, \boldsymbol{x}_{-i}, \lambda_i) = \pi(x_i, \boldsymbol{x}_{-i}) + \lambda_i \sum_{j \neq i} \pi(x_j, x_i).$$

Let us assume, in addition to Assumption 2, that for all $i \in \{1, ..., N\}$,

$$|\pi_1(x_i, \boldsymbol{x}_{-i}) + \lambda_i \sum_{j \neq i} \pi_2(x_j, x_i)| \leq \omega \;\; \text{for some positive } \omega$$

and

$$\sum_{j \neq i} \pi_2(x_j, x_i) \geq \delta \;\; \text{for } \delta > 2\nu\omega$$

hold, which respectively correspond to Assumptions 1 and 3 in the case of two-player games. These assumptions guarantee, again, that there exists a unique monotone pure-strategy equilibrium in the corresponding symmetric N-player Bayesian game $G'_N = (X, \Lambda, f, U_i^N)$, where individuals' altruism is independently drawn according to the density function f.[12]

Another change must be made to an individual's maximization problem. Suppose that an individual expects that his opponents' actions are distributed according to the density function $\varphi(x)$. Then, his maximization problem is now written as

$$\max_{x_i} \quad \int_{\boldsymbol{x}_{-i} \in X_{-i}} U_i^N(x_i, \boldsymbol{x}_{-i}, \lambda_i) \Pi_{j \neq i} \varphi(x_j) d\boldsymbol{x}_{-i},$$

where X_{-i} is the space of all the profiles of the other individuals' actions.

Given the above modifications, it is straightforward to check that the results obtained in the previous section continue to hold for N-player games under slightly modified assumptions.[13] Therefore, we only state the main result without proof:

Proposition 4. *Consider two societies with the same initial distribution of young children's altruism. In one society, an N-player supermodular game G_N is played, and in the other society, another N-player supermodular game $\hat{G}_N$ is played. If $\hat{G}_N$ has larger spillover effects than G_N, then the range of old individuals' altruism in society G_N is never included in that of society $\hat{G}_N$. If the equilibrium actions in these societies are*

[11]By simply rescaling λ_i, we could also consider the case in which each individual cares about the average material payoff of the other players,

$$U_i^N(x_i, \boldsymbol{x}_{-i}, \lambda_i) = \pi(x_i, \boldsymbol{x}_{-i}) + \lambda'_i \bar{\pi}(x_i, \boldsymbol{x}_{-i}),$$

where $\bar{\pi}(x_i, \boldsymbol{x}_{-i}) \equiv \sum_{j \neq i} \pi(x_j, x_i)/(N-1)$ and $\lambda'_i = (N-1)\lambda_i$.

[12]Mason and Valentinyi [19] first derived these assumptions for N-player Bayesian games.

[13]For the assumptions stated at the beginning of subsection 3.2, the derivatives of the payoff functions with respect to the second argument should be replaced by those with respect to k-th argument, where $k = 2, ..., N$. For example, $\pi_k < \hat{\pi}_k$ should hold instead of $\pi_2 < \hat{\pi}_2$.

sufficiently close to linear, the latter is included in the former. If, in particular, the equilibrium actions in both societies are linear, the distribution of altruism of society G_N is a mean-preserving spread of that of society $\hat{G}_N$.

5 Conclusion and Discussion

This paper attempted to find socio-economic determinants of people's altruism and its distribution. We identified supermodularity and the extent of positive spillover effects as some of these determinants. That is, if the interactions between individuals in a society can be represented by a supermodular game, then when spillover effects are relatively large, either the lowest degree of altruism in the society is higher, the highest degree of altruism in the society is lower, or both. As the equilibrium actions become close to linear with respect to the degree of altruism, it becomes more likely that both occur. If, in particular, the equilibrium actions are linear, the distribution of altruism under smaller spillover effects is a mean-preserving spread of that under larger spillover effects.

To our knowledge, this study is the first attempt to identify socio-economic determinants of the distribution of altruism. However, many important issues remain unaddressed in this paper. First, our analysis considers only one generation, and thus, the vertical transmission of altruism from parents to children is not made explicit. As a result, the long-run implications for the distribution of altruism are not examined. Second, we found two socio-economic determinants of altruism, but we have not fully characterized all the determinants. Searching for the other characteristics of a game that can affect altruism formation should be worth the effort. Third, although we consider a random matching model, it may also be of interest to analyze a model of repeated interactions and to compare the results obtained from these two models because interactions within a company, community, or village are often repeated within a closed group. The main results of this paper may or may not be strengthened depending on how reputation effects relate to the difference between individuals' equilibrium actions. Finally, many theoretical implications, including the ones obtained in this paper, await empirical scrutiny. These challenging yet important issues are left for future research.

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