Abstract

This paper analyzes foreign aid to a small open economy which is represented by one-sector overlapping generations model. Capital market of this recipient economy is open, but its domestic savings and capital formations are small compared with the rest of the world. Inflows and outflows of international capital transfer are determined such that domestic capital return is equalized to the exogenous world interest rate.

Data shows that aid recipients are in fact small open economies. Most developing countries record non-zero figures of foreign direct investment. Greater part of them indicate small amounts of GDP because they are developing.

Multiple modalities of foreign aid are examined such as grants for consumption or income compensation. Grants and concessional loans for capital investment are also analyzed. Criteria of effectiveness of foreign aid are consumption, income and welfare of individuals in a recipient country.

It is shown that capital loan aid is more effective than other aid policies in terms of economic growth and welfare. Also, both income and capital grants may induce capital flight.

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1 Introduction

Does foreign aid work or not? If it does, under what conditions does it work? It is now almost half a century since a number of economic studies first challenged the open-ended macroeconomic question of “aid effectiveness.” It was once believed that the literature\(^1\) as represented by Burnside and Dollar (2000), had finally answered this question. That is, their empirical analysis on cross-country panel data concludes that foreign aid works on economic growth only when the capability of the recipient government to accept and process aid is high. Therefore, they claim that foreign aid is more effective under a better political environment. However, Easterly, Levine, and Roodman (2004) cast serious doubt on the results owing to limited robustness of their data set. Rather, Roodman (2007a) concludes that no decisive answer exists, based on his description of a series of empirical studies on aid effectiveness as “the anarchy of numbers” because the diverse results\(^2\) suggest many are fragile.

Rajan and Subramanian (2008) bring some closure to this perplexing state of the empirical literature. Based on commonly available panel data, they thoroughly regress most possible estimation models of aid effectiveness, which cover some existing studies. They find little robust evidence for a positive or negative relationship between foreign aid inflows and economic growth of a recipient country. The data do not support any effect on economic growth by policy, geographical environment, or form of aid. Roodman (2007b) argues that a quantitative approach will probably continue to disappoint as much as it offers hope, since the biggest challenge is demonstrating causation when


documenting correlation.

In order for empirical studies to move forward, a solid theoretical framework that explains the mechanism of foreign aid is indispensable. While there are several econometric studies of aid effectiveness, there is little analytical research of foreign aid. As Easterly (2003) urges, economic researchers need more theoretical analysis of foreign aid based on dynamic macro models. Of course, there is some excellent literature\(^3\) on dynamic optimization that provides us with important analytical findings related to aid effectiveness. However, not much of it is applied directly to empirical studies because the models are so complicated as to be unsuitable for numerical simulation.

Furthermore, most analytical literature of foreign aid is in the form of two-country open macro models, which imply that both donors and recipients are large countries. However, most aid recipients are in fact small countries in many aspects, such as in terms of output, trade volumes, and financial flows. In order to resolve the mixed and scattered results on transfer paradox in the existing studies, Cremers and Sen (2008) succeed in showing that transfer paradox may easily occur under normal situations despite the rare theoretical possibilities predicted by static trade models. Unfortunately, since their analysis\(^4\) is based on a two-country overlapping generations model, both these countries are assumed to be large. Therefore, income transfer in this analysis may not be identical to foreign aid.

This study assumes the recipient is a small open economy with capital mobility under one-sector overlapping generations model. The model is a simple version of Cremers and Sen (2008) without trade, and has some advantages for analyzing foreign aid policies. Uses of foreign aid as an outside resource transfer are characterized by basic needs (consumption aid), income compensation (income aid) and capital investment (capital aid). Since real ODA expenses consist of not only grants but also concessional loans, this study could be useful for empirical research on multiple aid modalities (cf.,

\(^3\)The following are some examples of dynamic optimization by an infinitely lived representative agent; Kemp’s (1995) study is based on a neoclassical growth model; Benarroch and Gaisford’s (2004) study is based on an endogenous growth model; and Naito and Ohdoi (2010) invent a two-country dynamic open macro model with multiple goods and multiple forms of productive capital.


3
Clemens, Radelet, and Bhavnani, 2004).

Because of various objectives for ODA distributions (such as the millennium development goals of the United Nations), there should be multiple criteria for the effectiveness of foreign aid. Consumption by individuals is a main objective of aid when donors care about poverty measures, including the poverty ratio, to reflect calorie intake. Income level is an undisputable objective as aid recipients are classified by per capita gross national income (GNI) and are further classified as least developed countries, low income countries, low middle income countries, and so on. This categorization is used for aid recipients by the DAC, a subcommittee of major ODA donors among OECD members.

Economic growth and the welfare improvement of recipients are two objectives of ODA defined by the DAC, at least one of which is adopted by much of the economic literature on foreign aid as an efficiency criterion. This study utilizes as many criteria of effectiveness as possible when analyzing foreign aid policies. It finds that capital loan aid to our small open economy is better than other aid policies across most criteria, including growth and welfare.

This study is aimed at filling the gap between economic researchers and foreign aid practitioners, with our analytical results summarized as realistic policy direction given the multiple uses and criteria of aid. As Roodman (2007b) stresses, the vast majority of employees in international development and bilateral aid institutions do not seek the minute findings of highly technical econometric studies, but rather wait a simple way to understand foreign aid. With the help of simple dynamic frameworks, most aid researchers and workers would understand the mechanism of aid in a recipient economy. It is hoped that this study at least to contributes to decreasing the serious shortage of macroeconomic policy direction for foreign aid.

The organization of the remainder of this paper is as follows. Section 2 confirms aid recipients as small open economies with financial flows. Section 3 introduces a small open economy of overlapping generations as an aid recipient, and defines multiple uses of permanent inflows of foreign aid. Section 4 explains the effects of foreign aid policies and compares them by multiple criteria of the effectiveness of foreign aid. Section 5 discusses the effects of

\footnote{Our major results will not be altered even if the number of generations is extended beyond two. By extending to 50 generations (i.e., Kotlikoff and Summers, 1981), it may become more applicable for empirical and simulation analysis because 1 period is equivalent to almost 1 year.}
information asymmetry, capital control, and technological progress. Section 6 provides conclusions and direction for future research.

2 Aid recipients and small open economies

2.1 Recipients and donors

In order for one country to be a foreign aid recipient, its income or production level must be lower than that of the rest of the world. This is why most developing countries are on the DAC list of aid recipients. Therefore, aid recipients have smaller per capita GNI (income) and gross domestic product (GDP), compared to developed countries (OECD members).

Recently, many developing countries have experienced positive economic growth, with quite a few of them enjoying a rapid growth rate of around 10% a year. The Brazil, Russia, India, China, and South Africa (BRICS) group of developing countries has become a major economic power bloc because of high levels of production and international trade. China, in particular, has become the world’s second largest economy measured by GDP.

According to the World Development Indicators (WDI), most recipients of aid have received positive ODA disbursements over the past 40 years. This shows even after the recent uprising of developing economies that there still exists a large number of recipients and a small number of donors in the world. The number of recipients is even increasing, partly because per capita GNI of most newly born developing countries is relatively low, and partly because the graduation of aid recipients to aid donors seldom occurs (cf., South Korea).

2.2 Capital market and international financial flows

If domestic volume of savings or investment in a country is relatively small, this country can be termed an economy with small capital stock. Since the world’s total capital stock is the sum of savings or investment by all countries, one country is small in terms of saving or investment if its share of the total is small. According to the WDI, the savings or investment share of developing countries tends to be less than that of major donor countries, except for large developing countries such as those in the BRICS. Therefore, the greater part of aid recipients comprise economies with small capital
The capital market of a country is open if its financial inflows or outflows are positive. All aid recipients are open in terms of capital stock because their international inflows and outflows are positive, as the WDI show. Therefore, aid recipients are economies with small and open capital flows.

The smallness and openness of economies can be also measured by their share of capital inflows or outflows against the world total, just as outlined above. While it is true that the shares of large developing countries are not small, those of developing countries in general are low, compared with major DAC members. Thus, aid recipients are small open economies in terms of international financial flows.

2.3 Capital mobility

In this study, a recipient country is assumed to have perfect capital mobility. Thus, the question is whether capital flows are mobile for aid recipients. It is extremely difficult to determine the degree of capital mobility for each country. However, we may be able to calculate the ratio of capital outflows to domestic savings, or that of capital inflows to domestic investment, as possible measures of the WDI.

A country may have easy access to foreign capital markets if there is a high ratio of capital outflows to domestic savings. Similarly, foreign countries may have easy access to its domestic capital markets if there is a high ratio of capital inflows to domestic capital investment. Unfortunately, this generalization does not always hold. Even if data show relatively large foreign direct investment (FDI) inflows and outflows, this does not guarantee flexibility of capital flows. For example, FDI inflows may be large, not because the aid recipient’s capital markets are fully mobile, but because there are numerous and lucrative business opportunities.

In principle, capital mobility is defined in terms of laws and regulations imposed on any domestic and international financial flows. Chinn and Ito (2008) create an index of financial openness based on existing restrictions and controls by most countries in the world. The index is normalized to show the highest financial openness at 100 and the lowest at 0. If the index of a country is sufficiently high, the country can be regarded as an economy with flexible capital mobility. The openness of most OECD members is measured at more than 75 (the highest range).

However, these indexes show no clear pattern for non–OECD members.
The BRICS countries are relatively less mobile or more restrictive for international financial flows although their FDI inflows and outflows tend to be large. China, India, and South Africa are measured at less than 25 (the lowest range); Russia’s at between 25 and 50; and Brazil’s at between 50 and 75.

The results for smaller developing countries are similar to large developing countries. For example, Egypt, Guyana, Honduras, Liberia, Peru, Uganda, Uruguay, and Zambia are measured at more than 75. Bolivia, Cambodia, Ecuador, Indonesia, Kenya, Paraguay, and Sudan are measured at between 50 and 75. Other African and many Asian countries are measured at less than 25. Therefore, we must assume perfect capital mobility for an aid recipient because this paper is an analytical study of foreign aid.

3 The Model

3.1 One–sector economy of overlapping generations

A recipient is represented by an economy of two overlapping generations: the young \((L(t))\) and the old \((L(t+1))\) in each period. This one good \((Y(t))\), constant population (labor) economy \((L(t) = L(t+1) = 1)\) with one productive capital \((K(t))\), is featured by the neoclassical production function \((AF(L_1(t), K(t)))\) and utility function \((u(c_1(t), c_2(t+1)))\), where subscripts 1 and 2 represent the young \(L(t)\): \(L_1(t)\) and the old \(L(t)\): \(L_2(t)\). An exogenous parameter \((A > 0)\) shows total factor productivity (TFP) or technology level as the Solow residual.

Each young individual \((L_1(t))\) chooses his or her young \((c_1(t))\) and old consumption \((c_2(t+1))\) to maximize utility, given two budget constraints of his or her young and old income levels: \(Income[L_1(t)] = w(t) = c_1(t) + s(t)\) and \(Income[L_2(t)] = (1 + r(t+1)s(t)) = c_2(t+1)\) where \(w(t)\), \(r(t+1)\), and \(s(t)\) are wages, return of capital, and savings, respectively for the individual.
3.2 Equilibrium and perfect capital mobility

Domestic input markets are perfect such that \( w(t) = A(f(k(t)) - k(t)f'(k(t))) \) and \( r(t) = Af'(k(t)) \), where \( k(t) \) is individual capital stock\(^6\). Under normal assumptions\(^7\) plus a perfect output market and rational expectations, a unique dynamic optimal path is derived. Domestic savings are represented by an increasing and concave function of \( k(t) \): \( s(t) = s(k(t)), \frac{\partial s}{\partial k(t)} > 0, \frac{\partial^2 s}{\partial k(t)^2} < 0 \).

Capital is perfectly mobile between domestic and international markets. Since the recipient is a small open economy, the domestic amount of capital \( (k) \) becomes constant such that the domestic return on capital \( (r) \) is equalized to an exogenous world interest rate \( (r_W) \) as per interest parity conditions: \( r = Af'(k) = r_W \), where underlines represent Autarky levels without foreign aid. The recipient economy is stabilized as output, wages, young and old amounts of consumption, and the utility of each individual are all constant. This economy does not move to any steady state without any dynamic transition.

Any positive portion of domestic savings \( (s) \) may flow out of this small open economy \( (\text{FDI outflow: } ko) \), and that of domestic investment \( (k) \) may flow in from the rest of the world \( (\text{FDI inflow: } ki) \): \( 0 \leq ko \leq s \) and \( 0 \leq ki \leq k \). The domestic capital market equilibrium condition is given as:

\[
s - ko + ki = k. \tag{1}
\]

Equivalently, the net FDI flow is equalized to net domestic investment: \( ki - ko = k - s \). Domestic capital is determined by outside forces while domestic savings are determined by internal optimization\(^8\). As TFP shrinks and/or the world interest rate rises, domestic investment of capital stock decreases:

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\(^6\) The per capita production function \( (f(k)) \) is increasing and concave \( (f'(k) > 0 \text{ and } f''(k) < 0) \), satisfying the Inada conditions \( \lim_{k \to 0} f'(k) = \infty \text{ and } \lim_{k \to \infty} f'(k) = 0 \).

\(^7\) The consumption of both the young and old is of normal goods. For a unique optimal path, it is sufficient to assume that saving is non-decreasing function of return of capital: \( s(t) = s(w(t), r(t + 1)), \frac{\partial s(t)}{\partial w(t)} > 0, \frac{\partial s(t)}{\partial r(t+1)} \geq 0 \). According to Cremers and Sen (2008), savings are strictly assumed to be increasing \( (\frac{\partial s(t)}{\partial r(t+1)} > 0) \) in order to incorporate the intertemporal effects of foreign aid.

\(^8\) FDI inflows and outflows, and domestic savings and investment are not always equalized: \( ko \lessgtr ki \) and \( s \lessgtr k \), respectively. In the end, wages and domestic savings are determined as: \( w = A(f(k) - kf'(k)) \) and \( s = (w, r_W) \).
\[ k = k(A, r_W), \quad \frac{\partial k}{\partial A} > 0, \quad \frac{\partial k}{\partial r_W} < 0. \]  

(2)

3.3 Objectives of donors and criteria of effectiveness

Donors send one–time resource transfer \((m(t) > 0)\) as foreign aid among multiple modalities at period \(t\) to the young \((L_1(t))\) in a recipient country. Donors support the country by sending foreign aid if its domestic capital stock is too low to produce a sufficient amount of output owing to technological constraints. In order to help future and weaker generations, the first and second objectives of aid are consumption \((c_1)\) and income \((Income[L_1(t)])\) of the young (cf., Cremers and Sen, 2008). The third and fourth objectives are per capita output \((y = \frac{Y}{L})\) and the growth rate \((gr(t) = \frac{y(t)-y(t-1)}{y(t-1)}\) of the recipient economy because both (in particular, growth) are standard objectives for most macro–econometric studies on ODA panel data. According to static and dynamic trade literature, the fifth objective is the welfare \((u_t = u(c_1(t), c_2(t+1)))\) of present and future generations of the recipient country. Therefore, the effectiveness of foreign aid must answer the following question: which aid policy improves each of the above-mentioned objectives?

Definition 1 One–time foreign aid is effective

if \(\frac{dc_1(t)}{dm(t)} \geq 0, i \geq t\) for the consumption criterion,

if \(\frac{dIncome[L_1(i)]}{dm(t)} \geq 0, i \geq t\) for the income criterion,

if \(\frac{dy(i)}{dm(t)} \geq 0, i \geq t\) for the production criterion,

if \(\frac{dgr(i)}{dm(t)} \geq 0, i \geq t\) for the growth criterion,

if \(\frac{du}{dm(t)} \geq 0, i \geq t\) for the welfare criterion,

where inequality is strict in at least one period.

4 Foreign Aid

4.1 Consumption aid and income aid

Consumption aid \((\Delta m(t) = \Delta m_C(t) = \epsilon > 0)\) allows the young to consume more than their original level. Since this aid is assumed to work as emergency aid against starvation, it affects only the consumption of the young \((\frac{dc_1(t)}{dm_C(t)} = 1)\). However, this cannot improve all other economic conditions, including
domestic savings without affecting the optimization of the young. The young simply consume the entire aid amount, which is provided after their savings decisions.

Income aid \( \Delta m_I(t) = \epsilon > 0 \) allows the young to receive more income than their original level \( \left( \frac{dIncome[L_1(t)]}{dm_I(t)} = 1 \right) \). Since this aid is assumed to work as a wage compensation against poverty, it affects only the income of the young. The young \( (L_1(t)) \) consume and save more as their income increases.

**Proposition 1** The effectiveness of consumption aid and income aid to a small open economy is summarized as follows:

- a. Consumption aid is effective for consumption and welfare criteria.
- b. Income aid is effective for consumption, income, and welfare criteria.
- c. Neither is effective for production and growth criteria.
- d. Consumption aid is more effective than income aid for the consumption criterion.
- e. Consumption aid is less effective for income and welfare criteria.

**Proof.** The optimization problem with consumption aid is shown as:

\[
\max u(c_1(t), c_2(t+1)), \text{ such that } w(t) = c_1(t) + s(t) \text{ and } (1 + r_W)s(t) = c_2(t+1).
\]

From assumption, \( \frac{dc_1(t)}{dm_C(t)} = 1 \) and \( c_1(t) = c_1 + \Delta m_c(t) \).

So, \( \frac{du}{dm_C(t)} > 0 \).

Since \( k(t) \) is prefixed to \( k \), \( \frac{dk(t)}{dm_C(t)} = 0 \), and so,

\[
\frac{dw(t)}{dm_C(t)} = \frac{dy(t)}{dm_C(t)} = \frac{dr(t)}{dm_C(t)} = 0.
\]

Then, \( Income[L_1(t)] = w(t) = w \) and \( \frac{ds(t)}{dm_C(t)} = 0 \).

For perfect capital mobility, \( \frac{dr(t+j)}{dm_C(t)} = \frac{dk(t+j)}{dm_C(t)} = 0 \) for \( j \geq 1 \).

So, \( \frac{dw(t+j)}{dm_C(t)} = \frac{dy(t+j)}{dm_C(t)} = \frac{dr(t+j)}{dm_C(t)} = 0 \).

Since \( Income[L_1(t+j)] = w(t+j) = w \) for \( j \geq 1 \),

\[
\frac{dlIncome[L_1(t+j)]}{dm_C(t)} = \frac{ds(t+j)}{dm_C(t)} = 0.
\]

Then, \( \frac{dc_2(t+j)}{dm_C(t)} = 0 \) and \( \frac{du(t+j)}{dm_C(t)} = 0 \) for \( j \geq 1 \).

The optimization problem with income aid is shown as:

\[
\max u(c_1(t), c_2(t+1)), \text{ such that } w(t) + \Delta m_I(t) = c_1(t) + s(t) \text{ and } (1 + r_W)s(t) = c_2(t+1).
\]

Since \( k(t) \) is prefixed to \( k \), \( \frac{dk(t)}{dm_I(t)} = 0 \), and so,
\[
\frac{dw(t)}{dm_{1}(t)} = \frac{dy(t)}{dm_{1}(t)} = \frac{dgr(t)}{dw(t)} = 0.
\]

Then, \(Income[L_{1}(t)] = w + \Delta m_{1}(t)\) or \(\frac{dIncome[L_{1}(t)]}{dm_{1}(t)} = 1\).

Since both consumptions are normal,
\[0 < \frac{dc_{1}(t)}{dm_{1}(t)} < 1 \text{ and } 0 < \frac{ds(t)}{dm_{1}(t)} < 1.\]

So, \(\frac{dc_{2}(t+1)}{dm_{1}(t)} > 0\) and \(\frac{du_{t}}{dm_{1}(t)} > 0\).

Similarly to the consumption aid case above,
\(Income[L_{1}(t+j)] = w(t+j) = w \text{ for } j \geq 1.\)

Then, \(\frac{dr(t+j)}{dm_{1}(t)} = \frac{dk(t+j)}{dm_{1}(t)} = \frac{dw(t+j)}{dm_{1}(t)} = \frac{dc_{2}(t+j+1)}{dm_{1}(t)} = 0 \text{ for } j \geq 1.\)

So, \(\frac{dy(t+j)}{dm_{1}(t)} = \frac{dgr(t+j)}{dm_{1}(t)} = \frac{du_{t+j}}{dm_{C}(t)} = 0 \text{ for } j \geq 1.\)

Therefore, \(\frac{dc_{1}(t)}{dm_{C}(t)} > \frac{dc_{1}(t)}{dm_{1}(t)} \text{ and } \frac{dw(t)}{dm_{1}(t)} > \frac{dw(t)}{dm_{C}(t)}.\)

From the envelope theorem, \(\frac{du_{t}}{dm_{1}(t)} > \frac{du_{t}}{dm_{C}(t)}.\) \(\blacksquare\)

Income aid cannot improve the output level of the recipient because such aid does not affect domestic investment in this small open economy. Once any increased savings are invested domestically, capital return becomes lower than the world interest rate. Since the young choose to invest the additional amount of their savings outside, new capital outflow \((\Delta ko(t))\) occurs, which can be called capital flight. It is ironic that income aid causes capital flight.

**Lemma 1** Capital flight occurs if income aid is sent to a small open economy: \(0 < \Delta ko(t) = \Delta s(t) < \epsilon.\)

The existing young in the aid recipient country \((L_{1}(t))\) are assumed to consume all consumption aid while they can consume and save all income aid. This is why they are better off with income aid than with consumption aid. Of course, consumption aid is ideal if the young are suffering from starvation. Either way, neither forms of aid improve the welfare of later generations.

**Lemma 2** One-time consumption and income aid to a small open economy are effective for one period only owing to capital flight. Both aid policies must be repeated in perpetuity if all future generations are to be better off.

### 4.2 Capital grant aid

Capital grant aid \((\Delta m_{KG}(t) = \epsilon > 0)\) allows the young \((L_{1}(t))\) to receive additional capital stock for their optimization level. They are assumed to own this official financial transfer as capital assets. When they become old,
they \((L_2(t))\) consume all their income, which consists of the returns and principal sum of their savings and this aid\(^9\). Since the young predict this aid under rational expectations, they \((L_1(t))\) consume more and save less than their original levels, and have more secured income in their old age.

**Proposition 2** The effectiveness of capital grant aid to a small open economy is summarized as follows:

- **a.** Capital grant aid is effective for consumption, income, and welfare criteria.
- **b.** Capital grant aid is not effective for production and growth criteria.
- **c.** Capital grant aid is as effective as income aid for consumption and welfare criteria.

**Proof.** From the putty–clay assumption, the optimization problem with capital grant aid is shown as:

\[
\begin{align*}
\max u(c_1(t), c_2(t+1)), \text{ such that } \\
w(t) = c_1(t) + s(t) \text{ and } (1 + r_W)(s(t) + \Delta m_{KG}(t)) = c_2(t + 1).
\end{align*}
\]

Since \(k(t)\) is prefixed to \(k\),

\[
\frac{dk(t)}{dm_{KG}(t)} = \frac{dy(t)}{dm_{KG}(t)} = \frac{dgr(t)}{dm_{KG}(t)} = 0,
\]

\[
\frac{dw(t)}{dm_{KG}(t)} = \frac{Income[L_1(t)]}{dm_{KG}(t)} = 0, \text{ or } Income[L_1(t)] = w.
\]

However, \(Income[L_2(t)] = (1 + r_W)(s(t) + \Delta m_{KG}(t))\).

From rational expectations,

\[
Income[L_2(t)] > Income[L_2(t)] = (1 + r_W)z.
\]

Equivalently, \(-1 < \frac{ds(t)}{dm_{KG}(t)} < 0\) and \(\frac{dc_1(t)}{dm_{KG}(t)} > 0\).

In addition, \(\frac{dc_2(t+1)}{dm_{KG}(t)} > 0\) and \(\frac{du(t)}{dm_{KG}(t)} > 0\).

Still, for perfect capital mobility,

\[
\frac{dr(t+j)}{dm_{KG}(t)} = \frac{dk(t+j)}{dm_{KG}(t)} = \frac{dw(t+j)}{dm_{KG}(t)} = 0 \text{ for } j \geq 1.
\]

So,

\[
\frac{dIncome[L_1(t+j)]}{dm_{KG}(t)} = \frac{dc_2(t+j+1)}{dm_{KG}(t)} = \frac{du(t+j)}{dm_{KG}(t)} = \frac{dy(t+j)}{dm_{KG}(t)} = \frac{dgr(t+j)}{dm_{KG}(t)} = 0 \text{ for } j \geq 1.
\]

In addition, \(\frac{dc_3(t)}{dm_{KG}(t)} = 1 > \frac{dc_1(t)}{dm_{KG}(t)}\).

In this one-good model, all prices of \(c_1(t), s(t),\) and \(k(t+1)\) are equal to 1.

Let subscripts \(I\) and \(KG\) represent those levels.

\(^9\)Transformation of saved outputs into capital stocks in this model follows the “putty–clay” assumption. Capital grants \((m_{KG}(t))\) cannot be consumed within the same period \((t)\) and are used for investment: the clay assumption. However, that amount can be consumed at the end of the next period \((t+1)\) which consists of a part of the principal of total capital stock \((k(t+1))\): the putty assumption.
with income aid and capital grant aid in period $t$. Checking both optimization problems with income aid and capital grant aid ($\Delta m_I (t) = \Delta m_{KG} (t) = \epsilon$), it is clear that both amounts of total capital stock owned by $L_1 (t)$ are identical to each aid case:

\[
s_I (t) = s + \Delta s (t)_I = s_{KG} (t) = s + \Delta s (t)_{KG} + \Delta m_{KG} (t),
\]

where $\Delta s (t)_I > 0$, $\Delta s (t)_{KG} < 0$, and $\Delta m_{KG} (t) > 0$. Therefore, $\frac{dc_1(t)}{dm_I(t)} = \frac{dc_3(t)}{dm_{KG}(t)}$ and $\frac{dc_2(t+1)}{dm_I(t)} = \frac{dc_2(t+1)}{dm_{KG}(t)}$.

So, $\frac{du}{dm_I(t)} = \frac{du}{dm_{KG}(t)}$. ■

Capital grant aid cannot improve the output level of the recipient for the same reason as income aid: once any portion of capital given to the young is invested domestically, the capital return becomes lower than the world interest rate. Since the young choose to invest this additional amount of their asset holding outside, capital flight occurs. Even though this capital grant is invested domestically, the same amount of domestic savings moves outside instead. It is even more ironic that capital grant aid causes capital flight than the fact that income aid causes capital flight.

**Lemma 3** Capital grant aid to a small open economy causes the same amount of capital flight as does income aid: $0 < \Delta k_{I}(t) = \Delta s (t)_{KG} + \Delta m_{KG} (t) = \Delta s (t)_I < \epsilon$.

In spite of the traditional wisdom of foreign aid, both income and capital grant aid have equivalent effects on consumption and welfare. This is not because the recipient is small and open, but because of dynamic optimization under rational expectations. The young predict both forms of aid and their effects perfectly. Nonetheless, there is a clear difference in that the domestic savings increase under income aid but decrease under capital grant aid. Either way, capital grant aid does not improve the welfare of later generations.

**Lemma 4** One–time capital grant aid to a small open economy is effective for only one period owing to capital flight. This aid policy must be repeated in perpetuity if all future generations are to be better off.

### 4.3 Capital loan aid

Capital loan aid allows the recipient economy to hold more capital stock than its optimized level. Private investors (the young and foreign investors) are
assumed to receive an incentive of interest subsidies for domestic investment. They are compensated for their lost interest by a grant element \((\text{ge}(t))\):

\[
\text{ge}(t) = \frac{r_W - r(t+1)}{r_W}, 0 \leq \text{ge}(t) < 1. \tag{3}
\]

Under rational expectations, private investors invest more domestically, expecting higher returns for domestic capital with the higher grant element. For simplicity, donors are assumed to prefer that the young \((L_1(t))\) rather than foreign investors receive this aid. The young can invest as much as they want at home, and foreign investors can fill a gap, if any. The young \((L_1(t))\) consume less and save more in their youth, and have more income in their old age. The young in the next period \((L_1(t+1))\) receive more wages and consume more, benefiting from higher production with more domestic capital stock.

**Proposition 3** The effectiveness of capital loan aid to a small open economy is summarized as follows:

a. Capital loan aid is effective for income, production, growth, and welfare criteria.

b. Capital loan aid is not effective for the consumption criterion.

c. Capital loan aid improves the welfare of both present and next period generations, but has no effect after two periods.

**Proof.** The optimization problem with capital loan aid is shown as:

\[
\max u(c_1(t), c_2(t+1)), \text{ such that }
\]

\[
w(t) = c_1(t) + s(t) \text{ and } (1 + r_W) s(t) = c_2(t+1).
\]

Since \(k(t)\) is prefixed to \(k\),

\[
\frac{dk(t)}{d\text{ge}(t)} = \frac{dy(t)}{d\text{ge}(t)} = \frac{dgr(t)}{d\text{ge}(t)} = 0.
\]

So,

\[
\frac{dw(t)}{d\text{ge}(t)} = \frac{d\text{Income}[L_1(t)]}{d\text{ge}(t)} = 0.
\]

From Equation 3 and Footnote 7 \(\frac{\partial s(t)}{\partial r(t+1)} > 0\), \(\frac{dr(t+1)}{d\text{ge}(t)} < 0\).

For the interest parity condition, \(r(t+1) + r_W \text{ge}(t) = r_W\).

Then, \(s(t) = \text{ge} + \Delta k(t+1)\) and \(\Delta k(t+1) > 0\).

Thus, \(\text{Income}[L_2(t)] = (1 + r(t+1) + r_W \text{ge}(t)) s(t) = (1 + r_W)(\text{ge} + \Delta k(t+1))\).

From Equation 3, \(\frac{dk(t+1)}{d\text{ge}(t)} = -\frac{Af''(k(t+1))}{r_W} > 0\).

In addition, \(\frac{dy(t+1)}{d\text{ge}(t)} = -A^2f'(k(t+1))f''(k(t+1)) > 0\),

\[
\frac{dgr(t+1)}{d\text{ge}(t)} = -\frac{Af'(k(t+1))f''(k(t+1))}{f(k(t+1))} > 0,
\]

14
\[
\frac{dw(t+1)}{dge(t)} = A^2 k(t+1) f''(k(t+1)) > 0.
\]
Therefore, \( \frac{ds(t)}{dge(t)} > 0, \frac{dc_1(t)}{dge(t)} = -\frac{ds(t)}{dge(t)} < 0, \frac{dc_2(t+1)}{dge(t)} > 0. \)

By the envelope theorem, \( \frac{du_t}{dge(t)} > 0. \)

Finally, \( \frac{ds(t+1)}{dge(t)} > 0, \frac{dc_2(t+2)}{dge(t)} > 0, \text{ and } \frac{du_{t+1}}{dge(t)} > 0. \)

However, for perfect capital mobility,
\[
\frac{dr(t+j)}{dge(t)} = \frac{dk(t+j)}{dge(t)} = \frac{dw(t+j)}{dge(t)} = 0 \text{ for } j \geq 2.
\]
So, \( \frac{dIncome[L_1(t+j)]}{dge(t)} = \frac{dc_2(t+j+1)}{dge(t)} = \frac{du_{t+j}}{dge(t)} = \frac{dy(t+j)}{dge(t)} = \frac{dgr(t+j)}{dge(t)} = 0 \)
for \( j \geq 2. \)

It is natural that concessional loans should not be sent to those economies suffering from starvation because this aid policy may worsen the situation by reducing consumption. However, capital loan aid has an advantage in that it encourages the recipient economy to grow, even if only for one period. This is because the young choose to invest and save more at home. Contrary to other aid policies, capital loan aid realizes positive economic growth. It might seem counter-intuitive that concessional loans rather than capital grants improve the welfare of present and future young generations. Since the positive effects of one-time capital loan aid is limited to two periods, this aid policy should be repeated every second period for all future generations to be better off.

In the case of capital loan aid, the directions and amounts of capital inflows and outflows are more complex than those of capital grant aid. This is because both domestic amounts of capital and savings increase for different reasons. Capital stock is determined by interest parity while savings are determined by dynamic optimization. Increased amounts of capital are not necessarily equalized to those of savings under this aid. Besides, as Equation 1 shows, the amount of capital inflows are independent of those of outflows.

**Corollary 1** If capital loan aid is sent to a small open economy, capital flight may or may not occur, depending on the capital account positions before and after this aid.

**Proof.** Let subscript \( KL \) represent capital loan aid.

From Proposition 3,
\( k_{KL}(t+1) > \bar{k} \) and \( s_{KL}(t) > \bar{s}. \)
Consider a case: \( s_{KL}(t) \geq k_{KL}(t+1). \)
From Equation 1, \( \bar{k} - k_o = \bar{k} - \bar{s}. \)
Then, the net flow is non negative: \( k_{i_{KL}}(t) - k_{o_{KL}}(t) \leq 0. \)
The young \((L_1(t))\) invest \( k_{KL}(t+1) \) at home and
the rest outside: \( k_{0KL} (t) = s_{KL} (t) - k_{KL} (t + 1) \).

If \( k_{0KL} (t) - k_0 > 0 \), this difference is capital flight:
\[
\Delta k_{0KL} (t) = k_0 (t)_{KL} - k_0.
\]
If \( k_{0KL} (t) - k_0 < 0 \), capital outflow shrinks, and capital flight does not occur.

Consider the opposite case: \( s_{KL} (t) < k_{KL} (t + 1) \).
Then, the net flow is positive: \( k_{iKL} (t) - k_{0KL} (t) > 0 \).
The young invest \( s_{KL} (t) \) at home, and foreign investors make up the rest: \( k_{iKL} (t) = k_{KL} (t + 1) - s_{KL} (t) \).
Since net flow is positive, capital flight does not occur.

Donors may have to prepare for more expenses under capital loan aid than other forms of grant aid. The cost of loan aid increases exponentially as the grant element rises. This is because the gap between the world interest rate and the domestic return on capital increases exponentially with the decreasing return on capital. On the contrary, the cost of grant aid increases at a constant rate (one). Thus, donors face a trade-off between the economic growth of recipients and their own budgetary costs of aid.

**Corollary 2** Capital loan aid may be less cost effective than income aid and capital grant aid.

**Proof.** Compare capital loan aid with income or capital grant aid.
The total cost of capital loan aid is represented as:
\[
r_{Wge} (t) k (t + 1) = (r_{W} - r (t + 1)) k (t + 1).
\]
From Equation 3,
\[
\frac{d(r_{W} ge(t) k(t+1))}{d_{ge(t)}} = r_{W} k (t + 1) - Af'' (k (t + 1)) ge (t) > 0.
\]
Since the output price is 1,
\[
\frac{dm(t)}{d_{m(t)}} = \frac{dm(t)}{d_{mKG(t)}} = 1.
\]
Then, capital loan aid is more expensive
if \( \frac{d(r_{W} ge(t) k(t+1))}{d_{ge(t)}} > 1 \).
From the interest parity condition, \( r_{W} = Af' (\bar{k}) \).
In addition, \( k (t + 1) > k \) for \( ge (t) > 0 \).
If \( r_{W} \bar{k} = Af' (\bar{k}) \bar{k} > 1 \), it is sufficient that \( \frac{d(r_{W} ge(t) k(t+1))}{d_{ge(t)}} > 1 \).

Since the original amount of domestic capital is small for a typical developing country, the marginal product of capital \( (f' (\bar{k})) \) is high, and it is more
likely that $Af'(k)k > 1$. However, if TFP($A$) is extremely low, it could be true
that $Af'(k)k < 1$. Then, capital loan aid becomes the ideal form of aid for donors, although this is nothing more than a theoretical possibility.

5 Discussion

5.1 Information structure and the timing of aid

Grant aid could be sent to the young as a surprise without any prior information. In other words, it is dynamically equivalent that the young receive grant aid just after their optimization. In this extreme case of imperfect information, income grant aid becomes identical to consumption aid. All that the young can do is consume this aid resource in their youth.

Capital grant aid ($\Delta \tilde{m}_{KG}(t) = \epsilon$) might be sent to the young ($L_1(t)$) as a surprise or just after their optimization. Since this aid does not affect their optimization, they consume and save their original levels in their youth ($\frac{du(t)}{dm_{KG}(t)} = \frac{dc_1(t)}{dm_{KG}(t)} = 0$). Whether the domestic amount of capital in the next period increases or not depends on the timing of foreign investors knowing about this aid.

First, if foreign investors know about this aid after the present period ($t$), they cannot do anything. Domestic capital stock increases by the same amount of this capital grant ($\bar{k}_{KG}(t+1) = \bar{k} + \Delta \tilde{m}_{KG}(t)$), which means zero capital flight ($\tilde{m}_{KG}(t) = 0$). Since domestic return of capital drops in the next period ($r(t+1) < r_W$), foreign investors incur losses. Still, income and consumption levels of the old ($Income[L_2(t)] = (1 + r(t+1)) (s + \Delta \tilde{m}_{KG}(t)) = c_2(t+1)$) may or may not increase. This is because the return of capital becomes lower while they own more capital stock ($s + \Delta \tilde{m}_{KG}(t)$). Thus, this aid does not necessarily improve the welfare of $L(t)$. Since they cannot utilize any information on this aid, their welfare level is less than that of the fully informed income or capital grant case ($\frac{du(t)}{dm_{KG}(t)} = \frac{du(t)}{dm_{KG}(t)} > \frac{du(t)}{dm_{KG}(t)} > 0$).

However, the next generation of youth ($L_1(t+1)$) will enjoy higher incomes ($\frac{du(t+1)}{dm_{KG}(t)} > 0$) and welfare ($\frac{du(t+1)}{dm_{KG}(t)} > 0$) because of higher capital stock in the next period. Since they receive more income than that of the fully informed grant case ($\bar{w}_{KG}(t+1) > w_{KG}(t+1)$), their welfare becomes higher ($\frac{du(t+1)}{dm_{KG}(t)} > \frac{du(t+1)}{dm_{KG}(t)} = 0$).

Next, foreign investors predict this aid perfectly within the present period ($t$). Then, they will shift the same amount of this aid to the outside in order...
not to incur losses, which means full capital flight \((\Delta k_{0KG}(t) = \Delta \tilde{m}_{KG}(t))\). Since the domestic capital stock does not change \((\hat{k}_{KG}(t + 1) = \bar{k})\), the older generation enjoys higher income and consumption \((\text{Income}[L_2(t)] = (1 + r_W)(s + \Delta \tilde{m}_{KG}(t)) = c_2(t + 1))\). Although their welfare level improves, it is less than that of the fully informed grant case owing to no information on this aid \((\frac{du(t)}{dm_{KG}(t)} = \frac{du(t)}{dm_{I}(t)} > \frac{du(t)}{dm_{KG}(t)}> 0)\). It is unfortunate that the next young generation \((L_1(t + 1))\) cannot enjoy higher incomes or welfare \((\frac{du(t+1)}{dm_{KG}(t)} = 0)\).

**Lemma 5** If one–time capital grant aid is sent without prior knowledge, it may be effective for the welfare criterion, and the welfare of both present and next generations could improve.

Therefore, the recipient government faces a generational trade–off. The next generation is better–off under no information for foreign investors, but the present generation is better–off under full information. Although the reality might lie between these two extremes, the assumption of perfect capital mobility might make this case get closer to the latter of full information. In reality, the speed of information flow should be higher in relation to production aid for capital investment than in relation to emergency aid for food consumption. Either way, any dynamic effect of this one–time aid disappears after two periods.

### 5.2 Capital control and a closed economy

Since any grant aid to a small open economy cannot realize economic growth, a recipient government that in a position to do so may opt for better policies. If it can perfectly control capital flows, both income aid and capital grant aid enable its economy to expand production by imposing a capital outflow restriction. Although foreign investors will not transfer any new inflow of capital, the recipient economy can continue to grow by accumulating its own capital stock at home until it reaches a steady state \((s (j + 1) = k (j + 2) > s (j) = k (j + 1) \text{ for } j \geq t)\). After all, a closed economy can realize economic growth without any foreign aid or outside resource transfer\(^{10}\).

\(^{10}\)No information on capital grant aid in perpetuity will enable the recipient economy to grow continuously. This is equivalent to the perfectly restricted capital outflow case or the closed economy case. Still, the recipient government may not pursue this policy if its economy is technologically so underdeveloped that the production level of its non–trivial steady state itself is extremely low.
Although this capital outflow restriction seems to be attractive at a first glance, it becomes more difficult to impose because the interest gap between domestic and outside returns of capital \((r_W - r(j + 1) > r_W - r(j))\) for \(j \geq t + 1\) becomes larger and larger. The recipient government will come under greater pressure by domestic individuals to invest outside.

If a recipient economy can be kept closed, one time income and capital grants are effective for all the criteria of consumption, income, production, growth, and welfare. Capital grant aid \((dm_{KG}(t) = \epsilon)\) results in higher economic growth than the same amount of income aid \((dm_I(t) = \epsilon)\). This is mainly because the young \((L(t))\) consume a portion of income aid while they consume none of capital grant aid when it is transferred\(^{11}\).

The welfare level of the present young generation \((L(t))\) becomes higher with income aid than capital grant aid \((\frac{du(t)}{dm_I(t)} > \frac{du(t)}{dm_{KG}(t)} > 0)\) because they optimize their decision with more income. On the contrary, future generations become better off with capital grant aid than income aid \((\frac{du(j + 1)}{dm_I(t)} > \frac{du(j + 1)}{dm_{KG}(t)} > 0 \text{ for } j \geq t)\) because they receive more wages thanks to more domestic capital stock \((k_{KG}(j) > k_I(j))\) for \(j \geq t + 1\). The recipient government faces a serious generational trade-off between the present and all future generations.

### 5.3 Multiple modalities and mix of aid

Donors (developed countries and/or international institutions) provide multiple modalities of aid to a large number of recipients. It is not unusual for a donor to send a mix of grants, loans, and technical assistance to one recipient in one year, each of which has different purposes and is aimed at different sectors. Then, it may be useful for donors to consider an optimal mix of one-time aid policies with varying amounts of consumption, income, capital grant, and capital loan aid. One option is a combination of consumption aid and capital loan aid. A minimum amount of consumption aid alleviates poverty while capital loan aid encourages economic growth\(^{12}\).

\(^{11}\)When aid is transferred to a closed economy, capital grant aid is more effective than income aid for the growth criterion because the fungibility problem (Pack and Pack, 1990; 1993) does not occur. Also, this result is not affected by informational structure. Even if the young \((L(t))\) know about this capital aid in their youth \((t)\), the putty-clay assumption prevents them from consuming.

\(^{12}\)From Proposition 3, capital loan aid is effective for income, growth, and welfare criteria, but not for the consumption criterion. Since the young consume less in their youth...
If any one–time aid can improve the TFP of a recipient, this aid is the most effective compared with the four modalities of aid mentioned above. Such an aid policy is not only effective for all the criteria, but most of its good effects are expected to persist forever in our model. Once the country’s technological level \((A)\) becomes higher, all future capital stock and, hence, production levels remain higher in this small open economy.

Therefore, project aid for infrastructure investment in areas such as irrigation and transport, including roads, railways, and airports, may be ideal if such investment positively affect TFP in a small open economy. In the same way, production potential will be enhanced further if the recipient gains more human capital with technical assistance and/or project aid for school education. Program assistance for better government policy and an enhanced business environment could improve social capital and production capability. In order to analyze the effects of these aid policies on a small open economy, we need to incorporate endogenous growth factors into this model.

6 Conclusion

This paper summarized the analytical results of foreign aid to a small open economy. Based on an economy of two overlapping generations with perfect capital mobility, this study proposed a framework for analyzing the effects of multiple modalities of foreign aid: consumption, income, capital grants, and capital loans. The effectiveness of one–time aid was measured by the criteria of consumption and income of the young, the production and growth of the recipient economy, and the welfare of present and future generations. This simple analysis provides policy direction for donors and recipients, some of which we hope are useful and understandable for both researchers and practitioners of foreign aid.

First, consumption aid improves the consumption and welfare of the young while income aid improves their consumption, income, and welfare. Second, income aid is more effective than consumption aid for welfare, but consumption aid is more effective for the consumption of the young. Third, capital grant aid improves the consumption and welfare of the young, and its effectiveness for welfare is identical to that of income aid. Fourth, consumption, income, and capital grant aid to a small open economy cannot realize with capital loan aid, a small amount of consumption aid to cover this decrease could be realistic.
any positive economic growth. Fifth, capital loan aid improves the welfare of the present and next young generations, as well as income and production in the next period, which results in positive economic growth. Sixth, none of these one-time aid policies can prolong a growth effect after two periods. Seventh, capital grant aid may achieve positive economic growth if it is delivered without prior knowledge.

Since some of these policy directions may be counter-intuitive and controversial, they should be treated with caution. These results were derived by a theoretical model with normalizing and simplifying assumptions, including perfect output and input markets, and rational expectations. Besides, exogenous levels of population, production technology, education and training, and legal and business environment are certain to influence our analytical results.

An immediate revision or extension of this paper would be to endogenize population growth, total factor productivity, and human and social capital to analyze more modalities or purposes of foreign aid. Furthermore, an interesting direction could be provided by the inclusion of a greater variety of international capital flows, including temporary restrictions of capital inflows and/or outflows. Our information structure could be richer using asymmetry among donors and recipients, and uncertainty with some probability distributions. Nevertheless, this study should be applied to empirical analysis by incorporating these extensions, which could make a small but solid contribution to this field of research.

References


