Strategic Ambiguity with Probabilistic Voting

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Political ambiguity

• Political Ambiguity: Politicians prefer to use vague words and outline several policies in their electoral promises.
  • Candidates announce a lottery, and a voter chooses a candidate who announces a better lottery for themselves.

• One possible reason why candidates announce such vague promises is because voters have convex preference.
  • Many past studies recognize the convexity of a voter's utility function as one reason for the emergence of ambiguity.
Political Ambiguity with Convex Preference

• Zeckhauser (1969): The median policy can be defeated by a risky lottery when voters are risk-loving.

• Shepsle (1972): A Condorcet winner does not exist in the case of risk-loving voters.
  • However, both studies do not establish the existence of equilibria.

• Aragones and Postlewaite (2002) show political ambiguity as an equilibrium phenomenon, given risk-loving voters.
  • They assume that candidates need to provide a positive probability for their most preferred policy.

➢ No existing studies show that a candidate chooses an ambiguous promise in equilibrium because of convex preference, without any restriction on the candidate's choices.
This Study

• This study identifies the conditions under which candidates choose ambiguous promises in equilibrium when voters have a convex utility function and without any restriction on the candidate's choices.
  ➢ In order to have political ambiguity as an equilibrium phenomenon, voters need to have a convex utility functions and be polarized, and voting must be probabilistic.

• Note: I do not intend to say that this is the only reason why political ambiguity emerges.
Convex Issues

• Most past studies assume that voters have a concave utility function.

• **Osborne (1995):** “I am uncomfortable with the implication of concavity that extremists are highly sensitive to differences between moderate candidates (p. 275),” and “it is not clear that evidence that people are risk-averse in economic decision-making has any relevance here (p. 276).”

• **Kamada and Kojima (2014):** “Economic policy is arguably a concave issue, given the evidence that individuals are risk-averse in financial decisions. By contrast, voters may have convex utility functions on moral or religious issues (p.204).”
Concave Utility

Utility differs a lot with a small change of a position.
Convex Utility

A small change of a position is not so important.
Past Studies

• There are two main types of models

• **Voter-centered models**
  • Convex preference
  • Context-dependent voting: Callander and Wilson (2008)
  • Give politicians the discretion to adopt policies: Kartik, Van Weelden, and Wolton (2017)
  • Behavioral voters: Jensen (2009), Berliant and Konishi (2005)
Past Studies 2

• **Candidate-centered models**
  • **Direct (non-eletoral) Benefits**
    • Candidates don’t know which policy is most expedient: Aragones and Neeman (2000)
    • Flexibility to implement their own preferred policy: Alesina and Cukierman (1990)
    • Recruiting elites by allowing ideological diversity: Jensen and Lee (2017)
  • **Electoral Benefits**
    • Uncertain about the position of the median policy: Glazer (1990)
    • This is especially true in a primary election: Meirowitz (2005)
    • Follower’s advantage without a Condorcet winner: Kamada and Sugaya (2018).
Definitions of Political Ambiguity

• A lottery

• A set of policies (no discretion to decide the probability distribution)
    • Ambiguity exists when candidates do not announce anything
      • Meirowitz (2005) and Berliant and Konishi (2005)

• The reality should be between these two definitions.
  ➢ This paper supposes political ambiguity as a lottery to clearly investigate the strategic choices of candidates on the degree of ambiguity.
Convex Preference and Probability Voting

- Kamada and Kojima (2014)
  - They suppose that candidates can choose only a single policy with probabilistic voting.
  - Perfectly divergent policies with a convex utility function and a polarized voters.
- This study allows candidates to choose a lottery
  - Partially divergent (and partially ambiguous) policies with convex utility functions and polarized voters
  - Probability voting model with convex utilities is useful to show not only political polarization but also political ambiguity.
Condorcet winning lottery

• $X$: The set of policies
  • $g(x, y)$: The majority margin for $x, y \in X$. The number of voters who prefer $x$ to $y$ minus the number of voters who prefer $y$ to $x$.

• $\Delta X$: The set of probability distributions over $X$.
  • $g(p, q)$: The majority margin for $p, q \in X$.

AFP lottery $p$ is the Condorcet winning lottery when $g(p, q) \geq 0$ for all $q \in X$. 
Three policies/Three Groups

• Three policies: $X = \{L, M, R\}$

• $(p_L, p_M, p_R) \in \Delta X$ where $p_x$ is the probability that $x \in X$ occurs.
  • $p_L + p_M + p_R = 1$

• Voters are divided into three groups $l, m, r$, and no group constitutes a majority.

• The members of each group have the following preference relations such that there exists a Condorcet winner, $M$.

  $l: L > M > R$
  $m: M > L \sim R$
  $r: R > M > L$
These voters’ preference relations can be represented by the utility function $u: X \rightarrow \{0, v, 1\}$ with $v \in (0,1)$.

- The most preferred: $u = 1$.
- The second: $u = v$.
- The worst: $u = 0$.

Voter is risk averse if $v > 1/2$, risk neutral if $v = 1/2$, and risk lover if $v < 1/2$.

Lottery $M$: $(p_L, p_M, p_R) = (0,1,0)$
Proposition 1

A Condorcet winning lottery is $M$ when $\nu \geq 1/2$, and it does not exist when $\nu < 1/2$.

• If $M$ is chosen, the utilities of voters in $l$, $m$, and $r$ are $\nu$, 1, and $\nu$ respectively.

• If lottery $q_1$ with $(p_L, p_M, p_R) = \left(\frac{1}{2}, 0, \frac{1}{2}\right)$ is chosen, the utilities of voters in $l$, $m$, and $r$ are $1/2$, $\nu$, and $1/2$ respectively.

➢ If $\nu < 1/2$, voters in $l$ and $r$ prefer $q_1$ to $M$, and $M$ is defeated by $q_1$ in a pairwise election.
Cycle

• $q_1$ cannot be a Condorcet winning lottery.

• Lottery $q_2$ with $(p_L, p_M, p_R) = \left(\frac{2}{3}, \frac{1}{3}, 0\right)$ provides the utilities to voters in $l$, $m$, and $r$ are $(2 + v)/3$, $(1 + 2v)/3$, and $v/3$ respectively.
  • Voters in $l$ and $m$ prefer $q_2$ to $q_1$.

• $q_2$ is defeated by $q_3$ with with $(p_L, p_M, p_R) = \left(0, \frac{2}{3}, \frac{1}{3}\right)$.

➢ For any lottery, there is another lottery that will get the support of the majority.
Multiple policy space

- The sum of probabilities to choose each policy is one, so at least one group has the positive probability that its best policy is chosen.
- This probability can be divided between the remaining two groups' most preferred policies. Then, this new lottery can defeat the original one.

Such a preference cycle usually occurs when a policy space has multiple dimensions.

When we suppose that voters can choose a lottery instead of a single policy, a space of campaign promises has two dimensions: its position and degree of ambiguity.
Concave and Linear Utilities

• If voters have a concave utility function \( (\nu > 1/2) \), all voters prefer to have the least risk, that is, in making a certain choice.

• If voters have a linear utility function \( (\nu = 1/2) \), voters in \( l \) and \( r \) are indifferent, but voters in \( m \) prefer \( M \) to \( q_1 \).

- They all (weakly) prefer a less risky choice, so a dimension of a space can be considered as one.
- A Condorcet winning lottery exists if \( \nu \geq 1/2 \).
Convex Utilities

• If voters are risk loving ($\nu < 1/2$),
  • Voters in $l$ and $r$ prefer a higher degree of ambiguity.
  • Voters in $m$ still prefer $M$ to $q_1$.

➤ Promise space has two dimensions even though policy space has only one dimension.

• One method of finding an equilibrium when there are multiple policy dimensions is to introduce probabilistic voting (e.g., see Persson and Tabellini, 2000).
Prob. Voting: Settings

• A continuum of voters is distributed to each group according to a probability mass function \( f : G \rightarrow [0, 1/2) \)
  \[
  f(m) = \lambda \\
  f(l) = f(r) = (1 - \lambda)/2
  \]
  • \( \lambda \in [0, 1/2) \) represents the degree of centralization of the voter distribution.

• Two candidates 1 and 2 simultaneously determine their policies \( \Sigma_i = (\sigma_i^L, \sigma_i^M, \sigma_i^R) \in \Delta X \) before the election, where \( i = 1 \) or 2 and \( \sigma_i^L + \sigma_i^M + \sigma_i^R = 1 \)
  • \( \sigma_i^x \in [0,1] \): the probability that candidate \( i \) will implement policy \( x \in X \) after the election
Prob. Voting: Settings 2

• Candidate $i$ maximizes the share of voters: $\Pi(\Sigma_i, \Sigma_{-i}) = \sum_{g \in G} \left\{ f(g)\pi \left( \sum_{x \in X} \sigma_i^x u_g(x) - \sum_{x \in X} \sigma_{-i}^x u_g(x) \right) \right\}$

  • $\sigma_{-i}^x$ is the opponent's probability that $x$ will be implemented.

• $\pi: \mathbb{R} \rightarrow [0,1]$ is
  • strictly increasing ($\pi'(t) > 0$),
  • satisfying $\pi(t) + \pi(-t) = 1$ (thus, $\pi(0) = 1/2$), and
  • strictly concave ($\pi''(t) < 0$) for all $t \in [0, \infty)$.
Denote

- The difference of probabilities to implement \( L \) and \( R \).

\[
\sigma \equiv \sigma^L_i - \sigma^L_{-i}
\]

- The threshold of the degree of centralization

\[
\bar{\lambda} \equiv \frac{(1 - 2\nu)\pi'(\sigma)}{2(1 - \nu)\pi'(0) + (1 - 2\nu)\pi'(\sigma)}
\]
Proposition 2

Suppose \( v < \frac{1}{2} \). A strategy profile with \( \sigma_1^M = \sigma_2^M = 0 \) and \( \sigma \equiv \sigma_i^L - \sigma_{-i}^L \) is a Nash equilibrium when \( \lambda \leq \bar{\lambda} \).

- **Example:** \( (\sigma_i^L, \sigma_i^M, \sigma_i^R) = (1/2, 0, 1/2) \) for both \( i = 1 \) and 2.
  - If \( v < 1/2 \) and \( \lambda \leq \frac{1-2v}{3-4v} \)
  - Perfectly ambiguous

A strategy profile with \( \sigma_1^M = \sigma_2^M = 0 \) is a Nash equilibrium when \( v \geq \frac{1}{2} \) or \( v < \frac{1}{2} \) and \( \lambda \leq \frac{1-2v}{3-4v} \).
Implication 1

- **Concave or Linear Utilities**: Because all voters (weakly) prefer the lower degree of ambiguity, the candidates should converge to the median policy.

- **Convex Utilities**: Conflicts of interest on the degree of ambiguity do exist among voters
  - In many extensions of the Downsian model of electoral competitions, the winner is the candidate who wins the support of the median voter.
  - However, when (i) voters have convex utilities, (ii) voters are polarized, and (iii) candidates are allowed to announce a lottery as a policy platform, then candidates ignore the median voter's interest in order to win an election.
Implication 2

• Kamada and Kojima (2014): Perfect polarization with $\sigma_i^L = 1$ and $\sigma_i^R = 1$ in an equilibrium

• This paper: Candidates may combine policy divergence and political ambiguity in an equilibrium
  - $\sigma_1^M = \sigma_2^M = 0$ (divergence)
  - But $\sigma_i^L \neq 1$ and $\sigma_i^R \neq 1$ (ambiguous)

When (i) voters have a convex utility function and (ii) the distribution of voters' most preferred policies is polarized, candidates choose policy divergence, political ambiguity, or any combination of the two.
Additional Implications

• It is less likely to lead to an equilibrium with more divergence.
  • $\bar{\lambda}$ decreases with higher $\sigma$.

• If voters are more sensitive to differences between candidates, candidates tend to be converged.
  • Suppose two functions $\pi$ and $\hat{\pi}$ such that $\pi(t) < \hat{\pi}(t)$ for all $t \in [0, \infty)$, that is voters are more sensitive to policy divergence with $\hat{\pi}$ than $\pi$. 
Example: The Constitutional Reform in Japan

- The Constitution of Japan was enacted in 1947.
  - The Constitution was written by non-Japanese, even though many opinions of the Japanese were taken into account.
  - The constitutional reform has been discussed frequently since Japan gained independence.
  - Article 9 is the most controversial part because, simply put, it prohibits Japan from holding any military power.

➤ This issue is not related to the economy, so it may be a convex issue.
➤ The distribution of voters' opinions are polarized.
Since 1955, the Liberal Democratic Party of Japan (LDP) has held the government, except for 1993-1994 and 2008-2012.

In the early period of the LDP administration, many claimed that the Constitution should be written by the Japanese people.

However, since the 1960s, the LDP administrations have avoided discussing (and almost gave up on) this issue because public opinion was so divided and an intra-LDP faction hesitated in implementing the reform.

Recently, Prime Minister Shinzo Abe explicitly promised to reform the Constitution.

The 2017 LDP manifesto devoted about 2 pages (of 38 pages).

In the 2012 and 2014 elections, the LDP manifestos devoted only 1/6 to 1/2 a page (of 26 pages) to this issue.
日本の政治を改善することを目的とした「政治改革に関する法制度の見直しを図るための特別委員会」（委員長：中川邦夫）は、2021年9月に設置された。委員会は、日本の政治システムの現状を深刻に指摘し、改革の必要性を強調した。

1. 政治の透明化を図る
2. 倫理的な政治を推進する
3. 行政の効率化を図る

これらの改革を踏まえ、今後は具体的な措置を講じることが期待されている。
国民の幅広い理解を得て、憲法改正を目指します。

现行憲法の「国民主権」、「基本的人権の尊重」、「平和主義」の3つの基本原理は堅持しつつ、憲法改正を目指します。

この国の未来を切り拓く。
Conclusion

• In order to have political ambiguity as an equilibrium phenomenon, voters need to have a convex utility function and be polarized, and candidates must be uncertain about voters' preferences.

• Future Research
  • Identify the policy issues on which voters have a convex utility function.
  • Any extension of the Downsian model should generate additional (or different) implications by supposing a lottery instead of a single policy.
  • A multidimensional policy space.
  • Generalization to continuous policy space.